Prototype of a Mirror bender for Siam Photon Laboratory

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Abstract: This paper presents a design study of adjustable mirror bender system. This system is functioned to adjust beam size focusing on sample in synchrotron light beamline (BL3) at synchrotron light research institute, Thailand. The design utilizes COSMOS program to help in finite element method for simulating the relation of radius value of mirror in function of various action force applying on curvature mirror. Al Alloy(6061) and Zerodur size 40mm x 300mm x 15mm are used as study mirror. All of structures of mirror bender system are built by stainless steel. The simulation of system is to apply a forces to adjust the mirror radius in 50,000 - 60,000 mm range for using it in real circumstance in beamline system. The results of study show the forces in 100-150 N range for Al Alloy (6061) and 150-200 N range for Zerodur giving proper curvature radius values. In contrast, the apply force is more than 300 N calculating from 80% of stress value of mirror, that damages the system. Finally, a prototype of the system is designed and built by using Al Alloy(6061) as a bender mirror. The testing results of the prototype show that apply forces just in range of 50-100 N are suitable enough for the system. That means the apply force of the prototype is 50 N less than the simulation. Therefore, this mirror bender system is ready and suitable for installation in synchrotron light beamline.

Key-Words: synchrotron light beamline, mirror bender, Zerodur, COSMOS

1 Introduction

Presently, science technology is developed and far more advance. Many applications are used high performance and technology to test and investigate. One of high performance and technology is synchrotron light machine. Synchrotron light is the electromagnetic radiation emitted when electrons, moving at velocities close to the speed of light, are forced to change direction under the action of a magnetic field. The synchrotron light is unique in its intensity and brilliance and it can be generated across the range of the electromagnetic spectrum: from infrared to x-rays. Applications of synchrotron light are used in many aspects such as physical science, biological physic and so forth. The synchrotron light machine is composed of many important parts and one of the important parts is beamline system. This system comprises many parts such as vacuum tube, vacuum chamber and especially, optical equipments. The optical equipments are mirror grating and crystal etc. functioning to select wavelength, size and focusing of beam light that point at study sample. One of the important techniques in synchrotron applications is X-PEEM (X-rav Photoemission Electron Microscopy) applying nano-structure in spectroscopy and even in taking photo in micronano scale. The principle of X-PEEM is to use suitable soft X-rays or vacuum ultraviolet range to excite electron in atom of sample emitted from sample for studying its photoelectron. Photoelectron energy depends on energy levels of atom, type of atom and wavelength of exciting xrays. Photoelectrons having diameters about 2-100 microns are emitted collected and amplified by equipments that similar to use in SEM (scanning electron microscopy). Amplified electron is brought to image on screen as shown in figure 1 that able to measure in chemical analysis of minute area (<100 nanometer). Intensity or contrast of picture that images on screen is the characteristics of surface, type of atom, work function of material

and/or characteristic of magnetic domain.



Fig. 1 Diagram of PEEM at BL7.3.2 of Advanced Light Source, US.

New beamline system has been constructing in Siam Photon lab at SLRI (Synchrotron Light Research Institute). It will use synchrotron light produced from an undulator machine to apply for X-PEEM technique and PES (Photoemission spectroscopy) [1]. Optical equipments of the beamline shown in figure 2 consist of a mirror TO to focus synchrotron light into of monochromater. Part of the **S**1 monochromator is to start at inlet S1 to outlet S2. Behind monochromator, light is able to use either X-PEEM or PES technique by moving in and out of M2Cy mirror. This work points at focusing system of X-PEEM technique for focusing synchrotron light on sample in microscale and keeping most part of light is in right position of X-PEEM microscope for high performance.

Focusing system behind monochromater composed of M2V and M2H mirrors that have cylinder surface to focus light in vertical and horizontal, respectively. M2H has constant radius on surface so that size of beam on sample is about 150 microns. On the other hand, M2V is able to adjust curvature radius value for selecting the beam size in vertical. Required curvature radius is in between 50,000mm to 60,000mm. Therefore, all parts of light are on FOV (Field of view) of microscope XPEEM (2-150 microns). This technique is relative to beam size hitting to sample and amplified system of microscopy technique. Beam adjustment is significant so that mirror bender system is needed and used to change the focus of beam position on sample as shown in figure 3. Hence, beam adjustment on sample is worked by changing curvature radius of mirror related to FOV of X-PEEM.

