Research on Parameter Identification of Modified Friction LuGre Model Based Distributions Theory

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Abstract: - Based on distributions theory to continuous time systems with friction using the this paper presents a batch on-line method for the parameter identification of the modified LuGre friction model. Mainly it applies the results of the identification procedures based on distributions theory to continuous time systems with friction. There are defined the so called generalized friction dynamic systems (GFDS) as a closed loop structure around a smooth system with discontinuous feedback loops representing friction reaction vectors. Both GFDS with static friction models (SFM) and dynamic friction models (DFM), also the modified friction LuGre model is analyzed. The identification problem is formulated as a condition of vanishing the existence relation of the system. Then, this relation is represented by functionals using techniques from distribution theory based on testing function from a finite dimensional fundamental space. The advantage of representing information by distributions are pointed out when special evolutions as sliding mode, or limit cycle can appear.The proposed method is a batch on-line identification because identification results are obtained during the system evolution after some time intervals but not in any time moment. This method does not require the derivatives of measured signals for its implementation. Some experimental results are presented to illuminate its advantages and practical use. At last, the simulation results have shown effectiveness of the proposed method for friction parameter identification.

Keywords: - Identification; Distribution theory; Friction; the modified friction LuGre model.

1 Introduction

Motion in many mechanical, hydraulic or pneumatic systems is influenced by the so-called friction forces because of interactions with the environment or of the interaction between their components.

Friction is a complex phenomenon, not yet completely known, with many different physical causes, so it is a difficult task to model it. Such models contain some specific nonlinearity such as stiction, hysteretic, Stribeck effect, stick-slip, depending on velocity [1], [2], [3], [4]. These models depend on many parameters whose values can change during the system evolution or are influenced by some other causes as external temperature, quality of materials etc. In literature there are accepted a large variety of friction models as Coulomb friction model [1], LuGre model [24], Dahl model [5], [6], exponential model [7], bristle model [8], state variable model [9].

Ignoring friction in controlling such systems can lead to tracking errors, limit cycles, undesired stick-slip motion [2]. To avoid these difficulties, adaptive control strategies, named model-based friction compensation techniques [2], are recommended. Such adaptive strategies involve identification procedures of the controlled system, including identification of the model friction parameters.

Unfortunately friction models are nonlinear, involving a discontinuous dependence with respect to velocity. Because of this, many techniques, as identification based on time-discretized models fail to offer good results.

A survey of models, analysis tools and compensation methods for the control of machines with friction is presented in [11]. Furthermore, the application of classical identification methods for continuous time friction models requires the acceleration measurement that is not an easy task. A frequency domain approach to identification of mechanical systems with friction is developed in [12], which does not require the acceleration information but the procedure is available for periodic excitation input only.

Good results on continuous time system identification based on distribution theory are reported in [13], for linear systems, or in [14], for nonlinear systems.

Because of their discontinuities, identification of systems with frictions is much more difficult. One way is to perform continuous time domain identification transforming the system differential equations to an algebraic system that reveals the unknown parameters [15], [16], [17]. This can be done by using some modulating functions to generate functionals to avoid the direct computation of the input-output data derivatives [18], [19].

From computational point of view many advantages are obtained by using the classic methods based on orthogonal functions. The main disadvantages of them are the strong sensitivity to nonzero initial conditions and the fixed time interval for integrals.

A novel approach to identify the continuous-time system is the distribution-based approach, using deterministic distributions [13], or random distributions [20]. Through these approaches, derivatives are described in the sense of distribution theory and construct the input-output algebraic relationships using differential information produced in the distribution sense.

This paper extends the procedures of [13], [14[, based on distributions, for parametric identification in continuous time systems with friction. By this method, it is possible to perform identification of these systems, processing only information on position and the sign of the velocity in any consistent transient response.

The proposed method is a batch on-line identification method because identification results are obtained during the system evolution after some time intervals but not in any time moment. Even if it is based on the input-output measurements only, the method is insensitive to the initial state of any transient.

The paper is organized as follows: After introduction in the first section, Section 2 is dedicated to the generalized friction dynamic systems GFDS, as presented in [10]. Section 3, presents the main steps the of continuous time system identification based on distributions. Section 4 illustrates application for a friction mechanical system identification using the modified friction LuGre model . Section 5 presents experimental results, again conclusions are resumed in Section 6.

