Affine Group Linear Operator-based Channel Characterization for Mobile Radio Systems

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Abstract: - Mobile radio channel characterization using linear operators of affine group is presented. The foundation of this approach is primarily based on the natural extension of basic concepts of time-scale representations of wavelet-type to the analysis and representation of the nonstationarity associated with the real-time mobile radio channel. We provide an operator-based characterization initiated by the need to present an intuitive and physical approach towards describing and representing the various effects of the channel. The concepts of delay-scale spreading function and operator as well as delay-scale scattering function and operator is introduced. We established the relationship between the continuous wavelet transform and these functions/operators by employing the powerful tool of group theory. We also derived canonical expression for the corresponding channel by discretizing the channel using frame concept. The practical relevance of our approach is illustrated via deterministic examples and the associated simulations.

Key-Words: - Channel characterization, mobile radio channel, nonstationary channel, time-frequency operator, time-scale operator, Fourier kernel, wavelet transform

1 Introduction

The challenges of using the mobile radio channel are its excessive multipath propagation property and the dispersion in frequency caused by the movement of the mobile unit/scatteres. In essence, multipath propagation is absolutely necessary to ensure sufficient radio coverage for mobile radio communication; however it causes difficulties in radio transmission in numerous ways. The deviation in frequency caused by mobility also degrades the performance of the receiver when the offset is unable to be estimated and corrected. All kinds of transmission techniques may suffer from channel quality degradation due to this doubly influence of the mobile channel. And these phenomena become much maligned in system design for high data rate transmission and in high speed environment.

The process variation caused by the movement of the mobile unit/environment ultimately makes the mobile radio channel a nonstationary channel. Thus accurate channel characterization and representation based on this nonstationarity is crucial in system design by providing information on the parameters essential for optimal mobile communication transceiver design and network planning. Existing channel characterizations model the doubly channel effects using the delay-Doppler spreading function [1], [2] which provides basis for understanding the relationship between the physics of the problem and its time-varying scattering representation. This time-frequency characterization require the use of windowed Fourier transform which maps a function to a weighted sum of windowed tones with a fixed window size which results in poor time-frequency resolution . More also the transform fares badly in representing or analyzing nonstationary processes like the mobile channel. Thus such characterizations method may not be adequate in tracking channel variations especially in fast fading and frequency selective channels. The wavelet transform [3], [4], [5] on the other hand is a nonstationary linear transform method and provides an evolutionary channel mapping method which offers better time and frequency resolution with evolutional window capable of tracking channel variations. An inspirational insight into the application of wavelet theory to the characterization of signals and system is available in [6]. In [7] different methods of estimating the time-scale or wideband spreading function is discussed with consideration to radar imaging and parameter estimation of distributed objects using continuous wavelet transform (CWT).

The application of wavelet transform as a tool in wideband correlation processing is addressed by Weiss [8] with particular interest in sonar and radar. In [9], [10], the intimate connection between wideband spreading function characterization for linear time variant (LTV) systems and CWT is presented. The proportionality of the wideband density function to continuous wavelet transform is also addressed in [11]. In [32] Marggetts considered the effect of mobility on a wideband direct sequence spread spectrum communication system and study a joint scale-lag diversity technique in mobile wideband systems.

In this paper, we present the characterization and description of mobile radio channel using the affine group operator concept. The main idea is to project the channel as a linear operator [19] specifically of the affine kind with a view to providing a physical as well as intuitive approach toward describing and representing the influences of the channel on transmitted signals. In a broad sense the doubly effect of the mobile radio channel (MRC) can be characterized by joint variable operators either in the form delay-Doppler or delay-scale operators for different group representations. We introduce the general concept of channel operators and concentrate on the affine channel operator of wavelet type. We regard the channel in question to be wideband. However, the usual wideband and narrowband distinction made by the ratio of channel bandwidth and channel center frequency is not meaningful for radio propagation. The bandwidth of the system is the zoom factor of the system on the excess time delay axis of the response function. Thus the wideband-narrowband question must be answered with respect to how the system experiences multipath and frequency fades. Hence wideband condition can be viewed as the upper bound for a narrowband assumption and as such it is deemed that this presentation is general as long as the channel is regarded as being nonstationary and frequency selective. In any broadband and high data rate systems typical of mobile broadband communication systems like Ultra wideband (UWB), Acoustic Communications and mobile Wireless interoperability for microwave access (WiMax) channels, this method greatly suffices.

This paper is organized as follows. The mathematical preliminaries and notations associated with this paper are given in section 2. In section 3 the generalized operator-based system model is developed and then narrowed down to affine operator representation of the time-scale characterization presented in section 4. We discuss the derived delay-scale function and operator in section 5. The canonical model for the affine-based time-scale characterization is developed in section 6, and the second-order statistics of our model is presented in section 7. An example and accompanying simulations to demonstrate the application of our model with respect to a deterministic channel are presented in Section 8.

2 Preliminaries and Notations

We provide in this section some of the mathematical notations and definitions used in this paper. However some definitions are reserved to be introduced at appropriate points where it is deem such action will give better cohesion to this work. All operators in this discussion are assumed to be on Hilbert space. The concept of an operator or transformation on a Hilbert space is a natural generalization of the idea of a function of a real variable. Indeed it is fundamental in mathematics, science and engineering where linear operators on a Hilbert space are widely used to represent physical quantities and transformations, and hence provide better insight to phenomena associated with processes and systems. The term operator is here considered in the same manner as words like transformation, mapping and phenomenal effects associated with the wireless channel. All operators are in bold face letters.

Definition 2.1: Let $\Im$ be an operator, and let's assume that H_0 and K_0 are Hilbert spaces with norm $\|\cdot\|_{H_0}, \|\cdot\|_{K_0}$ and inner products $<.,.>_{H_0}, <.,.>_{K_0}$, respectively, and $\Im : H_0 \to K_0$

i. $\Im$ is linear if $\lambda_1\Im x + \lambda_2 \Im y \quad \forall\ x, y \in H_0$ and $\lambda_1, \lambda_2 \in \mathrm{C}$, where C denotes complex-valued number.

ii $\Im$ is injective if $\Im x \neq \Im y$ whenever $x \neq y$.

iii $\Im$ is surjective if $Range(\Im) = K_0$.

iv $\Im$ is bijective if it is both injective and subjective. This means that given any $y \in K_0$, there is exactly one $x \in H_0$ such that $\Im x = y$.

v The norm of $\Im$ is given by $\|\Im\| = Sup\{\|\Im x\|_{K_0} : x \in H_0 \text{ and } \|x\|_{H_0} = 1\}$

vi $\Im$ is bounded if there exist a number Q such that $\|\Im x\| \le Q\|x\| \quad \forall x \in H_0$.

vii $\Im$ is continuous at x_0 if $\|x_n - x_0\| \to 0$ implies $\|\Im(x_n) - \Im(x_0)\| \to 0$

viii The adjoint of $\Im$ is the unique operator $\Im^* : K_0 \to H_0$ such that $<\Im x, y>_{K_0} = <x, \Im^* y>_{H_0} \quad \forall x \in H_0, y \in K_0$

ix $\Im$ is an isometry if $\|\Im x\|_{K_0} = \|x\|_{H_0} \quad \forall \quad x \in H_0$.

