Motion Control and Trajectory Tracking Control for a Mobile Robot Via Disturbance Observer

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Abstract: - This paper investigates the tracking control of a wheeled mobile robot in the unknown environment. A disturbance observer is developed with utilization the integral filter. The proposed control scheme employs the disturbance observer control approach to design an auxiliary wheel velocity controller to make the tracking errors, which includes the velocity tracking error, the angular velocity tracking and the trajectory tracking error vector, as small as possible in consideration of unknown bounded disturbance in the kinematics of the mobile robot, and makes use of the disturbance controller to reject the unknown bounded disturbance. The approximation errors and the unknown bounded disturbance can be efficiently rejected by employing the integral filter. A major advantage of the proposed methods is that the position (or velocity/angular velocity) and the desired trajectory (or velocity/angular velocity) are no longer necessary. This is because the observer controller "tracking" both the mobile kinematics and the unknown bounded disturbance. Most importantly, all signals in the closed-loop system can be assured to be uniformly ultimately bounded. The system stability and convergence of the motion control and the trajectory tracking errors are proved using the Lyapunov stability theory. Simulation results are provided to verify the proposed control strategy. It is shown that the control strategy is feasible.

Key-Words: - Disturbance observer, Velocity tracking, Trajectory tracking, Time-varying disturbance, Mobile robot, Motion control, Lyapunov stability

1 Introduction

Motion controls of wheeled mobile robot (WMR) have attracted the attention of many researchers [1, 2, 3, 4, 5, 6]. Interest in such systems stems primarily from the WMR with loading capacity which is necessary in industry. However, WMR has nonholonomc nature and doesn't meet Brockett's condition, which is the necessary condition to make a smooth time-invariant control law.

Kanayama *et al.* [2] proposed an asymptotic motion controller which used continuous feedback control mode. However, this controller adopts local linearization using Lyapunov indirect stability theorem and cannot be globally stable. Fierro and Lewis [3] designed a controller for both motion control and point stabilization using backstepping mode. Bakir and Jasmin [7] proposed global asymptotic motion controller using backstepping. Coulaud *et al*. [8] and A.G. Lorence *et al*. [9] also proposed a globally asymptotically stable controller using image-processing algorithm. Anti-disturbance adaptive control was studied for the mobile robots using dual adaptive neural control where unknown network parameters are estimated in real time [10]. The tracking problem of the mobile robots has also attracted the attention of many researches [11]-[15]. Using Barbalet lemma or the backstepping method, some controllers have been proposed such that the mobile robots could globally follow the special paths such as circles and straight lines. W. Dong *et* *al.* [16] proposed controller ensures the entire state of the dynamic system asymptotically track the desired trajectory, considering unknown inertia parameters. When the input of the mobile robot appears saturation, Z.P. Jiang *et al.* [17] presented a control strategy to deal with the problem of global stabilization and global tracking control for the mobile robot. All these papers assumed that disturbances are to be constant or slow time-varying, or even without consideration. When the disturbances are fast time-varying, the performances of those control modes are unsatisfactory.

And, because of the difficulty in dynamic modeling, artificial intelligence controls using neural networks and fuzzy logic can be considered as an effective tool for nonlinear controller design. In [18] and [19], the observer using multilayer neural-network was developed for the mobile robot tracking control, but the controller structure and the neural-network learning algorithm are complicated, and it is computationally expensive. In [20], the observer using fuzzy method was presented to compensate the load disturbance that makes the tracking inaccuracy, furthermore, the method considered only the tracking error, and the real external disturbances in the velocity and the angular velocity of the mobile robot were not considered.

In the past, a novel disturbance observer [21], which use in the mobile application with arbitrarily fast time-varying disturbance, has been successfully used for hard disk drives. Based on the disturbance observer, we consider the situations where tracking control using disturbance observers are to obtain desired velocity and desired trajectory in unknown environment, as shown in Fig. 1. One control purpose of the mobile robot is that the actual velocity is equal to the desired velocity, and making sure the angular velocity of the mobile robot is desired one. Another control purpose is that the real trajectory of the mobile robot can be located quickly to the desired trajectory. To this effect, actuator dynamics is combined with the mobile robot and the input torques of two driving wheels. We propose a new control method using disturbance observers for the mobile robot, which reject bounded disturbances. The proposed schemes estimate unknown parameters, and control the mobile robot with desired posture, while having the characteristic of global stability. Besides, in presented scheme can reject external arbitrary fast time-varying disturbances.

The main contributions of this paper are listed as follows:

(1) Decoupled tracking and orientation control strategies are proposed for the WMB without imposing any restriction on the system dynamics;

(2) Controller design for the WMB with anti-disturbance;

(3) Disturbance observers design for arbitrarily fast time-varying disturbances in the WMB system; and

(4) Trajectory tracking based the control design is developed in unknown environment.

Simulation results are described in detail to show the effectiveness of the proposed controls.

The remainder of this paper is organized as follows. The model of a nonholonomic mobile robot is introduced in Section 2. The main problems of the formulation to position and orientation control are discussed in Section 3. The nonlinear observer and the controller design are presented in section 4. Simulation studies are showed in section 5. Concluding remarks are given in Section 6.

2 Model of a Nonholonomic Mobile Robot

The mobile robot shown in Figure 1 is a typical example of a nonholonomic mechanical system. It consists of a vehicle with two driving wheels mounted on the same axis, and a front passive wheel. The position and the orientation are achieved by independent actuators providing the necessary torques to the rear wheels. The two driving wheels have the same radius denoted by r and are separated by $2R$. Point C is located in center of mass of the mobile robot; point P is located in the intersection of the midline of the mobile base and the axis of the driving wheels. The distance between point C and point P is denoted by d. The position and the orientation of the robot in an inertial Cartesian frame $\{O, X, Y\}$ is completely specified by the vector $q=[x_c, y_c, \theta]^T$, where x_c, y_c are the coordinates of the center of mass of the vehicle, and θ is the orientation of mobile platform $\{C, X_c, Y_c\}$ measured from X axis.

