Study on effects of nonholonomic constraints on dynamics of a new developed quadruped leg-wheeled passive mobile robot

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Abstract: - A new passive wheel type leg-wheeled hybrid mobile robot based on surface motion principle was introduced. To produce the propulsion force, a passive wheel was installed at the end of the parallel mechanism structured leg connecting with the frame-body to make the wheel vertical to the ground at any time. With the inertia framework, the robot framework and some assumptions, two forms of Maggie Equation to model the nonholonomic constraint systems were derived from the Lagrangean Equation. To determine the effect of nonholonomic constraints on dynamics of the robot, the matrix method was used to calculate the Lagrangean multipliers together with the Routh Equation. Upon an Atmega8 MCU-based logic control system, the straight-line skating experiments and the turning experiments were conducted with the prototype machine and effects of nonholonomic constraints were analyzed. Last, some conclusions were drawn.

Key-Words: - QLWIS robot, nonholonomic constraint, dynamic analysis, Maggie equation

1 Introduction

As there exist different applications and terrains, robot technologies developed extensively and intens -ively and many legged, wheeled, tracked and articulated mobile robots had been designed around the world during the past years. Comparatively, the legged robots could accommodate all terrains but hard to control, and the wheeled robots were easy to control on some smooth floors or grounds with very limited terrain adaptive abilities, and the articulated mobile robots, i.e., the snake-like robots were easy to maintain but hard to control and the tracked robots were capable of carrying large loads with certain terrain adaptabilities in some papers.

Due to motion terrains, these mobile robots could not be lightweight, simple, easy to operate, stable, reliable and maintainable if only legs, wheels, tracks or articulated segments were used. To get good terrain adaptability, such hybrid mobile robots as leg-wheeled mobile robots mainly were designed in Japan, Germany and other countries for plenty of applications. According to the driving mode of motors, these leg-wheeled mobile robots could be classed into two types.

The first was the passive driving type. There are no driving DC motor, braking, steering and additional mechanisms installed at the ends of legs, i.e. Roller-walker [1-6], Rollerblader [7-9] and Skateboarding Robot [10] to get larger friction force under normal circumstances. The second was the active driving type, and DC motors drive wheels installed at end of legs directly with mechanical braking, steering and other mechanisms. Some active driving leg-wheeled mobile robots, i.e. ALDURO [11-13], CharoitII [14], Walk'n Roll [15], Workpartner [16,17], Biped type leg-wheeled robot [18,19], WS-2 [20], combined wheel-leg vehicle [21], the Mars Exploration Rovers, the Spirit Rovers and the Opportunity Rovers from NASA [22] were widely applied in mine areas, countryside farm, civil engineering, logging sites and star explorations etc [23]. They could be named leg-wheeled passive mobile robot and leg-wheeled active mobile robot respectively according to driving mode of motors installed at ends of legs. Though there were driven by friction forces between wheels and the ground in common.

In Hirose, Endo and Takeuchi [1-6], the structure, motion optimum method were discussed

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in details, in Chitta, Heger and Kumar [7-9], the nonholonomic dynamics modelling method were dealt with, in Müller J, Schneider and Hiller [11-13], the structure and motion control method were focused on, while in others [14-23], the locomotion and gait control problems especially for the active driving leg-wheeled mobile robots were concerned. However, few of them deal with dynamic analysis and effects of nonholonomic constraints on their dynamics up to now. Thus, we aim to discuss dynamic analysis and effects of nonholonomic constraints of this new leg-wheeled passive mobile robot mainly in this paper.

2 Principle and structure of QLWIS robot

Based on surface motion principle and the fact that the sliding friction force was greater than the rolling friction force generally, this robot was developed.

2.1. surface motion principle

It was known to all that when the wheels on the robot are in the surface contact condition, no matter their driving type. For active leg-wheeled or wheeled mobile robot, the wheels were the contact media between the robot and motion surface, the resultant force was the torque differences of the sliding friction forces and the rolling friction forces, which were in the same motion direction. But for the leg-wheeled passive mobile robot, the factor that the sliding friction force in the normal direction of the rolling wheel was greater than the rolling friction force in the tangent direction under normal circumstances must be taken into account as shown in Fig.1 when one wheel moves in surface contact condition.

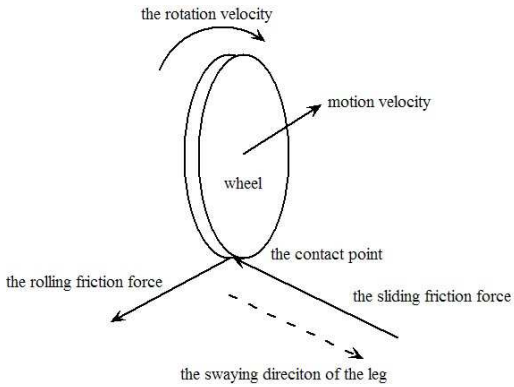


Fig.1. the wheel in surface contact condition

Unlike leg-wheeled active mobile robots, the sliding friction force only exists when the leg swayed within the outer and the inner limited ranges. If there were four or six legs installed and the related two legs sway symmetrically and simultaneo -usly, each component force f_i (i the index number corresponded to the wheel) of the i^{th} wheel could be combined into the total driving force f, as illustrated in Fig.2 (i.e. four legs).