A linear map $\Im$ is norm-preserving, if and only if $<\Im x, \Im y>_{K_0} = <x, y>_{H_0} \quad \forall x, y \in H_0$.

x $\Im$ is called unitary in the case $\Im^* \Im = \Im \Im^* = \mathbf{I}$ where $\mathbf{I}$ is an identity operator.

xi $\Im$ is self-adjoint or Hermitian if $\Im^* = \Im$.

Definition 2.2: Let $f \in \mathbb{C}$ denote a complex-valued function, and $\Re$ a real number line, given $1 \le p < \infty$, we define the Lebesgue space $L^p(\Re) = \left\{ f : \|f\|_p = \left(\int |f(t)|^p dt\right)^{\frac{1}{p}} < \infty \right\}$ as a Banach space and $L^2(\Re)$ as the Hilbert space with inner product $< f, g >$ where g in our case is a time function.

3 System Model

The mobile communications channel is dominantly characterized by the doubly influence of multipath phenomenon and shift/variation in frequency induced on the transmitted signal by mobility. To represent these phenomena require the use of appropriate joint time-frequency or as the case may be, a joint time-scale representation. In general we neglect the effect of additive white Gaussian noise, thus for a continuous time case, the effect of the mobile channel can be represented by a linear operator $\Im_G^{H_C}$ as shown in Fig.1,

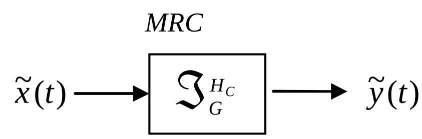

Fig.1: Generalized mobile channel model

such that

$$\tilde{y}(t) = (\Im_G^{H_C} \tilde{x})(t) \tag{1}$$

where $\tilde{x}(t)$ and $\tilde{y}(t)$ are the baseband representations of the transmitted and received signals respectively. We consider $\Im_G^{H_C}$ as a Hilbert-Schmidt (HS) operator [20] which is always bounded. This type of operator establishes a Hilbert space H_0 with inner product $<\Im_G^{H_C}, \mathbf{U}>$ and norm $\|\Im_G^{H_C}\|_{HS}^2 := <\Im_G^{H_C}, \Im_G^{H_C}>$, where $\mathbf{U}$ is a unitary operator that acts on the transmitted signal as will be explained shortly.

Definition 3.1: Let $\Im_G^{H_C}$ be a linear mapping $\Im_G^{H_C} : H_0 \to H_0$ in the Hilbert space H_0 which maps a function $\tilde{x} = \phi_{\tilde{x}} \tilde{g}$ to a function $\tilde{y} = \phi_{\tilde{y}} \tilde{q}$ where H^c is the channel parameterization with superscript c denoting either time-invariant or time-variant channel, and $\phi_{\tilde{x}}$ and $\phi_{\tilde{y}}$ are the bases of $\tilde{x}$ and $\tilde{y}$, with corresponding coefficients $\tilde{g}$ and $\tilde{q}$. If we define $\Im_G^{H_C}$ on a group G, then the relation with H^c is given by

$$\tilde{y}(t) = (\Im_G^{H_C} \tilde{x})(t) = (H^c \otimes_{G,\Pi} \tilde{x})(t) \tag{2}$$

where $\otimes_{G,\Pi}$ delineates the actions of the channel on $\tilde{x}(t)$. The subscripts G,Π denotes the channel operation under a particular group G and transform domain Π. For a linear time-invariant (LTI) channel, G is of the Heisenberg group and Π is in the Fourier domain, thus $\otimes_{G,\Pi}$ is equivalent to a convolution operator of the Fourier class, and for time-varying channel $\otimes_{G,\Pi}$ is group convolution operator. Hence depending upon the assumptions made regarding the channel and the entire system, several different channel operators' approximations and parameterizations suffice. Note that for simplicity, since we are considering a time-varying channel, we assume $\Im_G^{H_C} \equiv \Im_G$, and work on baseband level such that $\tilde{x}, \tilde{y} \overset{def}{=} x, y$.

In mobile radio system $\Im_G$ involves the combination of different operators which are generally time-delay, and frequency variation or time-dilation operators acting jointly to account for time-frequency and time-scale dispersion effects of the channel on the transmitted signal. The order of occurrences of the operators in the joint representations (commutativity); or if the actions are simultaneous, can be of importance in the accurate representation of the channel action by the associated mathematical representational or mapping tool employed. Joint Time-frequency and Time-scale characterization has been addressed in [12], [13], [14], [15]. In [16] the association of joint variable representation is presented. For $t \in \Re^d, \omega \in \Re^d$ and $s \in \Re^d$ we denote by $\mathbf{D}_\tau$ the time delay operator, $\mathbf{F}_\omega$ the frequency shift (Doppler) operator, and $\mathbf{S}_s$ the time scaling operator defined respectively by

$$\mathbf{D}_\tau : L^2(\Re^d) \to L^2(\Re^d), \quad (\mathbf{D}_\tau \tilde{x})(t) = \tilde{x}(t-\tau)$$
$$\mathbf{F}_\omega : L^2(\Re^d) \to L^2(\Re^d), \quad (\mathbf{F}_\omega \tilde{x})(t) = \tilde{x}(t)\, e^{j\omega t} \tag{3}$$
$$\mathbf{S}_s : L^2(\Re^d) \to L^2(\Re^d), \quad (\mathbf{S}_s \tilde{x})(t) = \frac{1}{\sqrt{|s|}} \tilde{x}\left(\frac{t}{s}\right)$$

where $\mathbf{D}_\tau$, $\mathbf{F}_\omega$, and $\mathbf{S}_s$ are considered as unitary, isometric and bijective operators and τ, ω, s are implicitly defined in the above expressions.

Theorem 1: Assume H_0 is a Hilbert space with norm $\|.\|_{H_0}$ and inner product $<.,.>_{H_0}$, and that $\mathbf{U}_m : H_0 \to H_0 \quad m = 1,2$, is a unitary operator

i. There exist joint operator $\mathbf{U}_1\mathbf{U}_2 \equiv \mathbf{U}_{1,2}$ and $\mathbf{U}_2\mathbf{U}_1 \equiv \mathbf{U}_{2,1}$
ii. $\mathbf{U}_{1,2}, \mathbf{U}_{2,1}$ are also unitary operators
iii. $\mathbf{U}_{2,1} = \mathbf{U}_{1,2}$ if and only if $\mathbf{U}_1$ and $\mathbf{U}_2$ are commutative
iv. If $\mathbf{U}_1$ and $\mathbf{U}_2$ are isometric, so are $\mathbf{U}_{1,2}$ and $\mathbf{U}_{2,1}$.

The corresponding proof is easy.

