

Theoretical and experimental determination of equivalent parameters of three-phase induction motor windings in case of power electronic converters supply

SORIN MUSUROI, VALERIU-NICOLA OLARESCU, DORU VATAU, CIPRIAN SORANDARU

Department of Electric Machines and Drives

POLITEHNICA University of Timisoara, Faculty of Electrical Engineering

Timisoara, Bd. Vasile Parvan nr.2, RO-300223

ROMANIA

sorin.musuroi@et.upt.ro, vali_olarescu@yahoo.com, doru.vatau@et.upt.ro,

ciprian.sorandaru@gmail.com, <http://www.et.upt.ro>

Abstract: - In the present paper is carried out a theoretical and experimental study of induction motor behavior in the presence of distorting regime (non-sinusoidal) due to the power electronic converter. The main aims are the determination of the relations of calculating the equivalent parameters of the machine winding supplied by power electronic converter and the practice of verifying them. Also proposed is a comparative study with case of sinusoidal supply, with emphasis on the influence of the presence of the electronic converter on the machine parameters. Issue is limited to three phase range of cage-induction machine of small and medium-sized powers, to 45 [kW], the most currently used in the usual adjustable speed drives. The tests were carried out using two three-phase induction motors having rated powers of 0.37 [kW] and 1.1 [kW] respectively.

Key-words: induction machines, converters, equivalent parameters, non-sinusoidal regime.

1 Introduction

In the context of rapid technical progress, the electrical drives have to ensure an operation with high demands on change and adjustment of speed, starting, braking and reversing, respectively a movements correlation of working mechanisms of the same production unit. All these technical requirements have created prospects for the development of complex drive systems, using power electronic converters based on semiconductor components that provide automatic management of production processes, with a low energy, turning to computer and microprocessor.

Today, adjustable speed electric drives made with electronic circuits, both the force and in the control part are one of the most important levers for boosting technical progress in a modern industry.

Now, the efficient changes of the induction machine speed, with keeping the technical-economic performance, is made through different control strategies which involve the supplying of the machines through electronic power convertors. In this non-sinusoidal supplying regime of the machine appears a change in parameters compared with the sinusoidal supplying case, as a presence of the superior frequency harmonics in voltage wave on the input of the machine. In this way, if PWM modulation is used, the current and voltage harmonics are of $v=Jm_f \pm k$ order, where m_f is the

frequency modulation factor. As the harmonics spectrum contain only v odd order differing of 3 and multiples of 3, because $(Jm_f \pm k)$ to be odd, an odd J establish a even k and vice versa. The positives harmonics goes in direct sense (+) and the negatives harmonics goes in opposite sense (-) of the magnetic field corresponding to the fundamental. These lead to a distorting regime in the induction machine. For operation study of the machine in this case, the non-sinusoidal voltage is decomposed into a lot of harmonics, and the influence of each harmonic is separately analyzed, and after that the effects of superposition principles is applied. The behavior of the induction machine could be analyzed regarding a series of motors, having the same shaft, with have the n speed, motors supplied with different voltages $U_{1(v)}$ and different frequencies $f_{1(v)}$ networks. In this paper is performed an experimental and theoretical study regarding the determination of the parameters of the medium and small power of the induction machine windings, when is supplied from a frequency converter (non-sinusoidal regime). In future reasoning, will be considerate the follow simplifying assumptions: the repression phenomena of the current in stator pole pitch is neglected, respectively will be taken into account only spatial fundamental harmonics of the winding. After these two assumptions is added another one: the neglecting of the saturation considering that the

parameters of the machine (resistance and inductance) are not influenced by the saturation phenomenon in function by the load, so, they are not time parameters through this. This assumption is practically valid on the load changes, in limits imposed by the normal running regime.

2 The determination of the equivalent parameters of the stator winding

For this case, of the medium and small power induction machine the values of the stator winding parameters are not practically affected by the skin effect, this affirmation is valid for fundamentals and also for the time superior harmonics v . So we can write the following relations [1]:

$$R_{1(1)} = R_1 = R_{1n} \quad (1)$$

$$X_{1(1)} = X_1 = aX_{1n} \quad (2)$$

$$R_{1(v)} = R_{1(1)} = R_1 = R_{1n} \quad (3)$$

$$X_{1(v)} = vX_1 = vaX_{1n} \quad (4)$$

where $a = \frac{f_1}{f_{1n}} = \frac{\omega_1}{\omega_{1n}} = \frac{n_1}{n_{1n}}$

The above mentioned equations allow the determination of the parameters of the stator windings in the situation when in the supplying voltage of the machine only the fundamental (the parameters have the index (1) - see equations (1) and (2)), or only a superior v time order harmonic (the parameters are identified through the index v , see equations (3) and (4)) are present. In the following, the fundamental and also the superior time harmonics are considered present in the supplying voltage of the motor. It is noted with $p_{Cu1(CSF)}$ the copper losses from the stator winding when the machine is supplied through a frequency converter. These losses are covered by a part of the power absorbed by the motor through the frequency converter from the network, $p_{1(CSF)}$. According to the principle of the superposition of the effects, it can be considered that:

$$\begin{aligned} p_{Cu1(CSF)} &= p_{Cu1(1)} + \sum_{v \neq 1} p_{Cu1(v)} = \\ &= 3R_{1(1)}I_{1(1)}^2 + 3\sum_{v \neq 1} R_{1(v)}I_{1(v)}^2 \end{aligned} \quad (5)$$

The resistance of the stator winding corresponding to the fundamental, $R_{1(1)}$, and also the resistances of the stator windings corresponding to all superior time harmonics, $R_{1(v)}$, are replaced through a single equivalent resistance, $R_{1(CSF)}$, corresponding to all harmonics, including the fundamental. The equivalence is made through the condition that in this resistance should be the same losses $p_{Cu1(CSF)}$, given by the equation (5), as in the case of considering of “ v ” resistances $R_{1(v)}$, the

current $I_{1(v)}$ going through each of them. This equivalent resistance $R_{1(CSF)}$ is determined at the fundamental's frequency and is covered by the $I_{1(CSF)}$ current (rated value).