The relation between the entrance armlength

(r, or the source distance), the exit armlength (r, or the image distance), the angle of incidence (α) and the radius of curvature of cylindrical surface (R) is given by the Equation 1

$$\frac{\cos^2\alpha}{r} + \frac{\cos^2\alpha}{r} = \frac{2\cos\alpha}{R}$$
(1)

Figure 3 shows that the smallest beam occurs when curvature radius of mirror is forced to focus beam in as same position as sample. Beam size is able to adjust by changing focusing point at before or behind sample. In general, mirror base for beam focusing in UV range is made from Zerodur or Silicon coated metallic thin film that is good in reflection such as gold or nickel. For M2V mirror will be built from Zerodur material which is mixed between glass and ceramic and has nearly zero thermal expansion ($\pm 0.15 \times 10^{-6}$ /°C) at room temperature. Moreover, curvature adjustment even depends on characteristic of specific material and design structure of mechanical movement.

Beam adjustment for minute size is complex and difficult for giving precise moment or accuracy force [2] that this is an important thing and interesting to design and build the mechanical movement in the first time in our country.



Source

Fig. 3 Beam adjustment on sample by changing curvature radius of mirror.

II. Design of two-arm mirror bender system

To design and develop two-arm mirror bender that SolidWorks program is used to design each part of its structure. When the whole parts are constructed, its system is tested and simulated by relative equation between force and changing curvature radius of mirror. By simulation, the mirror is Al Alloy(6061) compared to Zerodur material. Cosmos program combined with SolidWorks is used for finite element method and simulation of force to act on the system. Results from the simulation obtain and analyze for building a real system.

Theory and calculation

General equation for the moment at end of both sides of a mirror is set catch confiscated $M_1 = hF_1$ and $M_2 = hF_2$. The adjustment of the radius of curvature of a mirror bender can accurately be analyzed using the theorem of beam [3]. Changing the shape of the beam under the moments M_1 and M_2 , as shown in figure 4, can be explained by the differential equation given in Equation 2 for visual effects [4].

$$\frac{d^2 z}{dx^2} = \left(\frac{h}{EI}\right) \left[\frac{F_1 + F_2}{2} + \left(\frac{F_2 - F_1}{L}\right)x\right]$$
(2)

where *E* is the young modulus, *h* is the distance between perpendicular force and the ration center of the beam. $I = t^3(w/12)$ is the moment of inertia, where *t* is the thickness, *w* is the width and *L* is the length of mirror, *z* is the distance along axis of curve change and *x* is the length along axis of mirror.



Fig. 4 Diagram of structure of mirror bender system

Changing the radius of curvature of the mirror can obtained from the equation $R \approx 1/(d^2z/dx^2) \approx EI/hF$, In taking the moment between both systems allows the nucleus to change or spindle unalike. Therefore, a constant factor must be added into equation. The value of the bending loss factor is a constant which depends on the system design [5], available only from experiments. Therefore, equation 2 will be modified by multiplying the loss bending-factor.

Mechanical design

In the past, study of mechanical design of mirror

bender system for synchrotron application can be built in many methods such as, curvature adjustment by cam, wire-cut technique etc. This research concentrates to two-arm curvature adjustment for a prototype having dimension of 320mm x 160mm x 134mm. All components make in country. The advantage of the two-arm curvature adjustment is able to apply force freely in centric system. Software that assists in design the system is solid woks. Structures of the two-arm curvature adjustment are shown in figure 5. The curvature adjustment of the system counts on taking moment at edge of both sides of arm [6].



Fig. 5 Structure model of mechanism two-arm bender design.

The system designed to have a suitable size performance has possibility to create in real. Most of structures are made in machine shop at Synchrotron Light Research Institute (Public organization) since it is easy to change, add and develop all of components further. Figure 6 and table I show details and all of components of the system.

III Finite element method simulation

All simulation components of curvature adjustment system are designed and used COSMOSWorks to analyze in finite element method [9-12]. The test of mechanical system is to find the relation between various forces that are applied and variation of curvature radius of mirror to obtain. Furthermore, the apply force to make failure of the system takes into account. Properties and conditions of simulation of the system are described below.



