Mathematical Model Associated to Three-Phase Induction Servomotors in the Case of Scalar Control

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Abstract: - Scalar control of induction servomotors was implemented on their steady-state model. If the machine is powered via a frequency and voltage converter, due to the presence in the motor input voltage wave of higher time harmonics, both its parameters and functional characteristic values will be more or less different comparing to the case of sinusoidal supply. The presence of these harmonics will result in the appearance of a distorting regime in the machine, with adverse effects in operation. In this work is realized a theoretical study of the behavior of the asynchronous servomotor in the presence of distorting (non-sinusoidal) regime and also a mathematical model for its scalar control is proposed.

Key-Words: - Mathematical model, Asynchronous servomotor, Non-sinusoidal regime, Power converter, Scalar control.

1. Introduction

Three-phase asynchronous servomotors are now spread to a growing extent. Theses eliminate the disadvantages of the d.c. servomotors linked to the collector-brush system; moreover they are robust, having a simple construction, a lower friction and a lower cost price. Asynchronous servomotors also present a number of drawbacks in that efficiency and power factor are lower, dimension and weight are greater and control is more complicated than the d.c. servomotors

Compared to the usual three-phase asynchronous servomotors, which do not distinguish from the point of view of construction, at the servomotors of the same type on can remark:

- A higher length / diameter ratio relative to the rotor, which has the drawback that the heat losses transfer from the rotor is most difficult;

- A stronger strengthening of the stator insulation in order to resist to the often transient processes;

- Taking into account of the rotor heating which becomes important.

There is a high variety of schemes for automatic position control made with three-phase induction servomotors. Despite its simple and robust construction, the motion control for this type of servomotors should take into account the complexity of the dynamic model which is nonlinear and variable in time and that the physical parameters of the machine are not always known with great precision. Under these conditions the motion control means controlling the speed and/or the position, respectively torque control. As on obtain a faster torque response as the motion control is more efficient.

Mainly there are two control strategies: scalar control and vector control.

Scalar control or scalar regulation can be done in open or closed speed control loop and can be accomplished by connecting scalar values, for example, $u_s = f(f_1)$ or $i_s = f(f_2)$, where u_s and i_s are the stator voltage and current and f_1 and f_2 are the stator and rotor frequencies. Usually, it is necessary to impose the condition for keeping the stator flux constant and equal to the nominal one ($\psi_s = \psi_{sn}$ = const.). This strategy is based on a steady-state simple induction servomotor model. The advantage of the scalar control resides in the simplicity of the control circuits but has the drawback of generally obtaining variable speeds with low accuracy; the dynamic performances of the system are also low. It follows a satisfactory adjustment only when the machine works with stationary speed for long periods. If the fluctuations occur in tension, disturbances in load or if the servo-system requires fast accelerations or decelerations, the open loop control is unsatisfactory, and the closed loop control is necessary. As a conclusion we can say that the implementation of scalar control though simple, is limited by the accuracy of speed and torque response of the machine.

Usually, the electric machines are designed to be supplied in sinusoidal regime. If the servomotor is supplied through a static frequency converter, because of the higher harmonics (non-sinusoidal supply) from the input voltage wave of the motor, both its parameters and functional characteristic values will be more or less different from the case of sinusoidal power supply. The presence of these higher harmonics will have as result the appearance of a deforming regime in the machine, with adverse effects in general in its operation.

In similar conditions of load and rotation speed with the case with sinusoidal supply on find an increase of the machine losses, of the electrical power absorbed and therefore a reduced efficiency. There is also a greater heating of the machine and an electromagnetic torque which, at a given load, is not invariable but pulsating, compared with an average value corresponding to the load.

The appearance of the deforming regime in the machine is inevitable because any static frequency converter based on semiconductor technique produces voltages or currents, which contain, in addition to fundamental and harmonic, higher odd time harmonics.

Generalizing, the output voltage harmonics are grouped into families centered on frequencies:

$$f_j = Jm_f f_c = Jm_f f_1 \quad (\mathrm{J} = 1, 2, 3, ...), \qquad (1)$$

and the various harmonic frequencies in a family are:

$$f_{(\nu)} = f_j \pm kf_c = (Jm_f \pm k) f_c = (Jm_f \pm k) f_1, \; (2)$$

with

$$\nu = Jm_f \pm k \;\; . \qquad (3)$$

In the above relations, m_f represents the frequency modulation factor, f_1 is the fundamental's frequency and f_c is the frequency of the control modulating signal. Whereas the harmonic spectrum contains only ν order odd harmonics, in order that ($Jm_f \pm k$) to be odd, an odd J determine an even k and vice versa. The harmonic amplitudes of one family are symmetrical in relation to the central frequency harmonic and harmonic family separation is even more evident as the modulation frequency m_f is greater.

2. Mathematical model of the three-phase induction servomotor in the case of non-sinusoidal supply.

2.1. General considerations. Actual stage.

Induction servomotor behavior analysis and their performance evaluation in a steady-state regime in the condition of supplying by a static frequency converters can be done by three methods:

a). **Direct integration of machine equations.** The method allows, besides the calculation of the currents and torque in a steady-state regime, also the rotor speed variations due pulsating loads. The main disadvantage of this method is that, especially in the case of medium power machines due to higher time constants of the machine, the stabilization process of the integration on the computer can take a long time, which limits the direct use of this method.

b). **The use of Fourier analysis.** The method is based on the decomposition of the voltage signal from the motor input into a sum of ν harmonics sinusoidal signals, the overall effect being the sum of partial effects. The main advantages of this method are: its simplicity and the possibility to use the computing relationships from the design of electric machines theory for the sinusoidal regime (practically verified for thousands of motors). The disadvantage of the method consists in the decomposition in Fourier series of some signals which are not partly continuous and hence non-sinusoidal, involving few convergent series, taking into account a large number of terms. However, this disadvantage can be reduced by use of computer technology in solving the problem.

c). **The use of the state variables.** It is a classical method in systems theory, to determine the machine currents and torque directly, in analytical form. The method can be applied if the machine is considered an ideal one, unsaturated, isotropic, with constant parameters and unaffected by voltage, current or frequency value (which generally is not the case).

