Onsager-Casimir Antireciprocity Relations for the Hall Gyrators Analysis

DAN GEORGE TONȚ[1], LUIGE VLĂDĂREANU[2], GABRIELA TONȚ[1]
[1]Department of Electrical Engineering, Measurements and Electric Power Use,
Faculty of Electrical Engineering and Information Technology
University of Oradea
Universității st., no. 1, zip code 410087, Oradea,
ROMÂNIA,
dtont@uoradea.ro, gtont@uoradea.ro, http://www.uoradea.ro
[2]Institute of Solid Mechanics of Romanian Academy
C-tin Mille 15, Bucharest 1, 010141,
ROMANIA
luigiv@arexim.ro, http://www.acad.ro

Abstract: -. The paper focuses on the analysis of characteristics of driving point impedance for a lossy gyrator. For a gyrator, the ratio between short-circuit input resistance and open-circuit input resistance is always greater then unity. The antireciprocity of the Hall transducer is a natural phenomenon and its results from investigation of transfer resistances. Is known that for a certain value of magnetic induction, in general, transfer resistances are not equal. In other words, Hall transducer is a passive non-reciprocal circuit. In certain circumstances, however, when Hall plate does have voltage zero, (constructively, electrodes are disposed on the same equipotential surface with zero inductance) transfer resistances are equal in value, but have opposite sign. In these cases we are talking about behavior of Hall transducer as antireciprocal circuit, respectively as a gyrator. Gyrators are a class of antireciprocal resistive two-ports. The findings are concerning and the locus of the input equivalent impedance An assessment of Hall generator behavior regarded as a circuit element and particularly as a gyrator was performed by the means of experimentally validation Onsager-Casimir relation.

Key-Words: - Hall transducer, antireciprocal, gyrator, two-ports.

1 Summary

Gyrators, as they are referred to in technical applications, are in fact antireciprocal two-port networks for which the short-circuit transfer admittances, respectively the open-circuit transfer impedances have opposite signs. Therefore, in this paper, gyrators will be analyzed in the general theory of electrical two-port networks framework independently of their physical realizations.

A Hall gyrator operates resistively up to the range of MHz, whereas an operational amplifier (OA)-based gyrator behaves resistively up to kHz. Therefore, the two-port impedance parameters (*Z*) are, in fact, resistances (*R*), respectively the admittance parameters (*Y*) are conductances (*G*).

The Hall generator has the intrinsic property to be an antireciprocal quadripol, and when we refer to this property we speak of a Hall gyrator. Hall generator is an example of passive quadripolar circuit, which is nonreciprocal, more exactly antireciprocal. In figure 1 is presented a Hall generator operating in no-load regime. The command electrodes 1–1' are placed on the end bars of the plate, while the Hall electrodes 2–2' are placed laterally and are point wise. The Hall plate is flown by the command current I_1 and is placed in an exterior magnetic field with a flux density magnetic induction (B), supposed uniform and invariable in time. In these conditions between the electrodes 22' appears a potential difference, which is the Hall voltage (U_H).

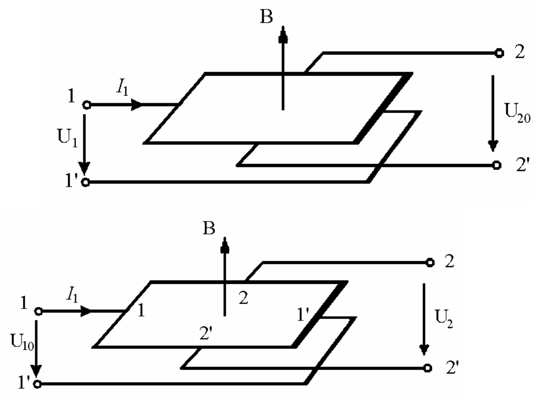


Fig. 1 Hall generator operating in no-load regime

The expression of the no-load Hall voltage can be expressed under the form:

$$U_{20} = S_0 B I_1 \quad (1)$$

where S_0 is the no-load sensitivity of the Hall generator [m^2/As] and represents the ratio factor between the Hall voltage and the product of the command magnitudes (BI_1).

Considering a Hall generator in no-load operation, supplied by terminals 11' (Fig.1a), the resistance between the terminals 11', $R_{11} = R_{10} = (U_1 / I_1)_{I2=0}$ and the no-load transfer resistance $R_{21} = (U_2 / I_1)_{I2=0}$ may be determined. If the Hall generator is in no-load operation, but is supplied through terminals 22' (Fig.1b), the no-load resistance for terminals 22', $R_{22} = (U_2 / I_2)_{I1=0}$ and the no-load transfer resistance $R_{12} = (U_1 / I_2)_{I1=0}$ may be determined.

2 Heuristic vs. Rigorous Formulations. Quadripolar Equations of a Hall generator

These parameters (R_{11}, R_{12}, R_{21}, R_{22}), determined in no-load operation are the quadripolar parameters in the Hall equations. For the determination of the Hall generator behavior toward terminals, respectively command electrodes and Hall electrodes, the application of the quadripolar circuit theory is a rational solution.

In figure 2 is shown the Hall generator in load operation. The association rule of currents and voltages is different for the two terminal pairs. The loading resistance is $R_S = -\underline{U}_2 / \underline{I}_2$.