We define the corresponding joint operators $\mathbf{V}_{\tau\omega}$ and $\mathbf{A}_{\tau,s}$ for delay-Doppler and delay-scale operators respectively by

$$\mathbf{V}_{\tau\omega} : L^2(\Re^d) \to L^2(\Re^d), \quad (\mathbf{V}_{\tau\omega}\tilde{x})(t) = (\mathbf{D}_\tau(\mathbf{F}_\omega \tilde{x}))(t) = \tilde{x}(t-\tau)\, e^{j\omega t} \tag{4}$$
$$\mathbf{A}_{\tau,s} : L^2(\Re^d) \to L^2(\Re^d), \quad (\mathbf{A}_{\tau,s}\tilde{x})(t) = (\mathbf{D}_\tau(\mathbf{S}_s \tilde{x}))(t) = \frac{1}{\sqrt{|s|}} \tilde{x}\left(\frac{t-\tau}{s}\right) \tag{5}$$

where $\mathbf{V}_{,\tau\omega} \overset{def}{\neq} \mathbf{V}_{\omega,\tau}$ and $\mathbf{A}_{\tau,s} \overset{def}{\neq} \mathbf{A}_{s,\tau}$, except if the joint operation is deemed to be accorded simultaneity condition (commutative). Thus $\mathbf{D}_\tau$ and $\mathbf{F}_\omega$, and, $\mathbf{D}_\tau$ and $\mathbf{S}_s$, do not commute. In fact the Heisenberg and affine group are generally regarded as non-commutative groups with unitary transformations that are non-commutative except when the underlying physics says otherwise. Of course $|\mathbf{V}_{,\tau\omega}| = |\mathbf{V}_{\omega,\tau}|$ and $|\mathbf{A}_{\tau,s}| = |\mathbf{A}_{s,\tau}|$. However for the rest of the discussion except otherwise stated, we assume that the joint channel operators are of the forms in (4) and (5). The expression in (5) is a joint channel operator of the affine group, and uses the term $1/\sqrt{|s|}$ to maintain the norm of the operator. This joint operator of affine group forms the bases of our discussions.

Classical results from group theory can be used to gain insight into properties of systems with joint variable functions. Group representation theory provides unified framework for the study of spreading function both for the Heisenberg group and the affine group which are used to represent time-varying propagations and scattering channels. Some applications of this theory as regards system representation are available in [9], [10], [11], [22]. We refer the reader to [18] for some background on group representations.

We now build our operator-based channel model mimicking the real mobile channel using three basic operators namely; unitary operator, inner product operator and integral operator of *simple kind*. The unitary operator has been defined in Definition 2.1. x.

Definition 3.2: The inner product operator $\mathbf{I}_G(.,.) \equiv <.,.>_G$ of group G Hilbert space H_0 maps two function pairs say (y, x) to produce a third which is a dependent on the group elements ϑ.

Definition 3.3: An integral operator $\mathbf{L}$ of group G on a Hilbert space H_0 given by

$\mathbf{L} \equiv \int_G \mathbf{I}\, d\, \mu(\vartheta) = \int_G \Im \Im^* \, d\, \mu(\vartheta)$ is called *simple kind* if and only if $\Im^* \Im = \Im \Im^* = \mathbf{I}$.

The transmitted signal on one hand undergoes a time-frequency/scale operation resulting in different copies of the $x(t)$ at the receiver end. We consider these copies as a function of a unitary operator denoted arbitrarily by $\mathbf{U}_\vartheta$, which is a unitary operator of group G and measure μ on each group element $\vartheta \equiv (\tau, \omega)$ and $\vartheta \equiv (\tau, s)$ for Weyl-Heisenberg and affine group on H_0 respectively. Thus we split this operator off $\Im_G$ as shown in Fig.2.

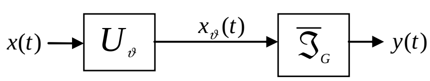

Fig.2: Compact model with the unitary operator split off the channel operator

We can express the received signal as

$$y(t) = (\overline{\Im}_G(\mathbf{U}_\vartheta \tilde{x}))_\mu(t) \qquad (6)$$

For this expression and representation to be valid, we regard $\overline{\Im}_G$ as also containing an operator that maintains and ensures the equivalence between the 'spread' copies of the transmitted signal and the received signal $y(t)$. We describe the weight of the coefficients associated with the unitary operator U_ϑ and generated by the operator $\Im_G$ in mapping $x(t)$ onto $y(t)$ as a density function defined by an inner product operator $\mathbf{I}_G$ of group G with respect to the pairs (y, x_ϑ) such that

$$\mathbf{I}_G(y, x_\vartheta) = < y, x >_\vartheta = d_\vartheta \qquad (7)$$

We consider an equivalent operator Ξ which acts on $x_\vartheta(t)$ or which acts on $x(t)$ (if we assume it is split off $\Im_G$) to produce the weight and whatever is the order is irrelevant since $\Xi \mathbf{U}_\vartheta = \mathbf{U}_\vartheta \Xi$. Hence

$$\mathbf{I}_G(y, x_\vartheta) = \Xi x_\vartheta = d_\vartheta \qquad (8)$$

We modify figure 2 accordingly as shown in Fig.3.

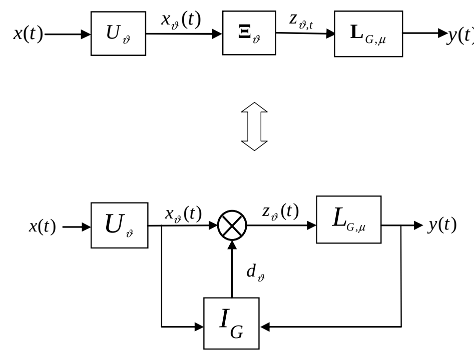

Fig.3: Equivalent mobile channel model comprising all three operators

The two figures above are equivalent; the upper figure is a real-time model of the mobile radio channel whereas the other is more of a synthesized channel in which the density function is estimated given that we have knowledge of the channel input and output. The operator $\mathbf{L}_{G,\mu}$ is an integral operator of simple kind defined on group G and measure μ, and emphasizes on the collective actions of all other operators discussed. Hence we can represent a typical mobile radio channel (MRC) by the operator-based expression given by

$$y(t) = (\mathbf{L}(\Xi(Ux))_\vartheta)_{G,\mu}(t) \qquad (9)$$

We maintain that all operators associated with (9) are HS operators where for clarity the bracket $(\Lambda g)(t)$ represents the result of operating on g with the operator Λ and then evaluating at the point t. This representation is a full and intuitive characterization of the mobile radio channel or any such system where mobility and scatters are involved. The choice of representation and its accuracy strictly depend on the physics of the problem, the group G and measure μ, and the group element ϑ. The operator $\mathbf{U}_\vartheta$ is equal to the joint time-frequency and time-scale expressions in (4) and (5) respectively and can be deduced from them considering the appropriate group G. On the other hand, Ξ_ϑ is an inner product operator and for an ideal channel equals a norm operator $< \mathbf{U}_\vartheta, \mathbf{U}_\vartheta >_G$.

