A Sequential Design Method for Multivariable Feedback Control Systems

TAIN-SOU TSAY
Department of Department of Aeronautical Engineering
National Formosa University
64, Wen-Hua Road, Huwei, Yunlin
TAIWAN
tstsay@nfu.edu.tw

Abstract: - In this paper, a sequential design method is proposed for analyses and designs of multivariable feedback control systems. The overall compensator is implemented systematic-cally with a cascaded layer-wrapped structure and diagonal compensators. The method can take into consideration of stability, integrity, diagonal dominance and performance in a sequential manner. Roots of stability-equations are used to select parameters of compensators in the parameter plane. Three 2x2 and one 4x4 multivariable examples are given and comparisons with the methods in current literature are made.

Key-Words: - **Sequential design, Integrity, Stability, Multivariable feedback control system.**

1 Introduction

There are many design techniques developed for multivariable feedback control systems [1-2]. Even for stable system, most modern control techniques based on various optimi-zation techniques, such as H_2, H_∞, L_1/l_1 norm based or μ optimization based designs tend to give unstable controllers. The industrial processes using unstable controllers have been limited. Furthermore, integrities [3, 4] of controlled systems for coping with sensor failures cannot be guaranteed.

The main purpose of this paper is to get stable compensators with integrity considerations for multivariable feedback control systems in a sequential manner. This implies that at each stage only one loop of a loop gain matrix is closed. The feedback loops can open or close for checking integrities. Sequential design methods have been used widely for design techniques in frequency domain of single-input single-output (SISO) systems can be easily applied to multi-input multi-output systems[5-19]. In general, constant precompensating matrices have been widely used to achieve diagonal dominance [5-19]. The concept of the diagonal dominance plays a central role in the frequency domain design methods. However, diagonal dominance with precompensating matrix cannot cope with plant uncertainties. The commonly used methods based on this concept are Nyquist array method, inverse Nyquist array method [5-10], and sequential return difference method [11-15].

The method used in this paper is to extend the stability-equation method [20-22] and to present a layer-wrapped precompensating matrix for the analysis and design of multivariable feedback control systems.

In this paper, for an N×N multivariable feedback system the constant pre-compensating matrix is decomposed into N cascaded constant matrices with N parameters in each matrix. Corresponding to these N matrices, there are N steps for design. In each step, there are N parameters and one diagonal compensator to be determined. The proper values of parameters for each specified diagonal compensator can be chosen by inspecting the stability boundaries and the constant-ω curves generated by the stability-equation method. The stability boundaries can be used to show that the proper choice of parameters in the boundaries can keep the compensated system stable even when some of the transducers are failing. Meanwhile, the relative difference among the constant-ω curves will show the relative damping characteristics of the system [20-22].

Similar procedure has been developed by Maine[11-14]. Under his approach, a constant precompensating matrix is first selected to achieve the diagonal dominance, and then close the loops systematically with diagonal-compensators in the diagonal elements. In comparison to Mayne's approach, the method proposed in this paper is to use different structure for the pre-compensating matrix and to achieve high integrity against transducer failures [3, 4]. In addition, since the stability-equation method is highly capable of handling multi-parameter problems, the overall compensator for achieving desirable system performance can be implemented easily.

A brief review of the stability-equation method is given in Section 2, then the concept of the layer-wrapped structure is presented in Section 3, and finally the detail design procedure is illustrated by the numerical examples given in Section 4.

2 The Basic Approach

Assume that the system characteristic equation is *F(S)* which can be decomposed into two parts concerning even and odd terms of s; i.e.,

$$F(s) = F_e(s) + F_o(s) \tag{1}$$

Let $s = j\omega$, then the stability-equations are

$$f_e(\omega) = F_e(j\omega)) \tag{2}$$

and

$$f_o(\omega) = F_o(j\omega)/j\omega \tag{3}$$

From reference 20, one has the following stability criterion.

***Stability Criterion*:** If the roots ω_{ei} and ω_{oj} (*i,j=1,2,.*) of the stability-equations $f_e(\omega)=0$ and $f_o(\omega)=0$, respectively, are all real and alternating in sequence, then the system with characteristic polynomial *F(s)* is stable.

For a system with two parameters (m_1 and m_2), the stability-equations can be written as

$$f_e(\omega) = \sum_{i=0}^{m} a_i \omega^{2i} \tag{4}$$

and

$$f_o(\omega) = \sum_{j=0}^{n} b_j \omega^{2j} \tag{5}$$

where the coefficients $a_i{}'s$ and $b_j{}'s$ are in the form of

$$a_i = A_{ei} + B_{ei} m_1 + C_{ei} m_2 \tag{6}$$

and

$$b_j = A_{oj} + B_{oj} m_1 + C_{oj} m_2 \tag{7}$$

where the *A's*, *B's* and C's are constants. By inserting Eqs.(6) and (7) into Eqs.(4) and (5), the result can be arranged as

$$\sum_{i=0}^{m} A_{ei}\omega^{2i} + m1\sum_{i=0}^{m} B_{ei}\omega^{2i} + \sum_{i=0}^{m} C_{ei}\omega^{2i} = 0 \tag{8}$$

and

$$\sum_{j=0}^{n} A_{oj}\omega^{2j} + m_1\sum_{j=0}^{n} B_{oj}\omega^{2j} + m_2\sum_{j=0}^{n} C_{oi}\omega^{2j} = 0 \tag{9}$$

for the even stability-equation, and for the odd stability-equation. From these two equations, the following two kinds of curves can be plotted:

***(1)The Stability-boundary Curve*:** By solving Eqs.(8) and (9) for sufficient number of suitable values of ω, the simultaneous solutions of m_1 and m_2 can be used to sketch a number of curves in a m_1 vs. m_2 plane. Then the curve for $\omega_{ei} = \omega_{oj}$ which constitutes the stability-boundary can be determined.

***(2)The Constant-ω Curves*:** By assigning sufficient number of values of ω to Eqs. (8) and (9) the constant-ω curves for even and odd stability-equations can be plotted in the m_1 vs. m_2 plane.

From references 20, 21 and 22, it has been shown that the differences among the magnitudes of the real roots(ω_{ei} and ω_{oj}) can be used as indications of damping characteristics approximately; therefore, the proper values of the parameters (m_1 and m_2) in a compensator can be chosen by inspecting the relative differences among the constant-ω curves[20-22]. This is the main approach of this paper.

3 Sequential Design using Cascaded Layer Wrapped Structure

The typical structures of the multivariable feedback control systems considered in this paper are shown in Fig.1(a) and 1(b). For convenient, this kind of structures are called "layer-wrapped structures".