$$I_{1(CSF)} = \sqrt{I_{1(1)}^2 + \sum_{v \neq 1} I_{1(v)}^2} \quad (6)$$

Thus,

$$\begin{aligned} p_{Cu1(CSF)} &= 3R_{1(CSF)} \cdot I_{1(CSF)}^2 = \\ &= 3R_{1(CSF)} \left(I_{1(1)}^2 + \sum_{v \neq 1} I_{1(v)}^2 \right) \end{aligned} \quad (7)$$

Making the relations (5) and (7) equal, is obtained:

$$\begin{aligned} 3R_{1(CSF)} \left(I_{1(1)}^2 + \sum_{v \neq 1} I_{1(v)}^2 \right) &= \\ &= 3R_{1(1)} \cdot I_{1(1)}^2 + 3\sum_{v \neq 1} R_{1(v)} \cdot I_{1(v)}^2 \end{aligned}, \quad (8)$$

or, taking in consideration relation (3):

$$\begin{aligned} 3R_{1(CSF)} \left(I_{1(1)}^2 + \sum_{v \neq 1} I_{1(v)}^2 \right) &= 3R_{1(1)} \left(I_{1(1)}^2 + \sum_{v \neq 1} I_{1(v)}^2 \right) = \\ &= 3R_{1(v)} \left(I_{1(1)}^2 + \sum_{v \neq 1} I_{1(v)}^2 \right) \end{aligned}, \quad (9)$$

from which results:

$$R_{1(CSF)} = R_{1(1)} = R_{1(v)} = R_1 = R_{1n} \quad (10)$$

Is noted with k_{R1} the ratio between $R_{1(CSF)}$ and R_1 . This represents a factor that highlights the modifications that “happen” to the stator phase resistance in the case of supplying of the machine through a frequency converter compared to a sinusoidal supply. It can be easily seen that:

$$k_{R1} = \frac{R_{1(CSF)}}{R_1} = 1, \quad (11)$$

which is normal regarding the conductors with which the stator winding is realized, having a diameter of (usually) $d=2$ [mm].

Applying the principle of superposition of the effects also for the reactive power absorbed by the stator winding ($Q_{Cu1(CSF)}$), is obtained:

$$\begin{aligned} Q_{Cu1(CSF)} &= Q_{Cu1(1)} + \sum_{v \neq 1} Q_{Cu1(v)} = \\ &= 3 \cdot X_{1(1)} I_{1(1)}^2 + 3\sum_{v \neq 1} X_{1(v)} I_{1(v)}^2 \end{aligned} \quad (12)$$

As in the previous case, the stator winding reactance corresponding to the fundamental $X_{1(1)}$ (determined at the fundamental's frequency $f_{1(1)}$) and the stator winding's reactance corresponding to all superior time harmonics $X_{1(v)}$ (determined at the $f_{1(v)}=v \cdot f_1$ frequency, where $v=Jm \pm k$), are replaced with an equivalent reactance $X_{1(CSF)}$, determined at the fundamental's frequency. This equivalent reactance

traveled by the current $I_{1(CSF)}$, gives the same reactive power $Q_{Cu1(CSF)}$, as in the case of considering of $X_{1(v)}$ reactance (each being determined at $f_{1(v)}$ frequency and being traveled by the current $I_{1(v)}$). After the equivalence, it can be written:

$$\begin{aligned} Q_{Cu1(CSF)} &= 3X_{1(CSF)}I_{1(CSF)}^2 = \\ &= 3X_{1(CSF)}\left(I_{1(1)}^2 + \sum_{v \neq 1} I_{1(v)}^2\right) \end{aligned} \quad (13)$$

Making relations (12) and (13) equal is obtained:

$$\begin{aligned} 3X_{1(CSF)}\left(I_{1(1)}^2 + \sum_{v \neq 1} I_{1(v)}^2\right) &= \\ = 3X_{1(1)}I_{1(1)}^2 + 3\sum_{v \neq 1} X_{1(v)}I_{1(v)}^2 \end{aligned} \quad (14)$$

or:

$$\begin{aligned} X_{1(CSF)}\left(I_{1(1)}^2 + \sum_{v \neq 1} I_{1(v)}^2\right) &= X_{1(1)}I_{1(1)}^2 + \sum_{v \neq 1} vX_{1(v)}I_{1(v)}^2 = \\ = X_{1(1)}\left(I_{1(1)}^2 + \sum_{v \neq 1} vI_{1(v)}^2\right) \end{aligned} \quad (15)$$

Is noted with $k_{X1} = \frac{X_{1(CSF)}}{X_1}$ - factor that shows the

modifications of the reactance value of a stator phase in the case of supplying the machine through a frequency converter, compared to the sinusoidal supplying, both being determined at the fundamental frequency. From the equation (15) results:

$$k_{X1} = \frac{X_{1(CSF)}}{X_1} = \frac{I_{1(1)}^2 + \sum_{v \neq 1} vI_{1(v)}^2}{I_{1(1)}^2 + \sum_{v \neq 1} I_{1(v)}^2} = \frac{1 + \sum_{v \neq 1} v\left(\frac{I_{1(v)}}{I_{1(1)}}\right)^2}{1 + \sum_{v \neq 1} \left(\frac{I_{1(v)}}{I_{1(1)}}\right)^2} \quad (16)$$

According [1]:

$$\frac{I_{1(v)}}{I_{1(1)}} = \frac{U_{1(v)}}{U_{1(1)}} \cdot \frac{1}{vf_{1r}} \cdot \frac{1}{x_{sc}^*} \quad (17)$$

where: $x_{sc}^* = \frac{X_{sc}}{Z_{(1)}}$ - is the short circuit impedance

reported, corresponding to the frequency $f_1=f_{1n}$ (usually $f_{1n}=50$ [Hz]) and f_{1r} is the reported frequency. With this, equation (17) becomes:

$$k_{X1} = \frac{X_{1(CSF)}}{X_1} = \frac{1 + \sum_{v \neq 1} \frac{1}{v} \left(\frac{1}{f_{1r} x_{sc}^*}\right)^2 \left(\frac{U_{1(v)}}{U_{1(1)}}\right)^2}{1 + \sum_{v \neq 1} \frac{1}{v^2} \left(\frac{1}{f_{1r} x_{sc}^*}\right)^2 \left(\frac{U_{1(v)}}{U_{1(1)}}\right)^2} \quad (18)$$

It can be observed that : $k_{X1} > 1$.