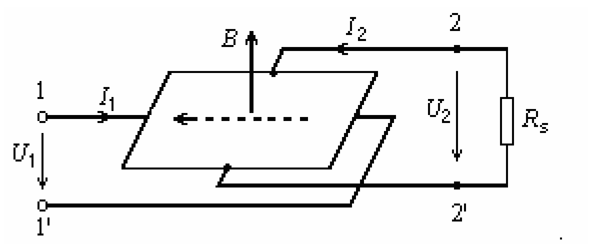


Fig. 2 Hall generator in load operation

Regading the quadripolar transfer parameters (R_{12} and R_{21}), at Hall generator interfere a particular characteristic, namely that a certain magnetic induction they are equal in value but have opposite signs, the result being mentioned by many researchers [5, 7, 9, 15]:

$R_{12} = -R_{21}$, respectively $(R_{t0})_2 = -(R_{t0})_1$ (2)

relations expressing even the antireciprocal condition. This result underpins Hall generator behavior as quadripolar antireciprocal circuitIf Hall plate has zero voltage, respectively between Hall electrodes is a voltage, and in the absence of the magnetic field in the quadripolar equations appear additional terms, with plus or minus as accordingly zero voltage is the same or opposite signs with tension Hall, which makes $|R_{12}| \neq |R_{21}|$.

Regarding signs that should be introduced into the quadripolar equations terms with resistance transfer is necessary to refer to some principled issues concerning the operation of the phenomenology task generator Hall. Thus, it notes that comparing the no load regime with operation regime (Fig. 2) the Hall circuit is flown by the current $\underline{I}_2$ and consequently we the might consider that in plate is revealed a secondary effect Hall. Specifically, the Hall plate is acting a supplementary electric field on the command circuit $C_H \overline{J}_2 \times \overline{B}$, in the oppositely with $\underline{I}_1$ (arrow with dotted line in Figure 2). As a result, in addition to obvious R_{11} equation (rel.3, rel.3') and will include $R_{12}\underline{I}_2$ with plus sign, which corresponds to this secondary effect Hall. This result is typical to Hall generator behaving as an antireciprocal quadripol.

The quadripolar equations of the Hall generator may be written as follows:

$$\begin{aligned} \underline{U}_1 &= R_{11}\underline{I}_1 + R_{12}\underline{I}_2, \\ \underline{U}_2 &= -|R_{21}|\underline{I}_1 + R_{22}\underline{I}_2 \end{aligned} \quad (3)$$

respectively

$$\begin{aligned} \underline{U}_1 &= R_{10}\underline{I}_1 + S_0 B\underline{I}_2 \\ \underline{U}_2 &= -|S_0 B|\underline{I}_1 + R_{20}\underline{I}_2 \end{aligned} \quad (3')$$

In which was taken into account that:

$$R_{12} = |R_{21}| = S_0 B$$

Informatively, we mentioned that, if the rule from the receivers were applied to the terminals 11', the rule from generators at terminals 22' and taking into account the physical significance of quadripolar parameters in the context of the reference sense association rules, the Hall generator quadripolar equations may be stated as follows:

$$\begin{aligned} \underline{U}_1 &= R_{11}\underline{I}_1 + R_{12}\underline{I}_2 \\ \underline{U}_2 &= R_{21}\underline{I}_1 + R_{22}\underline{I}_2 \end{aligned} \quad (4)$$

respectively

$$\begin{aligned} \underline{U}_1 &= R_{10}\underline{I}_1 + S_0 B\underline{I}_2 \\ \underline{U}_2 &= S_0 B\underline{I}_1 - R_{20}\underline{I}_2 \end{aligned} \quad (4')$$

the antireciprocity condition being now:

$R_{12} = R_{21}$, respectively $(R_{t0})_2 = -(R_{t0})_1$ (5)

and the load resistance is: $R_S = \underline{U}_2 / \underline{I}_2$.

Hall generator behavior in operating regime, at a certain impedance (resistance) connected to the exit terminals, is completely determined by knowing the four quadripolar parameters that intervene in equations (rel.3, rel.3'), of which only three are independent because the antireciprocity condition. These quadripolar parameters depend on semiconductor material properties, geometry of the plate, the dimensions of electrodes, but it is important to emphasize that they depend on and magnetic induction.

Therefore, if you analyze the functioning Hall generator at different values of magnetic induction, it is necessary to know and modify these parameters based on magnetic induction. To highlight the dependence quadripolar parameters with the value of magnetic induction, Hall generator equations are written under the form of:

$$U_1 = R_{11}(B)I_1 + R_{12}(B)I_2$$
$$U_2 = R_{21}(B)I_1 + R_{22}(B)I_2 \tag{6}$$

This dependency can be generally quite pronounced. In connection with dependence on the basis of magnetic induction of these quadripolar parameters can be mention the following: own resistances in no load regime ($R_{11}=R_{10}$, $R_{22}=R_{20}$) increase with magnetic induction, both because of the geometrically magneto-resistive and the physical effects and they do not depend on magnetic induction sense [6, 14].