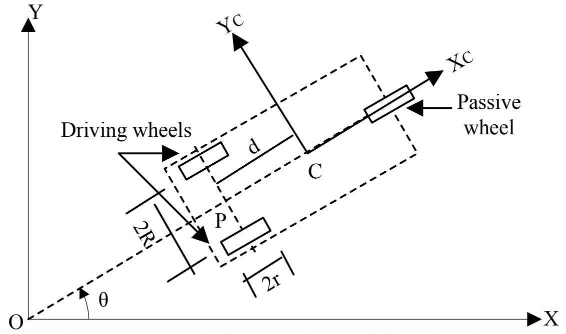

Fig.1 A nonholonomic mobile platform.

A nonholonomic mobile robot system having an n-dimensional configuration space L with generalized coordinates ($q_1, \ldots, q_n$) and subject to m constraints can be described by [22]

$$M(q)\ddot{q} + C_m(q,\dot{q})\dot{q} + D = B(q)\tau - A^T(q)\lambda \tag{1}$$

where $M(q) \in R^{n\times n}$ is a symmetric, positive definite inertia matrix, $C_m \in R^{n\times n}$ is the Centripetal and Coriolis matrix, $D \in R^{n\times 1}$ is the external disturbance vector, $B(q) \in R^{n\times r}$ is the input transformation matrix, $\tau \in R^{r\times 1}$ denotes the input vector, $A^T(q) \in R^{n\times m}$ is the matrix associated with the constraints, and $\lambda \in R^{m\times 1}$ is the vector of constraint forces.

All kinematic constraints of the mobile platform are independent of time, and can be expressed as

$$A(q)\dot{q} = 0 \tag{2}$$

With respect to the dynamics of mobile robot (1), the following properties are known [23].

Property 2.1: $M(q)$ is a symmetric and positive-definite matrix;

Property 2.2: The matrix $\dot{M} - 2C_m$ is skew-symmetric [24], that is, $x^T(\dot{M} - 2C_m)x = 0$, $\forall x \in R^n$.

Assume that $S(q)$ be a full rank matrix (n - m) formed by a set of smooth and linearly independent vector fields spanning the null space of $A(q)$, i.e.,

$$S^T(q)A^T(q) = 0 \tag{3}$$

Using (2) and (3), it is possible to find an auxiliary vector time function $v(t) \in R^{n\times m}$ such that, for all t [3], [25]

$$\dot{q} = S(q)v(t) \tag{4}$$

2.1 Kinematics and dynamics of a mobile platform

The pure rolling and nonslipping nonholonomic constraint states that the robot can only move in the direction normal to the axis of the driving wheels, and the mobile base satisfies this nonholonomic constraint [26, 27]. And the velocity component of the contact point with the ground, perpendicular to plane of the wheel is zero, namely

$$\dot{y}_c \cos\theta - \dot{x}_c \sin\theta - d\dot{\theta} = 0 \tag{5}$$

From (3) and (5), the constraint matrix of the mobile platform is expressed as

$$A(q) = [-\sin\theta \quad \cos\theta \quad -d] \tag{6}$$

Thus, matrix $S(q)$ can now be expressed as

$$S(q) = \begin{bmatrix} \cos\theta & -d\sin\theta \\ \sin\theta & d\cos\theta \\ 0 & 1 \end{bmatrix} \tag{7}$$

Therefore, it is easy to verify that the kinematic equations of tracking (4) can be expressed as

$$\dot{q} = \begin{bmatrix} \dot{x}_c \\ \dot{y}_c \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & -d\sin\theta \\ \sin\theta & d\cos\theta \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_0 \\ \omega \end{bmatrix} \tag{8}$$

where $v = [v_0 \quad \omega]^T$. v_0 and ω are bounded linear and angular velocities of the mobile robot respectively. Eq. (8) is called the steering system of the vehicle.

The Lagrange formalism is used to find the dynamic equations of the mobile robot. The dynamical equations of the mobile platform can be expressed in the matrix form (1), where

$$M(q) = \begin{bmatrix} m & 0 & md\sin\theta \\ 0 & m & -md\cos\theta \\ md\sin\theta & -md\cos\theta & I \end{bmatrix},$$

$$C_m(q,\dot{q}) = \begin{bmatrix} 0 & 0 & md\dot{\theta}\cos\theta \\ 0 & 0 & md\dot{\theta}\sin\theta \\ 0 & 0 & 0 \end{bmatrix},$$

$$B(q) = \frac{1}{r}\begin{bmatrix} \cos\theta & \sin\theta \\ \cos\theta & \sin\theta \\ R & -R \end{bmatrix}, \; D = \begin{bmatrix} d_1\cos\theta \\ d_1\sin\theta \\ d_2 \end{bmatrix},$$

$$\tau = \begin{bmatrix} \tau_r \\ \tau_l \end{bmatrix}, \; A^T(q) = \begin{bmatrix} -\sin\theta \\ \cos\theta \\ -d \end{bmatrix}. \tag{9}$$

where d_1 and d_2 are the disturbances of linear and angular velocities of the mobile robot, respectively;

τ_r and τ_l denote torques of right and left wheel respectively.