Definition 3.4: Let T be the torus group with unit circle in $\mathbb{C}$ i.e. $T = \{z \in \mathbb{C} : |z| = 1\}$, and $\hat{\Re}$ the real number line thought of as the frequency axis, we define a group $\mathsf{H} \quad \mathsf{T} = \Re\hat{\Re}$ the set acting on $L^2(\Re)$ as Weyl-Heisenberg group with unitary operator denoted by $\mathbf{V}_{T \times R\times \hat{R}}$

If we consider the group H and group element $\vartheta \equiv (\tau, \nu)$, the appropriate measure μ for the representation of the joint delay-Doppler $(\tau - \nu)$ action of the mobile channel is the Left Invariant measure $d\mu(\vartheta) = d\tau\, d\nu$ (see Appendix A in [22] for more details and definitions). Thus (9) is simplified as

$$y(t) = (\mathbf{L}(\Xi(\mathbf{U}_{\vartheta} x))_{\tau\nu})(t)\,|_{\mathsf{H},\mu} \qquad (10)$$
$$= (\mathbf{L}(\Xi(\mathbf{V}x))_{\tau\nu})(t)\,|_{\mathsf{H},\mu}$$

where (10) is an operator-based time-frequency representation with respect to the group H. An observation from our discussion shows that the operator Ξ_{ϑ} is obviously a pivot operator that is related in one way or another to all other operators defined as indicated by the expression

$$\Xi_{\vartheta} x_{\vartheta} = \mathsf{I}_G(\Im_G x, \mathbf{U}_{\vartheta} x) = \mathsf{I}_G(\overline{\Im}_G x, \mathbf{U}_{\vartheta} x) \qquad (11)$$

This operator therefore explicitly describes the dispersive behavior of the channel. The result of the operator on x_{ϑ} is the weighted function termed *spreading function* (in time-frequency theory) or *coherent state* (in Quantum Physics) [20], thus we can refer to Ξ_{ϑ} as the *spreading operator*. The connection between the coherent state and short-time Fourier transform (STFT) can be extended to the spreading function due to strong interrelation between time-frequency analysis and Quantum physics thus

$$\Xi_{\tau\nu} x = STFT(\tau\nu; w) = < k_{w,y}, V_{\tau\nu} x > \qquad (12)$$

where $k_{w,y}(\tau, t) = y(t).w(t-\tau)$ is called the kernel of the channel operator $\Im_G$ for a given window function $w(t)$. We denote the ensuing function d_{ϑ} by $\tilde{S}(\tau\nu)$ refer to as the *delay-Doppler spreading function* by Bello [21] and accounts for the joint delay and Doppler variation of the channel. And accordingly we define $\Xi_{\tau\nu}$ as the *delay-Doppler spreading operator*. The window term $w(t)$ is viewed as the snapshot of the mobile radio channel often modeled as a time-varying filter. This snapshot irrespective of the width is assumed to be stationary over the transmission interval in order to analyze the channel using the windowed transform. Such situation constrains the resolution of the operator Ξ_{ϑ}. Thus since the mobile radio channel is nonstationary in reality, any channel variation in time and frequency that does not fall within this window width is not accounted for. This fixed window orientation results in channel mapping with poor (fixed) time-frequency resolution as indicated in Fig.4.

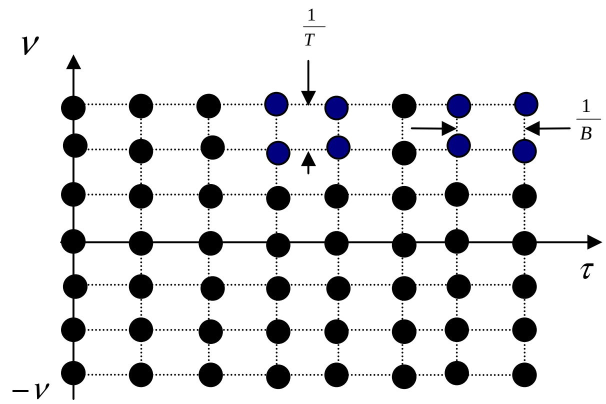

Fig.4:Delay-Doppler resolution plane as a function of fixed windowed transform

And for a frequency selective and fast fading channel, and even in the narrowband time-varying baseband equivalent channel, the delay-Doppler operator is not shift invariance; hence sinusoids are not eigenfunctions. We thus need eigenfunctions that will provide shift invariance spreading operator that adequately match the channel properties so mentioned thus providing optimal representation.

4 Affine Operator-Based Time-Scale Channel Characterization

The notion of Doppler variation can be considered as a form of time scaling (dilation) operation [22] where scale is given by

$$s = \frac{c - v}{c + v} \approx 1 + \frac{\omega_d}{\omega_c} \qquad (13)$$

where c, v, ω_c and ω_d are the speed of electromagnetic wave in free space, speed of mobile unit, operating frequency and associated Doppler shift respectively. Thus we can obtain a time-scale representation of the channel on temporal plane equivalent to time-frequency representation. Note the use of time-scale and time scaling; while the former refers to joint operator and representation, the latter is a single linear operation.

Definition 4.1: Let $A \quad \Re\hat{\Re}$ be a set acting on $L^2(\Re)$ which involves translation τ and dilation or time scaling s, and maps a function on $x \in L^2(\Re)$ into a new independent variable $\frac{x-\tau}{s}$. The set A is called the affine group with unitary operation denoted by $\mathbf{A}_{R\times R}$

The time-scale representation of (10) with respect to the affine group and unitary operator $\mathbf{A}_{R\times R} = \mathbf{A}_{\tau,s}$ is given by

$$y(t) = (\mathbf{L}(\Xi(\mathbf{A}x))_{\tau,s})(t)\,|_{\mathrm{A},\mu} \tag{14}$$

where τ is a time delay or translation parameter with respect to the delay operator $\mathbf{D}_\tau$, and s is a time scaling parameter with respect to time scaling operator $\mathbf{S}_s$ as a result of channel mobility. With reference to Fig.3, $x_{\tau,s}(t)$ can be written as

$$(\mathbf{A}_{\tau,s}x)(t) = x_{\tau,s}(t) = \frac{1}{\sqrt{|s|}}\,x\left(\frac{t-\tau}{s}\right) \tag{15}$$

The left Haar measures of the affine group is given by $d\mu(\vartheta) = \frac{ds\ d\tau}{s^2}$ [3]. This measure other than $d\mu(\vartheta) = ds\ d\tau$ ensures constant surface and is required for due energy conservation in mapping the input to output.

A similar coherent state relationship to the spreading operator $\Xi_{\tau\nu}$ can be extended to the affine spreading operator $\Xi_{\tau,s}$ which acts on $x_{\tau,s}(t)$ to yield the *delay-scale density* or *spreading function* analogous to the delay-Doppler spreading function of the Weyl-Heisenberg group. In general the goal in electronic communication is to recover the transmitted signal from the 'echoes' with the help of the information from this density function $d_{\tau,s}$. The estimation of $d_{\tau,s}$ is thus important in providing efficient communication services. The delay-scale spreading function can be estimated by formulating the inverse problem for obtaining $d_{\tau,s}$ with recourse to [7] and expressed as

$$y(t) = \mathbf{\Gamma} d_{\tau,s} \tag{16}$$

where

$$\mathbf{\Gamma}(.) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(.)\,\frac{1}{\sqrt{|s|}}\,x\left(\frac{t-\tau}{s}\right)\frac{d\tau\, ds}{s^2}$$

Thus

$$\Im_G x = \mathbf{\Gamma} d_{\tau,s}$$

This expression is an estimator where the knowledge of the operators and the response can aid in recovering x. Different methods of solving for $d_{\tau,s}$ have been addressed by the literature [7]. One of such methods is the wavelet transform technique. The importance of this method in mobile radio channel characterization is that it provides solution to the lapses of (12) in representing the spreading operator.