Concerning the transfer quadripolar parameters in no-load regime, $R_{12} = (R_{t0})_2$ and $R_{21} = (R_{t0})_1$ they vary almost linearly with magnetic induction in the considered magnetic induction field and changes the sign at once with the sense of magnetic field change. Because of the dependency between magnetic induction and quadripolar parameters, the Hall voltage characteristic depending on magnetic induction (B) cannot be linear. Starting from the quadripolar equations (rel.4, rel.4'), the Hall voltage can be written under the expression

$$U_2 = \frac{S_0(B)}{1 + \frac{R_{20}(B)}{R_S}} BI_1 = S(B)BI_1 \tag{7}$$

R_S is the resistance in no-load regime, and factor S(B) is called sensitivity generator Hall in no-load regime [m^2/As]. It sees that, for voltage U_2 to vary linearly with the BI_1 is necessary that sensitivity S(B) in operating regime to be constant, independent of magnetic induction (B). This is not possible, not only because even the sensitivity in no-load regime, $S_0(B)$, depends on magnetic induction (B), but also because its own resistance R_{20} which occurs in the relation (rel.7) is depending on magnetic induction (B) .

In this context, we note that at the Hall generator, the resistance in operating regime R_S can adapt the following operating conditions depending on the application considered: a - linearity; b – full power c - maximum yield.

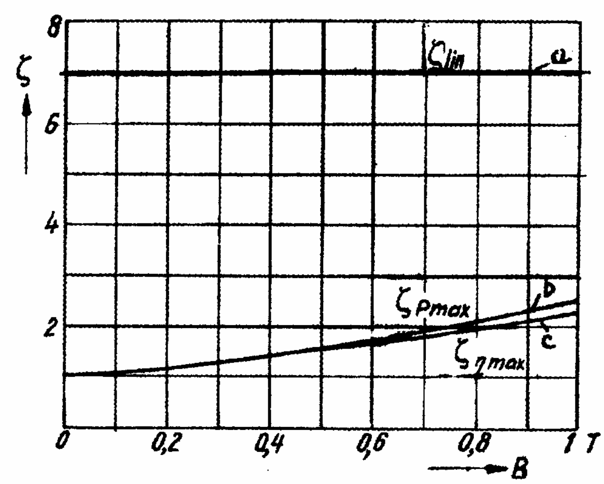


Fig. 3 Hall generator adaptation charactersitics corresponding to the conditions of: linearity (a) full (b), and maximum yield (c)

For example, if ζ is the results of division of resistance in no load regime and Hall plate resistance in no load regime R_{20} and represents the relative resistance depending on the magnetic induction, for a plate of InAs are obtain the curves in figure 3. From these curves can see that in the field of magnetic induction considered, to adapt to the condition of linearity does not depend on practical magnetic induction (B), in turn adaptation to the conditions for maximum power and maximum yield depends on magnetic induction.

Regarding the dependency of quadripolar parameters and the default dependence Hall voltage characteristic with magnetic induction are referring in general to generator Hall and had referred them to understand how to put this issue in the case Hall gyrator. In this case, this problem is easier that the generator operating as gyrator Hall, the magnetic induction (B) is usually constant in time. It is obvious (eq.7) that the magnetic induction and sensitivity Hall generator under constant load, the Hall voltage varies with I_1, obviously linear variation. Accordingly, it may be considered with sufficient accuracy Hall gyrator that works with a constant magnetic induction (B) is a liniary quadripolar element.

3 Expressions of equivalent input impedances (admitances)

The quadripol equivalent input impedance expression feed on the terminals 11', depending on the impedance quadripolar parameters is [9]:

$$\underline{Z}_{e1} = \frac{\underline{U}_1}{\underline{I}_1} = \underline{Z}_{11} + \frac{\underline{Z}_{12}\,\underline{Z}_{21}}{\underline{Z}_S - \underline{Z}_{22}} = \underline{Z}_{10} + \frac{\underline{Z}_{12}\,\underline{Z}_{21}}{\underline{Z}_S + \underline{Z}_{20}} \tag{8}$$

in which $\underline{Z}_S$ the load impedance

This expression implies the application of the association rule of reference senses from receivers to input terminals 11' and the rule of generators at the output terminals 22'. Input equivalent admittance depending on the admittance quadripolar parameters generates an analogous expression:

$$\underline{Y}_{e1} = \underline{Y}_{11} + \frac{\underline{Y}_{12}\,\underline{Y}_{21}}{\underline{Y}_S - \underline{Y}_{22}} = \underline{Y}_{1k} + \frac{\underline{Y}_{12}\,\underline{Y}_{21}}{\underline{Y}_S + \underline{Y}_{2k}} \tag{9}$$

in which $\underline{Y}_S = 1/\underline{Z}_S$.