In these conditions, the equivalent impedance of the stator winding $Z_{1(CSF)}$, corresponding to all harmonics and defined at the fundamental's frequency is:

$$Z_{1(CSF)} = R_{1(CSF)} + jX_{1(CSF)} \quad (19)$$

3 Determination of equivalent global change parameters for power rotor actuator by static frequency converter

Next is the question of determining the two coefficients in an aggregate form. In its analysis, the following simplifying assumptions are allowed:

a) The permeability of the magnetic core is considered infinitely large compared with that of the air, magnetic field lines are straight perpendicular to the slot axis.

b) Both the ferromagnetic core and rotor cage (bar + short-circuiting rings) are homogeneous and isotropic medium.

c) The marginal effect is neglected, the slot is considered very long on the axial direction.

d) The skin effect is taken into account in the calculus only in bars that are in the transverse magnetic field of the slot. For portions of the bar outside the slot and in the short circuit rings, the current density is considered constant throughout the cross section of the bar.

e) The change from the constant density zone into the variable density zone occurs abruptly.

f) The local variation of the magnetic induction and current density is considered as sine wave with respect to time for both fundamental and each v harmonic.

Following it is considered also a winding with multiple cages whose bars (in number of "c") are placed in the same slot of any form, electrically separated from each other (see Figure 1). These bars are connected at the front by short-circuiting rings (one ring may correspond to several bars slot). This "generalized" approach, pure theoretically in fact, has the advantage that by its applying the relations of the two equivalent factors $k_{r(CSF)}$ and $k_{X(CSF)}$, valid for any slot type and multiple cages, are obtained.

Rotor slot shown in Fig. 1 is the height h_c , and is divided into "n" layers (strips), each strip having a height $h_s = h_c / n$. The number of layers "n" is chosen such that the current density of each band can be considered constant throughout the height h_s (and therefore not manifest skin effect in the strip).

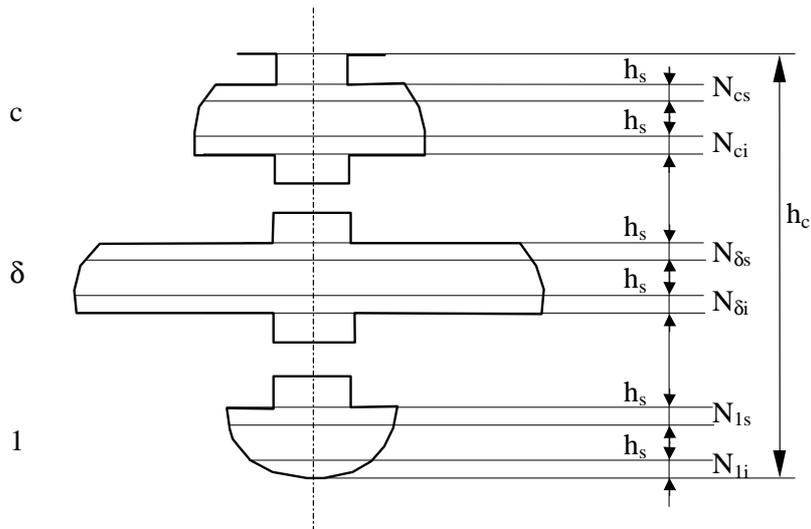


Fig. 1. Generalized slot for multiple cages.

Slot bars are numbered from 1 to c, from the bottom of slot. Lower layer of each bar is identified by the index "i" and the top layer by the index "s". Thus, for a bar with index δ characterized by a specific resistance ρ_δ and an absolute magnetic permeability μ_δ , the lower layer is noted with $N_{\delta i}$ and the extremely higher layer with $N_{\delta s}$. The current that runs through the bar δ is noted with $i_{c\delta}$ ($I_{c\delta}$ - rated value). The length of the bar, over which the skin effect occurs, is L.

For the beginning, let's consider that the only the presence of the fundamental in the power supply which corresponds to the supply pulsation, $\omega_{1(1)} = \omega_1 = 2\pi f_1$. Those two factors, the following relations are obtained [1]:

$$k_{r\delta(1)} = \frac{R_{\delta(1)\sim}}{R_{\delta-}} = \frac{1}{I_{c\delta(1)}^2} \cdot \sum_{\varepsilon=N_{\delta i}}^{N_{\delta s}} \frac{I_{\varepsilon(1)}^2}{b_\varepsilon} \cdot \sum_{\varepsilon=N_{\delta i}}^{N_{\delta s}} b_\varepsilon, \quad (20)$$

$$k_{x\delta(1)} = \frac{L_{\delta i\sigma(1)\sim}}{L_{\delta i\sigma-}} = \frac{\text{Re}[\Psi_{\delta i\sigma(1)}]}{\left(\sum_{\varepsilon=N_{\delta i}}^{N_{\delta s}} b_\varepsilon\right)^2} \quad (21)$$

$$= \frac{\sqrt{2}\mu_\delta L h_s I_{c\delta(1)} \cdot \sum_{\lambda=N_{\delta i}}^{N_{\delta s}} \frac{1}{b_\lambda} \left[\left(\sum_{\varepsilon=N_{\delta i}}^{\lambda-1} b_\varepsilon\right) \left(\sum_{\varepsilon=N_{\delta i}}^\lambda b_\varepsilon\right) + \frac{b_\lambda^2}{3} \right]}{\left(\sum_{\varepsilon=N_{\delta i}}^{N_{\delta s}} b_\varepsilon\right)^2}$$

where b_λ and b_ε are the width of λ and ε order strips and $\Psi_{\delta i\sigma(1)}$ is the δ bar flux corresponding to the fundamental of the own magnetic field, assuming that for the λ order strip, the magnetic linkage corresponds to a constant repartition of fundamental current density on the strip.