Namely, the driving force could be written in the following form:

$$f = \sum f_i = \sum \left(f_{ni} + f_{ti} \right) \quad f_{ni} \gg f_{ti} \qquad (1)$$

When the force f superimposes with or parallel to the motion direction of the robot, it then became the driving force. Related with the legs' swaying directions, the force f might drag the robot.

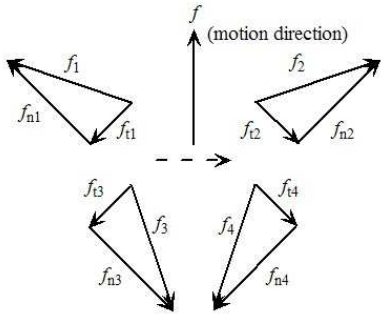


Fig.2. the component force of four wheels

What needed to point out was the lateral force in dot line shown in Fig.2 had been cancelled because the rolling friction forces corresponding to four wheels are symmetrical, the component force of the rolling friction forces could be neglected when the robot moved in straight line. It could be seen that the robot was based on surface motion principle. Because it can move like ice-skaters, it named Quad leg-wheeled Ice-skater Robot (abbr. QLWIS robot) accordingly.

2.2. structure of QLWIS robot

Based on the surface motion principle in Fig1 and Fig.2, some problems must be taken into account when the QLWIS robot was designed. The first was generation of the sliding friction force. Because the rolling friction forces were produced automatically when the robot moved, its generation mechanism can be ignored, as the installed wheel on the leg could be used as the rolling friction force generating mechanism in theory. The second was generation of its motion direction. According to the Newton Law, the motion direction must be defined to control motion of the robot. The third was generation of the resultant friction force. The last was the equilibrium control problem.

In normal circumstances, the rotation mechanism could be used as the sliding friction force generating device and the motion direction restriction device. To get better mobility, controllability and omnidirec -tional ability, the limited rotation leg mechanism

and the 360° rotation mechanism might be used to provide reliable and simple answers for the first and the second problems respectively. As shown in Eq.1, the resultant friction force was the vector sum of the sliding friction forces and the rolling friction forces; it must be generated with the coordinated control of these two devices. To get the natural equilibrium ability, four legs could be utilized in this robot, it was balanced in nature, and the last problem can be neglected herein.

When the limited rotation DOF within the limited ranges was used as the sliding friction force generati -on mechanism and the 360° rotation DOF as the motion direction restriction mechanism, and the parallel mechanism as legs to make wheels be vertical to motion surface at any time, four passive wheels installed at the ends of four legs as rolling unit, the mechanical leg was shown in Fig.3.

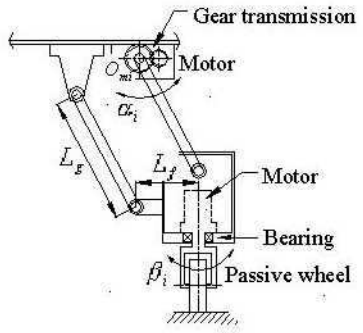


Fig.3. the leg structure

As a mobile platform, the frame-body must be designed. If the upper end of the leg driven by a motor with a 1:3 gear transmission connected with the body and a motor adjusting the orientation angle of the wheel was mounted at the other end, the quadruped prototype of QLWIS robot were shown in Fig4. Meanwhile, Some limited jigging switches are installed to detect the outer and the inner limited positions of legs, but the orientation motors feed back with potentiometers to determine orientation angles of wheels were not shown in it.

Fig.4. the prototype of QLWIS robot

Its main parameters were shown in Tab.1.To simplify the control system, the two front legs of the QLWIS robot were resting in their inner limited positions, and two rear legs and wheels moved between their inner and outer limited positions synchronously to produce the driving force, the robot would skate along the curve or in the straight-line defined by two front wheels. If the wheels and the legs moved simultaneously, it can be called "simultaneous mode gait". When legs moved after wheels adjusted the orientation angles and wheels moved after legs adjusted postures in different times, it would be named "independent mode gait" according to the motion sequence of legs and wheels. These two gaits conformed to the surface motion principle well.

Tab.1. Some main parameters of QLWIS robot

Parameter	Value
Leg length	$L_g = 20\ cm$
Wheel radius	$r_w = 4\ cm$
Rear-leg install angle	$\phi = \pm 45°$
Frame-body	$21cm \times 21cm$
Leg drive motors max. speed	$\dot{\alpha} = 0.25\pi\ r/s$
Wheel orientation motors max. speed	$\dot{\beta} = 2\pi\ r/s$
Max. outer angle of legs	$\alpha_{\max} = 30°$
Max. inner angle of legs	$\alpha_{\min} = -30°$
Max. outer angle of wheels	$\beta_{\max} = 45°$
Max. inner angle of wheels	$\beta_{\min} = -45°$
Offset width	$L_f = 8\ cm$
Total weight	$m \approx 20\ kg$

There were some characteristics of the QLWIS robot comparing to other wheeled and legged mobile robots. First, the robot was hybrid and of leg-wheel fusion type. The wheels and the legs must be installed simultaneously, where the wheels installed at the ends of legs were the rolling units contacting with the motion surface. Second, the motors could not produce the driving torque directly. It was passive mobile robot. Third, the legs must be driven to produce the sliding friction force because it was the main source of the driving friction force. Lastly, the wheels must be orientated to produce the driving friction force and the motion direction, the robot could not be driving if wheels were in "incorrect" or "wrong" orientations.

3 Dynamic modeling of QLWIS robot

As a mobile robot, the QLWIS robot had its unique kinematics and dynamic characteristics due to its passive driving wheels installed at the ends of legs and coordinated motion of legs and wheels. To

discuss its dynamics, some related assumptions must be made and some frame-works constructed.