If the rule from the receivers applies to both pairs of terminals, equivalent impedance and admittance expressions are [7]:

$$\underline{Z}_{e1} = \underline{Z}_{11} - \frac{\underline{Z}_{12}\,\underline{Z}_{21}}{\underline{Z}_S + \underline{Z}_{22}} = \underline{Z}_{10} - \frac{\underline{Z}_{12}\,\underline{Z}_{21}}{\underline{Z}_S + \underline{Z}_{20}} \tag{10}$$

and

$$\underline{Y}_{e1} = \underline{Y}_{11} - \frac{\underline{Y}_{12}\,\underline{Y}_{21}}{\underline{Y}_S + \underline{Y}_{22}} = \underline{Y}_{1k} - \frac{\underline{Y}_{12}\,\underline{Y}_{21}}{\underline{Y}_S + \underline{Y}_{2k}} \tag{11}$$

It notes that the two rules of association of the reference senses, input equivalent impedances (admittances) expressions differ by signs that interfere terms in these expressions. If you take into account the physics meanings of quadripolar parameters, in accordance with the association rule considered, is obvious that must result the same expression, i.e.:

$$\underline{Z}_{e1} = \underline{Z}_{10} - \frac{\left(\underline{Z}_{t0}\right)_1 \left(\underline{Z}_{t0}\right)_2}{\underline{Z}_S + \underline{Z}_{20}} \tag{12}$$

$$\underline{Y}_{e1} = \underline{Y}_{1k} - \frac{\left(\underline{Y}_{tk}\right)_1 \left(\underline{Y}_{tk}\right)_2}{\underline{Y}_S + \underline{Y}_{2k}} \tag{13}$$

Taking into account the resistive nature of antireciprocal quadripol studied in the paper, input equivalent impedance and admittance expressions may be written as follows:

$$\underline{Z}_{e1} = R_{10} - \frac{R_{12}\,R_{21}}{\underline{Z}_S + R_{20}} \tag{14}$$

and

$$\underline{Y}_{e1} = G_{1k} - \frac{G_{12}G_{21}}{\underline{Y}_S + G_{2k}} \tag{14'}$$

if the rules from the receivers are applied at both terminals and expressions (15), (15'):

$$\underline{Z}_{e1} = R_{10} + \frac{R_{12}\,R_{21}}{\underline{Z}_S + R_{20}} \tag{15}$$

and

$$\underline{Y}_{e1} = G_{1k} + \frac{G_{12}G_{21}}{\underline{Y}_S + G_{2k}} \tag{15'}$$

If is applied the association rule of reference senses from receivers to input terminals 11' and the rule of generators at the output terminals 22'. Also, we will take into account the antireciprocity condition expression of within each association rules of reference senses. In one case (rel.14) the antireciprocity is highlighted if we take into account $R_{12} \cdot R_{21} < 0$, respectively $G_{12}G_{21} < 0$, and in other case (rel.15) we take into account $R_{12} \cdot R_{21} > 0$, respectively $G_{12}G_{21} > 0$. It notes that, instead of relations (14) and (15), can write the equations:

$$\underline{Z}_{e1} = R_{10} + \frac{\left|R_{12}R_{21}\right|}{\underline{Z}_S + R_{20}} \tag{16}$$

and

$$\underline{Y}_{e1} = G_{1k} + \frac{\left|G_{12}G_{21}\right|}{\underline{Y}_S + G_{2k}} \tag{17}$$

which are independents from the association rule of reference senses that apply. In these relations is observed that occurs, in the second term, the module product parameters transfer quadripolar.

By a similar reasoning for reciprocal and resistive quadripols and taking into account, the appropriate reciprocity conditions we obtain the following equations:

$$\underline{Z}_{e1} = R_{10} - \frac{\left|R_{12}R_{21}\right|}{\underline{Z}_S + R_{20}} \tag{18}$$

and

$$\underline{Y}_{e1} = G_{1k} - \frac{\left|G_{12}G_{21}\right|}{\underline{Y}_S + G_{2k}} \tag{18'}$$

which also are usually independent of the association rule of reference senses for voltages and currents at terminals.

The advantage of writing impedance input equivalent equation into this form (relations 16 18), as an analyzed quadripol is antireciprocal respectively reciprocal, is that enables a fast and simple comparison between the two types of quadripols. It can seen that the difference between quadripols appears only by the sign that interferes in the module product of the transfer parameters.

The input equivalent impedance expression of a antireciprocal quadripol emphasizes clearly the property of reversing the impedances. In the case of gyrator lossless ($R_{10}=R_{20}=0$), in relation (16) by customization is obtaining the expression:

$$\underline{Z}_{e1} = \frac{|R_{12}R_{21}|}{\underline{Z}_S}$$

In this particular case may be added that $\underline{Z}_{e1} = \underline{Z}_{e2}$.

Taking into account the equations 16 and 17 and that $\underline{Z}_{e1} = 1/\underline{Y}_{e1}$ it results the form (19):

$$\frac{R_{11}\left(\underline{Z}_S + R_{20}\right)+|R_{12}R_{21}|}{R_{20}+\underline{Z}_S} = \frac{G_{2k}+\underline{Y}_S}{G_{11}\left(G_{2k}+\underline{Y}_S\right)++|G_{12}G_{21}|} \tag{19}$$

which reveals the relation between $|R_{12}\ R_{21}|$ and $|G_{12}\ G_{21}|$.

It can see that in case of a lossy gyrator this relation is relatively complicated. The association is noticeably simplified in the case of an ideal gyrator, in which case it becomes $R_{12}R_{21} = 1/G_{12}G_{21}$.