The precompensating matrix for an N×N multivariable feedback control system is implemented by cascading the determined columns $P_{ik}(i=1,2,...,N)$ in each stage. In stage j, the precompensating matrix P_j is represented by

$$P_j=\begin{bmatrix}P_{11} & 0 & .. & 0\\ P_{21} & 1 & .. & 0\\ \vdots & & \ddots & \\ P_{N1} & 0 & .. & 1\end{bmatrix}\begin{bmatrix}1 & P_{12} & .. & 0\\ 0 & P_{22} & .. & 0\\ \vdots & & \ddots & \\ 0 & P_{N2} & .. & 1\end{bmatrix}\cdots\begin{bmatrix}1 & 0 & P_{1j} & \cdots & 0\\ 0 & 1 & P_{2j} & \cdots & 0\\ \vdots & \vdots & & \ddots & \vdots\\ 0 & 0 & P_{Nj} & \cdots & 1\end{bmatrix} \quad (10)$$

$$(stage\,1) \qquad (stage\,2) \qquad (stage\;\; j)$$

the parameters $P_{ij}(i=1,2,...,N)$ together with the diagonal compensators $K_i(s)$ as shown in Figs.1(a) and 1(b) are determined to satisfy the performance and integrity against the failures of transducers. In other words, the selections of P_{ij} will maintain the stability of the system regardless whether the feedback loops denoted less than or equal to j are closed or opened.

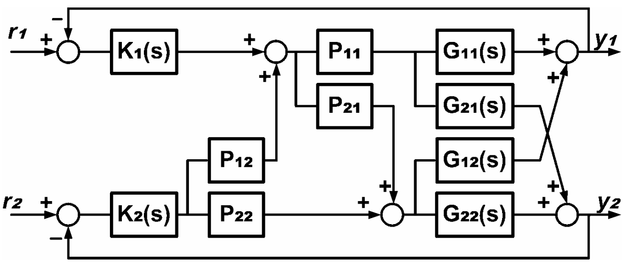

Fig.1(a). Block diagram of a 2×2 multivariable feedback control system.

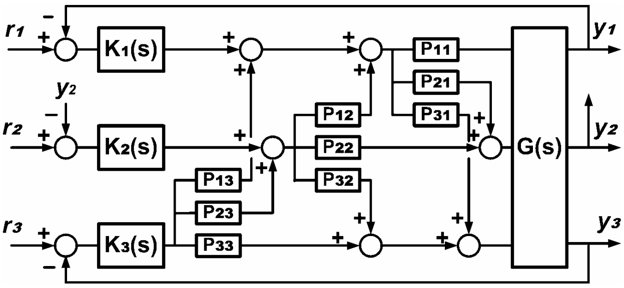

Fig.1(b). Block diagram of a 3x3 multivariable feedback control system.

The stability-equation method is used to analyze the layer-wrapped structure from stage 1 to stage N; the constant root loci and stability boundaries can be generated automatically by computer facilities. The types of diagonal compensators $K_j(s)$ are defined by the engineer with experience, and the proper choice of the parameters can be obtained *by* inspecting the results provided in the parameter plane by use of the stability-equation method.

Assume that a 2x2 multivariable feedback control system is considered. The transfer function matrix of the plant is

$$G(s)=\begin{bmatrix}g_{11}(s) & g_{12}(s)\\ g_{21}(s) & g_{22}(s)\end{bmatrix} \qquad (11)$$

The pre-compensating matrix is in the form of

$$P_2=\begin{bmatrix}P_{11} & 0\\ P_{21} & 1\end{bmatrix}\begin{bmatrix}1 & P_{12}\\ 0 & P_{22}\end{bmatrix} \qquad (12)$$

$$(stage\,1)\,(stage\,2)$$

and the matrix consists of the diagonal compensators is represented by

$$K(s)=\begin{bmatrix}K_1(s) & 0\\ 0 & K_2(s)\end{bmatrix} \qquad (13)$$

The steps for analysis and design are:

(1).Close loop-1 and define the diagonal compensator $K_1(s)$, then find the parameters P_{11} and P_{21} for satisfying the desirable performance.

(2).Close loop-2 and open loop-1 to define the diagonal compensator $K_2(s)$, then find the parameters P_{12} and P_{22} for satisfying the desirable performance.

(3).Close both loop-1 and loop-2 to check whether $K_2(s)$, P_{12} and P_{22} found in step(2) are acceptable. If not, then go back to step(2) to select other values of $K_2(s)$, P_{12} and P_{22} to satisfy desirable performance.

For step 1, the transfer function matrix of this subsystem can be written as

$$T_{if}'(s)=\frac{\begin{bmatrix}K_1(s)[g_{11}(s)P_{11}+g_{12}(s)P_{21}] & 0\\ K_1(s)[g_{21}(s)P_{11}(s)+g_{22}(s)P_{21}] & 0\end{bmatrix}}{1+K_1(s)[g_{11}(s)P_{11}+g_{12}(s)P_{21}]} \qquad (14)$$

For step 2, one has

$$T_i''(s)=\frac{\begin{bmatrix}0 & K_2(s)[\bar{g}_{11}(s)P_{12}+\bar{g}_{12}(s)P_{22}]\\ 0 & K_2(s)[\bar{g}_{21}(s)P_{12}(s)+\bar{g}_{22}(s)P_{22}]\end{bmatrix}}{1+K_1(s)[\bar{g}_{21}(s)P_{12}+\bar{g}_{22}(s)P_{21}]} \qquad (15)$$

Similarly, for step 3, the transfer function matrix of the overall system is

$$T_{if}''(s)=\frac{\begin{bmatrix}\begin{matrix}K_1(s)[\bar{g}_{11}(s)\\ +K_2(s)D_{\bar{g}}(s)P_{22}]\end{matrix} & K_2(s)[\bar{g}_{11}(s)P_{12}+\bar{g}_{12}(s)P_{22}]\\ K_1(s)\bar{g}_{21}(s) & \begin{matrix}K_2(s)\{\bar{g}_{21}(s)P_{12}\\ +[g_{22}(s)+K_1(s)D_{\bar{g}}(s)]P_{22}\}\end{matrix}\end{bmatrix}}{1+K_1(s)\bar{g}_{11}(s)+K_2(s)\{\bar{g}_{21}(s)P_{12}+[\bar{g}_{22}(s)+K_1(s)D_{\bar{g}}(s)]P_{22}\}} \qquad (16)$$

where the relations among $\bar{g}_{ij}(s)$) are defined by

$$\bar{G}(s)=\begin{bmatrix}\bar{g}_{11}(s) & \bar{g}_{12}(s)\\ \bar{g}_{21}(s) & \bar{g}_{22}(s)\end{bmatrix}=\begin{bmatrix}g_{11}(s) & g_{12}(s)\\ g_{21}(s) & g_{22}(s)\end{bmatrix}\begin{bmatrix}P_{11} & 0\\ P_{21} & 1\end{bmatrix} \qquad (17)$$

which represents the compensated plant after step 1; and $D_{\bar{g}}(s)$ is the determinant of the compensated plant.

