A Combination Model and Application for the Water Quality Evaluation

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Abstract: Water quality evaluation benefits monitoring the water quality, utilizing and developing water resources rationally, providing a basis for the planning of water pollution control strategies, and predicting the future trend of water environment scientifically. This paper proposes a combination approach that can be used to incorporate two or more water quality evaluation models. The application of this approach is to assess the water quality of the middle reach of the Yangtze River, which is the major resource of supplying drinking water, fishing, irrigating crops and generating energy in China.

Keywords: Water quality; Evaluation index; Combination evaluation; Fuzzy synthetic evaluation; Entropy-based evaluation; Optimal-graded matrix

1. Introduction

Fresh surface water in rivers and lakes is vitally important for human beings and many other terrestrial ecosystems. For example, many regions in China depend on the Yangtze River for their drinking water, growing crops, harvesting fish and generating electricity. However, many human activities and their by-products have the potential to pollute water. Water quality assessment and monitoring are necessary to maintain the water quality and control water pollutions.

Water quality evaluation is to comprehensively evaluate water quality grades according to some water quality criteria and indexes. Many methods of water quality evaluation and management have been addressed in the literature. A well-known evaluation method developed by the NSF (National Sanitation Foundation) is the WQI (Water Quality Index) approach [10, 20] which was one of the first attempts to study the water quality indices. In the WQI approach, if there are n types of constituents, the selected index can be obtained by using the following formula [2]: $WQI = \sum_i w_i q_i \times 100 / \sum_i w_i$, where q_i ($i = 1, 2, \ldots, n$) is the water quality parameter of concern and w_i is the corresponding weight of the quality parameter. The output of a WQI ranges from 0 to 100. A value of 100 indicates the perfect water quality condition, while a value of 0 represents the unacceptable quality of concern. Although widely used, this approach has its limitations because it does not consider the uncertainties in water quality assessment and management problems, such as the randomness associated with various input variables and the imprecision in the decision making of the output values.

Considerable studies have been contributed to uncertainty analysis in water quality assessments and environmental decision makings. These studies can be classified as follows. The statistical approaches address uncertainty by considering the randomness of variables and parameters of the water environment [22, 29, 31, 32]. Non-statistical approaches such as the fuzzy synthetic evaluation methods address the uncertainties using the fuzzy set theory [35], where the probability of an uncertain event is linked with fuzzy sets [3, 18, 24, 26]. Based on the Shannon entropy [25] and the principle of maximum entropy [11, 16] and the principle of minimum cross entropy [13], the entropy methods enable the least-biased decision makings and predictions when the data of a resource system is limited or incomplete [1, 14, 27, 28]. Based on the grey theory, the grey clustering method has been applied to the water environment where partial information is known and partial information is unknown or uncertain (i.e., a grey system) [15, 37, 39]. To avoid the incompatibility of evaluation results among individual water quality indices of real water, the interval evaluation methods propose the evaluation criteria of water qualities to be intervals [4, 9, 12, 33].

Chinese researchers have made considerable contributions to the literature in water quality evaluation, especially the fuzzy synthetic evaluation approaches [5, 17, 19, 30, 34, 36] and their applications to some river systems such as Fen River [6] and Huangshui River [34]. Based on fuzzy set theory, the fuzzy synthetic evaluation may properly describe the uncertainties of the water quality grading criteria or decision-making processes via some fuzzy membership functions. Thus, the evaluation results can be closer to the objective facts. Also, there are a few studies based on the entropy-based method, for example, C. K. Zhang [38] proposed a model using the principle of maximum entropy and applied it to the water quality assessment of Fen River. The advantage of the entropy measure is that it is derived from a theoretical basis and not subjective information, thus it usually provides unbiased assessments and predictions. In China, subjective information such as professional judgment or empirical rules has been commonly used in practice for water quality evaluation, although such information may have great value, many conventional models do not have the capability to accommodate subjective information. The entropy-based models are able to provide the least-biased solutions if the information is subjective.