4 Onsager-Casimir Relations. Antireciprocity analysis of the Hall generator

The antireciprocity of Hall generator is explained by decomposing in two parts the no-load transfer resistances R_a and R_b. In one transfer resistances appears the sum of these components, while in the other appears their difference [9]:

$$\begin{aligned} R_{12}(B) &= R_a + R_b \\ R_{21}(B) &= R_a - R_b \end{aligned} \tag{20}$$

In the case when the Hall plate has zero voltage R_a corresponds to the zero voltage due to the command current I_1, and is independent of the magnetic field sense, while R_b is corresponding only to that Hall effect which depends on the magnetic field sense. This decomposition is only theoretical.

If we don't have zero voltage, respectively R_a is zero and only R_b remains in (20), it results the antireciprocity condition $R_{12} = -R_{21}$. If zero voltage exists and $R_a<R_b$, the behavior of the Hall generator is of a gyrator, because $R_{12}R_{21}<0$.

To demonstrate the possibility of decomposition in the two components of transfer resistance (rel.20) we start with electrical conduction law in the presence of galvano-magnetical effects.

$$\frac{\overline{J}}{\sigma} = \overline{E} + C_H \overline{J} \times \overline{B} \tag{21}$$

Considering two electro-kinetical states set in Hall plate at a certain value of magnetic induction, characterized in a point by the magnitudes $\overline{E^{/}}, \overline{J^{/}}$ and $\overline{E^{//}}, \overline{J^{//}}$ we obtain the expressions:

$$\begin{aligned} &\overline{E^{/}}\ \overline{J^{//}} = \frac{\overline{J^{/}}\ \overline{J^{//}}}{\sigma} - C_H(\overline{J^{/}} \times \overline{B})\ \overline{J^{//}} \\ &\overline{E^{//}}\ \overline{J^{/}} = \frac{\overline{J^{//}}\ \overline{J^{/}}}{\sigma} - C_H(\overline{J^{//}} \times \overline{B})\overline{J^{/}} = \\ &= \frac{\overline{J^{//}}\ \overline{J^{/}}}{\sigma} + C_H(\overline{J^{/}} \times \overline{B})\overline{J^{//}} \end{aligned} \tag{22}$$

of which, by their comparison it results the failure of reciprocity theorem local form because $\overline{E^{/}}\ \overline{J^{//}} \neq \overline{E^{//}}\ \overline{J^{/}}$.

Integrating relations (22) for the entire plate volume it results:

$$\begin{aligned} &\int_{V_\Sigma} \overline{E^{/}}\ \overline{J^{//}} dv = \int_{V_\Sigma} \frac{\overline{J^{/}}\ \overline{J^{//}}}{\sigma} dv - \int_{V_\Sigma} C_H(\overline{J^{/}} \times \overline{B})\overline{J^{//}} dv = \eta_a - \eta_b \\ &\int_{V_\Sigma} \overline{E^{//}}\ \overline{J^{/}} dv = \int_{V_\Sigma} \frac{\overline{J^{/}}\ \overline{J^{//}}}{\sigma} dv + \int_{V_\Sigma} C_H(\overline{J^{/}} \times \overline{B})\overline{J^{//}} dv = \eta_a + \eta_b \end{aligned} \tag{23}$$

in which has been made the replacement:

$$\eta_a = \int_{V_\Sigma} \frac{\overline{J^{/}}\ \overline{J^{//}}}{\sigma} dv \ ; \quad \eta_b = \int_{V_\Sigma} C_H(\overline{J^{/}} \times \overline{B})\overline{J^{//}} dv \tag{23'}$$

Assuming the plate electric field is derived from a scalar potential, can be written:

$\overline{E^{/}} = -\nabla V^{/}$ and $\overline{E^{//}} = -\nabla V^{//}$

so that relations (23) become:

$$-\int_{v_\Sigma} \overline{J^{//}}\left(\nabla V^{/}\right) dv = \int_{v_\Sigma} \nabla\left(V^{/}\overline{J^{//}}\right) dv = \eta_a - \eta_b \tag{24}$$

$$-\int_{v_\Sigma} \overline{J^{//}}\left(\nabla V^{/}\right) dv = \int_{v_\Sigma} \nabla\left(V^{//}\overline{J^{/}}\right) dv = \eta_a + \eta_b \tag{24'}$$

in which has been taken account of the vectorial identity $\nabla\left(V\overline{J}\right)=\overline{J}\left(\nabla V\right)+V\left(\nabla\overline{J}\right)$ and that the considered regime is: $\left(\nabla\overline{J}\right)=0$.