2.2 Structural properties of a mobile platform

The system (1) can be transformed into a more appropriate form for control purposes. Using (4) substituting (1), and then multiplying by S^T, the constraint matrix $A^T(q)\lambda$ can be eliminated. Thus, the appropriate form of system (1) can be obtained. The complete equations of tracking of the nonholonomic mobile platform are given by

$$\dot{q} = S(q)v(t) \tag{10}$$

$$\overline{M}\dot{v} + \overline{C}_m v + \overline{D} = \overline{B}\tau \tag{11}$$

where $\overline{M} = S^T MS$, $\overline{C}_m = ST(M\dot{S} + C_m S)$, $\overline{D} = S^T D$, $\overline{B} = S^T B$. Eq. (11) describes the behavior of the nonholonomic system in a new set of local coordinates, i.e., $S(q)$ is a Jacobian matrix that transforms velocities in mobile base coordinates v to velocities in Cartesian coordinates q. Therefore, the properties of the original dynamics hold for the new set of coordinates.

Property 2.3: The matrix $\overline{M}(q)$ is symmetric positive definite;

Property 2.4: the matrix $\left(\dot{\overline{M}} - 2\overline{C}_m\right)$ is skew-symmetric; and

Property 2.5: the $\left\|\overline{D}\right\|$ are bounded.

Use of (9) in (11) yields

$$\overline{M} = \begin{bmatrix} m & 0 \\ 0 & I - md^2 \end{bmatrix}, \quad \overline{B} = \frac{1}{r}\begin{bmatrix} 1 & 1 \\ R & -R \end{bmatrix},$$

$$\overline{C}_m = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \overline{D} = \begin{bmatrix} d_v \\ d_\omega \end{bmatrix}$$

Finally, the decoupled system (11) can be expressed as

$$m_1\dot{v}_1 + d_1 = \tau_1 \tag{12}$$

$$m_2\dot{\omega} + d_2 = \tau_2 \tag{13}$$

where $m_1 = m$, $m_2 = I - md^2$, $\tau_1 = \left(\tau_r + \tau_l\right)/r$, $\tau_2 = (\tau_r - \tau_l)R/r$. Furthermore, (12) and (13) can be rewritten as

$$m_1 \frac{d^2}{dt^2}\left(\int_0^t v(s)ds\right) + d_1 = \tau_1 \tag{14}$$

$$m_2 \frac{d^2}{dt^2}\int_0^t \omega(s)ds + d_2 = \tau_2 \tag{15}$$

3 Problems Formulation to Motion Control

The overview of a mobile robot is shown in Fig. 1. Since the mobile robot works in an unknown environment, we should focus on the accuracy of the tracking and orientation control, which is accessed via corresponding sensors. The position error signal (PES) indicates the displacement of the center of mass of the mobile base from the desired tracking control location, and the orientation error signal (OES) represents the X_C axis deviation from the desired orientation control of X_C axis.

Using (14) and (15), we consider the double integrator model representing the mobile robot as follows:

$$m_1\ddot{y}_1 + d_1 = \tau_1 \tag{16}$$

$$m_2\ddot{y}_2 + d_2 = \tau_2 \tag{17}$$

where $y_1 = \int_0^t v(s)ds$, $y_2 = \int_0^t \omega(s)ds$. y_1, $\dot{y}_1$, $\ddot{y}_1$ are the position, velocity, and acceleration of the mobile robot respectively; y_2, $\dot{y}_2$, $\ddot{y}_2$ are angle, angular velocity, and angular acceleration of the mobile robot respectively.

Remark 3.1: In a standard servo control system, it is general practice and understanding that the positional signals including its position y_1, velocity $\dot{y}_1$, and sometimes its acceleration $\ddot{y}_1$, are available for feedback control design. For tracking control purpose, the desired trajectory y_{1d}, its first and second derivatives $\dot{y}_{1d}$ and $\ddot{y}_{1d}$, are also known bounded and continuous signals. Furthermore, the tracking control error $e_1 = y_1 - y_{1d}$ is easily computable. As such, these involved angular variables y_2, $\dot{y}_2$, $\ddot{y}_2$ and e_2 are available.

However, in a mobile robot system, the things will be changed. We can not get both the position of the center of mass of the mobile robot y_1, and the desired trajectory y_{1d}. As such, the actual and desired orientations are unavailable. The main objective of the track following servo is to maintain the center of mass on the track, at same time, to maintain the actual orientation of the mobile in required one. Since we do not know the exact shape of the servo track, we can only demodulate a signal using a sensor to tell us the relative distance between the center of mass and track center, which is PES. As PES can be measured quite accurately, its derivative can be estimated quite well and is generally assumed to be known for control system design. The case of corresponding angular variables is similar the positional one. Then PES and OES are expressed as

$$e_1 = y_1 - y_{1d} \tag{18}$$

$$e_2 = y_2 - y_{2d} \tag{19}$$

where e_1 and e_2 denote PES and OES respectively, though y_1, y_{1d}, y_2 and y_{2d} are unavailable for the mobile robot.

Assumption 3.1: The external disturbances $d_1(t)$ and $d_2(t)$ are bounded with an unknown bound.

Assumption 3.2: In a mobile robot system, both the PES signal e_1 and its first derivative $\dot{e}_1$, are available; the OES signal e_2 and $\dot{e}_2$ are available too.

Assumption 3.3: The track position y_{1d} and track orientation y_{2d}, and them first and second derivatives are bounded and continuous signals, though they are not available for feedback control in the mobile robot.

The control object is to present a control strategy to resist the external arbitrarily fast time varying disturbance and to make the PES and the OES ideally at zero in the tracking control.

4 Disturbance Observer Design and Simulation

4.1 Disturbance observer design

From (16), (17) and (18), the following tracking control error dynamics can now be expressed as

$$m_i \ddot{e}_i = \tau_i - d_i - m_i \ddot{y}_{id} \tag{20}$$

where $i = 1,2$. And the subscript *i* denotes 1 or 2 in the following text.