Comparing the advantages and disadvantages of the three methods described above can be synthesized as follows:

1. - Using the “a” method in solving constructive-technological design problems is difficult, the use the method being generally disadvantageous because of the longer duration of the integration process stabilization.

2. - Method "b", despites the drawback mentioned and whose effects can be mitigated by computer using, is imposed itself in the specific

constructive-technological design process of the servomotors supplied through power static frequency converter by the possibility of using many calculation relationships from the classical theory of induction machines fed from a sinusoidal power supply, with possible corrections.

3. The "c" method, because the working hypotheses underlying it is recommended to be applied in the study and modeling of automatic control systems with induction machines.

Based on these considerations, for the study from this work, the Fourier analysis method will be considered. Using this approach method requires as simplifying assumption the neglecting of the magnetic saturation.

This assumption does not deform the reality too much, whereas the present study aims especially the investigation of the non-sinusoidal steady-state regime of the machine, at a load almost equal to the nominal one. In this situation, the current that occurs is not much different from basic sinusoidal current value. This case shall remain valid even in the starting conditions.

In the literature there are known various mathematical models associated to induction machines fed by static frequency and voltage converters. The majority of these models are based on the association between an induction machine and an equivalent scheme corresponding to the fundamental and a lot of schemes corresponding to the various ν frequencies, corresponding to the Fourier series decomposition of the motor input voltage (see Figure 1) [3]. In this model the skin effect is not considered.

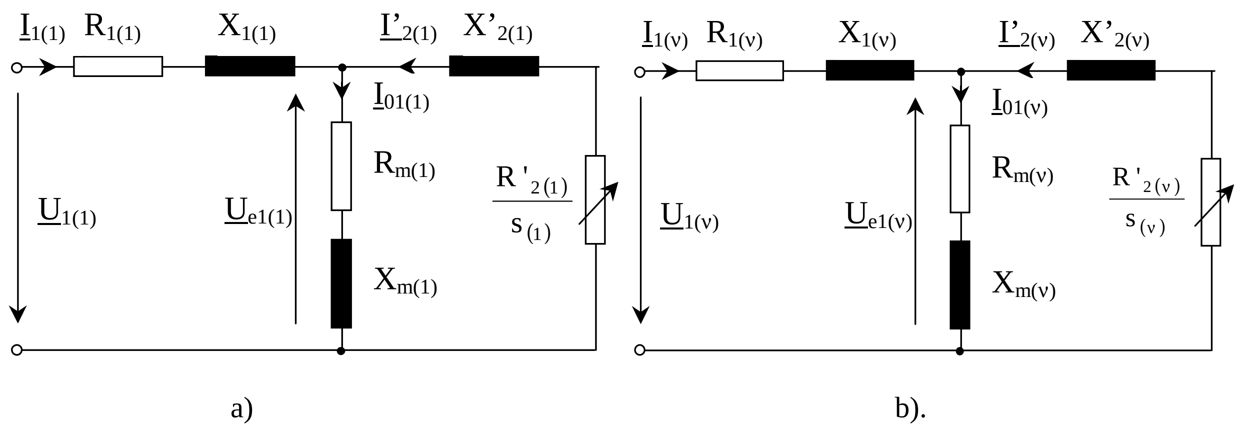

Fig.1. Equivalent scheme of the machine supplied through frequency converter:
a) for the case of fundamental; b) for the ν order harmonics (positive or negative sequence).

For the equivalent scheme in Figure 1.a, corresponding to the fundamental, the electrical parameters are defined as:

$$R_{1(1)} = R_1 = R_{1n}; \quad X_{1(1)} = X_1 = aX_{1n};$$
$$R'_{2(1)} = R'_2 = R'_{2n}; \quad X'_{2(1)} = X'_2 = aX'_{2n};$$
$$R_{m(1)} = R_m = a^2 R_{mn}; \quad X_{m(1)} = X_m = aX_{mn}; \quad (4).$$
$$\frac{R'_{2(1)}}{s_{(1)}} = \frac{R'_2}{s} = \frac{a}{c} R'_{2n}.$$

In relations (4), R_{1n}, X_{1n}, R'_{2n}, X'_{2n}, R_{mn}, X_{mn} represents the values of the parameters R_1, X_1, R'_2, X'_2, R_m and X_m in nominal operating conditions (fed from a sinusoidal power supply, rated voltage frequency and load) and

$$a = \frac{f_1}{f_{1n}} = \frac{\omega_1}{\omega_{1n}} = \frac{n_1}{n_{1n}};$$
$$c = \frac{n_1 - n}{n_{1n}} = \frac{n_1 - n}{n_1} \cdot \frac{n_1}{n_{1n}} = s \cdot a. \quad (5)$$

Corresponding to the equivalent scheme from Fig. 1a, the basic machine equations are:

$$\underline{U}_{1(1)} = \underline{Z}_{1(1)} \underline{I}_{1(1)} - \underline{U}_{e1(1)};$$
$$\underline{U}'_{e2(1)} = \underline{Z}'_{2(1)} \underline{I}'_{2(1)} = \underline{U}_{e1(1)}; \quad (6)$$
$$\underline{U}_{e1(1)} = -\underline{Z}_{m(1)} \underline{I}_{01(1)};$$
$$\underline{I}_{01(1)} = \underline{I}_{1(1)} + \underline{I}'_{2(1)}.$$

where:

$$\underline{Z}_{1(1)} = R_{1(1)} + jX_{1(1)} = R_{1n} + jaX_{1n};$$

$$\underline{Z}'_{2(1)} = R'_{2(1)} \frac{1}{s_{(1)}} + jX'_{2(1)} = \\ = R'_{2n} \frac{1}{s} + jaX'_{2n} = \frac{a}{c} R'_{2n} + jaX'_{2n}; \qquad (7)$$

$$\underline{Z}_{m(1)} = R_{m(1)} + jX_{m(1)} = a^2 R_{mn} + jaX_{mn}.$$

In the relations (5), f_1 and f_{1n} are random frequency of the rotating magnetic field, respectively the nominal frequency of the rotating magnetic field [(usually f_{1n} = 50 Hz) and U_1 and U_{1n} are the supply phase voltage, respectively the rated supply phase voltage.