To simplify the modeling procedure, it can be supposed that 1) the robot moved on horizontal surface, 2) the robot, including the plastic tire of wheels, is rigid and remained its original shape and dimension, 3) there was no slippage in normal directions of wheels and 4) wheels were in pure rolling condition in its tangential direction.

3.1. two relative coordinate frameworks

Similar to other wheeled robot, it was difficult to describe its dynamics because the position and the velocity of the robot must be defined in the inertial framework, while positions and postures of four wheels and legs be defined in the framework attached to the robot. Thus, the two coordinate frameworks, the inertia framework and the robot framework included, must be setup to illustrate postures of wheels and the relationship between postures (including positions and orientation angles) of four wheels and the velocity of the robot in Fig.5.

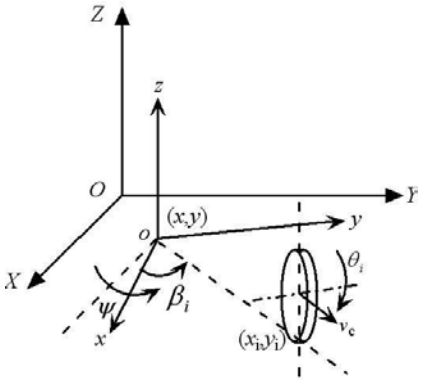


Fig.5 two kinematic coordinate frameworks

Two relative frameworks were

the inertia framework { OXYZ }: Y rightward horizontally, Z upward vertically, $Y = Z \times X$,

the robot framework { oxyz }: x *on the surface,* z *upward vertically and through the middle point of the top frame-body,*

where, the inertia framework was attached on the ground and the robot framework was fixed on the robot respectively, and the *xoy* plane was on the ground but it moved together with the robot. For simplicity, it could be supposed that the axis z coincided with the axis Z at beginning. While, the robot framework was the relative coordinate system and the inertia framework is the absolute one. As a result, the posture (including the position and the velocity) of the robot must be expressed in the inertia framework to describe its movement, transm -itting from the robot framework to the inertia framework.

As to the QLWIS robot, the relative postures of wheels in the robot framework { $oxyz$ } can be defined by the following:

1) the radius r_i and coordinates of centers of wheels (x_i, y_i) in the { $oxyz$ } framework,

2) the rotation angles θ_i around its horizontal axis,

3) the orientation angles β_i , that was the angles between the axis-x and the perpendicular plane

where i =1,2,3,4 was the index of wheels.

When the coordinate of o in the inertia framework was (x , y , 0) and the angle between the axis x and the axis X was ψ , the posture of o could be denoted as $\xi = (x \quad y \quad \psi)^T$ in the inertia framework and the position transmission matrix from the robot framework to the inertia framework is define by

$$\Omega = \begin{pmatrix} c_\psi & -s_\psi & 0 & x \\ s_\psi & c_\psi & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{2}$$

$c(\cdot)$ and $s(\cdot)$ is the sine and the cosine function respectively. Then, the full posture and the absolute motion of the QLWIS robot could be denoted by the generalized coordinate in the inertia framework composed of eleven vectors:

$$q = (\xi \quad \beta \quad \theta)^T \tag{3}$$

where $\beta = (\beta_1 \quad \beta_2 \quad \beta_3 \quad \beta_4)^T$ and $\theta = (\theta_1 \quad \theta_2 \quad \theta_3 \quad \theta_4)^T$

3.2. kinematic equations of wheels and robot

The assumptions 3) and 4) meant that velocities of the contact points between the ground and wheels were equal to zero in planes perpendicular to (normal direction) and parallel to (tangential direction) the plane of the wheels, and its connective motion $v_c = (\dot{x}_i \quad \dot{y}_i)$ in the robot framework it could be written as the following when the slight deviation of o was ignored:

$$\begin{cases} \dot{x}_i s_\beta - \dot{y}_i c_\beta = 0 \\ v_c = r_w \dot{\theta} \end{cases} \tag{4}$$

And $r_w = r_i$ was the radius of four wheels. According to the Mechanics Equation

$$v_a = v_r + v_c \tag{5}$$

Where, v_a, v_r and v_c were the absolute velocity in the inertia framework, relative velocity of the robot to the robot framework and the connective velocity of the robot framework to the inertia framework respectively. As a result, Eq.4 could be written in the following form in the normal direction and in the tangential direction of the i^{th} wheel in the inertia framework:

$$\begin{cases} \left(-s_{\psi+\beta_i} \quad c_{\psi+\beta_i} \quad x_i c_{\beta_i} + y_i s_{\beta_i}\right)\dot{\xi} = 0 \\ \left(c_{\psi+\beta_i} \quad s_{\psi+\beta_i} \quad x_i s_{\beta_i} - y_i c_{\beta_i}\right)\dot{\xi} - r_i\dot{\theta}_i = 0 \end{cases} \tag{6}$$

The kinematic constraints of the QLWIS robot could be formulated in the following forms if above equations of four wheels were collected:

$$\begin{cases} J_n(\psi+\beta)\dot{\xi} = 0 \\ J_t(\psi+\beta)\dot{\xi} - J_r\dot{\theta} = 0 \end{cases} \tag{7}$$

If $J(q)$ was called as the Jacobian velocity matrix of the QLWIS robot, the kinematic equation could be rewritten in the standard form:

$$J(q)\dot{q} = 0 \tag{8}$$

Where, $J(q) = \begin{pmatrix} J_n(\psi+\beta) & 0_{4\times4} & 0_{4\times4} \\ J_t(\psi+\beta) & 0_{4\times4} & -J_r \end{pmatrix}$ and $J_r = diag(r_i)$, ψ and β could be measured by such magnetometer sensors as HMC1001 and photoelectrical encoders or potentiometers etc.