Remark 4.1: In the standard servo setting where signals y_i, $\dot{y}_i$ and $\ddot{y}_i$ are available, we can design the following ideal certainty equivalent control

$$\tau_i = -(k_{ip} e + k_{id} \dot{e}) + m_i \ddot{y}_{id} + \hat{d}_i(t) \tag{21}$$

where k_{ip} and k_{id} are positive constants, $\hat{d}_i(t)$ is the disturbance observer for $d_i(t)$.

As mentioned earlier, $\ddot{y}_{id}$ is not available in the system. Compared with standard servo control setting, consider the following control based on available signals:

$$\tau_i = -(k_{ip} e + k_{id} \dot{e}) + \hat{d}_i(t) \tag{22}$$

where $k_{id} > 1/2$ .Substituting (22) into (20) yields a closed-loop system

$$m_i \ddot{e}_i = -(k_{ip} e + k_{id} \dot{e}) + \hat{d}_i(t) - \overline{d}_i \tag{23}$$

where $\overline{d}_v = -d_v - m_v \ddot{y}_{vd}$, $\overline{d}_w = -d_w - m_\omega \ddot{y}_{\omega d}$.

From the closed-loop dynamics (23), if we can design a disturbance observer such that

$$\left|\hat{d}_i(t) - \overline{d}_i\right| \le \alpha_i e^{-\beta_i t}$$

where α_i and β_i are positive constants, then the stability of the system (23) will be achieved easily. Consider the following differential equation:

$$\dot{\hat{d}}_i(t) - \dot{\overline{d}}_i + \gamma_i (\hat{d}_i(t) - \overline{d}_i) + e^{-\gamma_i t} \dot{\overline{d}}_i = 0 \tag{24}$$

where γ_i are positive constants. Its solution is $\hat{d}_i(t) = (1 - e^{-\gamma_i t})\overline{d}_i(t) + e^{-\lambda_i t}\overline{d}_i(0)$. It show that $\hat{d}_i(t)$ converge to their true value $\overline{d}_i(t)$ exponentially. However, because $\overline{d}_i(t)$ and $\dot{\overline{d}}_i(t)$ are not available, $\hat{d}_i(t)$ cannot be obtained from (24) directly.

Lemma 4.1: According to the listed below integral filters

$$\begin{aligned} \dot{z}_i(t) &= -\mu_i z_i(t) + (\mu_i - \upsilon_i) e^{-\nu_i t} z_i(0) \\ &\quad + \textstyle\int_0^t e^{-\mu_i (t-r)} \overline{d}_i \end{aligned} \tag{25}$$

$$\begin{aligned} \dot{\hat{z}}_i(t) &= -\mu_i \hat{z}_i(t) + (\mu_i - \upsilon_i) e^{-\nu_i t} \hat{z}_i(0) \\ &\quad + \textstyle\int_0^t e^{-\mu_i (t-r)} \hat{d}_i \end{aligned} \tag{26}$$

where μ_i , υ_i are positive constants, $z_i(0)$ and $\hat{z}_i(0)$ are initial values, the following conclusions can be obtained:

(i) The signal $\hat{d}_i(t)$ can converge to its true value exponentially, i.e.,

$$\left|\tilde{d}_i(t)\right| \le \alpha_i e^{-\beta_i t}$$

where $\tilde{d}_i = \overline{d}_i(t) - \hat{d}_i(t)$, α_i is positive constant, and β_i is positive design parameter;

(ii) The signal $\hat{d}_i(t)$ can be obtained from the following integral equation

$$\textstyle\int_0^t e^{\mu_i r} \hat{d}_i(r) dr = \mu_i e^{\mu_i t} \int_0^t e^{-\mu_i r} \psi_i(r) dr + \psi_i(t). \tag{27}$$

where

$$\begin{aligned}\psi_i(t) &= (\upsilon_i - \gamma_i)e^{(\mu_i+\gamma_i-\upsilon_i)t}(z_i(0) - \hat{z}_i(0)) \\ &+ e^{(\mu_i-\upsilon_i)t}\cdot \hat{z}_i(0) + (e^{\gamma_i t} - 1)\int_0^t e^{\mu_i r}E_i(r)dr \\ &+ (e^{\gamma_i t} - 1)\vartheta_i(t) + [\mu_i + (\gamma_i - \mu_i)e^{\gamma_i t}] \\ &\cdot \int_0^t [\vartheta_i(r) + \int_0^r e^{\mu_i s}E_i(s)ds]dsdr.\end{aligned} \tag{28}$$

are computable signals, with $V_i = k_{ip}e_i + k_{id}\dot{e}_i$, $\vartheta_i(t) = m_i(e^{\mu_i t}\dot{e}_i(t) - \dot{e}_i(0) - \mu_i\int_0^t e^{\mu_i r}\dot{e}_i(r)dr$, and constant $\gamma_i > 0$ is design parameter.

Proof: See Appendix A.

The stability of the system (23) is given in the following theorem.

Theorem 4.1: Assume the equation (23) consisting of system (14) and (15) satisfying Assumptions 4.1—4.3, the controller (22) and the observer (27). The external disturbance can be rejected exponentially, the PES and the OES can converge to zero, i.e., the estimated values of $\hat{d}_1$ and $\hat{d}_2$ globally exponentially converge to their true values respectively, the tracking control errors e_1, e_2, $\dot{e}_1$ and $\dot{e}_2$ are all converge to zero, as t is infinity.

Proof: Using the Lemma 4.1, we have $\tilde{d}_i(t)^2 \le \alpha_i{}^2 e^{-2\beta_i t}$. As $t \to \infty$, $\tilde{d}_i(t)^2 \to 0$, i.e., the estimated values $\hat{d}_i$ in (27) globally exponentially converge to their true values respectively.