Water quality assessment and management of a river system is generally characterized by different types of uncertainties such as the randomness associated with various components of the river system (for example, the river flow and the effluent flow) [24]. Since there are so many uncertainties involved in the water quality evaluation of a river system, different methods may have different quality criteria and measurements, and sometimes they may have conflicting objectives or produce incompatible results. To obtain a more reliable and effective result or at least a compatible or compromising result, sometimes we may want to combine two or more models of concern to minimize each model's limitations or incompatibilities. This paper attempts to combine two or more evaluation methods to form a combination evaluation approach, and then applies this approach to assess the quality of the middle reach of the Yangtze River. This paper is organized as follows. Considering the uncertainties in the sampling and analysis processes, section 2 constructs several fuzzy matrices for the water quality evaluation via some fuzzy membership functions. Section 3 applies the fuzzy synthetic model to determine the corresponding optimal-graded matrix where the subjective weighting method is used. Section 4 applies the entropy-based model to determine the corresponding optimal-graded matrix where the objective weighting method based on the entropy theory is used. Section 5 establishes the combination evaluation approach to determine the final optimal-graded matrix from the optimal-graded matrices obtained previously by two (or more) different evaluation methods such as the fuzzy synthetic method and the entropy-based method. Section 6 applies the combination evaluation model to evaluate the water quality of middle reach of Yangtze River. Section 7 provides a summary and conclusion of this paper.

2.Assessment Indexes and Matrices

Suppose there are n polluted water samples and m pollution indexes, and each pollution index has k different grades. Let C be a m × n matrix consisting of measured concentration levels, where each entry c_{ij} (i = 1, 2 …, m, j = 1, 2, …, n) denotes the measured concentration level of sample j with pollution index i. Also, let S be a m × k matrix consisting of standard concentration levels, where each entry s_{ih} (i = 1, 2 …, m, h = 1, 2, …, k) denotes the standard concentration level of pollution index i having grade h. These two matrices contain the basic data for the water quality evaluation.

$$C_{m\times n} = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1n} \\ c_{21} & c_{22} & \dots & c_{2n} \\ \dots & \dots & \dots & \dots \\ c_{m1} & c_{m2} & \dots & c_{mn} \end{bmatrix} = \left(c_{ij}\right)_{m\times n} \qquad (1)$$

$$S_{m\times k}= \begin{bmatrix} s_{11} & s_{12} & \dots & s_{1k} \\ s_{21} & s_{22} & \dots & s_{1k} \\ \dots & \dots & \dots & \dots \\ s_{m1} & s_{m2} & \dots & s_{mk} \end{bmatrix} = \left(s_{ih}\right)_{m\times k} \qquad (2).$$

The goal of the pollution control is to make the measured concentration level c_{ij} as close as possible to the standard level s_{i1} so that the water quality of sample j is improved with respect to the pollution index i, for all i and j. This goal can be viewed as a fuzzy goal since it is imprecisely defined. Also, sampling data is an uncertain process prior to making any observations and analysis. Thus, taking the uncertainties in the sampling and analysis processes into account, the sampling data and the goal of pollution control are represented as fuzzy sets, i.e., matrices C and S are to be converted to fuzzy matrices such that the value of each entry in these two matrices lies within the closed interval [0, 1].

To convert matrix S to a fuzzy matrix, for each pollution index i (i = 1, 2, …, m), let $e_{ih} = \frac{s_{ik} - s_{ih}}{s_{ik} - s_{i1}}$, h = 1, 2, …, k. (3)

Thus, e_{ih} = 1 if h = 1, e_{ih} = 0 if h = k, and e_{ih} lies between 0 and 1 if 1 < h < k. Using this fuzzy member function, matrix S is converted into a fuzzy matrix E as follows.

$$E_{mxk} = \begin{bmatrix} e_{11} & e_{12} & \dots & e_{1k} \\ e_{21} & e_{22} & \dots & e_{2k} \\ \dots & \dots & \dots & \dots \\ e_{m1} & e_{m2} & \dots & e_{mk} \end{bmatrix} = \left(e_{ih}\right)_{mxk} \qquad (4)$$

We can see that the entries in the first row of E have value 1 and the entries in the last row of E have value 0. Also, the h-graded standard water quality can be expressed by the vector $e_h = \left(e_{1h}, e_{2h}, \dots, e_{mh}\right)$.