For ν order harmonics order the scheme from Fig. 1.b is applicable.

The slip $s_{(\nu)}$, corresponding to the ν order harmonic is:

$$s_{(\nu)} = \frac{\nu n_1 \mp n}{\nu n_1} = 1 \mp \frac{n}{\nu n_1} = \\ = 1 \mp \frac{1}{\nu} \pm \frac{c}{a}\frac{1}{\nu}, \qquad (8)$$

where sign (-) (from the first equality) corresponds to wave that rotates within the sense of the main wave and the sign (+) in the opposite one.

For the case studied in this paper - that of small and medium power machines – the resistances $R_{1(\nu)}$ and reactances $X_{1(\nu)}$ values are not practically affected by the skin effect. In this case we can write:

$$R_{1(\nu)} = R_{1(1)} = R_1 = R_{1n}, \qquad (9)$$

$$X_{1(\nu)} = \omega_{1(\nu)} \cdot L_{1\sigma(\nu)} = \nu\omega_1 L_{1\sigma(\nu)}, \qquad (10)$$

where $L_{1\sigma(\nu)}$ is the stator dispersion inductance corresponding to the ν order harmonic.

If it is agreed that the machine cores represent the linear mediums (the machine is unsaturated), it results that the inductance can be considered constant, independent of the load (current) and flux, one can say that:

$$L_{1\sigma(\nu)} = L_{1\sigma(1)} = L_{1\sigma}. \qquad (11)$$

By replacing the expression inductance $L_{1\sigma(\nu)}$ from relation (11) in relation (10), we obtain:

$$X_{1(\nu)} = \nu\omega_1 L_{1\sigma} = \nu X_1 = \nu a X_{1n}. \qquad (12)$$

For rotor resistance and rotor leakage reactance, corresponding to the ν order harmonic, both reduced to the stator the following expressions were established:

$$R'_{2(\nu)} = R'_{2(1)} = R'_2 = R'_{2n}, \qquad (13)$$

$$X'_{2(\nu)} = \nu \cdot X'_2 = \nu \cdot a \cdot X'_{2n}, \qquad (14)$$

Magnetization resistance corresponding to the ν order harmonic, $R_{m\nu}$, is given by the relation:

$$R_{m(\nu)} = k_{K''} \cdot \nu^2 \cdot a^2 \cdot R_{mn}. \qquad (15)$$

where $k_{K''} = \frac{K''_{(\nu)}}{K''_{(1)}}$

K" is a coefficient dependent on iron losses and on the magnetic field variation.

The magnetization reluctantce corresponding to the magnetic field produced by the ν order harmonic is:

$$X_{m(\nu)} = k_{K'} \nu \cdot a \cdot X_{mn}. \qquad (16)$$

where $k_{K'} = \frac{K'_{(\nu)}}{K'_{(1)}}$

For K 'apply the same considerations as for K ".

Corresponding to the equivalent scheme shown in Figure the machine equations for the ν order harmonic are:

$$\underline{U}_{1(\nu)} = \underline{Z}_{1(\nu)} \underline{I}_{1(\nu)} - \underline{U}_{e1(\nu)}; \\ \underline{U}'_{e2(\nu)} = \underline{Z}'_{2(\nu)} \underline{I}'_{2(\nu)} = \underline{U}_{e1(\nu)}; \\ \underline{U}_{e1(\nu)} = -\underline{Z}_{m(\nu)} \underline{I}_{01(\nu)}; \\ \underline{I}_{01(\nu)} = \underline{I}_{1(\nu)} + \underline{I}'_{2(\nu)}; \qquad (17)$$

where:

$$\underline{Z}_{1(\nu)} = R_{1(\nu)} + jX_{1(\nu)} = \\ = R_{1n} + j \cdot \nu \cdot a \cdot X_{1n};$$

$$\underline{Z}'_{2(\nu)} = R'_{2(\nu)} \frac{1}{s_{(\nu)}} + jX'_{2(\nu)} = \\ = R'_{2n} \frac{1}{1 \mp \frac{1}{\nu} \pm \frac{c}{a} \cdot \frac{1}{\nu}} + j \cdot \nu \cdot a \cdot X'_{2n}; \qquad (18)$$

$$\underline{Z}_{m(\nu)} = R_{m(\nu)} + jX_{m(\nu)} = \\ = k_{K''} \cdot \nu^2 \cdot a^2 \cdot R_{mn} + j \cdot k_{K'} \cdot \nu \cdot a \cdot X_{mn}$$

2.2. The setting of the equivalent servomotor scheme in the case of non-sinusoidal supply

In the present study, the authors intend to establish a single mathematical model associated to induction servomotors, supplied by static voltage and frequency converter, which consists of a single equivalent scheme, and which to describe the machine operation, according to the presence in the input power voltage of higher time harmonics.

For this, the following simplifying assumptions are taken into account:

- the permeability of the magnetic core is considered infinitely large, comparing to the air and the magnetic field lines are straight perpendicular to the slot axis;

- both the ferromagnetic core and rotor cage (bar + short circuit rings) are homogeneous and isotropic mediums;

- the marginal effects are neglected, the slot is considered very long on the axial direction. The electromagnetic fields both the fundamental and the corresponding to the ν order harmonic, are considered, in this case plane-parallels. This assumption is accepted for theoretical calculations of most authors;

- skin effect is taken into account in the calculations only in bars that are in the transverse magnetic field of the slot. For portions of the bar outside of the slot, in ventilation channels (where appropriate), and in short circuit rings, current density is considered as constant throughout the cross section of the bar.