It can be seen fromEq.8 that the general velocities $\dot{q}$ were in the null space of the Jacobean velocity matrix $J(q)$, and it was the characteristics of the QLWIS robot because there were no active motors installed to drive the wheels.

3.3. dynamic modelling of QLWIS robot

According to the assumption 3) and Eq.4, it meant that the QLWIS robot was a nonholonomic dynamic system when it moved because Eq.4 was a nonholon -omic constraints applied on wheels, and some nonh -olonomic dynamic equations must be utilized to model its nonholonomic dynamics.

To model its nonholonomic dynamics, two equations were widely used. One was the Routh Equation, that is, the Lagrangean Equation with the multipliers λ_i (the index i was the number of the nonholonomic constraint, and $i = 1,2,\ldots k$). It was easy to model the nonholonomic system, but the equation number might be increased from n (n was number of independent coordinates of the nonholon -omic system) to $n+k$ with k undefined paramete -rs λ_i.

And the other was the Kane Equation, namely, the Kane method. It could be used to model both the holonomic systems and the nonholonomic systems, and there were not any integral and differential calculations in equations was its characteristics. But the quasi-coordinated could be selected random and freely, there were no unique forms deduced from the Kane Equation.

For a nonholonomic dynamic system, its degree-of-freedom and related independent coordinates were $n-k$. Depending on these $n-k$ degree-of-freedom, the nonholonomic dynamics could be expressed clearly with the least number of equations.

When a nonholonomic constraint dynamic system with redundancy coordinates were expressed in n dimension space with

$$q = \begin{pmatrix} q_1 & q_2 & \cdots & q_n \end{pmatrix}^T \quad (q \subset \Omega \subset R^n) \tag{9}$$

were subjected to holonomic constraints

$$f_j = f_j(q,t) = 0 \quad (j = 1,2,\cdots,k) \tag{10}$$

and independent nonholonomic constraints

$$\sum_{j=1}^{n} B_{ij}(q,t)\dot{q}_j + B_i(q,t) = 0 \quad (i = 1,2,\cdots,\kappa) \tag{11}$$

Then, the κ independent virtual displacements could be expressed with other $n-\kappa$ δq_j. For example, the κ coordinates can be expressed with them

$$\begin{bmatrix} B_{1,1} & \cdots & B_{1,\kappa} \\ \vdots & \ddots & \vdots \\ B_{\kappa,1} & \cdots & B_{\kappa,\kappa} \end{bmatrix}\begin{bmatrix} \delta q_1 \\ \vdots \\ \delta q_\kappa \end{bmatrix} + \begin{bmatrix} B_{1,\kappa+1} & \cdots & B_{1,n} \\ \vdots & \ddots & \vdots \\ B_{\kappa,\kappa+1} & \cdots & B_{\kappa,n} \end{bmatrix}\begin{bmatrix} \delta q_{\kappa+1} \\ \vdots \\ \delta q_n \end{bmatrix} = 0 \tag{12}$$

And it could also be rewritten as following

$$\begin{bmatrix} \delta q_1 \\ \vdots \\ \delta q_\kappa \end{bmatrix} = -\begin{bmatrix} B_{1,1} & \cdots & B_{1,\kappa} \\ \vdots & \ddots & \vdots \\ B_{\kappa,1} & \cdots & B_{\kappa,\kappa} \end{bmatrix}^{-1}\begin{bmatrix} B_{1,\kappa+1} & \cdots & B_{1,n} \\ \vdots & \ddots & \vdots \\ B_{\kappa,\kappa+1} & \cdots & B_{\kappa,n} \end{bmatrix}\begin{bmatrix} \delta q_{\kappa+1} \\ \vdots \\ \delta q_n \end{bmatrix} \tag{13}$$

Meanwhile, it might be simplified in concise form

$$\delta q_i = \sum_{j=\kappa+1}^{n} D_{ij}(q,t)\delta q_j \tag{14}$$

When it was substituted into the Lagrangean Equation, the next equation could be reduced

$$\sum_{i=1}^{\kappa} \Lambda_i \left(\sum_{j=\kappa+1}^{n} D_{ij}\delta q_j \right) + \sum_{j=\kappa+1}^{n} \Lambda_j \delta q_j = 0 \tag{15}$$

Because $n-\kappa$ $\delta q_j \left(j = \kappa+1, \kappa+2, \cdots, n\right)$ were utterly independent, the order can be exchanged into

$$\sum_{j=\kappa+1}^{n}\left(\sum_{i=1}^{\kappa}D_{ij}\Lambda_i+\Lambda_j\right)\delta q_j=0 \qquad (16)$$

And the Maggie Equation used to model nonholo -nomic system could be obtained

$$\sum_{i=1}^{\kappa}D_{ij}\Lambda_i+\Lambda_j=0 \quad (j=\kappa+1,\kappa+2,\cdots,n) \quad (17)$$

Where, $\Lambda_i=d/dt\,\partial T/\partial\dot{q}_i-\partial T/\partial q_i-Q_i$,