Applying the integrals transformation of Gauss-Ostrogradski and noting that the electric current intensity is null in all lateral areas uncovered by electrodes, and contact surfaces of electrodes are equi-potential, relations (24) become:

$$-\int_{\Sigma} V^{\prime}(B)\overline{J^{\prime\prime}}\,\overline{ds}=\eta_a-\eta_b \tag{25}$$

$$-\int_{\Sigma} V^{\prime\prime}(B)\overline{J^{\prime}}\,\overline{ds}=\eta_a+\eta_b \tag{25'}$$

of which, by dividing with $I_1^{\prime}I_2^{\prime\prime}$ we obtain the transfer resistance to a certain magnetic induction and certain sense:

$$R_{12}(B)=\frac{U_1^{\prime\prime}}{I_2^{\prime\prime}}=\frac{V_1^{\prime\prime}-V_{1'}^{\prime\prime}}{I_2^{\prime\prime}}=\frac{\eta_a+\eta_b}{I_1^{\prime}\cdot I_2^{\prime\prime}}=R_a(B)+R_b(B) \tag{26}$$

$$R_{21}(B)=\frac{U_2^{\prime}}{I_1^{\prime}}=\frac{V_2^{\prime}(B)-V_{2'}^{\prime}(B)}{I_1^{\prime}}=\frac{\eta_a-\eta_b}{I_1^{\prime}\cdot I_2^{\prime\prime}}= \tag{27}$$
$$=R_a(B)-R_b(B)$$

that is just the decomposition of the relation (20). If the two resistance transfer are known, out of relations (26) it result the R_a and R_b components: expressions

It specifies that the decomposition of resistance transfer in the two components is a result that may apply whatever form plate Hall. Also, it shows that decomposition can be done and when the magnetic field is not transversal [2].

It should be noted that analogously may be decomposed into two parts and transfer conductances in short of the Hall generator. If we reverse the magnetic field sense, we obtain expressions:

$$R_{12}(-B)=R_a(-B)+R_b(-B)=R_a(B)-R_b(B)$$
$$R_{21}(-B)=R_a(-B)-R_b(-B)=R_a(B)+R_b(B) \tag{28}$$

has been kept in mind that the R_b changes sign with the changing of the magnetic field sense: $R_b(-B)=-R_b(B)$.

Comparing relations 14 and 16 it shows that:

$$R_{12}(B)=R_{21}(-B),\quad R_{21}(B)=R_{12}(-B) \tag{29}$$

which can be written in the general form:

$$R_{jk}(B)=R_{kj}(-B) \tag{30}$$

This is known relationship of Onsager-Casimir [32]. The relations applies regardless of whether Hall generator has or not zero voltage, so are relations with general validity.

Therefore, it can be said that if a Hall generator transfer resistance is determined to a certain magnetic induction sense, and other resistance transfer Hall is determined to same induction value, but opposite sign transfer resistances fulfill the reciprocity condition expressed by the Onsager-Casimir relation has been verified with great precision experimentally having relative error range 10^{-4}-10^{-5}, [33]. One consequence of Casimir's Onsager relation refers to the input equivalent impedance invariance of Hall generator depending on the sense of the magnetic field. The expression of the impedances equivalent (rel.34), for a certain sense of magnetic induction is:

$$\underline{Z_{e1}}(B)=R_{10}(B)-\frac{R_{12}(B)R_{21}(B)}{\underline{Z_S}+R_{20}(B)} \tag{31}$$

and if the only sense of magnetic induction is changing we obtain expression:

$$\underline{Z_{e1}}(-B)=R_{10}(-B)-\frac{R_{12}(-B)R_{21}(-B)}{\underline{Z_S}+R_{20}(-B)} \tag{32}$$

The own resistances in no-load regime, R_{10} and R_{20}, are invariant to the changing of magnetic induction sense. Concerning the resistance transfer $R_{12}R_{21}$ is invariant, because, according to Onsager-Casimir relation $R_{12}(B)=R_{21}(-B)$ and $R_{21}(B)=R_{12}(-B)$, thus:

$$R_{12}(B)R_{21}(B)=R_{12}(-B)R_{21}(-B) \tag{33}$$

Consequently, and equivalent input impedances of the Hall generator are equal to a certain magnetic induction and opposite senses, ie

$$\underline{Z_{e1}}(B)=\underline{Z_{e1}}(-B) \tag{34}$$

5. About Hall generator (gyrator) yield

Taking into account the resistive character of the generator Hall, the maximum yield condition is the load resistance R_S to be equal to the impedance (resistance) Z_{2c} characteristic specifically, [7, 35]:

$$R_S=Z_{2c}=\sqrt{\frac{R_{20}}{R_{10}}}\sqrt{R_{10}R_{20}+\left|R_{12}R_{21}\right|}=R_{20}\sqrt{1+k} \tag{35}$$

in which has been kept in mind that a gyrator fullfil the condition $R_{12}\,R_{21}<0$.

In these situation, on the side of feeding source, the equivalent impedance input is equal to the impedance (resistance) characteristic Z_{1c}, whose expression is:

$$Z_{1c} = \sqrt{\frac{R_{10}}{R_{20}}}\sqrt{R_{10}R_{20} + |R_{12}R_{21}|} = R_{10}\sqrt{1+k} \qquad (36)$$

As to the transfer characteristic constants g_{1c} and g_{2c}, they will be equal only constants mitigation A_{1C} and A_{2c}, as the phase constants are null.