If in the motor power supply is considered only v order harmonic which corresponds to supply pulsation $\omega_{1(v)} = v\omega_1$ the relations (20) and (21) remain valid, with the following considerations:

index "1" is replaced with index "v" and the rotor phenomena are with the pulsation $\omega_{2(v)}$ given by the relation:

$$\omega_{2(v)} = s_{(v)} \cdot \omega_{1(v)} = \left(1 \mp \frac{1}{v} \pm \frac{s}{v}\right) \cdot v \cdot \omega_1, \quad (22)$$

Next is considered the real case, where in the δ bar are present both the fundamental and v order time harmonics. For this, the equivalent d.c. global factor of δ bar resistance modification is calculated with the relation [1]:

$$k_{r\delta(CSF)} = \frac{P_{\delta(CSF)\sim}}{P_{\delta(CSF)-}} = \frac{R_{\delta(CSF)\sim}}{R_{\delta(CSF)-}}, \quad (23)$$

where $p_{\delta(CSF)\sim}$ are total a.c. losses in δ bar (considering the appropriate skin effect for all harmonics) and $p_{\delta(CSF)-}$ are the bar δ total losses, without considering the repression phenomenon.

The a.c. total losses in δ bar are obtained by applying the effects superposition principle by adding the all δ bar a.c. losses caused by each v order time, including the fundamental. Therefore on obtain:

$$P_{\delta(CSF)\sim} = P_{\delta(1)\sim} + \sum_{v \neq 1} P_{\delta(v)\sim}, \quad (24)$$

The a.c. losses in δ bar, corresponding to the fundamental, $p_{\delta(1)\sim}$, are calculated with the following relation:

$$P_{\delta(1)\sim} = I_{c\delta(1)}^2 \cdot R_{\delta(1)\sim}, \quad (25)$$

or

$$P_{\delta(1)\sim} = I_{c\delta(1)}^2 \cdot k_{r\delta(1)} \cdot R_{\delta-}. \quad (26)$$

In the same way, the expression of δ bar a.c. losses produced by some v order time harmonic is obtained:

$$P_{\delta(v)\sim} = I_{c\delta(v)}^2 \cdot R_{\delta(v)\sim} = I_{c\delta(v)}^2 \cdot k_{r\delta(v)} \cdot R_{\delta-}. \quad (27)$$

By replacing relations (26) and (27) in relation (24) it results:

$$P_{\delta(CSF)-} = I_{c\delta(1)}^2 \cdot k_{r\delta(1)} \cdot R_{\delta-} + \sum_{v \neq 1} I_{c\delta(v)}^2 \cdot k_{r\delta(v)} \cdot R_{\delta-} = R_{\delta-} \left(I_{c\delta(1)}^2 \cdot k_{r\delta(1)} + \sum_{v \neq 1} I_{c\delta(v)}^2 \cdot k_{r\delta(v)} \right) \quad (28)$$

The δ bar losses without considering the repression phenomenon in the bar is calculated using the following relationship:

$$P_{\delta(CSF)-} = I_{c\delta(CSF)}^2 \cdot R_{\delta-}, \quad (29)$$

where:

$$I_{c\delta(CSF)} = \sqrt{I_{c\delta(1)}^2 + \sum_{v \neq 1} I_{c\delta(v)}^2} \quad (30)$$

is the rated value of the current which runs through the δ bar, in the case of a motor supplied by a frequency converter. By replacing the relation (30) in relation (29):

$$P_{\delta(CSF)-} = R_{\delta-} \left(I_{c\delta(1)}^2 + \sum_{v \neq 1} I_{c\delta(v)}^2 \right) \quad (31)$$

By replacing relations (28) and (31) in (23) on obtains the expression for the global equivalent factor of the a.c. increasing resistance in the bar δ , $k_{r\delta(CSF)}$, in case of presence of all harmonics in the motor power:

$$k_{r\delta(CSF)} = \frac{P_{\delta(CSF)-}}{P_{\delta(CSF)-}} = \frac{R_{\delta-} \left(I_{c\delta(1)}^2 \cdot k_{r\delta(1)} + \sum_{v \neq 1} I_{c\delta(v)}^2 \cdot k_{r\delta(v)} \right)}{R_{\delta-} \left(I_{c\delta(1)}^2 + \sum_{v \neq 1} I_{c\delta(v)}^2 \right)} = \frac{k_{r\delta(1)} + \sum_{v \neq 1} k_{r\delta(v)} \left(\frac{I_{c\delta(v)}}{I_{c\delta(1)}} \right)^2}{1 + \sum_{v \neq 1} \left(\frac{I_{c\delta(v)}}{I_{c\delta(1)}} \right)^2} \quad (32)$$

The global equivalent change of a.c. δ bar inductance modification has the expression:

$$k_{x\delta(CSF)} = \frac{q_{\delta(CSF)-}}{q_{\delta(CSF)-}} \quad (33)$$

where $q_{\delta(CSF)-}$ is the a.c. total reactive power, in δ bar, and $q_{\delta(CSF)}$ is the total reactive power for a uniform current distribution δ bar.

Applying the superposition in the case of a.c. total reactive power, the following relationship is obtained:

$$q_{\delta(CSF)-} = q_{\delta(1)-} + \sum_{v \neq 1} q_{\delta(v)-}, \quad (34)$$

A.c. reactive power corresponding to the fundamental is calculated using the following relationship:

$$q_{\delta(1)-} = \omega_{1(1)} L_{\delta i\sigma(1)-} \cdot I_{c\delta(1)}^2 = \omega_1 L_{\delta i\sigma(1)-} \cdot I_{c\delta(1)}^2, \quad (35)$$

or

$$q_{\delta(1)-} = \omega_1 \cdot k_{x\delta(1)} \cdot L_{\delta i\sigma-} \cdot I_{c\delta(1)}^2. \quad (36)$$

In the same way, the expression of a.c. reactive power in δ bar corresponding to v order harmonic is obtained:

$$q_{\delta(v)-} = \omega_{1(v)} L_{\delta i\sigma(v)-} \cdot I_{c\delta(v)}^2 = v \cdot \omega_1 \cdot k_{x\delta(v)} \cdot L_{\delta i\sigma-} \cdot I_{c\delta(v)}^2. \quad (37)$$