Because there were many differential calculations in nonholonomic system modelling, it was true that equations deduced form Eq.17 were hard to analyse mathematically. When the Maggie Equation was denoted in the form of

$$\sum_{i=1}^{n}h_{ij}(E_i(T))=\sum_{i=1}^{n}h_{ij}Q_i \quad (j=1,2,\cdots,n-\kappa) \quad (18)$$

Where, $E_i=d/dt\,\partial/\partial\dot{q}_i-\partial/\partial q_i$, or

$$\sum_{i=1}^{n}h_{ij}\frac{d}{dt}\frac{\partial T}{\partial\dot{q}_i}-\sum_{i=1}^{n}h_{ij}\frac{\partial T}{\partial q_i}=\sum_{i=1}^{n}h_{ij}Q_i$$
$$(j=1,2,\cdots,n-\kappa) \quad (19)$$

And h_{ij} were coefficients when the generalized velocity $\dot{q}_i$ was expressed with the quasi-velocity $\dot{\pi}_j$, and could be calculated with $h_{ij}=\partial\dot{q}_i/\partial\dot{\pi}_j$

When the generalized velocities $\dot{q}_i$ were selected as the quasi velocities $\dot{\pi}_j$ directly, that is to say,

$$\dot{\pi}_j=\dot{q}_j \quad (j=1,2,\cdots,n-\kappa) \qquad (20)$$

The coefficients h_{ij} could be denoted as

$$h_{ij}=\delta_{ij}=\begin{cases}1 & i=j\\ 0 & i\neq j\end{cases} \quad \forall\, i\le n-\kappa \qquad (21)$$

And

$$\dot{q}_{(n-k)+s}=\sum_{j=1}^{n-k}h_{(n-k)+s,j}\dot{q}_j$$
$$\forall n-\kappa<i\le n \quad (s=1,2,\cdots,\kappa) \qquad (22)$$

When Eq.21 and Eq.22 were substituted into Eq.19, another form of the Maggie Equation could be obtained

$$\Lambda_j+\sum_{i=(n-k)+1}^{n}h_{ij}\Lambda_i=0 \quad (j=1,2,\cdots,n-\kappa) \qquad (23)$$

It could be seen from Eq.23 that 1) the Maggie Equation could model the nonholonomic dynamics with the least number of equations without any undefined generalized coordinates, together with two-order differential calculations and 2) it meant that the sum of constraint forces related with the independent coordinates or degree-of-freedom and projections of other constraint forces on them was zero.

With Eq.23 and the generalized coordinates $\xi=(x\quad y\quad \psi)^T$, the nonholonomic equations of the QLWIS robot could be deduced easily.

4 Effect of nonholonomic constraints on dynamics

From what shown above, the nonholonomic constraints affected the dynamics of nonholonomic dynamic systems, because they were the one-order compatible function, say, Eq.4, in the velocity space. And there will be effects on dynamic modelling with the multipliers λ_i in the Routh Equation, but they were "invisible" in the Maggie Equation. The vectors of λ_i were the effect of nonholonomic constraints on the QLWIS robot dynamics.

It could be derived from Eq.23 that

$$\begin{Bmatrix}\Lambda_{n-k+1}\\ \Lambda_{n-k+2}\\ \vdots\\ \Lambda_n\end{Bmatrix}=\begin{bmatrix}\frac{\partial f_1}{\partial\dot{q}_{n-k+1}} & \frac{\partial f_2}{\partial\dot{q}_{n-k+1}} & \cdots & \frac{\partial f_k}{\partial\dot{q}_{n-k+1}}\\ \frac{\partial f_1}{\partial\dot{q}_{n-k+2}} & \frac{\partial f_2}{\partial\dot{q}_{n-k+2}} & \cdots & \frac{\partial f_k}{\partial\dot{q}_{n-k+2}}\\ \vdots & \vdots & \ddots & \vdots\\ \frac{\partial f_1}{\partial\dot{q}_n} & \frac{\partial f_2}{\partial\dot{q}_n} & \cdots & \frac{\partial f_k}{\partial\dot{q}_n}\end{bmatrix}\begin{Bmatrix}\lambda_1\\ \lambda_2\\ \vdots\\ \lambda_k\end{Bmatrix} \qquad (24)$$

With some related expressions shown or obtained above, the λ_i could be defined as

$$\begin{Bmatrix}\lambda_1\\ \lambda_2\\ \vdots\\ \lambda_k\end{Bmatrix}=\begin{bmatrix}\frac{\partial f_1}{\partial\dot{q}_{n-k+1}} & \frac{\partial f_2}{\partial\dot{q}_{n-k+1}} & \cdots & \frac{\partial f_k}{\partial\dot{q}_{n-k+1}}\\ \frac{\partial f_1}{\partial\dot{q}_{n-k+2}} & \frac{\partial f_2}{\partial\dot{q}_{n-k+2}} & \cdots & \frac{\partial f_k}{\partial\dot{q}_{n-k+2}}\\ \vdots & \vdots & \ddots & \vdots\\ \frac{\partial f_1}{\partial\dot{q}_n} & \frac{\partial f_2}{\partial\dot{q}_n} & \cdots & \frac{\partial f_k}{\partial\dot{q}_n}\end{bmatrix}^{-1}\begin{Bmatrix}E_{n-k+1}(T)-Q_{n-k+1}\\ E_{n-k+2}(T)-Q_{n-k+2}\\ \vdots\\ E_n(T)-Q_n\end{Bmatrix} \qquad (25)$$