The characteristic equations of $T'_{if}(s)$, $T''_{i}(s)$ and $T''_{if}(s)$ are, respectively

$$p_{c1}(s) = p_o(s)\{1 + K_1(s)[g_{11}(s)P_{11} + g_{12}(s)P_{21}]\} = 0 \quad (18)$$

$$p_{c2}(s) = p_{o1}(s)\{1 + K_2(s)[\bar{g}_{21}(s)P_{12} + \bar{g}_{22}(s)P_{22}]\} = 0 \quad (19)$$

and

$$p_{c3}(s) = p_{o2}(s)\{1 + K_1(s)\bar{g}_{11}(s) + K_2(s)[\bar{g}_{21}(s)P_{12} + (\bar{g}_{22}(s) + K_1(s)D_{\bar{g}}(s))P_{22}]\} = 0 \quad (20)$$

where $p_o(s)$ is the open loop characteristic equation of the plant *G(s)* with $K_1(s)$ only; and $p_{o1}(s)$ is the open loop characteristic equation of the plant with $K_2(s)$ only. Similarly $p_{o2}(s)$ represents the open loop characteristic equation of the plant with both $K_1(s)$ and $K_2(s)$. Eqs.(18) to (20) can be decomposed into two stability equations, then the analyses in the P_{11} vs. P_{21} and P_{12} and P_{22} planes can be performed.

Other objects for the design of multivariable feedback systems, such as low-interaction at low frequency and low-interaction at high frequency, can also be achieved systematically by use of the proposed method. For example, in Eq.(14) proper values of parameters P_{11} and P_{21} can be chosen to satisfy requirement of diagonal dominance for all frequencies; i.e.,

$$|g_{11}(j\omega)P_{11} + g_{12}(j\omega)P_{21}| >> |g_{21}(j\omega)P_{11} + g_{22}(j\omega)P_{21}| \quad (21)$$

Similarly, by use of Eq.(15) proper values of parameters P_{12} and P_{22} can be chosen to satisfy

$$|\bar{g}_{21}(j\omega)P_{12} + \bar{g}_{22}(j\omega)P_{22}| >> |\bar{g}_{11}(j\omega)P_{12} + \bar{g}_{12}(j\omega)P_{22}| \quad (22)$$

Finally, by use of Eq.(16), the requirement of column dominance [7,8] can be achieved by making the coefficients of the off-diagonal terms as small as possible. From Eqs.(14) to (17), it can be seen that the off-diagonal terms $(2,1)^{th}$ and $(1,2)^{th}$ elements of $T''_{if}(s)$ are the off-diagonal term $(2,1)^{th}$ of $T'_{if}(s)$ and the off-diagonal term $(1,2)^{th}$ of $T''_{i}(s)$, respectively. Therefore, if $T'_{if}(s)$ and $T''_{i}(s)$ are diagonal dominant (as defined by Eqs.(21) and (22)) then the overall system transfer function matrix $T''_{if}(s)$ is also diagonal dominant.

The design procedure and the application of the stability-equation method in each step are explained in the following three 2x2 examples and extended to a 4x4 example.

4 Numerical Examples

Example 1: Assume that the transfer function matrix of the 2x2 multivariable feedback control system shown in Fig.1(a) is[23]:

$$G_1(s) = \begin{bmatrix} \dfrac{10}{s(s+1)(s+2)} & \dfrac{-1}{s(s+1)} \\ \dfrac{3.5}{s(s+2)} & \dfrac{6}{s(s+1)} \end{bmatrix} \quad (23)$$

For compensation, two first order lead/lag diagonal compensators $K_1(s)$ and $K_2(s)$ are used.

In step 1, one of the diagonal-compensator $K_1(s)$ is defined as

$$K_1(s) = \frac{100(s+1)}{s+100}$$

The transfer function matrix of this subsystem is

$$T_{if}'(s) = \frac{\begin{bmatrix} 1000P_{11} - 100(s+2)P_{21} & 0 \\ 250(s+1)P_{11} + 600(s+2)P_{21} & 0 \end{bmatrix}}{s^3 + 102s^2 + 200s + 1000P_{11} - 100(s+2)P_{21}} \quad (24)$$

where P_{11} and P_{21} are two adjustable parameters. The negative summation of the characteristic roots is increased from 2 to 102 with the aid of $K_1(s)$. The characteristic equation is

$$s^3 + 102s^2 + 200s + 1000P_{11} - 100(s+2)P_{21} = 0 \quad (25)$$

which can be decomposed into the following two stability-equations

$$-102\omega^2 + 1000P_{11} - 200P_{21} = 0 \quad (26)$$

and

$$-\omega^2 + 200 - 100P_{21} = 0 \quad (27)$$

Then the stability boundary and the constant-ω curves can be plotted as shown in Fig.2(a). In order to have fine damping characteristics and to satisfy the diagonal dominance as defined in Eq.(21) suitable values of P_{11} and P_{21} are selected at 90 and -52.5, respectively; i.e., point Q_1 in Fig.2(a). By this selection, the roots ω_{e1} and ω_{o1} of the stability equations are approximately at 31 and 74, respectively. The characteristic roots of this subsystem are found at

-30.891, -25.554±j44.601.

This result justifies the description of the stability-equation method given in references 1 and 2; i.e., larger differences among ω_{e1} and ω_{o1} will give better damping characteristics of the compensated subsystem. In addition, from Eq.(24) it can be seen that the ratio of $P_{11}/P_{21} = 90/-2.5 = 600/-350$ will make the coefficients of off-diagonal term much smaller than that of the diagonal term; i.e., low-interaction is achieved because the coefficient of the term s^1 of the off-diagonal element is zero. In short, the rules for selecting proper values of P_{11} and P_{21} are (1) selecting the ratio of P_{11}/P_{21} to achieve low-interaction and (2) selecting the values of P_{11} and P_{21} to obtain desirable performance.