If we introduce as parameter variable the ratio (ξ) between the receptor load resistance R_S and no-load resistance R_{20} od Hall plate and taking into account the relationship of 34 the maximum yield is expressed as:

$$\xi = \frac{R_S}{R_{20}} = \sqrt{1+k} \qquad (37)$$

In these expressions can see that is explicitly appear the global parameter k, which has the expression:

$$k = \frac{|R_{12}R_{21}|}{R_{10}R_{20}} = \frac{|G_{12}G_{21}|}{G_{11}G_{22}} = \left(\frac{U_2}{U_1}\right)_{I_2=0} \cdot \left(\frac{U_1}{U_2}\right)_{I_1=0} \qquad (38)$$

: The appropriate constant transfer g_{1c} is:

$$g_{1c} = \frac{1}{2}\ln\frac{U_1 I_1}{U_2 I_2} = \ln\frac{\sqrt{R_{10}R_{20}} + \sqrt{R_{10}R_{20} + |R_{12}R_{21}|}}{R_{21}} \qquad (39)$$

In which is taken into account that:

$$e^{g_{1c}} = \sqrt{\frac{U_1\ I_1}{U_2\ I_2}}. \qquad (40)$$

The expression of Hall generator maximum yield η_M is so:

$$\eta_M = \frac{U_2 I_2}{U_1 I_1} = \frac{1}{(e^{g_{1c}})^2} =$$

$$= \frac{R_{21}^2}{\left[\sqrt{R_{10}R_{20}} + \sqrt{R_{10}R_{20} + |R_{12}R_{21}|}\right]^2} = \qquad (41)$$

$$= \frac{(S_0 B)^2}{\left[\sqrt{R_{10}R_{20}} + \sqrt{R_{10}R_{20} + (S_0 B)^2}\right]^2} =$$

If you take into account k expression (rel.38), the maximum yield (rel.41) becomes:

$$\eta_M = \frac{\sqrt{k+1} - 1}{\sqrt{k+1} + 1} \qquad (42)$$

Decomposing in series the term $(k+1)^{1/2}$ from this expression and neglecting the superior terms, which at the Hall generator means that we need to limit at magnetic induction relatively small, we obtain the following approximate expression, the maximum efficiency:

$$\eta_M \approx \frac{1}{4}k = \frac{1}{4}\frac{(S_0 B)^2}{R_{20}R_{10}} \qquad (43)$$

It is known that the yield Hall generator is very small, we can overcome the theoretical limit value of 0.172. This means that no mitigation constant of Hall generator cannot be less than 7.6 dB.

In connection with the possibility to increase the efficiency Hall generator is very interesting solution developed by S. Grützmann developed in his doctoral work [30]. Thus, Hall plates with more command circuit and more circuits Hall, separated galvanic outside of the plate (fig.4). Hall plate is generally provided me with pairs of electrodes command and n pairs of electrodes Hall, which meet all the many circuits and custom circuit Hall. For example, square plate in Figure 4 is provided with four command circuits and four Hall circuits. Considering the limit case $\theta \to \pi/2$ for each command current is obtained reported to a pair of electrodes a ratio of tensions in no load equal with 1. Hall plate having *m* command circuits and *n* circuits Hall, whose effects stood for maximum yield we obtaine in the expression general case [30]:

$$(\eta_M)_{\theta\to\pi/2} = \frac{\sqrt{m.n+1} - 1}{\sqrt{m.n+1} + 1} \qquad (44)$$

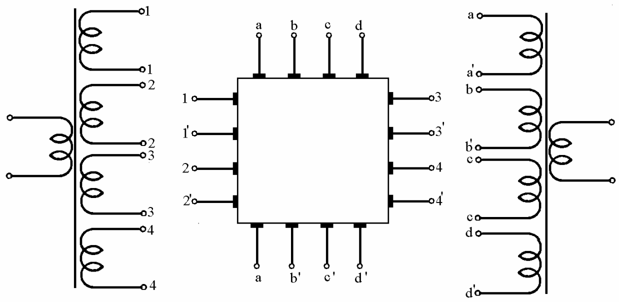


Fig.4 Placa Hall cu mai multe circuite de c*-da şi circuite Hall

For example, for the plate in fig.4 in which *m = n = 4 = 0.6* is obtained $\eta_M = 0.6$ and this yield increase with *m* and *n*. At the lower limit, when *m = n = 1,* to obtain value, determined prior to the usual construction of a Hall plate. The major disadvantage of this solution is the difficulty of achieving such plates Hall. In literature are mentioned and other solutions to the problem of increasing efficiency Hall [30].

6 Experimental results

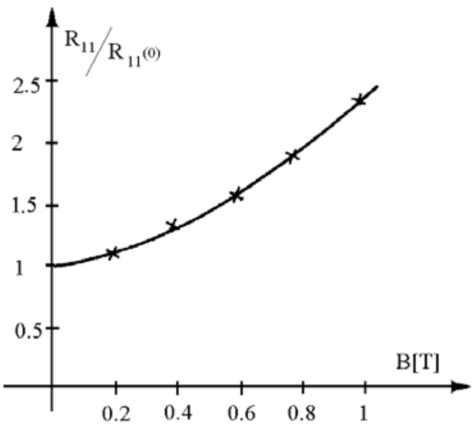
In case of a Hall plate made of InSb with practically point form electrodes, were determined the quadripolar resistances in no-load operation. The given results are presented in table 1 and table 2.