Similarly, matrix C can be converted to a fuzzy matrix F based on the following fuzzy member function. For each pollution index i and sample j (i = 1, …, m, j = 1, …, n),

$$f_{ij} = \begin{cases} 1, & c_{ij} \le c_{i1} \\ \dfrac{s_{ik} - c_{ij}}{s_{ik} - s_{i1}}, & s_{i1} < c_{ij} < s_{ik} \\ 0, & c_{ij} \ge s_{ik} \end{cases} \quad (5)$$

So measured concentration fuzzy matrix of pollution index is:

$$F_{m\times n} = \begin{bmatrix} f_{11} & f_{12} & \dots & f_{1n} \\ f_{21} & f_{22} & \dots & f_{2n} \\ \dots & \dots & \dots & \dots \\ f_{m1} & f_{m2} & \dots & f_{mn} \end{bmatrix} = (f_{ij})_{m\times n} \quad (6)$$

The jth water sample can be denoted by the vector $f_j = (f_{1j,} f_{2j}, ..., f_{mj})^T$.

To consider the weight of each pollution index, let w_i represent the weight of pollution index i (i = 1, 2, …, m), and let $W = (w_1, w_2, ..., w_m)$ where $\sum_i w_i = 1$. Also, since there are uncertainties involved in the grade classifications for the measured samples, the grade classifications are represented as a fuzzy set. Suppose U is a k × n fuzzy matrix where each entry u_{hj} (to be determined) represents the dependence degree of the measured sample j with respect to grade h (h = 1, 2, …, k, j = 1, 2, …, n), i.e., the probability that sample j is of grade h. That is,

$$U_{k\times n} = \begin{bmatrix} u_{11} & u_{12} & \dots & u_{1n} \\ u_{21} & u_{22} & \dots & u_{2n} \\ \dots & \dots & \dots & \dots \\ u_{k1} & u_{k2} & \dots & u_{kn} \end{bmatrix} = (u_{hj})_{k\times n} \quad (7)$$

Also, matrix U is subject to the following constraints:

$$\sum_{h=1}^{k} u_{hj} = 1, \quad j = 1, 2, \dots, n. \quad (8)$$

$$\sum_{j=1}^{h} u_{hj} > 0, \quad h = 1, 2, \dots, k. \quad (9)$$

Furthermore, the variation of the jth sample with respect to the h-graded standard water quality can be expressed by the weighted generalized distance (10) as:

$$d[e_h, f_j] = u_{hj}\left[\sum_{i=1}^{m}\left(w_i\left|e_{ih} - f_{ij}\right|\right)\right]. \quad (10)$$

Since the goal of pollution control is to make these variations as small as possible (i.e., make the measured concentration levels as close as possible to their corresponding standard levels), we need to determine matrix U such that this goal is achieved and the constraints (8) and (9) are satisfied. U is also called the optimal-graded matrix for a water quality criterion of concern and is used to determine which grade the sampled water belongs to. In the next two sections, we will use two different evaluation methods to determine matrix U, where the fuzzy synthetic evaluation method uses the subjective weighting method (i.e., W is obtained by subjective information such as experts' experiences) and the entropy-based method uses the objective weighting method based on entropy theory [11, 13, 16, 25].

3.Using the Fuzzy Synthetic Evaluation Method to Determine the Matrix U

The fuzzy synthetic evaluation model obtains indexes' weight vector W by subjective information such as experts' experiences and judgments. The objective is to minimize the sum of the squares of the weighted generalized distances defined by (10). That is,

$$\min\left\{\sum_{j=1}^{n}\sum_{h=1}^{k}\left[u_{hj}\left(\sum_{t=1}^{m}\left(w_i\left|e_{ih} - f_{ij}\right|\right)\right)\right]^2\right\}, \quad (11)$$

By the objective function (11) and constraints (8) and (9), we can construct a Lagrange function such that the constrained optimization problem is converted to an unconstrained optimization problem. By introducing the Lagrange multiplier λ, the Lagrange function is as follows.

$$L(u_{hj},\lambda)=\sum_{h=1}^{j}u^2{}_{hj}(\sum_{i=1}^{m}w_i\left|e_{ih}-f_{ij}\right|)^2-\lambda(\sum_{h=1}^{k}u_{hj}-1)$$
. (11-1)

To find the minimize this function, we first find the partial derivatives with respect to λ and with respect to u_{hj}, respectively, and then set each partial derivative to be 0.

$$\frac{\partial L(u_{hj},\lambda)}{\partial\lambda}=\sum_{h=1}^{k}u_{hj}-1=0\,. \quad (11\text{-}2)$$

$$\frac{\partial L(u_{hj},\lambda)}{\partial u_{hj}}=2u_{hj}(\sum_{i=1}^{m}w_i\left|e_{ih}-f_{ij}\right|)^2-\lambda=0\,. \quad (11\text{-}3)$$