Consider the Lyapunov function candidate

$$V_i = k_{ip}e_i{}^2 + m_i\dot{e}_i{}^2 \tag{29}$$

then, differentiating V_i with the time and integrating (23), and we have

$$\begin{aligned}\dot{V}_i &= 2k_{ip}e_i\dot{e}_i + 2m_i\dot{e}_i\ddot{e}_i = -2k_{id}\dot{e}_i^2 + 2\dot{e}_i\tilde{d}_i(t) \\ &\le -2(k_{id} - 1/2)\dot{e}_i^2 + \tilde{d}_i^2(t) \\ &\le -k_{i0}\dot{e}_i^2 - k_{i1}\dot{e}_i^2 + \tilde{d}_i^2(t)\end{aligned} \tag{30}$$

where $k_{id} - 1/2 = (k_{i0} + k_{i1})/2 > 0$ with $k_{i0} > 0$ and $k_{i1} > 0$. When $|\dot{e}_i| \ge |\tilde{d}_{i(t)}|/\sqrt{k_{i1}}$, we have $\dot{V}_i \le 0$. Therefore, we know $|\dot{e}_i| \le |\tilde{d}_{i(t)}|/\sqrt{k_{i1}}$. Noticing that $|\tilde{d}_i(t)| \to 0$ as $t \to \infty$, $|\dot{e}_i| \to 0$ as $t \to \infty$ can be obtained. Thus, we have $\ddot{e}_i \to 0$ as $t \to \infty$. It follows that $e_i \to 0$ as $t \to \infty$.

4.2 Simulations

The proposed controllers and observers in this section are verified with computer simulation using MATLAB. The parameter values of the mobile robot are taken as m = 9 kg, I = 5 kg.m^2, 2R = 0.306 m, r = 0.052 m. The parameters of the controllers are chosen as $k_{1p} = k_{2p} = 3.5$; $k_{1d} = k_{2d} = 0.6$. The parameters of the observers are chosen as $\mu_1 = \mu_2 = 6$; $\upsilon_1 = \upsilon_2 = 4$; $\gamma_1 = \gamma_2 = 8$. Initial velocity and angular velocity are taken as 0.1 m/s and 0 respectively; the target posture is taken as velocity and angular velocity are taken as $v(t) = 2$ m/s and $\omega(t) = 1$ rad/s respectively. The peaks, frequencies of two disturbances are optional. For the purpose of simulation, the parameters of two disturbances are taken as: $d_v = 0.1\sin(40\pi t)$ and $d_\omega = 0.15\cos(20\pi t)$. The simulation results for velocity tracking and angular velocity tracking are shown in Fig. 2 and Fig. 3.

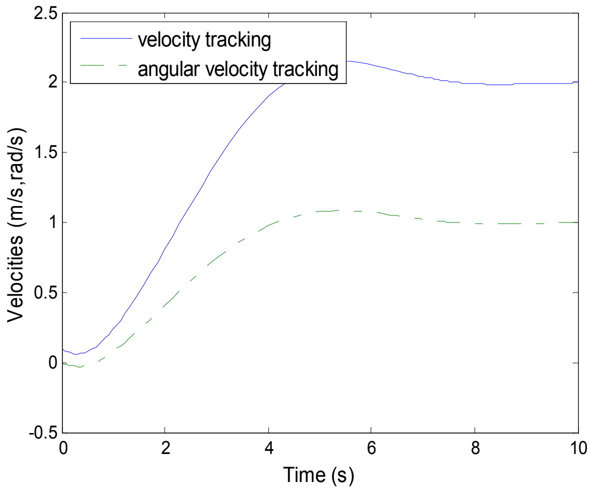

Fig. 2 Velocities.

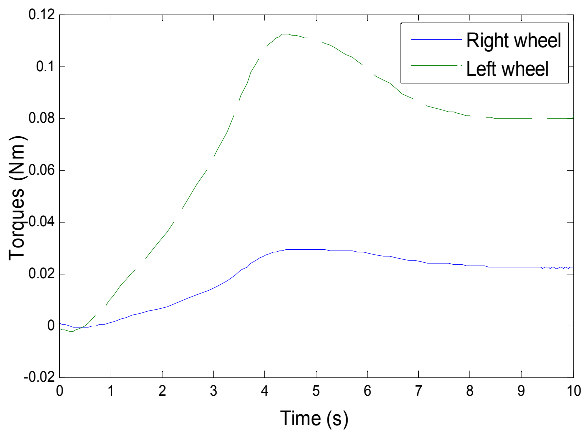

Fig. 3 The input torques.

Fig. 2 shows the tracking and the orientation control performance respectively. Fig. 3 shows the input torques of two driving wheels. The simulation results show that the tracking and the orientation of control tend to the desired values, which validates the effectiveness of the disturbance observers in Theorem 4.1. Under the proposed control mode, tracking control of the desired trajectory and desired orientation is achieved and this is mainly due to the "disturbance observer" mechanism. The simulation results demonstrate the effectiveness of the proposed disturbance observers in the presence fast time-varying external disturbances. Although fast time-varying external disturbances are introduced into the simulation model, the tracking/orientation control performance of system, under the proposed control, is not degraded. Different tracking/orientation control performance can be obtained by adjusting the values of design parameter.

Furthermore, this kind of disturbance observer also can track velocity with time-varying. The initial velocity and angular velocity are the same as above case. The target velocities are taken as $v(t) = t$ m/s and $\omega(t) = 0.5t$ rad/s respectively. The simulation results for velocity tracking and angular velocity tracking are shown in Figs. 4 and Fig. 5.