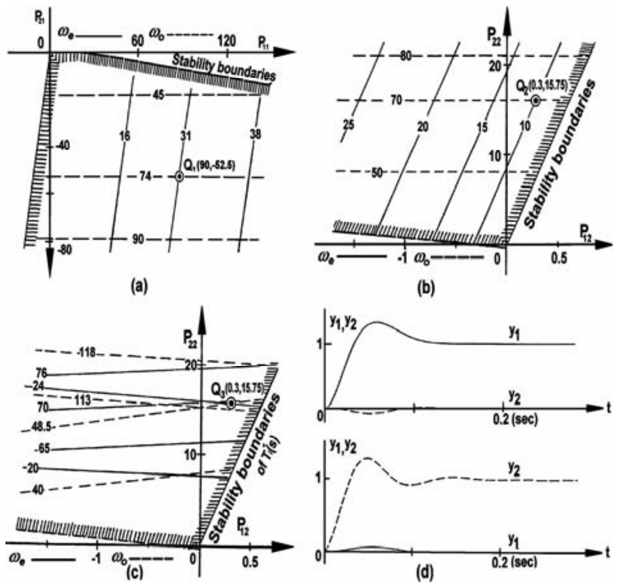

Fig.2. Parameter analyses in (a)Step 1; (b) Step 2; (c)Step 3; and (d)Time Responses of Example 1.

In step 2, the remaining diagonal-compensator $K_2(s)$ is defined as

$$K_2(s) = \frac{50(s+1)}{s+50}$$

Since loop-1 is open and loop-2 is closed, the transfer function matrix $T_i'(s)$ is

$$T_i'(s) = \frac{\begin{bmatrix} 0 & (2625s+505250)P_{12} - (50s+100)P_{22} \\ 0 & -15750P_{12} + (300s-600)P_{22} \end{bmatrix}}{s^3 + 52s^2 + 100s - 15750P_{12} + (300s+600)P_{22}} \quad (28)$$

where P_{12} and P_{22} are two adjustable parameters. The characteristic equation is

$$s^3 + 52s^2 + 100s - 15750P_{12} + (300s+600)P_{22} = 0 \quad (29)$$

and the stability equations are

$$-52\omega^2 + (-15750)P_{12} + 600P_{22} = 0 \quad (30)$$

and

$$-\omega^2 + 100 + 300P_{22} = 0 \quad (31)$$

Constant-ω curves are shown in Fig.2(b). Proper values of P_{12} and P_{22} are selected at 0.3 and 15.75, respectively; i.e., point Q_2 in Fig.2(b). The roots of the stability equations can be read out approximately; i.e., $\omega_{e1} = 10$ and $\omega_{o1} = 70$. The roots of the characteristic equation are found at

-0.9896, -25.5052±j64.2184

From Fig.2(b) and Eq.(28), it can be seen that the ratio $P_{12}/P_{22} = 0.3/15.75 = 50/2625$ will make the coefficient of the term s^1 to be zero.

In step 3, after loop-1 is closed the parameter analysis is performed in the P_{12} vs. P_{22} plane. The transfer function matrix of the overall system $T_{if}''(s)$ is represented by Eq.(16). Since P_{11} and P_{21} are selected at 90 and -52.5, respectively, the characteristic equation is

$$\begin{aligned} &s^5 + 152s^4 + 10550s^3 + 37300s^2 + 5025000s \\ &\quad + (-15750s^2 - 1575000s)P_{12} + (300s^3 + 30600s^2 \\ &\qquad + 1635000s + 28575000)P_{22} = 0 \end{aligned} \quad (32)$$

The stability equations are

$$\begin{aligned} &152\omega^4 - 37300\omega^2 + 15750\omega^2 P_{12} \\ &\quad + (-30600\omega^2 + 28575000)P_{22} = 0 \end{aligned} \quad (33)$$

and

$$\begin{aligned} &\omega^4 - 10550\omega^2 + 5025000 - 1575000P_{12} \\ &\quad + (-300\omega^2 + 1635000)P_{22} = 0 \end{aligned} \quad (34)$$

Constant-ω curves are shown in Fig.2(c). Note that the stability boundaries are derived from the analyses of the sub-system considered in step-2, for which loop-1 is open and loop-2 is close. This implies that the choices of P_{12} and P_{22} are constrained by the boundaries for achieving the integrity against transducer failure in loop-1 which may cause loop-1 to open.

It can be seen that the choice of point Q_2 in Fig.2(b) is corresponding to point Q_3 in Fig.2(c), and that the choice of Q_3 is satisfactory for the overall system. The roots of the stability equations are approximately at

$$\omega_{e1} = 24, \omega_{o1} = 48.5, \omega_{e2} = 70, \omega_{o2} = 113.$$

The overall compensator is

$$P_2K(s)=\begin{bmatrix}90 & 0\\ -52.5 & 1\end{bmatrix}\begin{bmatrix}1 & 0.3\\ 0 & 15.75\end{bmatrix}\begin{bmatrix}\frac{100(s+1)}{s+100} & 0\\ 0 & \frac{50(s+1)}{s+50}\end{bmatrix} \quad (35)$$

The transfer function matrix of the final compensated system is

$$T_{if}''(s)=\frac{1}{\Delta(s)}\begin{bmatrix}5250s^3+363000s^2 \\ +28931250s+450056250 & 13500s^2+13500s\\ -31500s^2-1575000s & 477225s^3+477225s^2 \\ & +25278750s+450056250\end{bmatrix} \quad (36)$$

where

$$\Delta(s)=s^5+152s^4+15275s^3+850225s^2+30303750s+4500566250$$

The characteristic roots are found at

-29.0899, -35.8683±j43.8834,
-25.5867±j64.5103

The step responses of the overall system are shown in Fig.2(d), which indicate that the designed system has nice damping characteristics as predicted.

Note that all the constant- ω curves are straight-lines; therefore locations of characteristics roots(i.e., damping characteristics) can be predicted approximately and easily by inspecting the differences among ω_{ei} and ω_{oj}.

Example 2: Consider the 30-plate distillation column in the UMIST pilot plant [5,7]. The transfer function matrix of the plant is

$$G_2(s)=\frac{1}{\Delta(s)}\begin{bmatrix}0.088+165s & 0.1825+353.138s\\ +4083.2s^2+24420s^3 & +28835s^2+253218.75s^3\\ 0.282+28.984s & 0.412+332.484s\\ +8641.61s^2+229054.5s^3 & +23593.44s^2+223098s^3\end{bmatrix} \quad (37)$$

where $\Delta(s)=1+26272s+1.5949\times10^6 s^2+1.3879\times10^8 s^3 + 2.7337\times10^9 s^4+1.503\times10^{10} s^5$.