From (11-3), we have

$$u_{hj}=\frac{\lambda}{2(\sum_{i=1}^{m}w_i\left|e_{ih}-f_{ij}\right|)^2}\,. \quad (11\text{-}4)$$

From (11-2) and (11-4), we have

$$\sum_{h=1}^{k}\frac{\lambda}{2(\sum_{i=1}^{m}w_i\left|e_{ih}-f_{ij}\right|)^2}=1\,.$$

Thus,

$$\lambda=\frac{2}{\sum_{h=1}^{k}\frac{\lambda}{(\sum_{i=1}^{m}w_i\left|e_{ih}-f_{ij}\right|)^2}}\,. \quad (11\text{-}5)$$

From (11-4) and (11-5), we have

$$u_{hj}=\frac{1}{(\sum_{i=1}^{m}w_i\left|e_{ih}-f_{ij}\right|)^2\sum_{h=1}^{k}\frac{1}{(\sum_{i=1}^{m}w_i\left|e_{ih}-f_{ij}\right|)^2}}\,.$$

If we change h to t in the second ∑ sign, we can move $(\sum_{i=1}^{m}w_i\left|e_{ih}-f_{ij}\right|)^2$ term into the second ∑ sign as follows.

$$u_{hj}=\frac{1}{\sum_{t=1}^{k}\left(\frac{\sum_{i=1}^{m}w_i\left|e_{ih}-f_{ij}\right|}{\sum_{i=1}^{m}w_i\left|e_{it}-f_{ij}\right|}\right)^2},$$

for h = 1, 2, …, k, j = 1, 2, …, n. (12)

Since each u_{hj} in matrix U can be determined by (12), we can determine the optimal-graded matrix U of the fuzzy synthetic evaluation method.

4.Using the Entropy-Based Evaluation Method to Determine the Matrix U

In this section, we first determine the weight of each pollution index, and then apply the entropy-based model to find the optimal-graded matrix U.

4.1 Determine the Weight Vector W

First, for each sample j of pollution index i, let P_{ij} represent the proportion of the measured concentration level c_{ij} to the sum of all measured concentration levels of sample j [21]. That is,

$$P_{ij}=\frac{c_{ij}}{\sum_{i=1}^{m}c_{ij}},\ i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n.$$

Second, we calculate the Shannon entropy of pollution index i by the formula [25, 27]

$r_i=-k\sum_{j=1}^{n}P_{ij}\ln P_{ij}$, i = 1, 2, …, m, where k is a positive constant.

Next, let g_i denote the distinction coefficient of index i. For a pollution index i, the greater the value of g_i is, the more important role it plays in the evaluation process, and the smaller its entropy is. Therefore, the distinction coefficient of index i can be expressed by $g_i = 1 - r_i$, i = 1, 2, …, m.

Finally, the weight of index i is determined by the formula

$$w_i = \frac{g_i}{\sum_{i=1}^{m} g_i}, \ i = 1, 2, \ldots, m.$$

4.2 The Entropy-Based Evaluation Model

An idea of entropy-based evaluation model is to construct a multiple objective optimization problem [3]. On one hand, minimize the sum of the weighted generalized distances. That is,

$$\min \sum_{j=1}^{n} \sum_{h=1}^{k} u_{hj} \left[\sum_{i=1}^{m} \left(w_i \left| e_{ih} - f_{ij} \right| \right) \right] \qquad (13)$$

On the other hand, apply the principle of maximum entropy [11, 16] to maximize the Shannon entropy [25] in order to obtain the least-biased result. That is,

$$\begin{cases} \max - \sum_{j=1}^{n} \left(\sum_{h=1}^{k} u_{hj} \ln u_{hj} \right) \\ \qquad (8), (9). \end{cases} \qquad (14)$$

To solve this dual objective optimization problem, by introducing a positive constant B to balance these two objectives, we can reformulate the problem as follows.

$$\min_{u_{hj}} \left\{ \sum_{j=1}^{n} \sum_{h=1}^{k} u_{hj} \left[\sum_{i=1}^{m} \left(w_i \left| e_{ih} - f_{ij} \right| \right) \right] + \frac{1}{B} \sum_{j=1}^{n} \sum_{h=1}^{k} u_{hj} \ln u_{kj} \right\}$$