To improve on the resolution of the spreading operator Ξ_ϑ we thus require a coherent state with better time-frequency/scale resolution. An alternative to windowed Fourier transform was proposed by Grossmann and Morlet [23] in which the rule for generating the 'basis functions' was changed. This is done by replacing the modulation operation defined on $\hat{\Re}$ by a scaling operation. This transform method is called continuous wavelet transform (CWT). On its merit CWT promises better time-frequency resolution due to the use of evolutionary 'window' in terms of variable scales. Basically continuous wavelet transform (CWT) is an inherently two dimensional transform that uses 'small waves' of finite support as the analyzing kernels generated by affine unitary operation on an arbitrary 'mother wavelet' $\varphi(t)$ such that

$$(\mathbf{A}_{\tau,s}\varphi)(t) = \varphi_{\tau,s}(t) = \frac{1}{\sqrt{|s|}}\varphi\left(\frac{t-\tau}{s}\right),\ t \in \Re \tag{17}$$

The wavelet $\varphi_{\tau,s}(t)$ can be viewed as a copy of the original wavelet $\varphi(t)$ shifted by time steps τ and rescaled in time by step size s. Given an analyzing wavelet $\varphi(t)$, the corresponding CWT is defined as follows:

Definition 4.2: Let $\varphi \in L^1(\Re)\cap L^2(\Re)$ be an analyzing wavelet, the continuous wavelet transform of an arbitrary finite-energy signal $f(t)$ is defined by the inner product operator $\mathbf{I}_G(.,.)$ such that

$$\mathbf{I}_G(f, \mathbf{A}_{\tau,s}\varphi) = \langle f, \mathbf{A}_{\tau,s}\varphi\rangle = d_{\tau,s} \tag{18}$$

The inverse transform is given by

$$\begin{aligned} f(t) &= (\mathbf{\Lambda}\varphi)(t) \\ &= C_\varphi^{-1}\left(\mathbf{L}(d_{\tau,s}.(\mathbf{A}_{\tau,s}\varphi))\right)_{A,\mu}(t) \end{aligned} \tag{19}$$

where $C_\varphi = \int_{-\infty}^{\infty}\frac{|\Psi(\omega)|^2}{|\omega|}d\omega < \infty$ is an admissibility condition described fully in [3] and $|\Psi(\omega)|$ is the Fourier transform of $\varphi(t)$.

A close observation on (9) and (18) reveals that both equations are equal except for the admissibility term. This means that we can assume that for a transmitted signal $x(t) \equiv \varphi(t) \in L^1(\Re)\cap L^2(\Re)$,

$\Im \equiv \Lambda$, there exist a unitary affine operator $\mathbf{A}_{\tau,s}$ which acts on it to produce delayed and rescaled versions of $x(t)$ which with associated spreading function $d_{\tau,s}$, are integrated to yield the received signal $y(t) \equiv f(t)$ as shown in Fig.5.

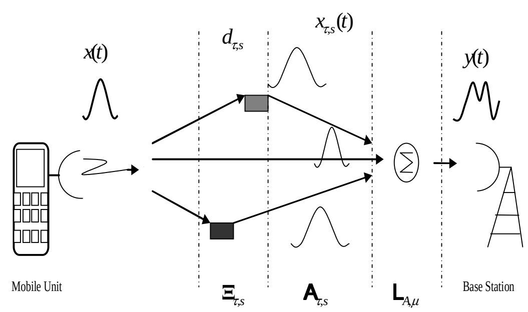

Fig.5: Illustration of the operators with respect to wavelet-type affine time-scale channel model

Thus we can extend the coherent state relation with Fourier transform to wavelet transform which implies and by induction that the response or associated spreading function of the affine time-scale channel representation is the wavelet transform of the received signal $y(t)$ with respect to the transmitted signal $x(t)$. We reframe (11) as

$$\Xi_{\tau,s} x_{\tau,s} = \mathbf{I}_G(y, \mathbf{A}_{\tau,s} x) = d_{\tau,s} \quad (20)$$

The function $d_{\tau,s}$ can be referred to as the *delay-scale spreading function* and $\Xi_{\tau,s}$ as the *delay-scale spreading operator*. However, $d_{\tau,s}$ is unique to a particular probe signal. It should be noted that a necessary requirement here is that the probe or transmitted signal $x(t)$ must be admissible so that

$$C_x = \int_{-\infty}^{\infty} \frac{|X(\omega)|^2}{|\omega|} d\omega < \infty.$$

If we let $\hat{y} = \frac{y(t)}{C_x}$, then

$$\hat{y}(t) = \left(\mathbf{L}(d_{\tau,s}.(\mathbf{A}_{\tau,s} x))\right)_{A,\mu}(t) \quad (21)$$

It can easily be shown that the corresponding delay-scale spreading function $\hat{d}_{\tau,s}$ is related to $y(t)$ by

$$\hat{d}_{\tau,s} = C_x^{-1}.\mathbf{I}_G\left(y, \mathbf{A}_{\tau,s} x\right) \quad (22)$$

Thus within some power constraints, we equivalently state that

$$\hat{d}_{\tau,s} \cong d_{\tau,s}$$

For an ideal channel, $d_{\tau,s}$ is the *auto-wavelet channel response*. It can be shown that when $d_{\tau,s}$ has support only along $s = 1$ and is perfectly concentrated along the τ axis, then (14) reduces to a linear time-invariant system and $d_{\tau,s} \to d_\tau$. While for $d_{\tau,s}$ with support only along $\tau = 0$ and perfectly concentrated along the s axis, we have a linear frequency-invariant` system and $d_{\tau,s} \to d_s$. If we consider $d_{\tau,s}$ as also a function time, then at $s = 1$, the resultant function, $d_{\tau,s,t} \to d_{\tau,t}$, provides a time-varying channel response.

The time-scale resolution of the delay-scale spreading function is shown in Fig.6, indicating a good resolution due to variable scales of the analyzing functions which corresponds to high frequency/low scale and low frequency/high scale representation and capable of tracking time-varying channels more accurately compared to the constant relative bandwidth of the STFT.

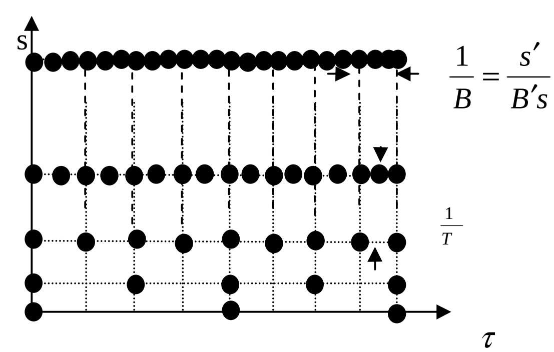

Fig.6: Time-scale resolution plane of wavelet transform

5 Delay-Scale Spreading Function and Operator

We observe that since continuous wavelet transform with respect to the invariant measure $\frac{d\tau ds}{s^2}$ is asymmetric, then $d_{\tau,s}$ is also asymmetric. The values of τ and s provides the support for the spreading function in delay and scale coordinates respectively. Let's define the total spread $\Delta^W_{total} = \tau_{\max} s_{\max}$ as the coherent rectangular area that determines the extension of the spreading function about the origin. The quantities $\tau_{\max}$ and $s_{\max}$ are the differences between the largest and smallest delay and scale respectively of significant multipath components. In actual systems, $\tau_{\max}$ determines the length of the operator $\Xi_{\tau,s}$ or system memory, and $s_{\max}$ determines the relative motion due to system fluctuations or the operator's time-variations. Quintessentially, if we have a proper knowledge of the maximum delay and scale values $\tau_{\max}$, $s_{\max}$ as well as the minimum delay and scale values $\tau_{\min}$, $s_{\min}$ respectively, and if we also have approximate statistics on the number of propagation paths L which we here assume is equivalent to the number of the received versions of the transmitted signal, then the range of the channel operator $\Im_G$ is implicitly determines by $\tau_{\min}$, $s_{\min}$ and $\tau_{\max}$, $s_{\max}$.