- the passing from the constant density zone into the variable density zone occurs abruptly;

- in the real electric machines the skin effect is often influenced by the degree of saturation (especially at startup), but simultaneous coverage of both phenomena in mathematical relationships easily applied in practice is very difficult, even precarious. Therefore, the simplifying assumption of neglecting the effects of saturation is allowed as valid in the establishing the relationships for equivalent parameters;

- the local variation of magnetic induction and of current density is considered sinusoidal in time, both for fundamental and for each ν harmonic.

- on take into account only the fundamental space of the e.m.f. harmonica;

Under these conditions of non-sinusoidal supply, the asynchronous servomotor may be associated with an equivalent scheme, corresponding to all harmonics.

The scheme operates in the fundamental's frequency $f_{1(1)}$ and is represented in Fig. 2.

The influence of higher harmonics is found in the particular values of the parameters valid for a given load.

According to this scheme, formally it can be considered that the servomotor, in the case of supplying through the power frequency converter (the corresponding parameters and dimensions of this situation is marked with index "CSF") behave as if they were fed in sinusoidal regime at fundamental's frequency, $f_{1(1)}$ with the following voltages system:

$$\begin{aligned} u_A &= \sqrt{2} \cdot U_{1(CSF)} \cdot \sin \omega_1 t \\ u_B &= \sqrt{2} \cdot U_{1(CSF)} \cdot \sin\left(\omega_1 t - \frac{2\pi}{3}\right), \\ u_C &= \sqrt{2} \cdot U_{1(CSF)} \cdot \sin\left(\omega_1 t + \frac{2\pi}{3}\right) \end{aligned} \tag{19}$$

where,

$$U_{1(CSF)} = \sqrt{U_{1(1)}^2 + \sum_{\nu \neq 1} U_{1(\nu)}^2} \ . \tag{20}$$

$U_{1(\nu)}$ is the phase voltage supply corresponding to the ν order harmonic.

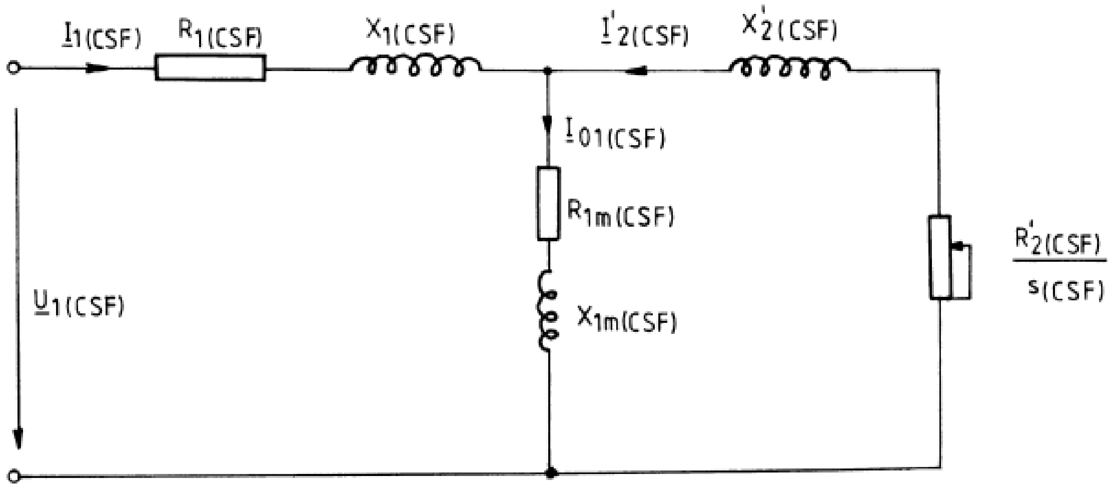

Fig. 2. The equivalent scheme of the asynchronous servomotor powered by a static frequency converter

Corresponding to the system supply voltages, the current system which crosses the stator phases is as follows:

$$\begin{cases} i_A = \sqrt{2} \cdot I_{1(CSF)} \cdot \sin\left(\omega_1 t - \varphi_{1(CSF)}\right) \\ i_B = \sqrt{2} \cdot I_{1(CSF)} \cdot \sin\left(\omega_1 t - \varphi_{1(CSF)} - \frac{2\pi}{3}\right) \\ i_C = \sqrt{2} \cdot I_{1(CSF)} \cdot \sin\left(\omega_1 t - \varphi_{1(CSF)} - \frac{4\pi}{3}\right) \end{cases} \quad (21)$$

where $I_{1(CSF)}$ is given by:

$$I_{1(CSF)} = \sqrt{I_{1(1)}^2 + \sum_{\nu \neq 1} I_{1(\nu)}^2} \quad (22)$$

Power factor in the deforming regime is defined as the ratio between active power and apparent power, as follows:

$$\Delta_{(CSF)} = \frac{P_{1(CSF)}}{S_{1(CSF)}} = \frac{P_{1(CSF)}}{U_{1(CSF)} I_{1(CSF)}} . \quad (23)$$

If we consider the real case in which the when in the supply of servomotor both fundamental and higher ν order time harmonics are present (non-sinusoidal regime), the active power absorbed by the machine $P_{1(CSF)}$ is defined, as in the sinusoidal regime, as the average in a period of instantaneous power. The following expression is obtained:

$$\begin{aligned} P_{1(CSF)} &= \frac{1}{T}\int_0^T p \cdot dt = \sum_{\nu=1} U_{1(\nu)} I_{1(\nu)} \cos\varphi_{1(\nu)} = \\ &= U_1 I_1 \cos\varphi_1 + \sum_{\nu \neq 1} U_{1(\nu)} I_{1(\nu)} \cos\varphi_{1(\nu)} \end{aligned} \quad (24)$$

Therefore, the active power absorbed by the servomotor when it is supplied through a power static converter is equal to the sum of the active powers corresponding to each harmonic (the principle of superposition effects is found).