(14-1)

By introducing the Lagrange multiplier λ, we can construct a Lagrange function of (14-1). That is,

$$L(u_{hj}, \lambda) = \sum_{j=1}^{n} \sum_{h=1}^{k} \left\{ u_{hj} \left[\sum_{i=1}^{m} (w_i \left| e_{ih} - f_{ij} \right|) \right] + \frac{1}{B} u_{hj} \ln u_{hj} \right\} + \lambda \left(\sum_{h=1}^{k} u_{hj} - 1 \right) \cdot$$

(14-2)

Similar to the algebra done in section 3, to solve this minimization problem, we first find the partial derivatives of (14-2) with respect to λ and u_{hj}, respectively, and then set each partial derivative to zero, and then solve for u_{ij} from the resulting two equations and simplify. The resulting formula for each u_{hj} (h = 1, 2, …, k, j = 1, 2, …, n) is as follows.

$$u_{hj} = \frac{\exp\left[-B \sum_{i=1}^{m} \left(w_i \left| e_{ih} - f_{ij} \right| \right) \right]}{\sum_{h=1}^{k} \exp\left[-B \sum_{i=1}^{m} \left(w_i \left| e_{ih} - f_{ij} \right| \right) \right]} . \qquad (15)$$

The optimal-graded matrix U by the entropy-based method can be obtained from (15).

5.Combination Evaluation Approach

In this section, we attempt to combine two (or more) evaluation models to form a combination evaluation model. Suppose there are q evaluation models (q ≥ 2) and the optimal-graded matrix of each of these q models is known. We want to determine the final optimal-graded matrix from these q optimal-graded matrices.

For each sample j (j = 1, 2, …, n), let H_r represent the vector consisting of dependence degrees of k grades in the rth evaluation model. That is, $H_r = \left(u_{1j}^{(r)}, u_{2j}^{(r)}, \ldots, u_{kj}^{(r)} \right)^T$, where $\sum_{h=1}^{k} u_{hj}^{(r)} = 1$, for r = 1, 2, …, q. Also, for each sample j (j = 1, 2, …, n), the deviation of vectors H_0 (to be determined) in the combination evaluation model and H_r in the rth evaluation model is

$$H_0 - H_r = \left(u_{1j}^{(0)} - u_{1j}^{(r)}, u_{2j}^{(0)} - u_{2j}^{(r)}, \ldots, u_{kj}^{(0)} - u_{kj}^{(r)} \right).$$

To obtain the optimal-graded matrix $U^{(0)}$ of the combination evaluation model, we minimize the sum of squares of the q deviations.

$$\begin{cases} \min \sum_{r=1}^{q} \| H_0 - H_r \|^2 = \sum_{r=1}^{q} \sum_{h=1}^{k} \left(u_{hj}^{(0)} - u_{hj}^{(r)} \right)^2 \\ \text{s.t.} \sum_{h=1}^{k} u_{hj}^{(0)} = 1 \end{cases}$$

(16)

Similar to the algebra done in sections 3 and 4, we can obtain the (unique) solution to the minimization model in (16) as follows.

$$u_{hj}^{(0)} = \frac{1}{q}\sum_{r-1}^{q} u_{hj}^{(r)} + \frac{1}{k}\left(1 - \frac{1}{q}\sum_{r=1}^{q}\sum_{h=1}^{k} u_{hj}^{(r)}\right).$$

(16-1)

If we let $\beta_h = \sum_{r=1}^{q} u_{hj}^{(r)}$, we can simplify (16-1) as follows.

$$u_{hj}^{(0)} = \frac{1}{k} + \frac{1}{kq}\left(k\beta_h - \sum_{h=1}^{k}\beta_h\right) \square h \square 1,2\ldots\square k. \quad (17)$$

The optimal-graded matrix $U^{(0)}$ of the combination evaluation model can be obtained from (17). We will apply this model to evaluate the water quality of the middle reach of the Yangtze River in the next section (where q = 2).

6. Application of the Combination Evaluation Model to the Middle Reach of the Yangtze River

In this section, the fuzzy synthetic evaluation model and the entropy-based model are combined to form the combination evaluation model discussed in section 5. Also, this model is applied to determine the water grade of the middle reach of the Yangtze River.