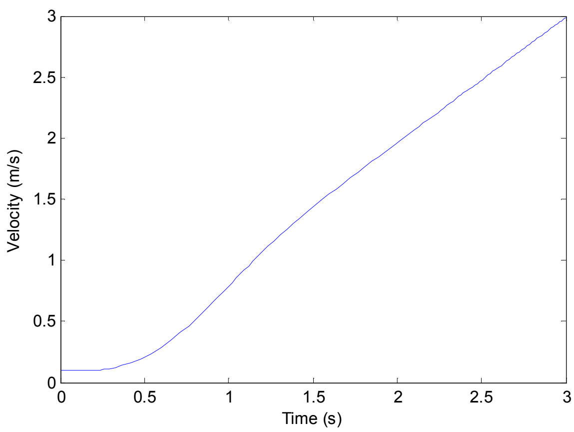

Fig. 4 The velocity

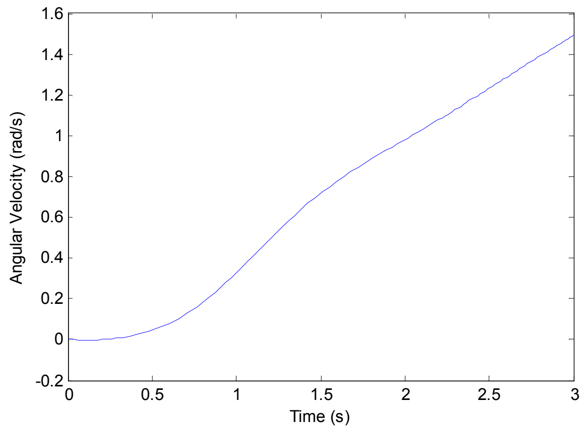

Fig. 5 Angular velocity

5 Tracking and Controller Design

5.1 Tracking errors

In this section, under the desired velocity, the tracking problem for mobile robot is presented. To validate the tracking, it is assumed that the reference trajectory (x_r, y_r, θ_r) can be expressed

$$q_r(t) = \begin{bmatrix} \dot{x}_r \\ \dot{y}_r \\ \dot{\theta}_r \end{bmatrix} = \begin{bmatrix} v_r \cos\theta_r \\ v_r \sin\theta_r \\ \omega_r \end{bmatrix} \tag{31}$$

where v_r and ω_r are desired reference velocity and angular velocity.

As in [28] and [29], the tracking error is expressed as

$$E_p = \begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix} \tag{32}$$

Clearly, for any value of θ, $(x_e, y_e, \theta_e) = 0$ if and only if $(x, y, \theta) = (x_r, y_r, \theta_r)$. The first derivative of E_p can be written as

$$\dot{E}_p = \begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\theta}_e \end{bmatrix} = \begin{bmatrix} \omega y_e - v + v_r \cos\theta_e \\ -\omega \dot{x}_e + v_r \sin\theta_e \\ \omega_r - \omega \end{bmatrix} \tag{33}$$

5.2 Proposed control law

To solve the tracking problem, the control laws are proposed as follows

$$v = v_r \cos\theta_e + \rho_1 \frac{2}{\pi} \arctan(x_e) \tag{34}$$

$$\omega = \omega_r + \frac{\rho_2 v_r y_e}{1 + x_e^2 + y_e^2} \frac{\sin \theta_e}{\theta_e} + \rho_3 \frac{2}{\pi} \arctan(\theta_e) \tag{35}$$

where ρ_1, ρ_2 and ρ_3 are positive design parameters.

Theorem 5.1: Assume that ω_r and v_r are bounded and uniformly continuous over [0, ∞). If either ω_r or v_r does not converge to zero, then the zero equilibrium of the closed-loop system (33)-(35) is globally asymptotically stable.

Proof: Consider the Lyapunov function candidate

$$V_3 = \rho_2 \log(1 + x_e^2 + y_e^2) + \theta_e^2 \tag{36}$$

then, differentiating V_3 with the time, considering (33)- (35), and we have

$$\dot{V}_3 = -\frac{4\rho_2 x_e \arctan(x_e)}{\pi(1 + x_e^2 + y_e^2)} - \frac{4\rho_3 \theta_e \arctan(\theta_e)}{\pi} \le 0 \tag{37}$$

Therefore, the trajectories $(x_e(t), y_e(t), \theta_e(t))$ are uniformly bounded on [0, ∞). According to [23],

$$\lim_{t\to\infty} \left[x_e \arctan(x_e) + \theta_e \arctan(\theta_e)\right] = 0 \tag{38}$$

which, in turn, we know

$$\lim_{t\to\infty} \left(|x_e(t)| + |\theta_e(t)|\right) = 0 \tag{39}$$

It remains to prove that $y_e(t) \to 0$ as $t \to \infty$. This can be established by method of arguments used in the proof of [30].

5.3 Simulation

The proposed controllers in this section are verified with computer simulation using MATLAB, considering the proposed controllers and the observers in section 4. The parameter values of the mobile robot and the disturbances of velocities are the same as ones in section 4.2. The trajectory tracking is based on the velocity tracking and the angular tracking. The parameters of three controllers are chosen as $k_p = 1$; $k_d = 0.6$. The parameters of three observers are chosen as $\mu = 4$; $\upsilon = 0.55$; $\gamma = 5$. The desired trajectory has been given to be $v_r = 2$ m/s, $\omega_r = 1$ rad/s, i.e. a circle. Tracking of the mobile robot with initial error vector is $E_p = [x_e \quad y_e \quad \theta_e]^T = [2 \quad 1 \quad 0.5]^T$. The simulation results of tracking are shown in Figs. 6 – 8. Fig. 6 shows the position tracking errors in *X* and *Y* coordinates. Fig. 7 shows the angular tracking error. Fig. 8 shows the input torques of two driving wheels. The simulation results show that the tracking errors tend to the desired values, which validates the effectiveness of the disturbance observer in Theorem 5.1. Under the proposed disturbance observer and the controller law, tracking problem can be achieved, and the tracking error vector exponentially converges to zero vector.