In relation (24), $\cos\varphi_{(1\nu)}$ is the power factor corresponding to the ν order harmonic having expression [4]:

$$\cos\varphi_{1(\nu)} = \frac{R_{1(\nu)} + \frac{R'_{2(\nu)}}{s_{(\nu)}}}{\sqrt{\left(R_{1(\nu)} + \frac{R'_{2(\nu)}}{s_{(\nu)}}\right)^2 + \left(X_{1(\nu)} + X'_{2(\nu)}\right)^2}} \quad (25)$$

It can see that the value of $\cos\varphi_{(1\nu)}$, given by relation (25) is very small, so the currents produced by higher harmonics are almost purely inductive, with all the consequences arising from this: reducing of the power factor, of the efficiency and of the maximum torque that can be developed by the servomotor.

Apparent power can be defined in the non-sinusoidal regime also, as the product of rated values of applied voltage and current:

$$S_{1(CSF)} = U_{1(CSF)} \cdot I_{1(CSF)} , \quad (26)$$

Taken into account the relations (24), (25) and (26), the relationship (23) becomes:

$$\Delta_{(CSF)} = \frac{U_1 I_1 \cos\varphi_1 + \sum_{\nu \neq 1} U_{1(\nu)} I_{1(\nu)} \cos\varphi_{1(\nu)}}{\sqrt{U_{1(1)}^2 + \sum_{\nu \neq 1} U_{1(\nu)}^2} \cdot \sqrt{I_{1(1)}^2 + \sum_{\nu \neq 1} I_{1(\nu)}^2}} \quad (27)$$

Because $\Delta_{(CSF)} \leq 1$, formally (the phase angle has meaning only in harmonic values), an angle $\varphi_{1(CSF)}$ can be associated to the power factor $\Delta_{(CSF)}$, as:

$$\cos\varphi_{1(CSF)} = \Delta_{(CSF)} \quad (28)$$

With this, the relation (27) can be written:

$$\cos\varphi_{1(CSF)} = \frac{U_1 I_1 \cos\varphi_1 + \sum_{\nu \neq 1} U_{1(\nu)} I_{1(\nu)} \cos\varphi_{1(\nu)}}{\sqrt{U_{1(1)}^2 + \sum_{\nu \neq 1} U_{1(\nu)}^2} \cdot \sqrt{I_{1(1)}^2 + \sum_{\nu \neq 1} I_{1(\nu)}^2}} \quad (29)$$

The relation (29) can be written also:

$$\cos\varphi_{1(CSF)} = \frac{\cos\varphi_1 + \sum_{\nu \neq 1} \frac{U_{1(\nu)}}{U_{1(1)}} \frac{I_{1(\nu)}}{I_1} \cos\varphi_{1(\nu)}}{\sqrt{1 + \sum_{\nu \neq 1}\left(\frac{U_{1(\nu)}}{U_{1(1)}}\right)^2} \cdot \sqrt{1 + \sum_{\nu \neq 1}\left(\frac{I_{1(\nu)}}{I_{1(1)}}\right)^2}} \quad (30)$$

If on take into account the relation [4]:

$$\frac{I_{1(\nu)}}{I_{1(1)}} = \frac{1}{\nu} \cdot \frac{1}{f_{1r} \cdot x_{sc}^*} \cdot \frac{U_{1(\nu)}}{U_{1(1)}} \quad (31)$$

where x^*_{sc} is reported short-circuit impedance, measured at frequency $f_1 = f_{1n}$ where f_{1n} is usually equal to 50 Hz, relation (30) becomes:

$$\cos\varphi_{1(CSF)} = \frac{\cos\varphi_1 + \sum_{\nu\neq1}\frac{1}{\nu}\cdot\frac{1}{f_{1r}\cdot x_{sc}^*}\cdot\left(\frac{U_{1(\nu)}}{U_{1(1)}}\right)^2\cos\varphi_{1(\nu)}}{\sqrt{\left[1+\sum_{\nu\neq1}\left(\frac{U_{1(\nu)}}{U_{1(1)}}\right)^2\right]\cdot\left[1+\sum_{\nu\neq1}\left(\frac{1}{\nu}\cdot\frac{1}{f_{1r}\cdot x_{sc}^*}\cdot\frac{U_{1(\nu)}}{U_{1(1)}}\right)^2\right]}} \quad (32)$$

The equivalent parameters of the scheme have been calculated at the fundamental's frequency, under the presence of all harmonics in the supply voltage.

Under these conditions, is denoted $p_{Cu1(CSF)}$ the losses that occur in the stator winding when the servomotor is supplied through a power frequency converter. These losses are, in fact, covered by some active power absorbed by the machine from the network, through the converter, $P_{1(CSF)}$. According to the principle of superposition effects, it can be considered [5]:

$$\begin{aligned} p_{Cu1(CSF)} &= p_{Cu1(1)} + \sum_{\nu\neq1} p_{Cu1(\nu)} = \\ &= 3R_{1(1)}I_{1(1)}^2 + 3\sum_{\nu\neq1} R_{1(\nu)}I_{1(\nu)}^2 \end{aligned} . \quad (33)$$

Further, the stator winding resistance corresponding to the fundamental, $R_{1(1)}$ and stator winding resistances corresponding to the all higher time harmonics $R_{1(\nu)}$, are replaced by a single equivalent resistance $R_{1(CSF)}$, corresponding to all harmonics, including the fundamental. The equalization is achieved by the condition that in this resistance occur the same losses $p_{Cu1(CSF)}$, given by relation (33), as if considering the "ν" resistances $R_{1(\nu)}$, each of them crossed by the current $I_{1(\nu)}$. This equivalent resistance, $R_{1(CSF)}$, determined at fundamental's frequency, is crossed by the current $I_{1(CSF)}$ (rms), with the expression given by (19).