Following the same design procedure and discussions stated in Example 1, the constant- ω curves of two subsystems with transfer matrices $T_{if}'(s)$ and $T_i''(s)$ and the overall system with transfer function matrix $T_{if}''(s)$ are shown in Figs. 3(a) to 3(c). The diagonal compensators $K_1(s)$ and $K_2(s)$ are defined as

$$K_1(s)=\left(1+\frac{1}{T_1 s}\right)$$

in step 1, and

$$K_2(s)=\left(1+\frac{1}{T_2 s}\right)$$

in step 2, where T_1 and T_2 are selected at 33.33 and 125 seconds, respectively. The parameters $P_{ij}(i, j=1,2)$ are chosen at

$P_{11}=-38, P_{21}=26$, point Q_4 in Fig.3(a),

$P_{12}=-0.35, P_{22}=20$, point Q_5 in Fig.3(b) and point Q_6 in Fig.3(c)

The overall compensator is

$$P_2K(s)=\begin{bmatrix}-38 & 0\\ 26 & 1\end{bmatrix}\begin{bmatrix}1 & -0.35\\ 0 & 20.0\end{bmatrix}\begin{bmatrix}1+.03/s & 0\\ 0 & 1+.008/s\end{bmatrix} \quad (38)$$

The characteristic roots of the final compensated system are found at

-0.0664, -0.0651, -0.013, -0.005222, -0.09415, -0.001438, -0.0005406, -0.0991, -0.003081 ± j0.006183, -0.006369 ± j0.008972

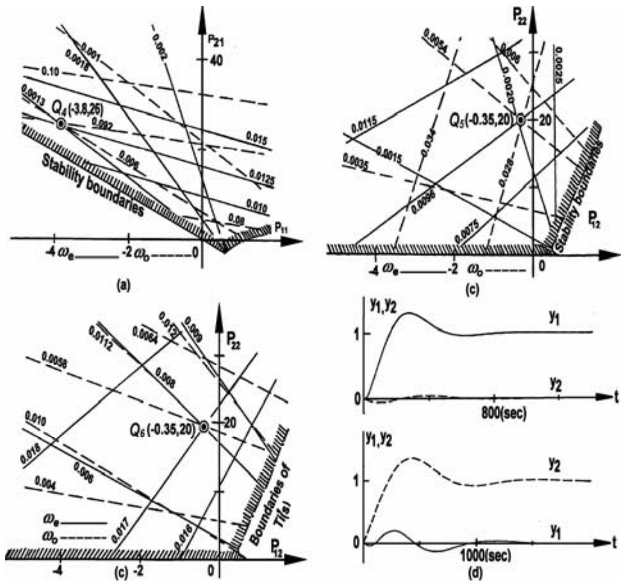

Fig.3. Parameter analyses in (a)Step 1; (b) Step 2; (c)Step 3; and (d)Time Responses of Example 2.

Step responses of the compensated system are shown in Fig.3(d). The same system has been considered by Stojic[7] utilizing the inverse Nyquist array method[5-10]. In his approach, a precompensating matrix is first selected to achieve diagonal dominance and then find coefficients of the PI controllers in the diagonal elements by use of the inverse Nyquist array method. In comparison to Stojic's approach, the advantages of the method proposed in this paper are that the selections of the parameters $P_{ij}'s$ are more straightforward and that the dominance is achieved in each step.

Example 3: Consider an aircraft gas turbine engine with plant transfer function matrix [8,15]

$$G_3(s) = \frac{1}{\Delta(s)}\begin{bmatrix} 2533+1515.33s+14.9s^2 & 1805947+1132094.7s+95150s^2 \\ 12268.8+8642.68s+85.2s^2 & 252880+1492588s+124000s^2 \end{bmatrix} \quad (39)$$

where $\Delta(s) = 2525+3502.7s+1357.3s2+113.22s^3+s^4$. For convenient, the plant is first multiplied by a scaling matrix

$$P_0 = \begin{bmatrix} 1 & 0 \\ 0 & 0.001 \end{bmatrix} \quad (40)$$

The diagonal-compensators $K_i(s)(i=1,2)$ are selected to be

$$K_1(s) = \frac{s+4}{s+0.2}$$

in step 1; and

$$K_2(s) = \frac{s+2.5}{s+5.0}$$

in step 2. Similar to Example 1, the constant-ω curves of two subsystems and the overall system are shown in Figs.4(a) to 4(c). Proper values of $P_{ij}'s$ are selected as

$P_{11} = -4, P_{21} = 19.5$, point Q_7 in Fig.4(a),
$P_{12} = 1.5, P_{22} = 40$, point Q_8 in Fig.4(b) and
point Q_9 in Fig.4 (c) .

The overall compensator is

$$P_2K(s) = \begin{bmatrix} 1 & 0 \\ 0 & .001 \end{bmatrix}\begin{bmatrix} -4 & 0 \\ 19.5 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1.5 \\ 0 & 40 \end{bmatrix}\begin{bmatrix} \frac{s+4}{s+.2} & 0 \\ 0 & \frac{s+2.5}{s+5.0} \end{bmatrix} \quad (41)$$

The characteristic roots of the compensated system are found at

-1.3371,-1.8823,-2.4907,-4.8665,-9.9989,
-9.06867±j15.30685, -46.4665±j24.4423

Step responses of the compensated system are shown in Fig.4(d). Similar results have been obtained by Chuang [15] utilizing the sequential return difference method[10-15]. However, the method proposed in this paper is much simpler.

Example 4: consider the 4x4 boiler furnace control system with transfer function matrix [22]

$$G_4(s) = \begin{bmatrix} \frac{1}{1+4s} & \frac{0.7}{1+5s} & \frac{0.3}{1+5s} & \frac{0.2}{1+5s} \\ \frac{0.6}{1+5s} & \frac{1}{1+4s} & \frac{0.4}{1+5s} & \frac{0.35}{1+5s} \\ \frac{0.35}{1+5s} & \frac{0.4}{1+5s} & \frac{1}{1+4s} & \frac{0.6}{1+5s} \\ \frac{0.2}{1+5s} & \frac{0.3}{1+5s} & \frac{0.7}{1+5s} & \frac{1}{1+4s} \end{bmatrix} \quad (42)$$

Combing diagonal controllers and re-compensating matrices with lead/lag sub-compensators, the overall compensator is in the form of

$$C(s) = P_1(s)P_2(s)P_3(s)P_4(s)$$

$$= \underbrace{\begin{bmatrix} \frac{s+b_{11}}{s+d_1}\bar{p}_{11} & 0 & 0 & 0 \\ \frac{s+b_{21}}{s+d_1}\bar{p}_{21} & 1 & 0 & 0 \\ \frac{s+b_{31}}{s+d_1}\bar{p}_{31} & 0 & 1 & 0 \\ \frac{s+b_{41}}{s+d_1}\bar{p}_{41} & 0 & 0 & 1 \end{bmatrix}}_{Stage1}\underbrace{\begin{bmatrix} 1 & \frac{s+b_{12}}{s+d_2}\bar{p}_{12} & 0 & 0 \\ 0 & \frac{s+b_{22}}{s+d_2}\bar{p}_{22} & 0 & 0 \\ 0 & \frac{s+b_{32}}{s+d_2}\bar{p}_{32} & 1 & 0 \\ 0 & \frac{s+b_{42}}{s+d_2}\bar{p}_{42} & 0 & 1 \end{bmatrix}}_{Stage2} \times$$