Tables 1 and 2 contain the measured concentration levels and the standard concentration levels of the Three Gorges Reservoir Area, and the middle reach and the lower reach of the Yangtze River, respectively, from the Environmental Quality Report. The data of the six pollution indexes CODmn, BOD, ammonia nitrogen, volatile phenol, arsenic and chromium of valence 6 was collected from the pollutant samples of four representative sections of the middle reach of the Yangtze River (note that the measurement unit is mg/l).

Table 1 The measured concentration levels C (m = 6 and n = 4)

index / section	CODmn	BOD	ammonia nitrogen	volatile phenol	arsenic	chromium of valence 6
Wuqi Dock	2.23	1.29	0.15	0.002	0.15	0.04
Cheng Lingji	2.64	1.69	0.37	0.001	0.06	0.04
Yanggang	2.78	1.76	0.34	0.001	0.08	0.08
Three Gorges	2.81	1.68	0.52	0.001	0.08	0.04

Table 2 The standard concentration levels S (m = 6 and k = 5)

grade / standard value / parameter	□	□	□	□	□
CODmn	2	4	6	8	10
BOD	3	3	4	6	10
ammonia nitrogen	0.5	0.5	1	2	2
volatile phenol	0.002	0.002	0.005	0.01	0.1
arsenic	0.05	0.05	0.05	0.1	0.1
chromium of valence 6	0.01	0.05	0.05	0.05	0.1

From the above data in the two tables, we can get $E_{m\times k}$ by (3) and (4) discussed in section 2 and $F_{m\times n}$ by (5) and (6) discussed in section 2.

$$E_{m\times k} = \begin{bmatrix} 1, & \frac{3}{4}, & \frac{1}{2}, & \frac{1}{4}, & 0 \\ 1, & 1, & \frac{6}{7}, & \frac{4}{7}, & 0 \\ 1, & 1, & \frac{2}{3}, & 0, & 0 \\ 1, & 1, & 0.969, & 0.918, & 0 \\ 1, & 1, & 1, & 0, & 0 \\ 1, & \frac{5}{9}, & \frac{5}{9}, & \frac{5}{9}, & 0 \end{bmatrix}, F_{m\times n} = \begin{bmatrix} 0.971, & 0.92, & 0.903, & 0.899 \\ 1, & 1, & 1, & 1 \\ 1, & 1, & 1, & 0.987 \\ 0.99, & 1, & 1, & 1 \\ 0, & 0.8, & 0.4, & 0.4 \\ 0.666, & 0.666, & 0.222, & 0.666 \end{bmatrix}$$

Using the subjective information provided by the experts, we can obtain the indexes weight vector $W = (0.05, 0.05, 0.3, 0.25, 0.1, 0.25)$. Also, if we choose k = $(\ln 4)^{-1}$ = 0.72135, the indexes' weight vector W can be obtained by the entropy-based method discussed in 4.1. That is, $W = (0.0185, 0.0311, 0.3578, 0.2450, 0.1026, 0.2450)$.

The optimal-graded matrix U_1 of the fuzzy synthetic evaluation method discussed in section 3 and matrix U_2 of the entropy-based method discussed in section

4 can be obtained by using (12) and (15), respectively, where we let B = 10 in (15).

$$U_1 = \begin{bmatrix} 0.2831, & 0.1963, & 0.1921 & 0.2489 \\ 0.4978, & 0.7235, & 0.5655, & 0.5918 \\ 0.1430, & 0.0675, & 0.1667, & 0.1210 \\ 0.0610, & 0.0098, & 0.0521, & 0.0302 \\ 0.0151, & 0.0029, & 0.0236, & 0.0081 \end{bmatrix}$$

$$U_2 = \begin{bmatrix} 0.3117, & 0.3173, & 0.2092, & 0.3083 \\ 0.5201, & 0.5403, & 0.6164, & 0.5291 \\ 0.1399, & 0.1369, & 0.1585, & 0.1493 \\ 0.0278, & 0.0053, & 0.0138, & 0.0130 \\ 0.0005, & 0.0002, & 0.0016, & 0.0003 \end{bmatrix}$$

Due to (17), we can determine the final optimal-graded matrix of the combination evaluation model discussed in section 5.

$$U = \begin{bmatrix} 0.2974, & 0.2568, & 0.2009, & 0.2786 \\ 0.5090, & 0.6319, & 0.5910, & 0.5605 \\ 0.1414, & 0.1022, & 0.1626, & 0.1351 \\ 0.0444, & 0.0076, & 0.0329, & 0.0216 \\ 0.0078, & 0.0015, & 0.0126, & 0.0042 \end{bmatrix}$$

From the optimal-graded matrix U of the combination evaluation model and the principle of maximum dependence degree, we conclude that the water quality of the four sections of the middle reach of the Yangtze River can be classified as grade II water.