2 Generalized Friction Dynamic Systems

As presented in [12], a generalized dynamic friction system (GFDS) is a system characterized by the state equation of the form

$$\dot{x} = f(x,u,r_1,..,r_i,..,r_p) \quad (1)$$

where x, is the state vector and u is the input vector. The vectors r_i are called friction reaction vectors. They depend on x and u through a specific operator $\Psi_i\{\}$, called friction operator,

$$r_i = \Psi_i\{x,u\}, i = 1:p \quad (2)$$

There are two categories of friction models: static friction models (SFM) and dynamic friction models.(DFM). For SFM, we deal with only in this paper, the operator (2) is a non-dynamic mapping with a specific structure as follows. For any $i = 1:p$,there are two functions $v_i = \nu_i(x,u)$, which determines the so called generalized velocity vector v_i, and $a_i = \alpha_i(x,u)$,expressing the so called active component of the velocity vector v_i. In SFM, the non dynamic mapping (2) can be expressed as a function of v_i, and a_i only, as depicted in Figure 2, $r_i = F_i\{x,u\}$.

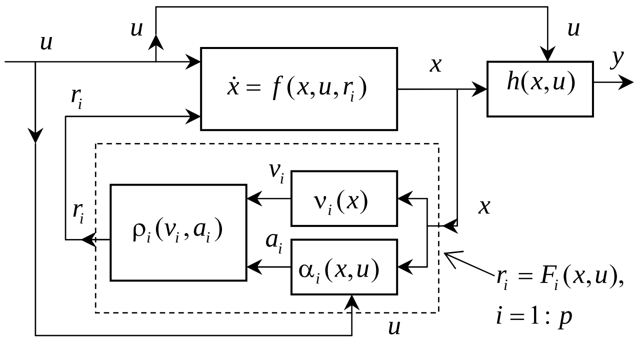


Fig.1, The feedback structure of a GDFS with SFM.

Inspired from mechanics [11], the function ρ_i is explicitly defined for $v_i = 0$ and for $v_i \neq 0$. As a result, two components of the friction reaction vectors $r_i, i = 1:p$ can be defined: static friction reaction r_i^s and cinematic friction reaction r_i^c, where, particularly, $r_i = r_i^s + r_i^c$,

$$r_i^s = \rho_i^s(v_i,a_i), v_i = 0;\ r_i^s = 0, v_i \neq 0; \quad (3)$$

$$r_i^c = \rho_i^c(v_i,a_i), v_i \neq 0;\ r_i^c = 0, v_i = 0 \quad (4)$$

The adjective static and dynamic, for the friction reaction vectors $r_i, i = 1:p$, must be understood with respect to the velocity vector v_i only. Also, for a vector $v_i \in \mathrm{R}^{m_i}, i = 1:p$, it is defined the function $sgn(v_i)$ as $sgn(v_i) = v_i / \|v_i\|$,where $\|v_i\|$ is the Euclidian norm of R^{m_i}. In this norm the function $sgn(v_i)$is a discontinuous function in the point $v_i = 0$. It is observed that

$$\|sgn(v_i)\| = sgn(\|v_i\|) = 1,\ v_i \neq 0,\ sgn(\|0\|) = 0 \quad (5)$$

If $m_i = 1$, v_i is a scalar variable, then (12) can be presented by using inequalities. Because of (3) and (4), the system state vector evolution $x(t)$ is characterized by a status of two values, related to each friction reaction vectors $r_i, i = 1:p$,

1. Evolution inside a surface characterized by zero value of the velocity vector v_i, $x(t) \in \mathrm{S}_i$, where $\mathrm{S}_i = \{x \in \mathbf{X}, v_i = 0\} = \{x \in \mathbf{X}, v_i(x) = 0\}$.

2. Evolution with nonzero value of the velocity vector v_i, that means outside the surface S_i, $x(t) \notin \mathrm{S}_i$.