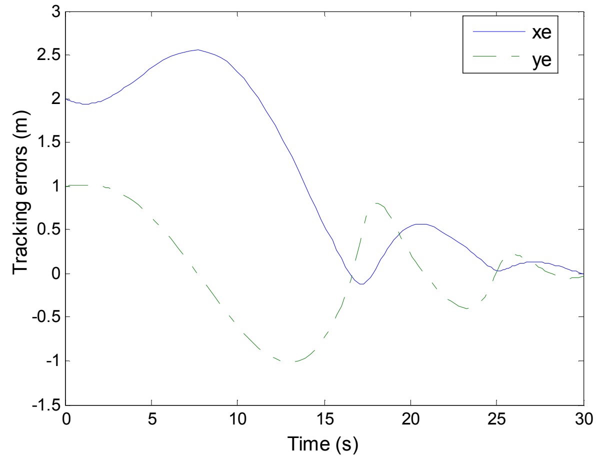

Fig. 6 Position tracking errors.

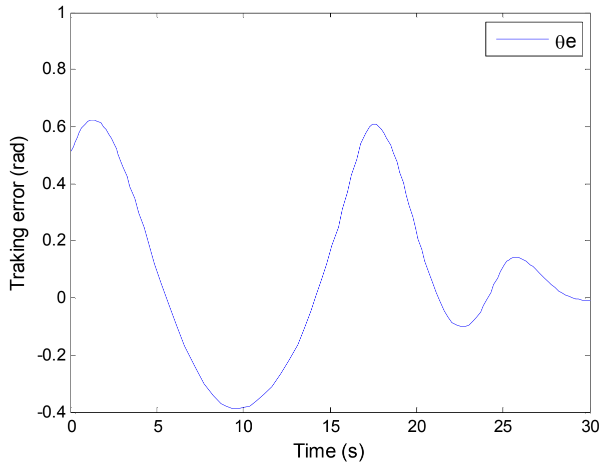

Fig. 7 Angle tracking.

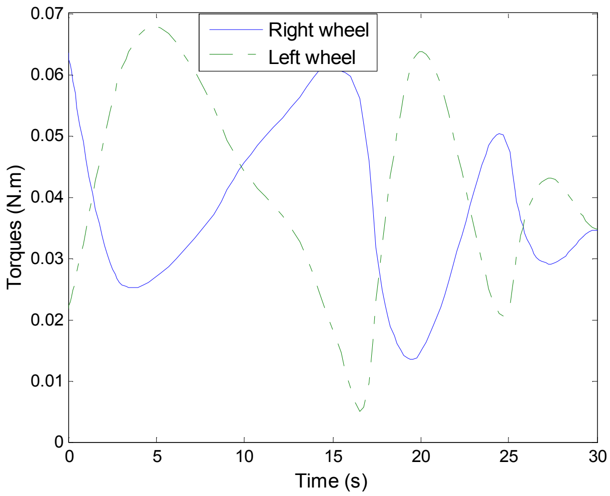

Fig. 8 The input torques.

6 Conclusion

In this paper, effective disturbance observer based on the series of integral filters has been presented systematically to velocity/angular velocity tracking and trajectory tracking for the mobile robot with unknown environment. For the controller, the stability and error boundedness is proved using Lyapunov stability theory. The proposed observer requires no information on the system. Simulation studies have verified the effectiveness of the proposed observer.

Appendix

A. *Proof for Lemma 4.1*

Proof (i) Consider the following differential equation

$$\dot{\hat{z}}_i(t) - \dot{z}_i + \gamma_i(\hat{z}_i(t) - z_i) + e^{-y_i t} z_i = 0 \tag{40}$$

Its solution is

$$\hat{z}_i(t) = (1 - e^{-\gamma_i t}) z_i(t) + e^{-\gamma_i t} z_i(0)$$

It can be found that $\hat{z}(t)$ converges to $z(t)$ exponentially. From (25) and (26), one has

$$z_i(t) = e^{-\mu_i t} z_i(0) + e^{-\mu_i t} \cdot \int_0^t e^{\mu_i r} \cdot \left[(\mu_i - \upsilon_i) e^{-\upsilon_i r} z_i(0) + \int_0^r e^{-\mu_i (r-s)} \overline{d}_i(s) ds\right] dr \tag{41}$$

$$\hat{z}_i(t) = e^{-\mu_i t} \hat{z}_i(0) + e^{-\mu_i t} \cdot \int_0^t e^{\mu_i r} \cdot \left[(\mu_i - \upsilon_i) e^{-\upsilon_i r} \hat{z}_i(0) + \int_0^r e^{-\mu_i (r-s)} \hat{d}_i(s) ds\right] dr \tag{42}$$

Substituting (41) and (42) into (40) results in

$$\int_0^t e^{\mu_i r} \tilde{d}_i(r) dr + (\gamma_i - \mu_i) \cdot \int_0^t \int_0^r e^{\mu_i s} \tilde{d}_i(s) ds dr = \phi_i(t) \tag{43}$$

where

$$\begin{aligned} \phi_i(t) = {} & (\upsilon_i - \gamma_i) e^{(\mu_i - \upsilon_i)t} \left[\hat{z}_i(0) - z_i(0)\right] \\ & + \upsilon_i e^{(\mu_i - \upsilon_i - \gamma_i)t} z_i(0) - e^{-\gamma_i t} \int_0^t e^{\mu_i r} \overline{d}_i(r) dr \\ & + u_i e^{-\gamma_i t} \int_0^t \int_0^r e^{\mu_i s} \overline{d}_i(s) ds dr. \end{aligned} \tag{44}$$

Define the following variable

$$\chi_i(t) = \int_0^t \int_0^r e^{\mu_i s} \tilde{d}_i(s) ds dr \tag{45}$$

Differentiating both sides of (45) and combining (43) yields $\dot{\chi}_i(t) = -(\gamma_i - \mu_i)\chi_i(t) + \phi_i(t)$.