7. Summary and Conclusion

There are a lot of uncertainties involved in the water quality evaluation and management including the water grades classification. This paper represents this vague event by a fuzzy matrix and uses it to determine the grade of the sampling water. Two methods based on different quality criteria and index weighting methods are addressed. The fuzzy synthetic method applies subjective weighting method and the quality criterion is to minimize the sum of the square of the weighted generalized distances. The entropy-based method applies objective weighting method based on the entropy theory and the quality criterion is to minimize the sum of the weighted generalized distances and maximize the Shannon entropy. Since different evaluation methods have different strengths and weaknesses and may have different or conflicting objectives, sometimes we may want to combine these methods to avoid discrepancies and conflicting criteria among them and see if we can get a better or at least a compromising result. This paper attempts to combine two or more methods of concern to form a combination evaluation method of water quality evaluation. A case study of the middle reach of the Yangtze River is presented. This combination evaluation method is adaptable to various environmental systems such as air quality evaluation and pollution control.

References:

[1]K. Awumah and I. C. Goulter and S. K. Bhatt. “Assessment of Reliability in Water Distribution Networks Using Entropy-Based Measures”. Stochastic Hydrol. And Hydraulics, 4, 309-320, 1990.

[2]L. W. Canter. “Environmental Impact of Water Resources Projects Inc.”. Lewis Publishers, Chelsea, Michigan, 1985.

[3]N. B. Chang and H. W. Chen and S. K. Ning. “Identification of River Water Quality Using the Fuzzy Synthetic Evaluation Approach”. Journal of Environmental Management, 63, 293-305, 2001.

[4]S. Chen. “Theory and Application for Engineering Fuzzy Sets”. Chinese National Defense Industry Press, Beijing, 1998.

[5]S. Y. Chen. “Fuzzy Hydrology and Fuzzy Optimize Theory of Water Resource System”. Dalian Technology University Press, 51-66, Dalian, 1990.

[6]S. Y. Chen and X. B. Chen. “Theory and Model of

Water Quality Fuzzy Evaluation". Environmental Science Journal, 11(1), 1-8, 1991.

[7]Y. Duan and K. Ye and E. P. Smith. "Evaluating Water Quality Using Power Priors to Incorporate Historical Information". Environmetrics, 17, 95-106, 2006.

[8]Y. Hong and J. Cao. "Primary Study on Urban Water Resources Index System of Comprehensive Assessment". Shanghai Environmental Sciences, 19(6), 269-271, 2000.

[9]M. Hu and D. Guo. "Fuzzy Neural Network Method for Evaluation of Lake Water Quality Rich Nutrition". Chinese Environment Science Research, 4, 2-40, 1998.

[10]H. Inhaber. "Environmental Indices". John Wiley and Sons, New York, 1976.

[11]E. T. Jaynes. "Information Theory and Statistical Mechanics, I". Physical Review, 106, 620-630, 1957.

[12]J. L Jin and S. Wang and Y. M. Wei. "Ideal Interval Method Based Model for Water Quality Evaluation". Natural and Science, 2(1), 24-28, 2004.

[13]S.Kullback and R. A. Leiblerra. "On Information and Sufficciency". Annals of Math and Statistics, 22, 79-86, 1951.

[14]A. Kusmulyono and I. Goulter. "Entropy Principles in the Prediction of Water Quality Values at Discontinued Monitoring Stations". Stochastic Environmental Research and Risk Assessment, 8(4), 301-317, 1994.

[15]W. H. Lan. "Application of Grey Clustering Method to Atmospheric Environmental Assessment and Comparing with other Methods". Arid Environmental Monitoring, 9(3), 147-150, 1995.

[16]R. D. Levine and M. Tribus. "The Maximum Entropy Formalism". The MIT Press, Cambridge, Massachusetts, page 498, 1978.

[17]H. M. Li and G. P. Shang. "Researches on Mathematical Models of Water Quality Evaluation". Chinese Environmental Science, 11(5), 356-360, 1991.