Fig. 6 Component of curvature adjustment system

Table	I.	Detail	of	mechanical	parts	of	curvature
adjusti	me	nt syste	m				

No.	Detail		
1	Mirror		
2	Top clamper		
3	Arm bender		
4	Arm combiner		
5	Shaft		
6	Right plate for bender		
7	Base plate for bender		
8	Left plate for clamper		
9 Right Mirror clamper			
10	Left Mirror clamper		
11	Pulling up bar		
12 I bridge			
13	Bender handle		
14	Bracket for SP		
15	BS 6267 RBB-1015-Full,DE ,AC,Full_68		

COSMOSWorks characteristics

This project uses COSMOSWorks engineering program to use FEA (finite element analysis) in analysis of characteristics of mirror such as strength and curvature of mirror by applying force to the system. Solidworks and COSMOSWorks program work together and link information between. All components of the system create on solid work and analyze with COSMOS works. Analysis process starts from meshing geometry into small elements linked together. FEA uses partial differential equation rendering and finding the approximating system. Steps of process divide into three basic processes as following below

1. Preprocessing is to assign suitable material properties, and apply boundary conditions in the form of restraints and loads.

2. Solution is to calculate and solve the resulting set of equations.

3. Post processing is to view the results in forms of plots, contour diagrams etc.

In FEA simulation, properties of mirror defined are Al Alloy(6061) and Zerodur [7] as shown in table II and table III, respectively. Both materials have a dimension of 40mm x 300mm x 15mm. However, this research especially considers simulating the holder set of mirror and mirror shown in figure 7. Table II Properties of Al Alloy (6061).

Property	Value	Unit
Elastic Modulus	69,000	N/mm ²
Poisson's Ratio	0.33	-
Shear Modulus	26,000	N/mm ²
Thermal Expansion	2.4×10^{-5}	-
Coefficient		
Density	0.0027	g/mm ³
Thermal	170	Ŵ/m K
Conductivity		
Specific Heat	1,300	J/Kg K
Tensile Strength	124.084	N/mm ²
Yield Strength	55.1485	N/mm ²



Fig. 7 Simulation of system by COSMOSWorks. Simulation results.

Property	Value	Unit
Abbe Constant	66	-
Dispersion (n _f -n _c)	0.00967	-
Density	2,530	kg/m ³ @ 25 C [.]
Young's' Modulus	9.1x10 ⁹	N/m ²
Poisson's Ratio	0.24	-
Specific Heat	2.99329	J/Kg K
Coefficient of	0.05+-	/C [.] (20-30 C [.])
Thermal	0.10x10 ⁻⁶	
Expansion Maximum		C.
Temperature	600	

Table III Properties of Zerodur.

From simulation results, various forces plotted are shown in figure 8 and 9. The results from simulation are brought to math lab to find curvature radius and indicate that the more increasing force, the more decreasing curvature radius. The relation between applied force and variation of curvature radius is shown in table IV and figure 10. Half of structure is used as there is symmetric shape and it saves time when running simulation. Figure 10 shows the results from two equations (eq. 3 and eq.4) that input of load force set 50-350 N is in function of invitation of curvature radius [8].



Fig. 8 Relationship between Al-Alloy (6061) position and gaining distance.



- Fig. 9 Relationship between Zerodur position and gaining distance.
- Table IV Variation of curvature radius to loading force applying to system between Al-Alloy(6061) and Zerodur.

Al All	oy (6061)		Zerodur
Force	Radian	Force	Radian (mm)
(N)	(mm)	(N)	
10	556,051.00	10	731,806.00
50	138,868.00	50	182,034.00
100	71,633.30	100	93,859.80
150	48,283.50	150	63,239.80
200	36,411.00	200	47,679.70
250	29,230.70	250	38,259.20
300	24,411.30	300	31,954.60
350	20,957.20	350	27,437.70



Fig. 10 Relationship between loading force and variation of curvature radius.

$$R_{A1 Alloy(6061)} = -3E - 07x^{5} - 0.181x^{3} + 42.63x^{2} - 5221x + 31371$$
(3)

$$R_{Zerodur} = 1E-09x^{6} - 2E-06x^{5} + 0.001x^{4} - 0.400x^{3} + 75.88x^{2} - 8,034x + 43,720$$
(4)

where x is the value of the force to a system in Newton unit and R is the radius of curvature change is in millimeter unit.

From the results shown in table IV, it is indicated that acting force at 100 - 150 Newton and 150 - 200 Newton suit for Al Alloy (6061) and Zerodur, respectively and apply to gain their expected curvature radius. The limited acting forces are known; therefore, to collect more reliable data, each step of force of 5 Newton takes into account. The step of 5 Newton is defined in simulation process for Al Alloy (6061) and Zerodur between 100-150 Newton and 150 - 200 Newton, respectively since previous simulation is set at 50 Newton each step. Table V and figure 11 are shown the results from the simulation.

Table V Variation of curvature radius in the function of applying force for Al alloy (6061) and Zerodur in the range of 100-150 Newton and 150 -200 Newton respectively.