$$\hat{y}(t) = \left(\mathbf{L}(d_{\tau s}.(\mathbf{A}_{\tau,s}x))\right)_{A,\mu}(t) \quad ,\tau_{\min} \le \tau \le \tau_{\max}, s_{\min} \le s \le s_{\max} \tag{23}$$

In general, we define the following:

$|\alpha_{\max}| = s_{\max} - s_{\min} \rightarrow$ Maximum scale spread

$\gamma_{\max} = \tau_{\max} - \tau_{\min} \rightarrow$ Maximum delay spread

For any Lth received versions of $x(t)$, we can write

$$\left.\begin{aligned} s_{l+1} &= s_{\min} \pm |\alpha_{l+1}| \\ \tau_{l+1} &= \tau_{\min} \pm |\gamma_{l+1}| \end{aligned}\right\} l = 0,1,2,.....,L-1 \tag{24}$$

where s_{l+1}, τ_{l+1} and α_{l+1}, γ_{l+1} are instantaneous scale, instantaneous delay, and the associated scale spread, delay spread respectively. With fairly accurate knowledge of $s_{\max}$, $s_{\min}$, $\tau_{\max}$, and $\tau_{\min}$, the channel response $d_{\tau,s}$ can be estimated as the group correlation of $y(t)$ with $x_{\tau,S}(t)$ as shown in Fig.7.

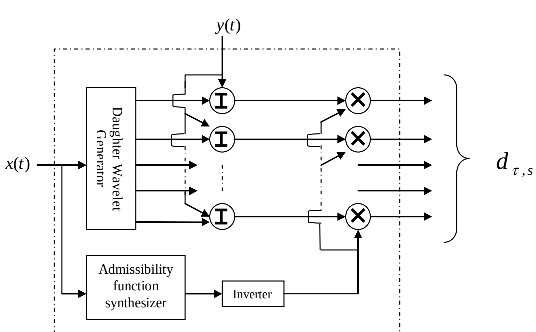

Figure 7: Estimator for channel response to wavelet input

In practice, $d_{\tau,s}$ has finite support. The minimum delay is zero ($\tau_{\min} = 0$) thus $\gamma_{\max} = \tau_{\max}$ which corresponds to a line-of-sight-propagation. For a signaling rate $\frac{1}{T_s}$, $T_s > \tau_{\max}$ signifies fast fading and $T_s < \tau_{\max}$ is slow fading. If the mobile unit is moving towards the transmitter, then the minimum and maximum (single-sided) scale values are given by $s_{\min(1-sided)} = 1$, $s_{\max(1-sided)} = 1 + \frac{v^+_{\max}}{c}$ where $v^+_{\max}$ is the maximum velocity towards the transmitter and assumes parallel arrival of copies of transmitted signal with respect to the receiver. For two-sided case, the scale dilation and contraction caused by the movement of the mobile unit towards or away from the transmitter is asymmetric about the unit value (implies a zero shift in frequency), thus $s_{\min(2-sided)} = 1 - \frac{v^-_{\max}}{c}$ and $s_{\max(2-sided)} = 1 + \frac{v^+_{\max}}{c}$ where $v^-_{\max}$ is the maximum velocity away from the transmitter. If we define the associated total spread for the time-frequency representation in (12) as $\Delta^F_{total} = \tau_{\max} \nu_{\max}$ [24, pp. 2-6], $\nu_{\max}$ being the Doppler spread, then Δ^W_{total} and Δ^F_{total} are related for the one sided case by

$$\Delta^W_{total} = \tau_{\max} + \frac{\Delta^F_{total}}{f_c} \tag{25}$$

where f_c is the operating frequency. Note the difference between the symbols associated with Doppler shift ν and velocity v. For any L^{th} received versions of $x(t)$ the delay spread at that scale is

$$\alpha_{l+1} = |s_{l+1} - 1| \tag{26}$$

In general, if we assume the presence of mobile scatterers and mobile transmitter/receiver, then

$$\alpha_{l+1,n} = \left|\sum_n s_{l+1,n} - 1\right| \tag{27}$$

where n represents the number of all contributions to the scaling of the propagating signal along a path L. Equations (26) and (27) are equivalent when only the mobile terminal is considered.

The broadening, or extension, of the spreading function about the origin of the (τ, s) plane provides a global characterization of the time-scale shifts introduced by the operator $\mathbf{A}_{\tau,s}$. If the spreading function is concentrated about some point ($(\tau', s') \neq (0,0)$), this corresponds to an offset time-scale variation which can be split off the operator resulting in an operator whose spreading function is concentrated about the origin. We defined the *minimum resolvable delay* τ_r to be the smallest $\tau > 0$ such that $d_{\tau_r, s_{\min}} \cong 0$, and the *minimum resolvable scale* s_r to be the smallest $s > s_{\min}$ such that $d_{0,s_r} \cong 0$ [32].

6 Canonical Affine Time-Scale Characterization

In essence, all practical channels and signals have effective finite number of freedom due to restrictions on time duration, fading rate, bandwidth, etc. These restrictions allow a simplified representation of linear time varying channels in terms of canonical elements or building blocks. The discrete representation form of (10) in terms of sampled time and frequency shifts was developed by Bello [21] based on time and bandwidth constraints [3]. A similar formation was also obtained by Sayeed et al [25], [26] in order to design the transmission signaling and obtain multipath and Doppler diversity. The discrete time-scale model of Mellin type for wideband time-varying system was developed by Jang and Papandreou-Suppappola [27]. In the same vein we develop the discrete time-scale model for the affine type expressed in (20). Our time-scale model is based on sampling the time delay τ and time scaling s parameters by utilizing the frame theory based approach.

The natural way to sample τ and s is to use a logarithmic discretization of the scale s and link this, in turn, to the size of steps taken between τ locations. To link s to τ, we move a discrete step to each new location τ' which is proportional to the scale s'. We choose $s = s_0^m$, $s_0 > 1$ and $\tau = n\,\tau_0 s_0^m$, $\tau_0 > 0$ [3]. The parameters τ_0 and s_0 are fixed delay and scale sizes respectively and determines the density of the discrete lattice and implicitly the resolution of the time-scale model. Accurate choices for τ_0 and s_0 are very important as will be discussed shortly. The integers n and m control the actual delay and scale values respectively.