When the robot moved, the nonholonomic constraints on the four wheels were

$$\dot{\theta}_i=\left(\rho\dot{\gamma}\cos\beta_i+L_g\dot{\alpha}_i\sin\beta_i\right)/r_w \qquad (26)$$

Where, the $\dot{\theta}_i$ was the rotation velocity of the i^{th} wheel, and $i=1,2,3,4$ corresponded to the rear-left wheel, the rear-right wheel, the front-left wheel and the front-right wheel individually, $\dot{\alpha}_i$ were the swinging velocity of the legs, β_i were the orientati -on angles of four wheels, $\dot{\gamma}$ was the turning angular velocity of the robot, and ρ was the turning radius and could be calculated with

$$\rho=l_w\left(\tan|\beta_3|+\tan|\beta_4|\right)/\left(\tan|\beta_4|-\tan|\beta_3|\right) \qquad (27)$$

and $2l_w$ is the distance between centres of two front wheels.

So, the λ_i can be calculated from Eq.26 were

$$\begin{Bmatrix}\lambda_1\\ \lambda_2\\ \lambda_3\\ \lambda_4\end{Bmatrix}=\begin{bmatrix}-r_w & 0 & 0 & 0\\ 0 & -r_w & 0 & 0\\ 0 & 0 & -r_w & 0\\ 0 & 0 & 0 & -r_w\end{bmatrix}^{-1}\begin{Bmatrix}I_w\ddot{\theta}_1+r_wF_{t1}\\ I_w\ddot{\theta}_2+r_wF_{t2}\\ I_w\ddot{\theta}_3+r_wF_{t3}\\ I_w\ddot{\theta}_4+r_wF_{t4}\end{Bmatrix}$$
$$=\begin{Bmatrix}-F_{t1}-\frac{I_w\ddot{\theta}_1}{r_w} & -F_{t2}-\frac{I_w\ddot{\theta}_2}{r_w} & -F_{t3}-\frac{I_w\ddot{\theta}_3}{r_w} & -F_{t4}-\frac{I_w\ddot{\theta}_4}{r_w}\end{Bmatrix}^T \quad (28)$$

Where,

$$\ddot{\theta}_i=\frac{\rho\ddot{\gamma}\cos\beta_i-\rho\dot{\gamma}\dot{\beta}_i\sin\beta_i+L_g\ddot{\alpha}_i\sin\beta_i+L_g\dot{\alpha}_i\dot{\beta}_i\cos\beta_i}{r_w},$$

I_w was the moment of inertia of wheels, and F_{ti} were the rolling friction force

Together with the dynamic equations derived from Eq.23 or Eq.17, the effect of nonholonomic constraints can be scalar determined and defined.

5 MCU based logic-control system

It could be seen from Eq. 27 and Eq.28 that the orie -ntation angles and velocities of four wheels and the swing velocities of two rear legs must be controlled to determine four Lagrangean multipliers and the eff –ect of nonholonomic constraints on its dynamics. And there were many MCUs from ADI, Freescale, Microchip, TI, Silicon, Atmel, NXP and Intel etc could be used as the main controller of the control system. The ATmega8 8-bit MCU from Atmel was selected as the controller, taking into such factors as the ISP program, C/C++ support, Capture/Compare/ PWM etc considerations

As an outstanding μcontroller, the Atmega8 MCU featured advanced RISC structure, 8k programmable Flash, two 8-bit T/Cs and one 16-bit T/C with independent prescaler, comparing and capturing unit, two programmable USART and SPI in M/S mode, 8-ch 10-bit ADCs and onchip analogue comparer, C/C++ language supporting and JTAG ISP capability etc. When the ADCs are used for the resistor feedback and the T/Cs for PWM function, the block of the Atmega8 based control system was shown in Fig.6.

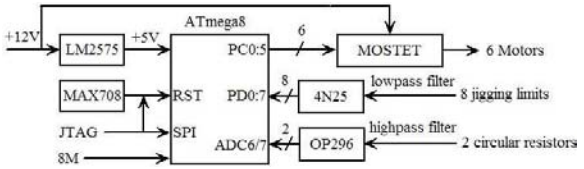


Fig.6 MCU based control system of QLWIS robot

In the MCU control system, the main Atmega8 ran in 8Mhz according to the user manual from Atmel. And the MAX708 from MAXIM was the reset and watchdog chip used for manual reset of Atmega8, the LM2575 from NS was the power management chip converting +12V DC to +5DC for the control system. At the same time, the 4N25s were the optocoupler with lowpass filter composed of operational amplifiers, resistors and capacitors to detect eight limited positions of two rear legs and wheels. The OP296s from ADI constituted the voltage follower to detect orientation angles of two front wheels via the highpass filter made up of operational amplifiers also, resistors and capacitors, and the MOSFET of Fairchild was the motor driving chip to drive two front wheels' driving motors, two rear wheels' driving motors and two rear legs' driving motors with PWM signals generated with digital timers when two front legs are resting in their inner limited positions. The ISP port originated from the SPI and reset (RST) pins were also utilized to enhance the programming function in this control system. With two onchip A/D converters, the orientation angles of two front wheels could be detected in rear-time.

With this control system, the logic control method was designed to study effects of four nonholonomic constraints on dynamics of the QLWIS robot. In independent mode gait, the logic control method coming from its motion principle were designed with the quasi-pc language as following as to the rear-left wheel in one motion cycle.