A	Al Alloy (606	51)	Zerodur		
Force (N)	Radius (mm)	Max von Mises Stress (MPa)	Force (N)	Radius (mm)	Max von Mises Stress (MPa)
100	71,633.30	16.580	150	63,239.80	24.500
105	68,646.50	16.970	155	61,260.00	24.900
110	65,574.30	17.771	160	59,366.00	25.690
115	62,745.80	18.573	165	57,614.00	26.480
120	60,181.30	19.374	170	55,940.00	27.270
125	57,809.90	20.176	175	54,388.70	28.060
130	55,619.80	20.978	180	52,876.70	28.850
135	53,593.60	21.779	185	51,489.10	29.640
140	51,691.80	22.581	190	20,156.20	30.430
145	49,940.80	23.382	195	48,886.90	31.220
150	48,283.50	24.670	200	47,679.70	32.00

The results from above can build the function of equations in the range of 100-150 Newton and 150-200 Newton for Al Alloy(6061) and Zerodur, respectively.





$$R_{A1 Alloy(6061)} = 3.148x^2 - 1,336x + 1.704E + 5$$
(5)

$$R_{\text{Zerodur}} = 1.739 x^2 - 918.1 x + 1.618 \text{E} + 5 \tag{6}$$

where x is the value of the force to a system in newton unit and R is the radius of curvature change is in millimeter unit.

IV. Simulation Results

From simulation of system by using applying force between 100-350 Newton is indicate that curvature radius of both Al Alloy (6061) and Zerodur can change in range of 556, 051.00-20,957.20 mm and 731,806.00-27437.70 respectively. mm, Nevertheless, design of beamline determines to vary curvature radius between 50,000 -60,000 mm for expecting size of beam. The results from table 3.4 show that applying force can cover the expecting curvature radius both Al Alloy (6061) and Zerodur. Furthermore, the whole system and applying force of system are considered to find Max Von Mises Stress for happening damage point. The simulation found that the damage point starts at applying force of 300 Newton for AL Alloy (6061) and Zerodur by calculating at 80% of stress of Al Alloy (6061). The results from table IV are shown that the applying forces are added more frequency of step to 5 Newton per step. Therefore, the forces applying to reach the expecting adjustment of curvature radius are shown more accuracy of force range. The forces applying for Al Alloy (6061) and Zerodur are 120-145 Newton and 155-195 Newton, respectively. When taking fit curve to find function of equations, there are two equations of second degree polynomial to be shown in equation 5 and 6. From the results and data

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analysis, the possibility of this system can be applied and built in practical situation and developed further.

Preparation and construction of a prototype

From study of design and simulation, the output results are satisfied and able to built a prototype for testing in real conditions. All mechanical parts are fabricated in machine shop at Synchrotron Light Research Institute. Curvature adjustment system is constructed and assembled as shown in figure 12. All moving parts are controlled by stepping motor through interfacing program to develop by LabView connected to microcontroller to control the stepping motor by instructions. Figure 13 show the window interface of programming LabView.



Fig. 12 Prototype of mirror bender set.



Fig. 13 Program control development by LabView.

Measuring instrument preparation and installation

Before testing, system preparation is significant and all components are composed of many parts as follow

- 1. dual gauge
- 2. driving part of curvature measurement
- 3. controller system
- 4. mirror bender set
- 5. temperature measurement
- 6. computer for interfacing
- 7. force gauge
- 8. moving part of force driving set



Fig. 14 Testing operation of prototype.

Mirror bender in moving part set is driven by 3 sets of five-phase stepping motor composed of two sets of force driving system and a set of curvature measurement. Controlling system is to use programming software from LabView interfacing to controller set for driving motors by demands. In part of acquiring curvature data is stored automatically and plot to make a graph. Moreover, the acquiring curvature data is brought to do curve fitting for finding variation of curvature to various loading force. In curvature measurement, 160 mm long from centre of top and bottom mirror is measured by centering at the middle of mirror in 80 mm long as shown in figure 15.

Table VI. Relationship between force and curvature

radius values.



Fig. 15 Curvature measurement position.

Principle of measurement is to shift each position following by length of material at 5 mm per step. When movement of dual gauge takes place each position, data is stored relative to loading force of system. Loading force starts from 0 N to 600 N until reaching to 160 mm of expecting point. Acquiring data is saved in Excel program that data from dual gauge can transfer to.

Results from prototype testing

The results from experiment, various forces are applied to the system and acquiring curvature data obtains to be plotted as shown in figure 16. In addition, data and graph are brought to find curvature radius by fit curve function of MATLAB to show in table VI and relation of force and curvature radius are shown in figure 17.