The mobile channel expression (20) can be explicitly written as

$$y(t) = \int_{s_{\min}}^{s_{\max}} \int_{\tau_{\min}}^{\tau_{\max}} d_{\tau,s} \frac{1}{\sqrt{|s|}} x\left(\frac{t-\tau}{s}\right) \frac{d\tau ds}{s^2} \qquad (28)$$

and

$$d_{\tau,s} = \int_{-\infty}^{\infty} y(t) \frac{1}{\sqrt{|s|}} x^*\left(\frac{t-\tau}{s}\right) dt \qquad (29)$$

Applying the discretization operation on (28) and (29) yields the canonical expression given by

$$y'(t) = \sum_{m\in Z}\sum_{n\in Z} d_{n,m}\, s_0^{-\frac{m}{2}}\, x\left(s_0^m t - n\tau_0\right) \qquad (30)$$

and

$$d_{\tau,s} = \int_{-\infty}^{\infty} y'(t)\, s_0^{-\frac{m}{2}}\, x^*\left(s_0^m t - n\tau_0\right) dt \qquad (31)$$

An operator based expression for (30) and (31) can be obtained by defining a simple group summation operator $\mathbf{Z}_G$ and affine-type frame unitary operator $\tilde{\mathbf{A}}_{n,m} : \tau, s \to n, m$ such that

$$y'(t) = \left(\mathbf{Z}_G\ (d_{n,m}.(\tilde{\mathbf{A}}_{n,m} x_{\tau,s}))\right)(t) \qquad (32)$$

and

$$d_{n,m} = \mathbf{I}_G\left(y', \tilde{\mathbf{A}}_{n,m} x_{\tau,s}\right) \qquad (33)$$

which represents a continuous function of time by a countable sets of coefficients $d_{n,m}$. The resultant compact canonical affine-type (wavelet) time-scale model corresponding to (30) is shown in figure 8

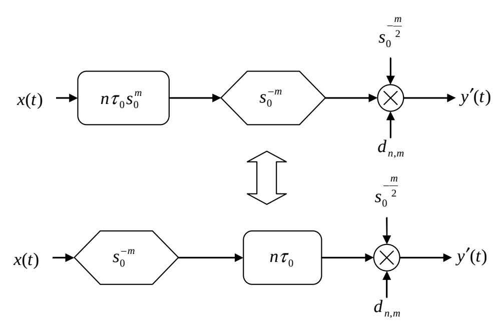

Figure 8: Canonical time-scale model

The outputs $y(t)$ and $y'(t)$ are related by considering the frame conditions given by

$$A\|y\|^2 \le \sum_m \sum_n \left| d_{n,m}{}^2 \le B\|y\|^2 \right| \qquad (34)$$

where A and B are energy bounds of $y(t)$. If $y'(t)$ falls within this acceptable energy range, it implies that

$$y(t) = k\, y'(t) \qquad (35)$$

where $k = \dfrac{2}{A+B}$ [6], is an error margin with which the sampled model differs from $y(t)$. For the purpose of this work, and in most practical situations, we assume that the ratio $\dfrac{B}{A}$ is unity, thus $y(t) \approx y'(t)$ upholds. The parameters τ_0 and s_0 are chosen to provide good resolution and acceptable sparseness to the model. This is ensured if the lattice structure is as close as possible to the continuous grid as shown in figure 9. A practical choice is to take the minimum resolvable delay and scale as the values for τ_0 and s_0.

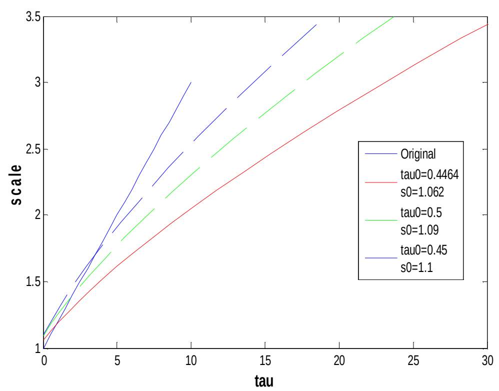

Figure 9: Lattice structure of an original delay-scale plane and the corresponding structure with respect to different values of τ_0 and s_0

For instance, let the delays associated with a particular channel be given as 0, 0.2, 0.8, 1.6, 2.4, and 5.0 μs and the corresponding scales, $s_l = 1 + \dfrac{v_l}{c}$, $l = 0,1,2,...,L-1$ where $v_{l=L-1}$ is the maximum velocity $v_{\max} = 120 km/hr$ and $L = 6$.Using the lattice structure $s = s_0^m$, $s_0 = 1.000000022$ we arrive at discrete levels $m = 0,1,...,5$ corresponding to approximately discrete step velocity of $23.76 km/hr$. The delay structure given above leads to a reasonable choice of sampling interval of $T_s = 0.2\mu s$, thus for the lattice structure $\tau = n\tau_0 s_0^m$, $n = 0,1,...,25$, with $\tau_0 = 0.199999956$ we arrive at a relatively sparse lattice density close to the original grid as shown in Fig.10.

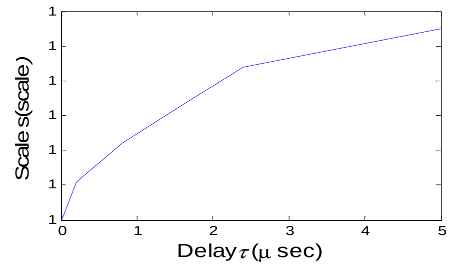

Fig.10a: Original delay-scale grid structure

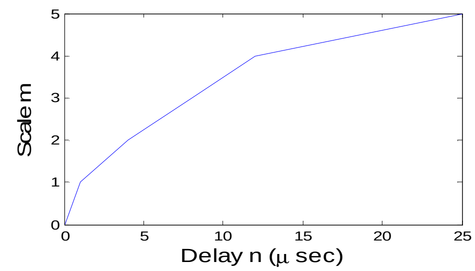

Fig.10b: Lattice structure for the corresponding $n-m$ grid for $\tau_0 = 0.199999956$ and $s_0 = 1.000000022$

7 Statistical Channel Characterization

In the foregoing discussion, our analysis has been based on deterministic models which invariably are of first order statistics. The spreading operator $\Xi_{\tau,s}$ is thus a deterministic operator. In reality, deterministic characterization of the mobile radio channel is not feasible and because the randomness of the channel, we have to resort to finding the appropriate statistical characterization. A scattering function as well as operator equivalent to the delay-Doppler Scattering function [2] of the Heisenberg group is required.

The delay-scale spreading function $d_{\tau,s}$ can be regarded as the *deterministic* scattering function of the low-pass equivalent received signal $y(t) \equiv y'(t)$. Statistical characterization are basically applied to stochastic processes, however to gain insight to statistical quantities like probability density function, the level-crossing rate, and the average duration of fades, the behaviour of deterministic processes as a funtion of random time t can be studies. The ith moment of such process is defined by

$$m_{d,i}(\tau,s) = E\left[d^i\right] \qquad (36)$$

where the first moment is the statistical mean, the second moment is the quadratic mean or the total power associated with the spreading function, and the third moment can be used to measure the asymmetry of function. The variance which captures the time-varying power is given by

$$V_d(\tau,s) = E\left[(d - m_{d,1}(\tau,s))^2\right] \quad (37)$$

The second order statistic that completely describes the random channel is the autocorrelation function (ACF). Depending on the underlying process, the ACF can be given many definitions. The process itself can be deterministic, or stochastic stationary or nonstationary (time-varying). Let $\{y(t)\}$ be a stochastic process with zero mean and finite variance, and lets $\mathbf{E}$ be the expectation operator, we define the ACF of $\{y(t) \equiv y'(t)\}$ by

$$R_y(t,t') = \mathbf{E}\left[y(t)y^*(t)\right] \quad (38)$$

If $\{y(t)\}$ is a stationary process, the correlation is invariant under any time shift and the ACF depends only on the time difference $t' - t = \Delta t$, thus

$$R_y(\Delta t) = \mathbf{E}\left[y(t)y^*(t + \Delta t)\right] \quad (39)$$

If however $\{y(t)\}$ is nonstationary, then the correlation varies as a function of time. A practical application will to adopt the philosophy of time-dependent power spectra [28] and then define a Local Scattering function (LSF) of the nonstationary process or channel [29].