Therefore:

$$\begin{aligned} p_{Cu1(CSF)} &= 3R_{1(CSF)}\cdot I_{1(CSF)}^2 = \\ &= 3R_{1(CSF)}\left(I_{1(1)}^2 + \sum_{\nu\neq1} I_{1(\nu)}^2\right) \end{aligned} . \quad (34)$$

Making the relations (33) and (34) equal, it results:

$$\begin{aligned} &3R_{1(CSF)}\left(I_{1(1)}^2 + \sum_{\nu\neq1} I_{1(\nu)}^2\right) = \\ &= 3R_{1(1)}\cdot I_{1(1)}^2 + 3\sum_{\nu\neq1} R_{1(\nu)}\cdot I_{1(\nu)}^2 \end{aligned} , \quad (35)$$

or

$$\begin{aligned} &3R_{1(CSF)}\left(I_{1(1)}^2 + \sum_{\nu\neq1} I_{1(\nu)}^2\right) = 3R_{1(1)}\left(I_{1(1)}^2 + \sum_{\nu\neq1} I_{1(\nu)}^2\right) = \\ &= 3R_{1(\nu)}\left(I_{1(1)}^2 + \sum_{\nu\neq1} I_{1(\nu)}^2\right) \end{aligned} \quad (36)$$

from which:

$$R_{1(CSF)} = R_{1(1)} = R_1, \quad (37)$$

Applying the principle of superposition effects for the reactive absorbed power by the stator winding ($Q_{Cu1\ (CSF)}$), the following expression is obtained [5]:

$$\begin{aligned} Q_{Cu1(CSF)} &= Q_{Cu1(1)} + \sum_{\nu\neq1} Q_{Cu1(\nu)} = \\ &= 3\cdot X_{1(1)}I_{1(1)}^2 + 3\sum_{\nu\neq1} X_{1(\nu)}I_{1(\nu)}^2 \end{aligned} . \quad (38)$$

As in the previous case, the stator winding reactance corresponding to the fundamental, $X_{1(1)}$ (determined at fundamental's frequency $f_{1(1)}$) and the stator winding reactances corresponding to all higher time harmonics $X_{1(\nu)}$ (determined at frequencies $f_{1(\nu)}=\nu\cdot f_1$ where $Jm_f \pm k$) are replaced by an equivalent reactance, $X_{1(CSF)}$, determined at fundamental's frequency. This equivalent reactance, crossed by the current $I_{1(CSF)}$, conveys the same reactive power, $Q_{Cu1(CSF)}$ as in the case of considering of "ν" reactances $X_{1(\nu)}$, (each of them determined at $f_{1(\nu)}$ frequency and crossed by the current $I_{1(\nu)}$). Following the equalization, the following expression can be written:

$$\begin{aligned} Q_{Cu1(CSF)} &= 3X_{1(CSF)}I_{1(CSF)}^2 = \\ &= 3X_{1(CSF)}\left(I_{1(1)}^2 + \sum_{\nu\neq1} I_{1(\nu)}^2\right) \end{aligned} . \quad (39)$$

Making the relations (38) and (39) equal, it results:

$$\begin{aligned} &3X_{1(CSF)}\left(I_{1(1)}^2 + \sum_{\nu\neq1} I_{1(\nu)}^2\right) = \\ &= 3X_{1(1)}I_{1(1)}^2 + 3\sum_{\nu\neq1} X_{1(\nu)}I_{1(\nu)}^2 \end{aligned} , \quad (40)$$

or

$$\begin{aligned} &X_{1(CSF)}\left(I_{1(1)}^2 + \sum_{\nu\neq1} I_{1(\nu)}^2\right) = \\ &= X_{1(1)}I_{1(1)}^2 + \sum_{\nu\neq1} \nu X_1 I_{1(\nu)}^2 = X_1\left(I_{1(1)}^2 + \sum_{\nu\neq1} \nu I_{1(\nu)}^2\right) \end{aligned} \quad (41)$$

On note the following:

$k_{X1} = \frac{X_{1(CSF)}}{X_1}$ the factor that highlights the changes that the reactants of stator phase value suffers in the case of a machine supplied through a power frequency converter, compared to sinusoidal supply, both calculated at fundamental's frequency.

From relation (41) follows [5]:

$$k_{X1} = \frac{X_{1(CSF)}}{X_1} = \frac{I_{1(1)}^2 + \sum_{\nu \neq 1} \nu I_{1(\nu)}^2}{I_{1(1)}^2 + \sum_{\nu \neq 1} I_{1(\nu)}^2} = \frac{1 + \sum_{\nu \neq 1} \nu \left(\frac{I_{1(\nu)}}{I_{1(1)}} \right)^2}{1 + \sum_{\nu \neq 1} \left(\frac{I_{1(\nu)}}{I_{1(1)}} \right)^2} \quad (42)$$

By replacing the relation (31) in (42) results:

$$k_{X1} = \frac{X_{1(CSF)}}{X_1} = \frac{1 + \sum_{\nu \neq 1} \nu \left(\frac{1}{f_{1r} x_{sc}^*} \right)^2 \cdot \frac{1}{\nu^2} \left(\frac{U_{1(\nu)}}{U_{1(1)}} \right)^2}{1 + \sum_{\nu \neq 1} \left(\frac{1}{f_{1r} x_{sc}^*} \right)^2 \cdot \frac{1}{\nu^2} \left(\frac{U_{1(\nu)}}{U_{1(1)}} \right)^2} = \quad (43)$$

$$= \frac{1 + \sum_{\nu \neq 1} \frac{1}{\nu} \left(\frac{1}{f_{1r} x_{sc}^*} \right)^2 \left(\frac{U_{1(\nu)}}{U_{1(1)}} \right)^2}{1 + \sum_{\nu \neq 1} \frac{1}{\nu^2} \left(\frac{1}{f_{1r} x_{sc}^*} \right)^2 \left(\frac{U_{1(\nu)}}{U_{1(1)}} \right)^2}.$$