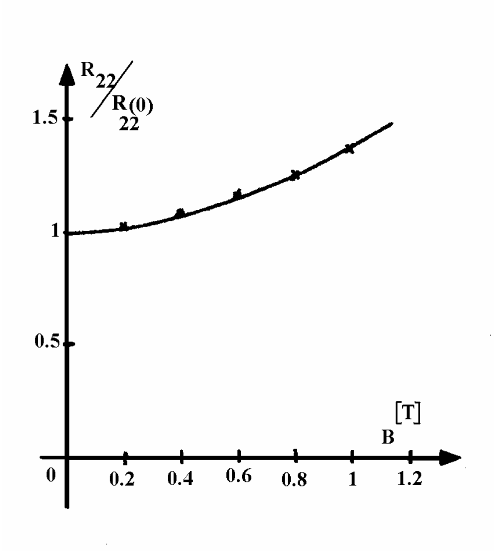
Table 1

B [T]	$R_{11}(+B)$ [Ω]	$R_{12}(+B)$ [Ω]	$R_{21}(+B)$ [Ω]	$R_{22}(+B)$ [Ω]
0	0.39	0	0	1.16
0.2	0.42	0.09	-0.09	1.19
0.4	0.51	0.17	-0.18	1.25
0.6	0.61	0.26	-0.26	1.38
0.8	0.73	0.36	-0.36	1.46
1	0.83	0.45	-0.45	1.60

Table 2

B [T]	$R_{11}(-B)$ [Ω]	$R_{12}(-B)$ [Ω]	$R_{21}(-B)$ [Ω]	$R_{22}(-B)$ [Ω]
0	0.39	0	0	1.16
0.2	0.41	-0.09	0.09	1.20
0.4	0.50	-0.18	0.17	1.26
0.6	0.60	-0.26	0.26	1.37
0.8	0.73	-0.36	0.36	1.46
1	0.83	-0.45	0.45	1.60





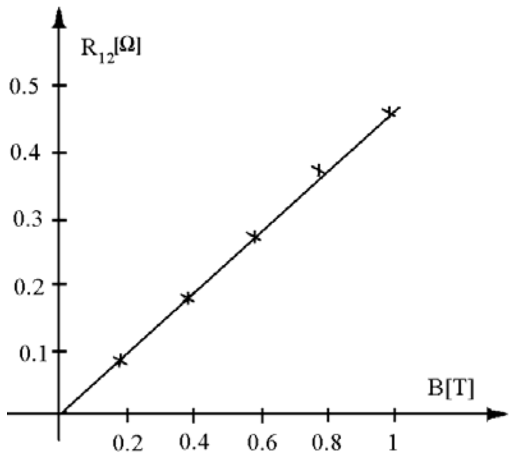


Fig. 5 Variation of parameters $R_{11}/R_{11}(0)$, $R_{22}/R_{22}(0)$ and R_{12} depending on magnetic induction

In fig. 5 are shown diagrams of variation of the parameters $R_{11}/R_{11}(0)$, $R_{22}/R_{22}(0)$ and R_{12}, as functions of the magnetic flux density

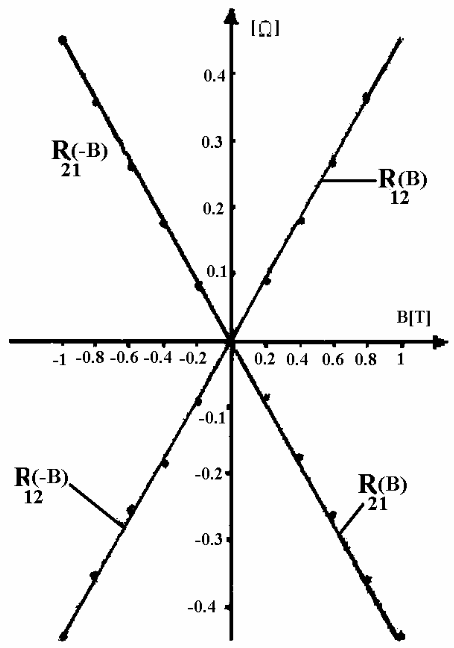


Fig.6 Variation in no load regime for transfer resistances depending on magnetic induction for a plate INSb

Conclusion

Based on the previous considerations, was verified the relation of Onsager-Casimir experimentally. This assessment is important for the behavior of the Hall generator regarded as a circuit element, and particularly as a gyrator.

From the tables 1 and 2, at zero fields we have null transfer resistances, verifying the absence of zero voltage of the Hall plate. For other values of the magnetic field, we observed that the absolute values of the transfer resistances are identical.

The investigation performed has made possible a study of circumstances in which the Hall plate is a nonreciprocal, antireciprocal or reciprocal circuit.

A common feature of all gyrator realization is that in their frequency operating range they are resistive two-ports, even if those ranges can be very different from one type to another. For example, a Hall gyrator operates resistively up to MHz, whereas a OA-based gyrator behaves resistively up to KHz. Therefore, the two-port impedance parameters (Z) are, in fact, resistances (R), respectively the admittance parameters (Y) are conductance's (G).

The general conditions for a two-port network to be an antireciprocal two-port, that is a gyrator, are stated as follows:

$$R_{12}R_{21}<0, \text{ respectively } G_{12}G_{21}<0 \tag{45}$$

where: R_{12}, R_{21} are the open circuit transfer resistances and G_{12}, G_{21} are the short-circuit transfer conductances.

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