By replacing relations (36) and (37) in relation (34), the expression for calculating the total a.c. reactive power in δ bar is obtained:

$$q_{\delta(CSF)-} = \omega_1 \cdot k_{x\delta(1)} \cdot L_{\delta i\sigma-} \cdot I_{c\delta(1)}^2 + \left(k_{x\delta(1)} \cdot I_{c\delta(1)}^2 + \sum_{v \neq 1} v \cdot k_{x\delta(v)} \cdot I_{c\delta(v)}^2 \right) = \omega_1 \cdot L_{\delta i\sigma-} \left(k_{x\delta(1)} \cdot I_{c\delta(1)}^2 + \sum_{v \neq 1} v \cdot k_{x\delta(v)} \cdot I_{c\delta(v)}^2 \right) \quad (38)$$

The total reactive power for a uniform current repartition in δ bar, in the case of motor supplied through a frequency converter is calculated by the relationship:

$$q_{\delta(CSF)-} = q_{\delta(1)-} + \sum_{v \neq 1} q_{\delta(v)-}, \quad (39)$$

where $q_{\delta(1)-}$ is the reactive power corresponding to the fundamental, in case of a uniform current distribution $I_{c\delta(1)}$ in δ bar, and $q_{\delta(v)-}$ is the reactive power corresponding to the v harmonic in case of a uniform current distribution $I_{c\delta(v)}$ in δ bar:

$$q_{\delta(1)-} = \omega_{1(1)} L_{\delta i\sigma-} \cdot I_{c\delta(1)}^2 = \omega_1 \cdot L_{\delta i\sigma-} \cdot I_{c\delta(1)}^2. \quad (40)$$

Similarly, for the reactive power corresponding to v harmonic, in the case of a uniform current $I_{c\delta(v)}$ repartition in δ bar, the following relationship is obtained:

$$q_{\delta(v)-} = \omega_{1(v)} \cdot L_{\delta i\sigma-} \cdot I_{c\delta(v)}^2 = v \cdot \omega_1 \cdot L_{\delta i\sigma-} \cdot I_{c\delta(v)}^2. \quad (41)$$

By replacing relations (40) and (41) in relation (39), the expression for the total reactive power for a uniform current distribution in δ bar becomes:

$$q_{\delta(CSF)-} = \omega_1 \cdot L_{\delta i\sigma-} \cdot I_{c\delta(1)}^2 + \sum_{v \neq 1} v \cdot \omega_1 \cdot L_{\delta i\sigma-} \cdot I_{c\delta(v)}^2 = \omega_1 \cdot L_{\delta i\sigma-} \left(I_{c\delta(1)}^2 + \sum_{v \neq 1} v \cdot I_{c\delta(v)}^2 \right) \quad (42)$$

By replacing relations (38) and (42) in relation (33), the expression for the global equivalent factor of the a.c. modifying inductance is obtained:

$$\begin{aligned}
 k_{x\delta(CSF)} &= \frac{q_{\delta(CSF)}^-}{q_{\delta(CSF)}^-} = \\
 &= \frac{\omega_1 L_{\delta\sigma} \left(k_{x\delta(1)} \cdot I_{c\delta(1)}^2 + \sum_{v \neq 1} v \cdot k_{x\delta(v)} \cdot I_{c\delta(v)}^2 \right)}{\omega_1 L_{\delta\sigma} \left(I_{c\delta(1)}^2 + \sum_{v \neq 1} v \cdot I_{c\delta(v)}^2 \right)} = \quad (43) \\
 &= \frac{k_{x\delta(1)} + \sum_{v \neq 1} \left[v \cdot \left(\frac{I_{c\delta(v)}}{I_{c\delta(1)}} \right)^2 \cdot k_{x\delta(v)} \right]}{1 + \sum_{v \neq 1} \left[v \cdot \left(\frac{I_{c\delta(v)}}{I_{c\delta(1)}} \right)^2 \right]}
 \end{aligned}$$

4 Determination of equivalent parameters of the winding rotor, considering the current repression

The rotor winding’s parameters are affected by the skin effect, at the start of the motor and also at the nominal operating regime. For establishing the relations that define these parameters, considering the skin effect, the expression of the rotor phase impedance reduced to the stator is used. For this, the rotor with multiple bars is replaced by a rotor with a single bar on the pole pitch [2]. Initially is considered only the fundamental present in the power supply of the motor. The rotor impedance reduced to the stator has the equation:

$$Z'_{2(1)} = \frac{R'_{2(1)}}{s(1)} + jX'_{2(1)} \quad (44)$$

Knowing that the induced e.m.f. by the fundamental component of the main magnetic field from the machine in the pole pitch bars is:

$$U_{e(1)} = I'_{2(1)} \cdot Z'_{2(1)}, \quad (45)$$

where, for the general case of multiple cages is valid the relation [2]:

$$I'_{2(1)} = \sum_{\delta=1}^c I'_{c\delta(1)} = \frac{U_{e(1)}}{\Delta_{(1)}} \sum_{\delta=1}^c \Delta_{\delta(1)}. \quad (46)$$

In the relation (22), the number of the cages and respectively the rotor bars/ pole pitch is equal with “c”. In the case of motors with the power up to 45 [kW], that are the subject of this paper, c=1 (simple cage or high bars) or c=2 (double cage). $\Delta_{(1)}$ is the determinant corresponding to the equation system [2], [3]:

$$U_{e(1)} = \sum_{\epsilon=1}^c R_{\delta\epsilon(1)} \cdot I_{c\epsilon(1)}, \quad \epsilon=1, 2, \dots, c. \quad (47)$$

having the expression [2], [3]:

$$\Delta_{(1)} = \begin{vmatrix} \underline{R}_{11(1)} & \dots & \underline{R}_{1n(1)} \\ \vdots & & \vdots \\ \underline{R}_{n1(1)} & \dots & \underline{R}_{nn(1)} \end{vmatrix}. \quad (48)$$

$\Delta_{\delta(1)}$ is the determinant corresponding to the fundamental obtained from $\Delta_{(1)}$, where the column δ is replaced by a column of 1 [2], [3]:

$$\Delta_{\delta(1)} = \begin{vmatrix} \underline{R}_{1(1)} \dots \underline{R}_{1,\delta-1(1)} & 1 & \underline{R}_{1,\delta+1(1)} \dots \underline{R}_{1n(1)} \\ \vdots & & \vdots \\ \underline{R}_{n(1)} \dots \underline{R}_{n,\delta-1(1)} & 1 & \underline{R}_{n,\delta+1(1)} \dots \underline{R}_{nn(1)} \end{vmatrix}. \quad (49)$$