Outside the surface S_i, r_i is a vector opposite to $v_i \neq 0$ but inside the surface S_i, r_i is a vector opposite to a_i. There is a closed subset $\mathrm{S}_i^0(u) \subseteq \mathrm{S}_i$, called sticky area (SA), which keeps the system state inside . This means

$$\tfrac{d}{dt} v_i(x(t)) = [\tfrac{d}{dx} v_i(x)]^T \cdot \dot{x}(t) = 0, \forall x \in \mathrm{S}_i^0(u). \quad (6)$$

Inside the SA $r_i = -a_i$. Because the input u can change the SA position the state x can be forced to be out of $\mathrm{S}_i^0(u)$, crossing its border. For any admissible u, the function $r_i = F_i(x,u)$ is continuous with respect to $\forall x \in \mathrm{S}_i^0(u)$. Because of this, when the system state $x(t)$ arrives on or leaves out $\mathrm{S}_i^0(u)$ the friction reaction $r_i(t)$ is a continuous time function. Condition c, is called the smooth sticky condition (SSC). However, when $x(t) \in \mathrm{S}_i \setminus \mathrm{S}_i^0(u)$, $r_i(t)$ has a discontinuity and $\frac{d}{dt} v_i(x(t)) \neq 0$. In this case $x(t)$ passes from one side to other of $\mathrm{S}_i \setminus \mathrm{S}_i^0(u)$, as a switching mode or as a sliding mode. For example, expressions as (7) and (8) of (3) and (4) respectively, satisfy the above conditions, where by a_i it must understand $a_i = a_i(x,u)$,

$$r_i^s = \rho_i^s(v_i, a_i) = -max\{Q_i, \|a_i\|\} \cdot sgn(a_i) \cdot [1 - sgn(\|v_i\|)] \quad (7)$$

$$r_i^c = \rho_i^c(v_i, a_i) = -[Q_i + K_{vi} \cdot \|v_i\| + B_i \cdot (e^{\beta_i \|v_i\|} - 1)] \cdot sgn(v_i) \quad (8)$$

As it can be observed, the cinematic reaction r_i^c is a sum of three components, $r_i^{cc}, r_i^{cv}, r_i^{cs}$ expressing respectively Coulomb friction, viscous friction and the so called Stribeck effect, [4], [11],

$$r_i^c = r_i^{cc} + r_i^{cv} + r_i^{cs}. \quad (9)$$

For $m_i = 1$, all r_i, a_i, v_i are scalar variables so the static reaction (7), r_i^s, is illustrated in Fig.2a and the cinematic reaction (8), r_i^c, in Fig.2b.

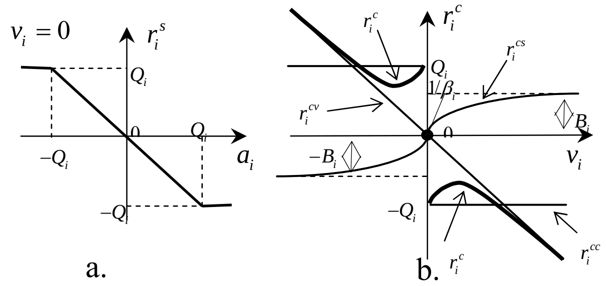


Fig.2, Static and cinematic components of a scalar friction reaction.

A friction reaction vector r_i, as above defined, has a sticky characteristic which means there is a subset $\mathrm{S}_i^{\mathrm{S}}(u) \subseteq \mathrm{S}_i$, called sticky set (SS), such

$$\dot{v}_i(t) = d/dt\{v_i(x(t))\} = 0, \forall x(t) \in \mathrm{S}_i^{\mathrm{S}}(u(t)) \subseteq \mathrm{S}_i \quad (10)$$