On can see that

$$k_{X1} > 1 \quad (44)$$

With the equivalent resistance given by (37) and the equivalent reactance resulting from the relationship (43) we can now write the relation for the equivalent impedance of stator winding, $\underline{Z}_{1(CSF)}$ covering all frequency harmonics and including the fundamental:

$$\underline{Z}_{1(CSF)} = R_{1(CSF)} + jX_{1(CSF)} = R_{1(CSF)} + jk_{X1} X_1 \quad (45)$$

Similarly, the equivalent resistance and equivalent reactance of one rotor phase reported to the stator, determined at the fundamental's frequency and appropriate to all harmonics, including the fundamental, it obtains the following expressions [4], [5]:

$$R_{2(CSF)}^{'} = k_{R_2^{'}} \cdot R_2^{'} \quad (46)$$

$$X_{2(CSF)}^{'} = k_{X_2^{'}} \cdot X_2^{'} \quad (47)$$

Where $k_{R_2^{'}}$ and $k_{X_2^{'}}$ are defined by:

$$k_{R_2^{'}} = \frac{\frac{k_{r(CSF)}}{k_r} + \frac{R_{2i}^{'}}{R_{2c}^{'}} \cdot \frac{1}{k_r}}{1 + \frac{R_{2i}^{'}}{R_{2c}^{'}} \cdot \frac{1}{k_r}}, \quad (48)$$

$$k_{X_2^{'}} = \frac{\frac{k_{x(CSF)}}{k_x} + \frac{X_{2i}^{'}}{X_{2c}^{'}} \cdot \frac{1}{k_x}}{1 + \frac{X_{2i}^{'}}{X_{2c}^{'}} \cdot \frac{1}{k_x}}, \quad (49)$$

where $R_{2c}^{'}$ is the resistance at fundamental's frequency of the part of rotor phase winding placed in the slots and reported to the stator, $R_{2i}^{'}$ is the resistance, the fundamental's frequency of the part of rotor winding with neglecting skin effect relative to the stator, $X_{2c}^{'}$ is the reactance of the part of the rotor winding placed in slots and reduced to the stator in which the skin effect occurs and $X_{2i}^{'}$ is the reactance of the part of the rotor phase winding in which the skin effect can be neglected, $k_{r(CSF)}$ and $k_{x(CSF)}$ are the global equivalent factors for rotor resistance respective rotor reactance changing (corresponding to all harmonics, including the fundamental) and k_r and k_x are the factors for rotor resistance growing respectively diminution of rotor reactance in the case of sinusoidal supply. For more details the literature [5] is recommended.

With these, the expression for the rotor phase impedance relative to the stator, when the servomotor is fed through a power frequency converter became:

$$\underline{Z}_{2(CSF)}^{'} = \frac{R_{2(CSF)}^{'}}{s_{(CSF)}} + jX_{2(CSF)}^{'}, \quad (50)$$

Where

$$s_{(CSF)} = \frac{R_{2(CSF)}^{'} I_{2(CSF)}^{'}}{U_{e1(CSF)}}. \quad (51)$$

In relation (51) we have:

$$U_{e1(CSF)} = \sqrt{U_{e1(1)}^2 + \sum_{\nu \neq 1} U_{e1(\nu)}^2}. \quad (52)$$

In the following, the expressions of the equivalent parameters for the magnetic circuit will be set (corresponding to all harmonics).

Thus, to determine the equivalent resistance of magnetization $R_{1m(CSF)}$, we have to take into account that this is determined only by the ferromagnetic stator core losses which are covered directly by the stator power, without making the transition through the stereo-mechanical power [1].

By approximating that $I_{01(CSF)} \approx I_{\mu(CSF)}$, for $R_{1m(CSF)}$ is obtained:

$$R_{1m(CSF)} = \frac{p_{z1(CSF)} + p_{j1(CSF)}}{3 I^2_{\mu(CSF)}}, \qquad (53)$$

where $p_{z1(CSF)}$ and $p_{j1(CSF)}$ are global losses occurring in the stator teeth respectively yoke due to the supplying of the servomotor through the frequency converter. The determination of these losses is presented in detail in [4].

In determining the total magnetization current $I_{\mu(CSF)}$, the principle of superposition effects is applied (the ferromagnetic core is considered linear):

$$I_{\mu(CSF)} = \sqrt{I^2_{\mu(1)} + \sum_{\nu \neq 1} I^2_{\mu(\nu)}}\,. \qquad (54)$$

The magnetization current corresponding to the ν order harmonic is calculated using the magnetization ampere-turn also corresponding to the ν order harmonic, $\theta_{\mu(\nu)}$, from the relationship:

$$\theta_{\mu(\nu)} = \frac{2\sqrt{2}}{\pi} 3 \cdot N_1 \cdot k_{q1} \cdot k_{y1} \cdot I_{\mu(\nu)}\,. \qquad (55)$$

In writing te relation (55) were taken into account that: $k_{q1(\nu)} = k_{q1}$ and $k_{y1(\nu)} = k_{y1}$ (k_{q1} is zone factor and k_{y1} is shortening factor).

The magnetization ampere-turn corresponding to the fundamental, $\theta_{\mu(1)}$, has the expression [1]:]

$$\theta_{\mu(1)} = \frac{2\sqrt{2}}{\pi} \cdot 3 \cdot N_1 \cdot k_{q1} \cdot k_{y1} \cdot I_{\mu(1)} = \theta_{\mu}\,, \qquad (56)$$

where is θ_μ is the magnetization ampere-turn when the servomotor is supplied from the mains.