Because in the first phase the steady-state regime is under focus, the phenomenon from the rotor corresponding to the fundamental has the pulsation $\omega_{2(1)} = s\omega_1$, where s is the motor slip for the sinusoidal power supply, in the steady-state regime. If the relation (22) is introduced in (21), the expression of the equivalent impedance of the rotor phase reduced to the stator, corresponding to the fundamental, valid when considering the skin effect is obtained:

$$Z'_{2(1)} = \frac{\Delta_{(1)}}{\sum_{\delta=1}^c \Delta_{\delta(1)}}. \quad (50)$$

Thus, the expressions for the rotor phase resistance and inductance reduced to the stator, corresponding to the fundamental, both affected by the skin effect can be written.

$$\frac{R'_{2(1)}}{s(1)} = \Re e \left[Z'_{2(1)} \right], \quad (51)$$

$$X'_{2(1)} = \Im m \left[Z'_{2(1)} \right]. \quad (52)$$

By considering in the motor power supply of the v harmonic only, similar expressions are obtained for the corresponding rotor parameters. Thus:

$$Z'_{2(v)} = \frac{\Delta_{(v)}}{\sum_{\delta=1}^c \Delta_{\delta(v)}}, \quad (53)$$

$$\frac{R'_{2(v)}}{s(v)} = \Re e \left[Z'_{2(v)} \right], \quad (54)$$

$$X'_{2(v)} = \Im m \left[Z'_{2(v)} \right]. \quad (55)$$

Following we consider the real case of an electric induction machine powered by a frequency converter and it follows the determination of the rotor winding equivalent parameters (corresponding to all harmonics, including the fundamental). For the beginning, the case of simple cage respectively high bars induction motors will be analyzed. Thus, a

rotor phase resistance corresponding to the fundamental, $R'_{2(1)}$, and rotor phase resistance corresponding to higher order harmonics $R'_{2(v)}$ ($v=Jm_f \pm k$) are replaced by an equivalent resistance $R'_{2(CSF)}$, which dissipates the same part of active power as in the case of “v” resistances. This equivalent resistance is defined at the fundamental’s and is crossed by $I'_{2(CSF)}$ current:

$$I'_{2(CSF)} = \sqrt{I_{2(1)}'^2 + \sum_{v \neq 1} I_{2(v)}'^2} \quad (56)$$

Whereas in the determination of global equivalent factor of a.c. resistance change, that $k_{r(CSF)}$, the principle of conservation of active power has been used, it is sufficient that for the deduction of the rotor equivalent resistance expression to start from the global factor expression (see relation 32)). Thus, for the rotor phase equivalent resistance reduced to the stator, corresponding to all harmonics, defined at fundamental’s frequency, on can write:

$$R'_{2(CSF)} = k_{r(CSF)} \cdot R'_{2c} + R'_{2i} \quad (57)$$

were: R'_{2c} is the resistance, considered at fundamental’s frequency, of a part from rotor phase winding from slots and reported to the stator; R'_{2i} is the resistance considered at fundamental’s frequency, part of rotor winding with neglecting skin effect, reported to the stator; $k_{r(CSF)}$ is the global modification factor of rotor winding resistance, having the expression given by the relation (32).

To track the changes that appear on the resistance of the rotor winding when the machine is supplied through a frequency converter, compared with the case when the machine is fed in sinusoidal regime, the $k_{R'2}$ factor is introduced:

$$k_{R'_2} = \frac{R'_{2(CSF)}}{R'_2} \quad (58)$$

where R'_2 is the rotor winding resistance reported to the stator, when the machine is fed in sinusoidal regime:

$$R'_2 = k_r R'_{2c} + R'_{2i} \quad (59)$$

where k_r is the modification factor of the a.c. rotor resistance, in the case of sinusoidal: $k_r \cong k_{r(1)}$. If the relations (33) and (36) are inserted in (35), is obtained:

$$k_{R'_2} = \frac{k_{r(CSF)} R'_{2c} + R'_{2i}}{k_r R'_{2c} + R'_{2i}} \quad (60)$$

If both the nominator and the denominator of second member on the relation (60) are divided with k_r and then with R'_{2c} , the following expression is obtained:

$$k_{R'_2} = \frac{\frac{k_{r(CSF)}}{k_r} + \frac{R'_{2i}}{R'_{2c}} \cdot \frac{1}{k_r}}{1 + \frac{R'_{2i}}{R'_{2c}} \cdot \frac{1}{k_r}} = \frac{k_{kr} + r_2 \cdot \frac{1}{k_r}}{1 + r_2 \cdot \frac{1}{k_r}} \quad (61)$$

where:

$$r_2 = \frac{R'_{2i}}{R'_{2c}} \cong const. \quad (62)$$

which is constant for the same motor, at a given fundamental’s frequency. For $c=1$, $k_{kr} > 1$, results that $k_{R'2} > 1$, which means that $R'_{2(CSF)} > R'_2$ also.

The procedure is similar for the reactance. The rotor phase reactance, corresponding to the fundamental, $X'_{2(1)}$, and also the reactance corresponding to the superior harmonics, $X'_{2(v)}$, are replaced with an equivalent reactance $X'_{2(CSF)}$. As in case of rotor resistance, we can write:

$$k_{X'_2} = \frac{X'_{2(CSF)}}{X'_2} \quad (63)$$

where $X'_{2(CSF)}$ is the equivalent reactance of the rotor phase, reduced to the stator, corresponding to all harmonics, including the fundamental, on the fundamental’s frequency:

$$X'_{2(CSF)} = k_{X(CSF)} X'_{2c} + X'_{2i} \quad (64)$$

and X'_2 is the reactance of the rotor phase reduced to the stator, witch characterized the machine when it is fed in sinusoidal regime:

$$X'_2 = k_X X'_{2c} + X'_{2i} \quad (65)$$

In relation (41) and (42), was noted:

X'_{2c} -the reactance of the rotor winding part from the slots, reduced to the stator, in witch the skin effect is present; X'_{2i} - the reactance of the rotor winding phase, part, where the skin effect can be neglected. $k_{X(CSF)}$ - is defined in relation (43), where $c \cong 1$.