The position of SS depends on input vector u. When the system state $x(t)$ approaches $\mathrm{S}_i^{\mathrm{S}}(u)$, generated by a vector r_i, it remains inside of that SS till the input $u(t)$ changes the position of $\mathrm{S}_i^{\mathrm{S}}(u)$, forcing $x(t)$ to be outside of it. Substituting (4) into (1) and denoting

$$\mathbf{f}(x,u) = f(x,u,F_1(x,u),..,F_i(x,u),..,F_p(x,u)) \quad (11)$$

the GDFS takes the compact form

$$\dot{x} = \mathbf{f}(x,u),\ x(t_0) = x_0, t \geq t_0. \quad (12)$$

This is a differential system with a discontinuous function on right side so for its analytical description, special mathematical approaches are necessary. For example approaches describing the solution in the Charatheodory sense [4], using the Filippov approach or differential inclusions and differential inequalities. However, for the identification it is supposed a solution exist for (28) and are available as measurements the input variable u and the output variable y where

$$y = h(x,u). \quad (13)$$

3 Continuous Time System Identification Based on Distributions

This section presents the main results on continuous time system identification based on distribution, as have been presented in [11]. Let Φ_n be the fundamental space from distribution theory [17] of the real testing functions, $\varphi: \mathbb{R} \to \mathbb{R}, t \to \varphi(t)$, having continuous derivatives at least up to the order n, with compact support T for any of the above derivative. The linear space Φ_n is organized as a topological space considering a specific norm [17]. A distribution is a linear, continuous real functional on Φ_n, $F:\Phi_n \to \mathbb{R}$, $\varphi \to F(\varphi) \in \mathbb{R}$.

Let $q:\mathbb{R} \to \mathbb{R}$, $t \to q(t)$ be a function that admits a Riemann integral on any compact interval T from $\mathbb{R}$. Using this function, a unique distribution $F_q:\Phi_n \to \mathbb{R}$, $\varphi \to F_q(\varphi) \in \mathbb{R}$ can be build by the relation $F_q(\varphi) = \int_{\mathbb{R}} q(t)\cdot\varphi(t)\cdot dt$, $\forall \varphi \in \Phi_n$.

Considering, at least, $q \in C^0(\mathbb{R})$, the following important equivalence take place [18],

$$F_q(\varphi) = 0, \forall \varphi \in \Phi_n \Leftrightarrow q(t) = 0, \forall t \in \mathbb{R} \tag{14}$$

The m-order derivative of a distribution is a new distribution, $F_q^{(m)} \in \Phi_n^{/}$ uniquely defined by the relations,

$$F_q^{(m)}(\varphi) = (-1)^m \cdot F_q(\varphi^{(m)}), \forall \varphi \in \Phi_n \tag{15}$$

When $q \in C^m(\mathbb{R})$, then

$$F_q^{(m)}(\varphi) = F_{q^{(m)}}(\varphi) = \int_{\mathbb{R}} q^{(m)}(t)\varphi(t)dt$$

that means the k-order derivative of a distribution generated by a function $q \in C^m(\mathbb{R})$ equals to the distribution generated by $q^{(m)}$, the k-order time derivative of the function q. If $q \in C^m(\mathbb{R})$, from (13), (16) one can write, $\forall \varphi \in \Phi_n$

$$F_q^{(m)}(\varphi) = \int_{\mathbb{R}} q^{(m)}(t)\varphi(t)dt = (-1)^m \int_{\mathbb{R}} q(t)\cdot\varphi^{(m)}(t)\cdot dt \tag{16}$$

Let us consider a dynamical continuous time system expressed by a differential operator,

$$q_{\theta/(u,y)} = F_C(u,y,\theta) \tag{17}$$

whose expression depends on a vector of parameters

$$\theta = [\theta_1 \ldots \theta_i \ldots \theta_p]^T. \tag{18}$$

It represents a family of models with a given structure in constant parameters. A special case is the model (17) expressing a linear relation in the parameters

$$q_{\theta/(u,y)} = F_C(u,y,\theta) = \sum_{i=1}^{p} w_i \cdot \theta - v = w^T \cdot \theta - v, \tag{19}$$

where w_i and v represent a sum of the derivatives of some known, possible nonlinear, functions ψ_i^j, ψ_0^j, with respect to the input and output variables,

$$w_i = \sum_{j=1}^{p_i} [\psi_i^j(u,y)]^{(n_i^j)}, i = 1:p, \tag{20}$$

$$v = \sum_{j=1}^{p_0} [\psi_0^j(u,y)]^{(n_0^j)}. \tag{21}$$

where parameters p_i, n_i^j, p_0, n_0^j are given integer numbers. In [11] the existence and uniqueness conditions for a problem of distribution based continuous time system identification are presented. Suppose that it is possible to record the functions (u, y) in an the time interval $T \subset \mathbb{R}$, called observation time interval or just time window. The restriction of the functions (u,y) to the time interval T is denoted by (u_T, y_T) respectively. If no confusion would appear, then we may drop the subscript T.