Fig. 16 Changing position of Al-Alloy (6061) to varies load force of system.

Force (N)	Radius (mm)
10	66,470.20
50	62,583.48
100	53,318.87
150	40,172.55
200	33,481.43
250	26,809.21
300	24,038.26
350	20,441.46
400	19,081.11
450	18,189.89
500	15,176.76
550	14,186.76
600	13,050.87



Fig. 17 Relationship between force and current radius from experiment.

Comparison of simulation and practical results

The simulation and practical results are compared as shown in table VII and figure 18 by considering force at 100-350 N. It indicates that the more applied force increases, the more curvature radius decreases following relation of equation 7 and 8 for simulation and practical testing, respectively.

Force	S1m.	Exp.
(N)	Radius (mm)	Radius (mm)
10	556,051.00	66,470.20
50	138,868.00	62,583.48
100	71,633.30	53,318.87
150	48,283.50	40,172.55
200	36,411.00	33,481.43
250	29,230.70	26,809.21
300	24,411.30	24,038.26
350	20,957.20	20,441.46
400	31,575.10	19,081.11

Table VII. Comparison of loading force and curvature radius from simulation and experiment.

Relation of equation is to obtain from Fit curve between loading force and curvature radius at input force 100-350 N getting exponential equation at level 5 where R-square = 1

$$\begin{split} R_{Sim} &= -8.315 E{-}08x^5 + 0.0001128x^4 - 0.06205x^3 \\ &+ 17.67x^2 - 2743x + 2.209 E{-}05, \\ R{-}square = 1 \end{split}$$

where x is the value of the force to a system in Newton unit and R is the radius of curvature change is in millimeter unit.



Fig. 18 Graph of comparison between force and current radius from simulation and experiment at load force 100-350N

Error values from the simulation and experiment results are found a equation to have differential coefficient as below

Define:

$$R = P1x^5 + P2x^4 + P3x^3 + P4x^2 + P5x + P6$$

Differential coefficient is P1 = 4.2165E-07 P2 = 4.607E-04 P3 = 0.19025 P4 = 36.29 P5 = 3,018P6 = 6.9E-06

For solving equation 7 to have the result as close as equation 8, factor values have to be found for the design of system by considering the results from both simulation and experiment testing. The factor values for two-arm mirror bender system are deliberated on coefficient of equation shown in table VIII

Table VIII. Factor value force design of mirror bender system.

Coefficient	Factor	
P1	0.1647187005	
P2	0.1966870096	
P3	0.2459373761	
P4	0.3274647887	
P5	0.476132615	
P6	0.761986892	

This research presents the development of mirror curvature adjustment system for synchrotron light to apply in beamline 3 at Synchrotron light research institute (Public organization). Design processes have complexity since knowledge of engineering has to be applied such as mechanical, electrical, computer and physics etc. This design system is two-arm curvature adjustment which has the advantage of adjusting force freely to both sides of arm. In a case of system assembly or installation might shift the center its system out from position making the ease of focusing beam hitting on the most of Field of view of X-PEEM technique for high accuracy and performance. The simulation and experiment results of curvature adjustment system are compared and suitable for using in curvature radius of 50,000-60,000 mm range. In this range, it is able to focus all parts of beam light hitting on field of view of X-PEEM measurement. In addition, loading force at 50-100 N are suitable for Al Alloy (6061) material having dimension of 300mm x 40mm x 15mm. The loading force in real experiment result is less than 70 N in FEA simulation results that helping less system damage. The different result of both results occurs since all mechanical parts of the system operate and some forces passes through them as virtual force to start initial stage of the system. Factor values that are significant to system design are relative to error values of the system as well as shown in table VIII. The error values occur from system assembly and environment conditions. In the experiment, Factor values is common multiple to relation of equation in equation 8 obtaining results as same as simulation results. Therefore, curve fitting by MATLAB is related to relation between force and curvature radius of order 5 of polynomial at R-square = 1. Practical testing by using Al Alloy (6061) as a mirror to prove that it is able to develop for two-arm curvature adjustment and adjust the curvature radius in required range. The results of Al Alloy(6061) is close to the results from Zerodur mixed between glass and ceramic materials. In fact, Zerodur that is used in beamline 3 at Siam Photon Lab. have special properties, for instance, the extremely low thermal expansion and long-term dimensional stability. This study and research prove that it is able to work in real conditions and apply to other system developments in the future. This is the beginning of development of mirror curvature adjustment using in synchrotron beamline in Thailand.

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