Since $\{y(t)\}$ is a stochastic process, the associated spreading function and operator are also stochastic processes at a given scale $s > 0$. The ACF of $\{d_{\tau,s}\}$ is given by

$$R_d(\tau,s;\tau',s') = \mathbf{E}[d_{\tau,s}\ d_{\tau',s'}] \quad (40)$$

$$= \frac{1}{\sqrt{|s's|}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_y(t,t')\, x\left(\frac{t-\tau}{s}\right) x^*\left(\frac{t'-\tau'}{s'}\right) dt\,dt'$$

This expression is valid as long as the condition [30]

$$\mathbf{E}\left[\int_R \left|y(t)x\left(\frac{t-\tau}{s}\right)\right| dt\right]^2 \le \left\{\int_R \{R_y(t,t)\}^{\frac{1}{2}} \left|x^*\left(\frac{t-\tau}{s}\right)\right| dt\right\}^2 < +\infty \quad (41)$$

is satisfied. This condition ensures the existence of the second order process of $d_{\tau,s}$. Thus

$$R_d(\tau,s;\tau',s') = \frac{1}{\sqrt{|s's|}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_y(t,t')\, R_x(\tau,s,t;\tau',s',t')\, dt\,dt' \quad (42)$$

represents the generalized ACF of the affine time-scale process. We can define the stationary process by considering correlation with respect to the same scale i.e. $s = s'$ in which a nonstationary process reduces to an ordinary wide sense stationary (WSS) process [31]. If we let $t = 0$, and $t' = t + \Delta t$, then

$$R_d(\tau,s;\tau',s') = \frac{1}{\sqrt{|s's|}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_y(\Delta t)\, x\left(\frac{t-\tau}{s}\right) x^*\left(\frac{\Delta-\tau'}{s'}\right) dt\,d\Delta t \quad (43)$$

$$= \left\langle R_y, x^*_{\tau',s',\Delta t}\right\rangle \left\langle 1, x_{\tau,s,t}\right\rangle$$

where $b_{\tau,s} = \left\langle R_y, x^*_{\tau',s',\Delta t}\right\rangle$ is called the *Delay-scale Scattering function* (DSSF), and $\left\langle 1, x_{\tau,s,t}\right\rangle$ can easily be shown to be an approximation of $\delta(\tau-\tau')\delta(s-s')$ which implies that at each different scale the distribution is simply a scaled version of the transmitted signal. If we consider (34), then the corresponding expression is

$$R_d(n,m;n',m') \cong \left\langle R_y, x^*_{n',m',\Delta t}\right\rangle \left\langle 1, x_{n,m,t}\right\rangle \quad (44)$$

The delay-scale scattering function can also be referred to as the *delay-scale affine power spectrum* analogous to the delay-Doppler power spectrum of Heisenberg group.

8 Simulation Results and Discussions

To illustrate the practicality of our method, we provide a delay-scale spreading function (deterministic channel) example and simulations. The mobile radio system considered is assumed to operate at carrier frequency of 3.5GHz with a maximum velocity of $120km/hr$. We consider the transmission of an admissible probe signal in this case a Morlet wavelet shown in Fig.11 and given by

$$\tilde{\varphi}(t) = \frac{1}{2\pi} e^{-\frac{t^2}{2}} \cos(2\pi v t) \quad (45)$$

and which we assumed lasts for about 10seconds, with $v = 0.9$. We employ a typical urban setting as the test environment with a propagation path of $L = 6$. Let the delay be given by 0, 0.2, 0.8, 1.6, 2.4, and 5.0 μs and the corresponding scales, $s_l = 1 + \frac{v_l}{c}$, $l = 0,1,2,\ldots,L-1$ where $v_{l=L-1}$ is the maximum velocity $v_{\max}$. All motions away from the transmitter are not considered. The respective s_l correspond to a delay in the order stated above with $s_{v_{\max}}$ corresponding to the delay at $5\mu s$ and $s_{v=0}$ for zero delay. We use the sampling interval $T_s = 0.002$ s for the time grid. The resultant delay-scale spreading function is shown in Fig12.

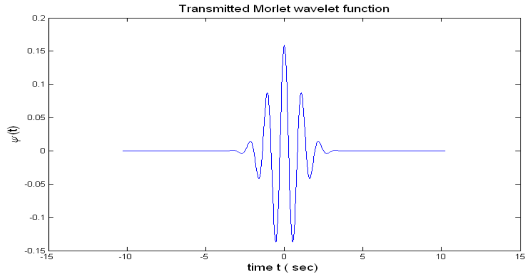

Fig.11: The transmitted Morlet wavelet

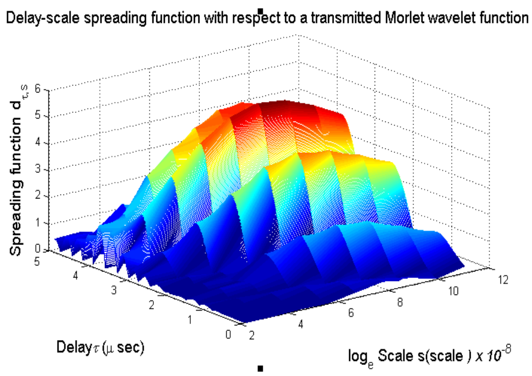

Fig.12: Delay-scale spreading function for transmitted morlet wavelet in time and frequency dispersive channel

Figure 12 clearly shows dispersion in time and scale with varying supports for the delay-scale-spreading function. The maximum delay spread $\tau_{\max}$ is about $5.0\,\mu$sec and the maximum scale spread $s_{\max}$ is about 11×10^{-8} on the natural log scale. The total spread is $\ln(\Delta^{W}_{total}) = -12.20607254$ which by applying (25) corresponds to total delay-Doppler spread of $\Delta^{F}_{total} = 1.925\times10^{-6}$ at $f_c = 3.5GHz$ .It can be seen that the scale spread is wholly dependent on the velocity of the mobile unit. A further research on deriving the associated parameters like power profile and scale (Doppler) spectrum both for the deterministic and the stochastic cases are being conducted.

9 Conclusion

We presented mobile radio channel characterization by means of compactly supported joint spreading operators. The concept of operator provides a more physical approach to channel characterization for both the deterministic and stochastic cases. The affine group representation in the wavelet domain allows for the characterization of the time-variant channel in purely time domain and provides better time and frequency resolution compared to conventional Fourier domain methods. Since scale is related to frequency, this allows for a simultaneous mapping of the delay and frequency on the time-scale plane. And because the scale spread is wholly dependent on the velocity of the mobile unit any variation in time and frequency associated with the channel can be accounted for due to the evolutional nature of the analyzing wavelet 'window' with respect to changes in scale.

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