If *the rear-leg not in the inner limited position*
Then *adjust it to the inner limited position*
Else if *the rear-left wheel not in the outer limited position*
Then *adjust to the outer limited position*
The rear-left leg swing from the inner to the outer limited position
If *it is in the outer limited position*
Then *it stops there*
Else *adjust it*
The rear-left wheel rotates from the outer to the inner limited position
If *it is in the inner limited position*
Then *it stops there*
Else *adjust it*
The rear-left leg swing from the outer to the inner limited position
If *it is in the inner limited position*
Then *it stops there*
Else *adjust it*
The rear-left wheel rotates from the inner to the outer limited position

If *it is in the outer limited position*
Then *it stops there*
Else *adjust it*

Back ground on the surface motion principle, the flow chart of the robot in the independent mode gait in one motion cycle was shown in the next figure.

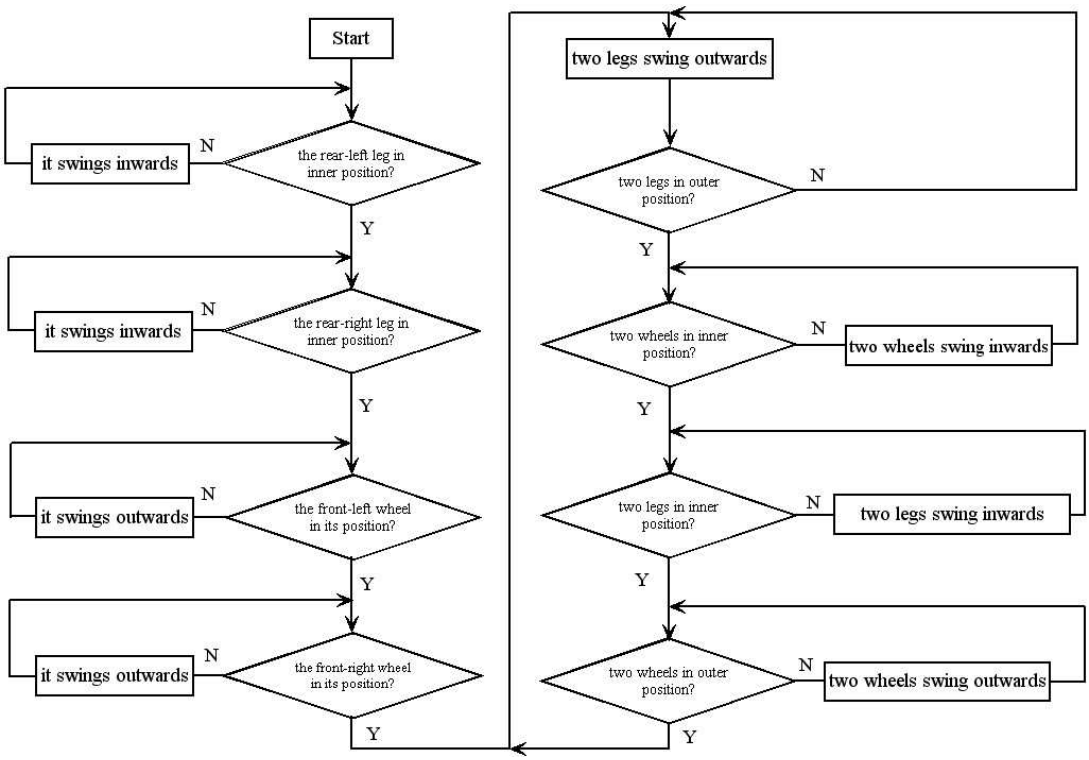


Fig.7. Logic-control flowchart of the QLWIS robot in one motion cycle

Because this flowchart and control method was based on the limited position feedback and the position logic, it might be called logic-control method or the Bang-Bang control method from point view of modern control engineering.

There were many development soft wares, i.e., the WinAVR, ICCAVR, Basic AVR and the AVR Studio from Atmel can be used to debug the control code. But, the free WinAVR and the ICCAVR were widely used to develop the control software codes. For instant, as the rear-left leg swings outwards the program can be expressed with C language if the low voltage signal will be generated when it reaches the inner limited position,

```
if (bit_is_set(PIND,7))
      //the signal is low? If not,
PORTC&= ~_BV(PC7);
      //reset the pin PC7, the motor drives it on
PORTC |= _BV(PC7);
      //set the pin PC7, the motor and the leg stop
Rear_leg_flag = 1;
      //the flag is set
```

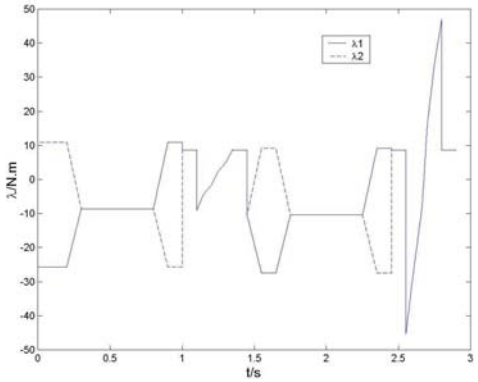
where, the *bit_is_set(.,.)* was the bit set function in the WinAVR program software, *_BV()* and *~_BV()* were the bit set and the bit clear functions, &= and |= were the and-not and the or-and functions in C/C++ language respectively. With the codes above, the MCU based logic-control method coule be developed easily in the WinAVR program.