[18]R. S. Lu and S. L. Lo and J. Y. Hu. " Analysis of Reservoir Water Quality Using Fuzzy Synthetic Evaluation". Stochastic Environmental Research and Risk Assessment, 2000.

[19]X. B. Mei and F. G. Wang and J. F. Cao. "The Application of Fuzzy Comprehensive Evaluation on Water Quality and Discussion". World Geology, 19(2), 172-177, 2000.

[20]W. R. Ott. "Water Quality Indices: A Survey of Indices Used in the United States". EPA-600/4-78-005, US Environmental Protection Agency, page 128, Washington, DC, 1978.

[21]Y. X. Peng. "Theory and Method of Systematic Evaluation". Chinese Economic Press, Beijing, 1995.

[22]K. H. Reckhow. "Water Quality Simulation Modeling and Uncertainty Analysis for Risk Assessment and Decision Making". Ecological Modelling, 72, 1-20, 1994.

[23]K. Sasikumar and P. P. Mujumdar. "Fuzzy Optimization Model for Water Quality Management of a river system". Journal of Water Resources Planning and Management, 124(2), 79-84, 1998.

[24]K. Sasikumar and P. P. Mujumdar. "Application of Fuzzy Probability in Water Quality Management of a River System". International Journal of Systems Science, 31(5), 575-591, 2000.

[25]C. E. Shannon. "A Mathematical Theory of Communications, I and II". Bell System Technology Journal, 27, 379-443, 1948.

[26]H. I. Sii and J. H. Sherrard and T. E. Wilson. "A Water Quality Index Based on Fuzzy Sets Theory". Proceedings of the 1993 Joint ASCE-CSCE National Conference on Environmental Engineering, 253-259, Montreal, Quebec, Canada, July 12-14, 1993.

[27]V. P. Singh. "The Entropy Theory as a tool for modeling and Decision Making in Environmental and Water Resources". Water SA, 26(1), 1-11, 2000.

[28]B. Srdjevic and Y. D. P. Medeiros and A. S. Faria. "An objective Multi-Criteria Evaluation of Water Management Scenarios". Water Resources

Management, 18(1), 35-54, 2004.

[29]A. K. Takyi and B. J. Lence. "Surface Water Management Using a Multiple-Realization Chance Constraint Method". Water Resour. Res., 35(5), 1657 – 1670, 1999.

[30]J. H. Tian and L. Qiu and F. X. Chai. "Application of Fuzzy Recognition in Comprehensive Evaluation of Water Quality". Journal of Environmental Sciences, 25(7), 950-954, 2005.

[31]A. Tyagi and C. T. Haan. "Reliability, Risk and Uncertainty Analysis Using Generic Expectation Functions". Journal of Environmental Engineering, 127(10), 938-945, 2001.

[32]V. R. S. Vemula and P. P. Mujumdar and S. Ghosh. "Risk Evaluation in Water Quality Management of a River System". Journal of Water Resources Planning and Management, 130(5), 411-423, 2004.

[33]J. Xia and B. Q. Hu. "The Interval Extension Evaluation Model of Water Environmental Quality and Its Application". Environmental Informatics Archives, 2, 297-307, 2004.

[34] L. Yang and X. L. Li. "Application of Fuzzy Mathematics to HuangShui River Water Quality Evaluation". Environmental Monitoring in China, 16(6), 49-52, 2000.

[35]L. A. Zadeh. "Fuzzy Sets". Information and Control, 8, 338-353, 1965; "Probability Measures and Fuzzy Events". Journal of Mathematical Analysis and Applications, 23, 421-427, 1968.

[36]M. R. Zeng and C. H. Wang. "Application of Fuzzy Mathematics to Water Quality Evaluation". Fujian Environment, 16(5), 7-9, 1999.

[37]Z. J. Zha. "Application of grey Clustering Method to Water Environment Quality Assessment". Sichuan Environment, 16(3), 49-51, 1997.

[38]C. K. Zhang. "Entropy-Based Water Quality Fuzzy Evaluation Model and Application". Theory and Practice of Systematic Engineering, 18(6), 80-85, 1998.

[39]L. F. Zhou and S. G. Xu. "Application of grey Clustering Method in Eutrophication Assessment of Wetland". Journal of American Science, 2(4), 52-57, 2006.