Its solution is

$$\chi_i(t) = e^{-(\gamma_i - \mu_i)t} \int_0^t e^{(\gamma_i - \mu_i) r} \phi(r) dr \tag{46}$$

Differentiating both sides of (46) twice yields

$$\begin{aligned} \ddot{\chi}_i(t) = {} & (\gamma_i - \mu_i)^2 e^{-(\gamma_i - \mu_i)t} \int_0^t e^{(\gamma_i - \mu_i) r} \phi_i(r) dr \\ & - (\gamma_i - \mu_i)\phi_i(t) + \dot{\phi}_i(t), \end{aligned} \tag{47}$$

where

$$\begin{aligned} \dot{\phi}_i(t) = {} & (\mu_i - \upsilon_i)(\upsilon_i - \gamma_i) e^{(\mu_i - \upsilon_i)t} \left[z_i(0) - \hat{z}_i(0)\right] \\ & + \upsilon_i(\mu_i - \upsilon_i - \gamma_i) e^{\mu_i - \upsilon_i - \gamma_i)} \hat{z}_i(0) \\ & - e^{(\mu_i - \gamma_i)t} \overline{d}_i(t) - \gamma_i \mu_i e^{-\gamma_i t} \int_0^t \int_0^r e^{\mu_i s} \overline{d}_i(s) ds dr \\ & + (\mu_i + \gamma_i) e^{-\gamma_i t} \int_0^t e^{\mu_i r} \overline{d}_i(r) dr. \end{aligned} \tag{48}$$

Differentiating both sides of (47) twice yields

$$\ddot{\chi}_i(t) = e^{\mu_i t} \tilde{z}_i(t) \tag{49}$$

Comparing (47) and (49), one has

$$\begin{aligned} \tilde{d}_i(t) = {} & (\gamma_i - \mu_i)^2 e^{-\gamma_i t} \int_0^t e^{(\gamma_i - \mu_i) r} \phi_i(r) dr \\ & - e^{\mu_i t} (\gamma_i - \mu_i) \phi_i(t) + e^{-\mu_i t} \dot{\phi}_i(t) \end{aligned} \tag{50}$$

Substituting (44) and (48) into (50) yields

$$\tilde{d}_i(t) = c_{i1} e^{-\upsilon_i t} + c_{i2} e^{-\gamma_i t} + c_{i3} e^{-(\upsilon_i + \gamma_i)t}$$

where c_{i1}, c_{i2} and c_{i3} are constants. Obviously, there will exist positive constants α_i and β_i such that $\left|\tilde{d}_i(t)\right| \le \alpha_i e^{-\beta_i t}$ with $\beta_i = \min(\upsilon_i, \gamma_i)$.

(ii) From (23), one has

$$\overline{d}_i(t) = m_i \ddot{e}_i + V_i + \hat{d}_i(t) \tag{51}$$

Define

$$\int_0^t e^{\mu_i r} m_i \ddot{e}_i(r) dr = E_i(t)$$

Then, one has

$$E_i(t) = m_i e^{\mu_i t} \dot{e}_i(t) - m_i \dot{e}_i(0) - m_i \mu_i \int_0^t e^{\mu_i r} \dot{e}_i(r) dr$$

Furthermore, we do not need $\ddot{e}_i(t)$ signal. Substituting (51) into (42) yields

$$\int_0^t e^{\mu_i r} \hat{d}_i(r) dr - \mu_i \int_0^t \int_0^r e^{\mu_i r} \hat{d}_i(s) ds dr = \psi_i(t) \tag{52}$$

where

$$\begin{aligned}\psi_i(t) &= (\upsilon_i - \gamma_i)e^{(\mu_i+\gamma_i-\upsilon_i)t}(z_i(0) - \hat{z}_i(0)) \\ &+ \left[\mu_i + (\gamma_i - \mu_i)e^{\gamma_i t}\right]\int_0^t \left[\vartheta_i(r) + \int_0^r e^{\mu_i s}V(s)ds\right]dr \\ &+ (e^{\gamma_i t} - 1)\int_0^t e^{\mu_i r}V_i(r)dr + (e^{\gamma_i t} - 1)\vartheta(t) \\ &+ \upsilon e^{(\mu_i-\upsilon_i)t}\hat{z}_i(0).\end{aligned}$$

To simplify (52), we define the following variable

$$\varsigma_i = \int_0^t\int_0^r e^{\mu_i r}\hat{d}_i(s)dsdr \tag{53}$$

Its first derivative is

$$\dot{\varsigma}_i = \int_0^t e^{\mu_i r}\hat{d}_i(r)dr \tag{54}$$

Substituting (53) and (54) into (52) yields

$$\dot{\varsigma}_i - \mu_i\varsigma_i = \psi_i \tag{55}$$

Its solution is

$$\varsigma_i = e^{\mu_i t}\int_0^t e^{-\mu_i r}\psi_i dr \tag{56}$$

Substituting (56) to (54) yields

$$\int_0^t e^{\mu_i r}\hat{d}_i(r)dr = \mu_i e^{\mu_i t}\int_0^t e^{-\mu_i r}\psi_i(r)dr + \psi_i(t) \tag{57}$$

This is completed the proof.

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