$$\underbrace{\begin{bmatrix} 1 & 0 & \frac{s+b_{13}}{s+d_3}\bar{p}_{13} & 0 \\ 0 & 1 & \frac{s+b_{23}}{s+d_3}\bar{p}_{23} & 0 \\ 0 & 0 & \frac{s+b_{33}}{s+d_3}\bar{p}_{33} & 0 \\ 0 & 0 & \frac{s+b_{43}}{s+d_3}\bar{p}_{43} & 1 \end{bmatrix}}_{\text{Stage }3}\underbrace{\begin{bmatrix} 1 & 0 & 0 & \frac{s+b_{14}}{s+d_4}\bar{p}_{14} \\ 0 & 1 & 0 & \frac{s+b_{24}}{s+d_4}\bar{p}_{24} \\ 0 & 0 & 1 & \frac{s+b_{34}}{s+d_4}\bar{p}_{34} \\ 0 & 0 & 0 & \frac{s+b_{44}}{s+d_4}\bar{p}_{44} \end{bmatrix}}_{\text{Stage }4} \quad (43)$$

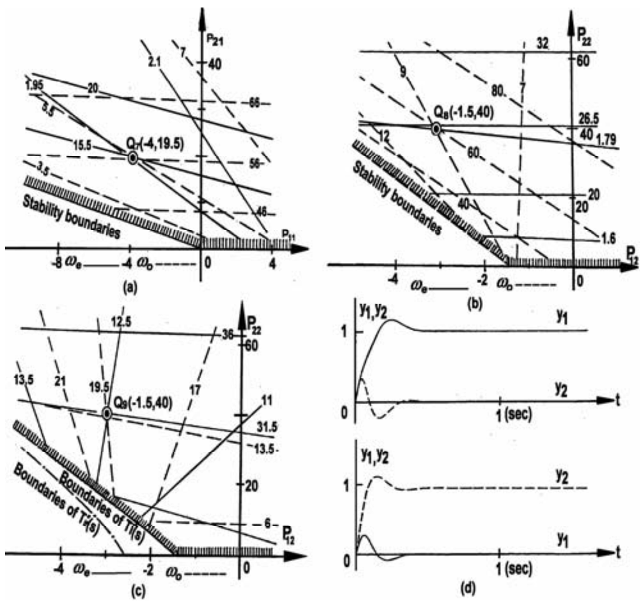

Fig.4. Parameter analyses in (a)Step 1; (b) Step 2; (c)Step 3; and (d)Time Responses of Example 3.

where $d_j, b_{ij}, \bar{p}_{ij} (i, j = 1,2,3,4)$ are adjustable parameters. Four steps are used for analyses and designs of this 4x4 multivariable system to find $P_j(s)(j = 1,2,3,4)$ systematically. Note that diagonal compensator and precompensating matrix are merged together in each step.

Step 1: In step 1, the transfer function matrix is in the form of

$$T^{(1)}(s) = \frac{\begin{bmatrix} p_0(s)\sum_{i=1}^{4}(s+b_{i1})g_{1i}(s)\bar{p}_{i1} & 0 & 0 & 0 \\ p_0(s)\sum_{i=1}^{4}(s+b_{i1})g_{2i}(s)\bar{p}_{i1} & 0 & 0 & 0 \\ p_0(s)\sum_{i=1}^{4}(s+b_{i1})g_{3i}(s)\bar{p}_{i1} & 0 & 0 & 0 \\ p_0(s)\sum_{i=1}^{4}(s+b_{i1})g_{4i}(s)\bar{p}_{i1} & 0 & 0 & 0 \end{bmatrix}}{p_0(s)\left[(s+d_1)+\sum_{i=1}^{4}(s+b_{i1})g_{1i}(s)\bar{p}_{i1}\right]} \quad (44)$$

where $p_0(s) = s^2 + 0.45s + 0.05$. There are nine parameters to be found and can be reduced to three adjustable parameters by diagonal dominance manipulation. One approach is to make coefficients of the highest order and lowest exponents of off-diagonal terms of Eq.(44) approach zero. The ratio of $\bar{p}_{i1}(i = 1,2,3,4)$ is found as

$$\bar{p}_{11} : \bar{p}_{21} : \bar{p}_{21} : \bar{p}_{41} = 31.089 : -13.663 : -4.813 : -1.00 \equiv \hat{p}11 : \hat{p}21 : \hat{p}31 : \hat{p}41$$

where $\bar{p}_{i1} = k_1\hat{p}_{i1}(i = 1.2.3.4)$. The ratio $b_{i1}(i = 1,2,3,4)$ is found as

$$b_{11} : b_{21} : b_{31} : b_{41} = 1 : 1.275 : 1.1887 : 3.0127 \equiv \hat{b}_{11} : \hat{b}_{21} : \hat{b}_{31} : \hat{b}_{41}$$

where $b_{i1} = k_2\hat{b}_{i1}(i = 1.2.3.4)$. Then the characteristic equation of $T^{(1)}(s)$ can be written as

$$\begin{aligned} F_{c1}(s) &= (s+d_1)p_0(s) + p_o(s)\sum_{i=1}^{4}(s+\hat{b}_{i1}k_2)g_{1i}(s)\hat{p}_{i1}k_1 = 0 \\ &= (s+d_1)p_0(s) + \left[sp_0(s)\sum_{i=1}^{4}g_{1i}(s)\hat{p}_{i1}\right]k_1 \\ &\quad + \left[p_0(s)\sum_{i=1}^{4}g_{1i}(s)\hat{b}_{i1}\hat{p}_{i1}\right]k_1k_2 = 0 \end{aligned} \quad (45)$$

where k_1 and k_1k_2 are considered as two adjustable parameters to be analyzed for a specified value of d_1. Then, design procedures for 2x2 multivariable feedback control systems can be applied to this 4x4 multivariable feedback control system. The parameter analyses are shown in Fig.5(a) for $d_1 = 1$. The constant-X curves represent the negative sum of the characteristic roots. In general. the larger the value of X, the better damping characteristics of the system will be. A suitable choice is made at $Q_{10}(0.8,1.6)$ in Fig.5(a), for which the roots of the stability-equations are at

$$\omega_{e1} = 0.498 \text{ and } \omega_{o1} = 3.0833.$$

Corresponding to ratios $\hat{b}_{i1}$ and $\hat{p}_{i1}(i = 1,2,3,4)$ found above and the choice of $(k_1, k_1k_2) = (0.8,1.6)$, the $P_1(s)$ is in the form of

$$P_1(s) = \begin{bmatrix} 24.871\frac{s+2.00}{s+1} & 0 & 0 & 0 \\ -10.93\frac{s+2.55}{s+1} & 1 & 0 & 0 \\ -3.85\frac{s+2.277}{s+1} & 0 & 1 & 0 \\ 0.80\frac{s+6.026}{s+1} & 0 & 0 & 1 \end{bmatrix} \quad (46)$$