By dividing the relations (55) and (56) is obtained:

$$\frac{\theta_{\mu(\nu)}}{\theta_{\mu(1)}} = \frac{I_{\mu(\nu)}}{I_{\mu(1)}} = \frac{I_{\mu(\nu)}}{I_{\mu}}\,, \qquad (57)$$

that the relation (54) can be made in the form:

$$I_{\mu(CSF)} = I_{\mu} \cdot \sqrt{1 + \sum_{\nu \neq 1} \left(\frac{\theta_{\mu(\nu)}}{\theta_{\mu(1)}} \right)^2}\,. \qquad (58)$$

The magnetization ampere-turn $\theta_{\mu(\nu)} = U_{H(\nu)}$. It is calculated for each ν harmonic in part, as well as for the fundamental (see e.g. [1]).

For the equivalent magnetizing reactance, corresponding to all harmonics, determined at the fundamental's magnetization frequency $f_{1(1)}$, we obtain:

$$X_{1m(CSF)} \cong \sqrt{\left(\frac{U_{1(CSF)}}{I_{\mu(CSF)}} \right)^2 - \left(R_{1(CSF)} + R_{1m(CSF)} \right)^2} \qquad (59)$$

For the equivalent impedance of the magnetization circuit it can be written:

$$\underline{Z}_{1m(CSF)} = R_{1m(CSF)} + j \cdot X_{1m(CSF)}\,. \qquad (60)$$

To determine the ratio $\frac{R'_{2(CSF)}}{s_{(CSF)}}$, the conservation law of the active powers received by the equivalent rotor is written: equivalent active power corresponding to all harmonics, including fundamental one, received by the equivalent rotor of the servomotor in the situation of supplying through the power frequency converter is equal to the sum of active power corresponding to the fundamental and the powers corresponding to each ν harmonic separately. Thus on can write:

$$3 \frac{R_{2(CSF)}}{s_{(CSF)}} I^2_{2(CSF)} = 3 \frac{R_{2(1)}}{s_{(1)}} I^2_{2(1)} + 3 \sum_{\nu \neq 1} \frac{R_{2(\nu)}}{s_{(\nu)}} I^2_{2(\nu)} \qquad (61)$$

or in reduced values:

$$3 \frac{R'_{2(CSF)}}{s_{(CSF)}} I'^2_{2(CSF)} = 3 \frac{R'_{2(1)}}{s_{(1)}} I'^2_{2(1)} + 3 \sum_{\nu \neq 1} \frac{R'_{2(\nu)}}{s_{(\nu)}} I'^2_{2(\nu)} \qquad (62)$$

From where, it results after calculation:

$$\frac{R'_{2(CSF)}}{s_{(CSF)}} = \frac{R'_2}{s} \cdot \frac{1 + \sum_{\nu \neq 1} \frac{s_{(1)}}{s_{(\nu)}} \cdot \frac{R'_{2(\nu)}}{R'_{2(1)}} \left(\frac{I'_{2(\nu)}}{I'_{2(1)}} \right)^2}{1 + \sum_{\nu \neq 1} \left(\frac{I'_{2(\nu)}}{I'_{2(1)}} \right)^2} \qquad (63)$$

Where the slip corresponding to the ν order harmonic is:

$$s_{(\nu)} = 1 \mp \frac{1}{\nu} \pm \frac{s}{\nu}\,. \qquad (64)$$

The pair of signs "- +" (located at the top) corresponds to wave that rotates in the sense of the main wave and the pair "+ -" (located at the bottom) where the opposite.

Given these assumptions and considering that the equivalent parameters were calculated reduced to the fundamental's frequency (in the conditions of a sinusoidal regime), may formally accept the calculation in complex quantities. Corresponding to the unique scheme shown in Figure 2, the servomotor equations are:

$$\begin{gathered} \underline{U}_{1(CSF)} = \underline{Z}_{1(CSF)} \cdot \underline{I}_{1(CSF)} - \underline{U}_{e1(CSF)}; \\ \underline{U}'_{e2(CSF)} = \underline{Z}'_{2(CSF)} \cdot \underline{I}'_{2(CSF)} = \underline{U}_{e1(CSF)}; \\ \underline{U}_{e1(CSF)} = -\underline{Z}_{m(CSF)} \cdot \underline{I}_{01(CSF)}; \\ \underline{I}_{01(CSF)} = \underline{I}_{1(CSF)} + \underline{I}'_{2(CSF)}. \end{gathered} \tag{65}$$

With this, all parameters corresponding to the equivalent scheme of asynchronous three-phase servomotor in the case of supplying through a power frequency converter were determined.

3 Conclusions

This paper aims to study the theoretical behavior of asynchronous three-phase servomotor in the case of supplying through a power frequency converter.

The analysis of the induction servomotors behavior and the evaluation of their steady-state regime performances under the condition of power frequency converter are mainly focused on the use of Fourier analysis. The main advantages of this method are its simplicity and the possibility to use in many situations the relations from the electric machines design theory for the sinusoidal regime, expressions which were practically verified. Using this method of approach requires as a simplifying assumption the neglecting of the variable magnetic saturation. This assumption does not deform the reality too much, whereas the present study primarily aims to analyze the non-sinusoidal steady-state regime of the machine at a nominal charge when the saturation degree has a determined value. In this situation, the current that occurs is not much different from the value of the basic sinusoidal current. This hypothesis remains valid even in the starting condition, if this is done through a power frequency converter.

Through this study sought to develop theory of the asynchronous three-phase servomotor in the non-sinusoidal powered regime, to serve as a starting point in optimizing the design methodology of constructive-technological, as economic conditions as favorable. Given that the asynchronous three-phase servomotor is powered through a static frequency converter, the machine operation in the presence of the higher harmonics of time in the supply voltage can be described by a single mathematical model. The model consists of a single equivalent scheme corresponding to all harmonics and is defined at the fundamental's frequency.

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