Taking account of the relations (64) and (65), relationship (63) becomes:

$$k_{X'_2} = \frac{k_{X(CSF)} X'_{2c} + X'_{2i}}{k_X X'_{2c} + X'_{2i}} = \frac{\frac{k_{X(CSF)}}{k_X} + \frac{X'_{2i}}{X'_{2c}} \cdot \frac{1}{k_X}}{1 + \frac{X'_{2i}}{X'_{2c}} \cdot \frac{1}{k_X}} \quad (66)$$

$$= \frac{k_{k_X} + x_2 \frac{1}{k_X}}{1 + x_2 \frac{1}{k_X}}$$

where:

$$x_2 = \frac{X'_{2i}}{X'_{2c}} \quad (67)$$

is a constant for the same motor at a given fundamental's frequency $k_{kX} < 1$, with the consequences $k_{X'2} < 1$ and $X'_{2(CSF)} < X'_{2}$ [2]. With this, the impedance of a rotor phase reported to the stator in the case of a machine supplied by a power converter, receives the form:

$$\underline{Z}'_{2(CSF)} = \frac{R'_{2(CSF)}}{s_{(CSF)}} + jX'_{2(CSF)}, \quad (68)$$

where:

$$s_{(CSF)} = \frac{R'_{2(CSF)} I'_{2(CSF)}}{U_{el(CSF)}}. \quad (69)$$

where:

$$U_{el(CSF)} = \sqrt{U_{el(1)}^2 + \sum_{v \neq 1} U_{el(v)}^2}. \quad (70)$$

In the case of double cage induction motors, the rotor parameters are necessary to be determined for both cages. The principle of calculation keep its validity from the above presented case, the induction motors with simple cage, respectively cage with high bars, with one remark: in the the relations for determination of $k_{r(CSF)}$ respectively $k_{x(CSF)}$, is considered $c=2$ (for $\delta=1$ results the working work cage, and for $\delta=c=2$ results the startup cage).

The complex structure of the used algorithm, and its component computing relationships synthetic presented in the paper, request a very high volume of calculation. This makes that the presence of the computer in solving this problem is absolutely necessary. In the Laboratory of systems dedicated to control the electrical servomotors from the

Politehnica University of Timișoara has designed the software calculation CALCMOT, which allows the determination and analysis of factors $k_{r(CSF)}$ respectively $k_{x(CSF)}$, and the parameters of the equivalent winding machine induction in the non-sinusoidal regime. This analysis allows to conclusion in quantitative - qualitative influence non-sinusoidal regime due to power converter machine by the skin effect compared to the sinusoidal supply.

5 Experimental validation

The induction machines which have been tested are: MAS 0,37 [kW] x 1500 [rpm] and MAS 1,1 [kW] x 1500 [rpm]. To validate the experimental studies of the theoretical work, tests were made both for the operation of motors supplied by a system of sinusoidal voltages, and for the operation in case of static frequency converter supply. With test of the short cut at variable frequency were determined parameters of stator and rotor windings at different frequencies. The values resulting from the tests are presented in tables 1 (sinusoidal supply) and 2 (supply through converter) -for 0.37 [kW] machine) and tables 3 (the sinus) and 4 (the non-sinusoidal) – for 1.1 [kW] machine).

For the inductivity separation was considered the value of L_1 , calculated at 50 Hz. The value of $L_{1(CSF)}$ was equal to the product of k_{X1} (factor calculated using the CALCMOT program) and L_1 , and the result is also constant.

Table 1. The results of the shortcut probe with variable frequency supply, for the induction motor with 0.37 [kW] x 1500 [rpm].

Nr.	$f_1=f_2$ [Hz]	P_{sc} [W]	I_{sc} [A]	U_{1sc} [V]	$R_1+R'_2$ [Ω]	R_1 [Ω]	R'_2 [Ω]	$X_1+X'_2$ [Ω]	$L_1+L'_2$ [H]
1.	59,98	132,817	1,021	64,526	42,470	25,9	16,57	46,801	0,124
2.	50,58	145,214	1,071	62,433	42,199	25,9	16,29	40,217	0,126
3.	39,96	143,719	1,070	56,656	41,843	25,9	15,94	32,447	0,129
4.	30,03	114,901	0,963	46,611	41,300	25,9	15,4	25,23	0,133
5.	25,95	164,591	1,156	53,773	41,055	25,9	15,155	21,869	0,134

Table 2. The results of the shortcut probe with variable frequency, supplied through power converter, for the induction motor with 0.37 [kW] x 1500 [rpm].

Nr.	$f_{1(1)}=f_{2(1)}$ [Hz]	$P_{sc(CSF)}$ [W]	$I_{sc(CSF)}$ [A]	$U_{1sc(CSF)}$ [V]	$R_{1(CSF)}+R'_{2(CSF)}$ [Ω]	$R_{1(CSF)}$ [Ω]	$R'_{2(CSF)}$ [Ω]	$X_{1(CSF)}+X'_{2(CSF)}$ [Ω]	$L_{1(CSF)}+L'_{2(CSF)}$ [H]
1.	25	160,869	1,12	53,135	42,748	26	16,748	20,577	0,131
2.	30	168,975	1,15	56,291	42,59	26	16,59	24,127	0,128
3.	40	188,491	1,21	64,532	42,914	26	16,914	31,667	0,126
4.	50	181,729	1,18	68,908	43,505	26	17,505	38,955	0,124
5.	60	180,238	1,17	74,699	43,889	26	17,889	46,369	0,123

Table 3 The results of the shortcut probe with variable frequency supply, for the induction motor with 1.1 [kW] x 1500 [rpm].