The transfer function matrix is

$$T^{(1)}(s) = \frac{1}{F_{c1}(s)}\begin{bmatrix} 4.488s^2 + 8.988s + 1.423 & 0 & 0 & 0 \\ -1.2569s & 0 & 0 & 0 \\ -0.4096s & 0 & 0 & 0 \\ 0.231s & 0 & 0 & 0 \end{bmatrix} \quad (47)$$

where $F_{c1}(s) = s^3 + 5.9386s^2 + 9.4884s + 1.4725$ .The characteristic roots of $T^{(1)}(s)$ are found at

$$-0.1735, -2.8825 \pm j0.4235.$$

Using the found $P_1(s)$, the open-loop transfer function of the plant is in the form of

$$G^{(1)}(s) = G(s)P_1(s) = \begin{bmatrix} g^{(1)}{}_{11}(s) & g^{(1)}{}_{12}(s) & g^{(1)}{}_{13}(s) & g^{(1)}{}_{14}(s) \\ g^{(1)}{}_{21}(s) & g^{(1)}{}_{22}(s) & g^{(1)}{}_{23}(s) & g^{(1)}{}_{24}(s) \\ g^{(1)}{}_{31}(s) & g^{(1)}{}_{31}(s) & g^{(1)}{}_{33}(s) & g^{(1)}{}_{34}(s) \\ g^{(1)}{}_{41}(s) & g^{(1)}{}_{42}(s) & g^{(1)}{}_{43}(s) & g^{(1)}{}_{44}(s) \end{bmatrix} \quad (48)$$

Step 2: Now, the transfer function matrix $T^{(2)}(s)$ of this step is in the form of

$$T^{(2)}(s) = \frac{\begin{bmatrix} 0 & p_{01}(s)\sum_{i=1}^{4}(s+b_{i2})g^{(1)}{}_{1i}(s)\bar{p}_{i2} & 0 & 0 \\ 0 & p_{01}(s)\sum_{i=1}^{4}(s+b_{i2})g^{(1)}{}_{2i}(s)\bar{p}_{i2} & 0 & 0 \\ 0 & p_{01}(s)\sum_{i=1}^{4}(s+b_{i2})g^{(1)}{}_{3i}(s)\bar{p}_{i2} & 0 & 0 \\ 0 & p_{01}(s)\sum_{i=1}^{4}(s+b_{i2})g^{(1)}{}_{4i}(s)\bar{p}_{i2} & 0 & 0 \end{bmatrix}}{p_{01}(s)\left[(s+d_2)+\sum_{i=1}^{4}(s+b_{i2})g^{(1)}{}_{2i}(s)\bar{p}_{i2}\right]} \quad (49)$$

where $p_{01}(s) = s^3 + 1.45s^2 + 0.50s + 0.05$. Similar to analyzed and designed procedures in step1. The ratios of $b_{i2}(i = 1,2,3,4)$ and $\overline{p}_{i2}(i = 1,2,3,4)$ are

$$b_{12} : b_{22} : b_{32} : b_{42} = -1.8386 : -2.4114 : -3.2656 : -1 \equiv \hat{b}_{12} : \hat{b}_{22} : \hat{b}_{32} : \hat{b}_{42}$$

and

$$\overline{p}_{12} : \overline{p}_{22} : \overline{p}_{32} : \overline{p}_{42} = 1 : -37.446 : 10.488 : 3.1136 \equiv \hat{p}_{12} : \hat{p}_{22} : \hat{p}_{32} : \hat{p}_{42}$$

respectively. The characteristic equation in this step is in the form of

$$\begin{aligned} F_{c2}(s) &= (s + d_2)p_{01}(s) + p_{o1}(s)\sum_{i=1}^{4}(s + \hat{b}_{i2}k_2)g^{(1)}{}_{2i}(s)\hat{p}_{i2}k_1 = 0 \\ &= (s + d_2)p_0(s) + \left[sp_{01}(s)\sum_{i=1}^{4} g^{(1)}{}_{2i}(s)\hat{p}_{i2}\right]k_1 \\ &+ \left[p_{01}(s)\sum_{i=1}^{4} g^{(1)}{}_{2i}(s)\hat{b}_{i2}\hat{p}_{i2}\right]k_1k_2 = 0 \end{aligned} \tag{50}$$

The parameter plane analyses are shown in Fig.5(b) for $d_2 = 1.25$. A suitable choice is made at $Q_{11}(-0.5, 0.375)$ in Fig.5(b), for which the roots of the stability-equations are at

$$\omega_{e1} = 0.3123, \omega_{e2} = 3.893 \text{ an } \omega_{o1} = 1.2738$$

Corresponding to ratios $\hat{b}_{i1}$ and $\hat{p}_{i1}(i = 1,2,3,4)$ found above and the choice of $(k_1, k_1k_2) = (0.8, 1.6)$, the found $P_2(s)$ is in the form of

$$P_2(s) = \begin{bmatrix} 1 & -0.5\dfrac{s+1.379}{s+1.25} & 0 & 0 \\ 0 & 18.723\dfrac{s+1.809}{s+1.25} & 0 & 0 \\ 0 & -5.244\dfrac{s+2.449}{s+1.25} & 1 & 0 \\ 0 & -15.57\dfrac{s+0.75}{s+1.25} & 0 & 1 \end{bmatrix} \tag{51}$$

The characteristic equation is in the form of

$$F_{c2}(s) = s^4 + 6.8522s^3 + 15.2532s^2 + 11.1173s + 1.4782$$

in this step. The characteristic roots are found at

$$-0.1694, -1{,}0829, -2.8 \pm j0.4659.$$

Step 3: In this step, only loop 3 with $P_1(s)$ and $P_2(s)$ are closed; i.e., $G^{(2)}(s) = G(s)P_1(s)P_2(s)$. The same design procedures in steps 1 and 2 are extended. The details of this step are omitted. The parameter analyses are shown in Fig.5(c) for $d_3 = 1.75$ and ratios of $b_{i3}(i = 1,2,3,4)$ and $\overline{p}_{i3}(i = 1,2,3,4)$ are

$$b_{13} : b_{23} : b_{33} : b_{43} = -1 : -1.0373 : -1.49 : -1.862 \equiv \hat{b}_{13} : \hat{b}_{23} : \hat{b}_{33} : \hat{b}_{43}$$

and

$$\overline{p}_{13} : \overline{p}_{23} : \overline{p}_{33} : \overline{p}_{43} = -1 : -1.172 : 119.377 : -66.851 \equiv \hat{p}_{13} : \hat{p}_{23} : \hat{p}_{33} : \hat{p}_{43}$$