When $m_w \approx 0.5kg$, $\ddot{\alpha}_1 = \ddot{\alpha}_2 \simeq 20\ rad/s^2$, the rolling friction coefficient $f_t \approx 0.1$ and the sliding friction coefficient $f_n \approx 0.5$, the straight-line skating and the rightwards turning experiments in independent mode gait were conducted .

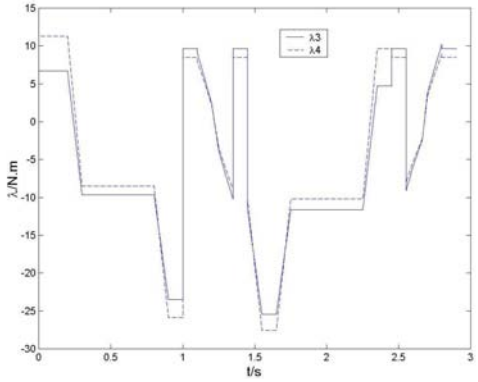
1) the rightwards turning experiment

During these experiments, the orientation angle of the front-left wheel $\beta_3 \approx -35.2°$, the orientation angle of the front-right wheels $\beta_4 \approx -46.0°$. Thus, the turning radius $\rho = 50cm$, $\rho_3 = 72.8cm$ and $\rho_4 = 58.4cm$ of two rear wheels.

Using Eq.28, the effects of four nonholonomic constraints corresponding to four wheels on the nonholonomic dynamics of the QLWIS robot in the first motion cycle were illustrated in Fig.8, which were calculated and drawn with the Matlab v6.3.



(a)

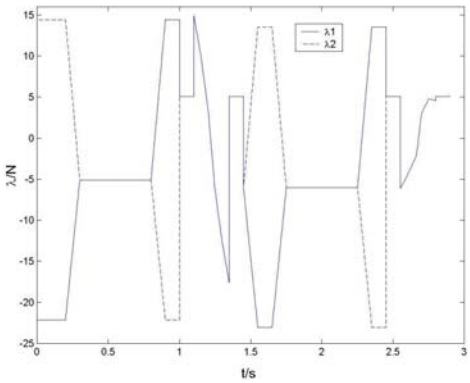


(b)

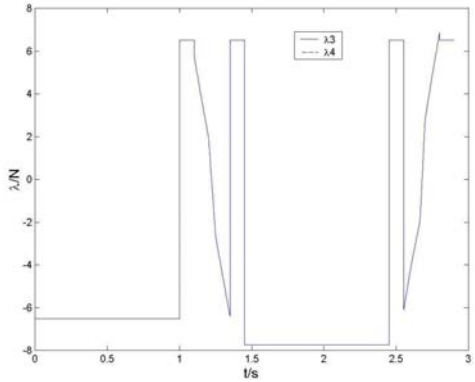
Fig.8. Effects of nonholonomic constraints on dynamics when robot turned

2) the straight-line skating experiment

When the two front wheels were in zero orientati -on angles, the QLWIS robot would skate in the straight-line defined by the initial postures of the robot because the turning radius ρ defined in Eq.27 was infinite. The effects of four nonholonomic constraints on the dynamics of the QLWIS robot in the first motion cycle were illustrated in the following figures using Eq.28, calculated and drawn in the Matlab V6.3 also.



(a)



(b)

Fig.9 Effects of nonholonomic constraints on dynamics when robot skated in straight-line

It can be seen from Fig. 8 and Fig.9 and further investigations, 1) the effects of four nonholonomic constraints on dynamics when the robot turns are larger than that when the robot skates in straight line, 2) there are more effects on the two rear wheels producing the driving friction force than two front wheels defining the motion direction, 3) the effects concern with the orientation angles of two front wheels, the λ is larger when the wheels on the side of turning is in bigger orientation angles and 4) the effects is negative correlation relating with the turning radius, the λ is larger if the radius is smaller and so on. When the turning radius is zero, the effects of four nonholonomic constraints will be infinite and they will hinder motion of the QLWIS robot absolutely in theory, and this conforms to the mathematical theory and the surface motion principle at the same time. These are the unique dynamic characteristics of the leg-wheeled passive mobile robot contrasting to other robots.

6 Conclusions

Upon the surface motion principle, a leg-wheeled passive QLWIS robot prototype was designed. Based on some assumptions and the surface motion principle, two forms of the Maggie Equation model -ing nonholonomic system were derived. With the Routh Equation, the Lagrangean multipliers were defined scalar to determine the effects of nonholono -mic constraints on the nonholonomic dynamics of the QLWIS robot. Back ground on the straight-line skating and the turning experiments conducted with the prototype machine and the Atmega8 MCU-based logic control system, the Lagrangean multipli -ers λ_i and effects of the nonholonomic constraints on the dynamics were illustrated in figures.

From the experiments and the analysis above, some conclusions can be drawn:

1) although there are no direct driving motors, the QLWIS robot also can generate the resultant propulsion force when legs and wheels move in sequence and co-ordinately, the robot conforms to the surface motion principle well.

2) according to its unique motion gait, there are nonholonomic constraints applied on the robot and the robot becomes a nonholonomic dynamic system, and the dynamic equations must be derived from nonholonomic equations. With the selected quasi-velocities and the Lagrangean Equation, two forms of the Maggie Equation are deduced, it can derive the dynamic equations with least number of equations and the degree-of-freedom directly.

3) together with the Routh Equation, the Lagrangean multipliers λ_i can be calculated with the matrix salary and the effect of the nonholomic constraint on the nonholonomic dynamics can also be determined accordingly with them.

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