Nr.	$f_1=f_2$ [Hz]	P_{sc} [W]	I_{sc} [A]	U_{Isc} [V]	$R_1+R'_2$ [Ω]	R_1 [Ω]	R'_2 [Ω]	$X_1+X'_2$ [Ω]	$L_1+L'_2$ [H]
1.	60,24	70,515	1,41	31,130	11,822	5,53	6,292	18,646	0,049
2.	56,38	67,155	1,384	29,163	11,686	5,53	6,156	17,534	0,049
3.	50,67	64,673	1,368	26,923	11,519	5,53	5,989	15,957	0,050
4.	44,65	63,613	1,368	24,841	11,330	5,53	5,8	14,190	0,050
5.	36,09	77,460	1,525	24,582	11,102	5,53	5,572	11,686	0,051
6.	30,81	68,332	1,442	21,587	10,95	5,53	5,42	10,208	0,052
7.	19,96	61,084	1,383	17,687	10,64	5,53	5,11	7,095	0,056
8.	15,66	69,514	1,488	17,811	10,46	5,53	4,93	5,81	0,059
9.	10,87	65,654	1,464	16,308	10,21	5,53	4,68	4,454	0,065
10.	6,92	59,761	1,416	14,887	9,93	5,53	4,4	3,453	0,079

Table 4. The results of the shortcut probe with variable frequency, supplied through power converter, for the induction motor with 1.1 [kW] x 1500 [rpm].

Nr.	$f_{1(1)}=f_{2(1)}$ [Hz]	$P_{sc(CSF)}$ [W]	$I_{sc(CSF)}$ [A]	$U_{Isc(CSF)}$ [V]	$R_{1(CSF)}+R'_{2(CSF)}$ [Ω]	$R_{1(CSF)}$ [Ω]	$R'_{2(CSF)}$ [Ω]	$X_{1(CSF)}+X'_{2(CSF)}$ [Ω]	$L_{1(CSF)}+L'_{2(CSF)}$ [H]
1.	20	73,356	1,45	19,342	11,63	5,55	6,08	6,534	0,052
2.	30	70,170	1,42	21,393	11,6	5,55	6,05	9,613	0,051
3.	40	68,628	1,39	23,871	11,84	5,55	6,29	12,440	0,0495
4.	50	64,517	1,34	25,804	11,977	5,55	6,427	15,079	0,048
5.	60	74,707	1,42	30,8	12,35	5,55	6,8	17,831	0,0473

In Tables 5 and 6 are presented theoretical values (obtained by running the calculation program) and the results of measurements, for $k_{R'2}$ and $k_{X'2}$, factors, respectively the calculation errors of, for both motors tested.

Table 5. The theoretical and experimental values of factors $k_{R'2}$ and $k_{X'2}$, respectively the errors of calculation, corresponding to 0.37 [kW] x 1500 [rpm] MAS.

Nr.	$f_{1(1)}$ [Hz]	$k_{R'2} = \frac{R'_{2(CSF)}}{R'_2}$ (calculated)	$k_{R'2}$ (measured)	$\epsilon_{k_{R'2}}$ [%]	$k_{X'2} = \frac{X'_{2(CSF)}}{X'_2}$ (calculated)	$k_{X'2}$ (measured)	$\epsilon_{k_{X'2}}$ [%]
1.	25	1,048	1,11	5,58	0,863	0,894	3,6
2.	30	1,026	1,077	4,97	0,912	0,857	-6,03
3.	40	1,021	1,061	3,77	0,944	0,884	-6,35
4.	50	1,014	1,075	6,01	0,967	0,897	-7,23
5.	60	1,011	1,079	6,82	0,975	0,914	-6,25

Table 6. The theoretical and experimental values of factors $k_{R'2}$ și $k_{X'2}$, respectively the errors of calculation, corresponding to 1.1 [kW] x 1500 [rpm] MAS.

Nr.	$f_{1(1)}$ [Hz]	$k_{R'2} = \frac{R'_{2(CSF)}}{R'_2}$ (calculated)	$k_{R'2}$ (measured)	$\epsilon_{k_{R'2}}$ [%]	$k_{X'2} = \frac{X'_{2(CSF)}}{X'_2}$ (calculated)	$k_{X'2}$ (measured)	$\epsilon_{k_{X'2}}$ [%]
1.	20	1,098	1,185	7,92	0,812	0,821	1,108
2.	30	1,041	1,120	7,58	0,886	0,916	3,386
3.	40	1,034	1,106	6,96	0,926	0,891	-3,77
4.	50	1,023	1,089	6,45	0,956	0,863	-9,72
5.	60	1,018	1,082	6,28	0,966	0,871	-9,83

6 Conclusions

1. In the case of the machine supplied through a power converter it can consider that the motor presents an equivalent resistance of the stator winding $R_{1(CSF)}$, defined by the fundamental frequency, whose value does not differ from the case of sinusoidal supply. Stator winding, so the resistance $R_{1(CSF)}$ which characterizes it, is crossed by current $I_{1(CSF)}$. Differences in the table between R_1 and $R_{1(CSF)}$ is because during the tests where the machine was supplied through the converter, the servomotor has warmed more than in the case of supplying it direct from network. This supplemental heat is due to the presence in the wave of supply voltages of higher harmonics generated by the inverter. (the current "harmonics").

2. In the case of the machine supplied by the power converter it can consider that the stator winding is characterized by an equivalent reactance $X_{1(CSF)}$, crossed by current $I_{1(CSF)}$ and determined at the fundamental frequency. This reactance, compared to the sinusoidal supply suffers a growing, the factor which considers this increase quantitatively, k_{X1} , having a over unit value (the result appears as normal, taking into account that the stator winding for the considered powers in the paper, the phenomenon of repression is neglected).

3. Applying the principle of conservation of active and reactive powers, expressions for calculating the equivalent resistance $R'_{2(CSF)}$, respectively equivalent reactance $X'_{2(CSF)}$, corresponding to the rotor phase in the condition of repression and a non-sinusoidal supply were determined. Both parameters were determined as function of fundamental frequency. Comparing these equivalent parameters of the machine supplied by power converter with the corresponding parameters of a sinusoidal supply, it is found that resistance of the rotor phase suffer an increase, and the reactance suffers a decrease in comparison with the case of sinusoidal regime. Changes are even more important as the motor's power rating is higher. The explanation resides in the fact that as the driving power increases, the skin effect which is present in rotor bars is more pronounced (the height of slots increase).

4. Parameters of the winding machine supplied by the power converter can be calculated with errors less than 10 [%]. The main cause of errors is the assumption of saturation neglect. Even in this case the results can be considered satisfactory, which leads to validate the theoretical study carried out in the paper.

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