A suitable choice is made at $Q_{12}(0.175, -0.3)$;i.e., $(k_1, k_1k_2) = (0.175, -0.3)$ in Fig.5(c) for roots of the stability equations are selected at

$$\omega_{e1} = 0.251, \omega_{e2} = 1.98 \text{ and } \omega_{o1} = 0.8287, \omega_{o2} = 4.9639.$$

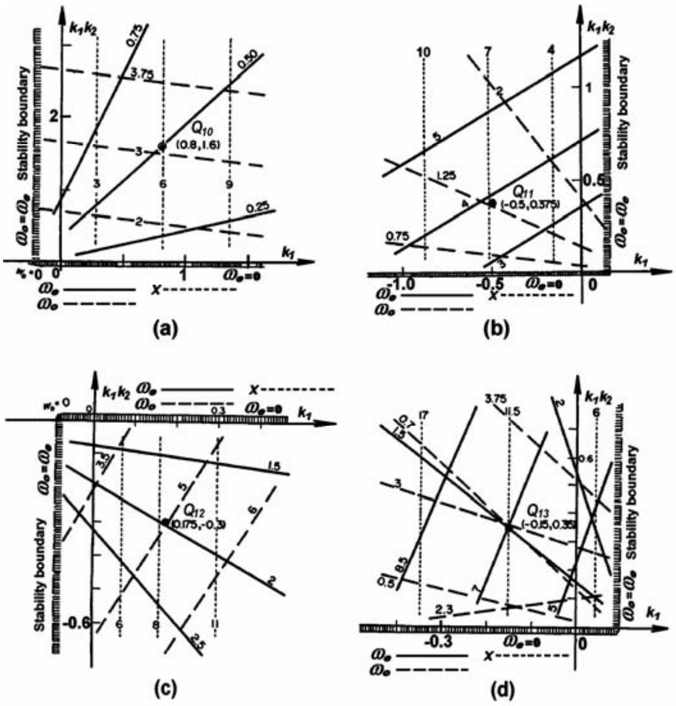

Fig.5.Parameter analyses in (a)Step 1;(b)Step 2; (c)Step 3;(d)Step 4.

The results of this step are given as follows: the found $P_3(s)$ is in the form of

$$P_3(s) = \begin{bmatrix} 1 & 0 & -0.175\dfrac{s+1.714}{s+1.75} & 0 \\ 0 & 1 & -0.2053\dfrac{s+1.778}{s+1.75} & 0 \\ 0 & 0 & 20.891\dfrac{s+2.554}{s+1.75} & 0 \\ 0 & -0 & -11.699\dfrac{s+3.192}{s+1.75} & 1 \end{bmatrix} \tag{52}$$

and characteristic roots of the $T^{(3)}(s)$ are at

$$-0.1701, -1, -1.273, -2.9126 \pm j0.9739.$$

Step 4: In this step, only loop 4 with $P_1(s)$, $P_2(s)$ and $P_3(s)$ is closed; i.e., $G^{(3)}(s) = G(s)P_1(s)P_2(s)P_3(s)$. The same design procedure is extended. The details of this step are omitted. The parameter analyses are shown in Fig.5(c) for $d_4 = 2.75$ and ratios of $b_{i4}(i = 1,2,3,4)$ and $\bar{p}_{i4}(i = 1,2,3,4)$ are

$$b_{14} : b_{24} : b_{34} : b_{44} = -1 : -1.619 : -1.3691 : -1.2677 \equiv \hat{b}_{14} : \hat{b}_{24} : \hat{b}_{34} : \hat{b}_{44}$$

and

$$\bar{p}_{13} : \bar{p}_{23} : \bar{p}_{33} : \bar{p}_{43} = 1 : 1.8918 : 3.5261 : 112.214 \equiv \hat{p}_{13} : \hat{p}_{23} : \hat{p}_{33} : \hat{p}_{43}$$

A suitable choice is made at $Q_{13}(0.15,-0.33)$ in Fig.5(d) for roots of stability equations are selected at

$$\omega_{e1} = 0.231, \omega_{e2} = 1.496, \omega_{e3} = 1.493 \text{ and } \omega_{o1} = 0.6983, \omega_{o2} = 0.6983.$$

The results of this step are given as follows: the found $P_4(s)$ is in the form of

$$P_4(s) = \begin{bmatrix} 1 & 0 & -0 & -0.15\dfrac{s+2.333}{s+2.75} \\ 0 & 1 & 0 & -0.284\dfrac{s+2.711}{s+2.75} \\ 0 & 0 & 1 & -0.529\dfrac{s+1.714}{s+3.195} \\ 0 & 0 & 0 & 16.832\dfrac{s+2.958}{s+2.75} \end{bmatrix} \quad (53)$$

and characteristic roots of the $T^{(4)}(s)$ are at

$$-0.1737, -1.005, -1.2495, -3.375 \pm j0.6531.$$

Step responses of the closed loop system with found compensator $C(s) = P_1(s)P_2(s)P_3(s)P_4(s)$ are shown in Fig.6. It can be seen that results are satisfactory for the considered system and interactions among all loops are very small.

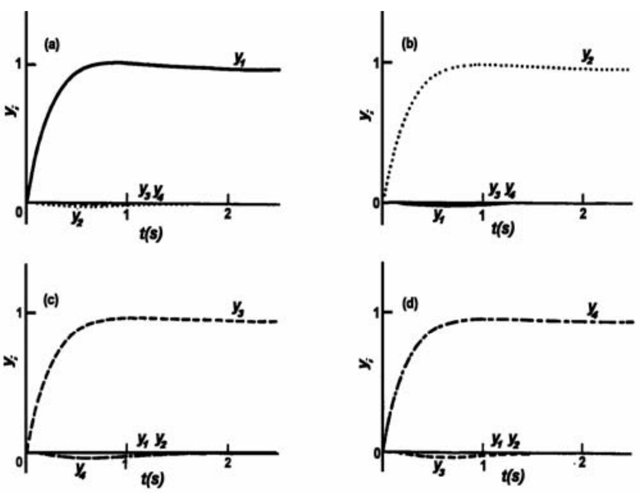

Fig.6.Time responses of Example 4.

5. Conclusions

A sequential design technique using stability-equation method has been extended and applied to the analysis and design of multivariable feedback control systems. By use of the proposed layer-wrapped structure together with the stability-equation method to design the overall compensator, it can be seen that the system characteristics, such as stability, integrity and damping characteristics can be considered systematically easily; thus it is a useful tool for analysis and design of multivariable feedback control systems.

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