An identification problem means to determine the parameter $\theta = \hat{\theta}$, given the priori information on the model structure F_C, (17), and a set of observed input-output pairs (u_T, y_T), $\hat{\theta} = \hat{\theta}(u_T, y_T, F_C)$ in a such a way that,

$$q_{\hat{\theta}/(u_T,y_T)}(t) = 0, \forall t \in \mathbb{R} \tag{22}$$

This condition involves,

$$q_{\hat{\theta}/(u,y)}(t) = 0, \forall t \in \mathbb{R}, \forall (u,y) \in \Omega \times \Gamma \tag{23}$$

for any input-output pair (u,y) observed to that system.

Let us consider two families of regular distributions, F_{w_i}, $i = 1:p$, and $F_v(\varphi)$ created based on the functions (20), (21),

$$\begin{aligned} F_{w_i}(\varphi) &= \sum_{j=1}^{p_i} \int_{\mathrm{R}} [\psi_i^j(t)]^{(n_i^j)} \varphi(t)dt = \\ &= \sum_{j=1}^{p_i} (-1)^{n_i^j} \int_{\mathrm{R}} [\psi_i^j(t)] \varphi^{(n_i^j)}(t)dt \end{aligned} \tag{24}$$

which determines the row vector,

$$F_w^T(\varphi) = [F_{w_1}(\varphi), \ldots, F_{w_i}(\varphi), \ldots, F_{w_p}(\varphi)] \in \mathbb{R}^p. \tag{25}$$

$$\begin{aligned} F_v(\varphi) &= \sum_{j=1}^{p_0} \int_{\mathrm{R}} [\psi_0^j(t)]^{(n_0^j)} \varphi(t)dt = \\ &= \sum_{j=1}^{p_0} (-1)^{n_0^j} \int_{\mathrm{R}} [\psi_0^j(t)] \varphi^{(n_0^j)}(t)dt \end{aligned} \tag{26}$$

Any input-output pair (u,y) observed from the system (27) is described by a pair of regular distribution (F_w, F_v) for any $\varphi \in \Phi_n$, [11]. The problem of the system (17) parameter identification can be represented now by distributions. For example, the regular distribution generated by the continuous function $q_{\theta/(u,y)}$ from (17), is related to the parameter vector θ, $\forall \varphi \in \Phi_n$ as (27)

$$\begin{aligned} F_{q_\theta}(\varphi) &= F_{q_\theta/(u,y)}(\varphi) = \sum_{i=1}^{p} F_{w_i}(\varphi)\theta_i - F_v(\varphi) = \\ &= F_w^T(\varphi)\theta - F_y(\varphi) \end{aligned}$$

If a triple (u^*, y^*, θ^*) is a realization of the model (17), then the identity (39) takes place,

$$F_{q_{\theta^*}}(\varphi) = F_{q_{\theta^*}/(u^*,y^*)}(\varphi) = 0, \forall \varphi \in \Phi_n \tag{28}$$

and vice versa, if an input-output pair (u^*, y^*) of the family of models (27), with unknown parameter θ, generates a distribution

$$F_{q_\theta}(\varphi) = F_{q_\theta/(u^*,y^*)}(\varphi) = \sum_{i=1}^{p} F_{w_i}(\varphi)\cdot\theta_i - F_v(\varphi) \quad (29)$$

which satisfies

$$F_{q_\theta}(\varphi) = F_{q_\theta/(u^*,y^*)}(\varphi) = 0, \forall\varphi\in\Phi_n \Rightarrow \theta = \theta^*, \quad (30)$$

As θ has p components it is enough a chose (utilize) a finite number $N \geq p$ of fundamental function $\varphi_i, i = 1:N$ and to build an algebraic equation,

$$\mathbf{F}_w \cdot \theta = \mathbf{F}_v \quad (31)$$

where $\mathbf{F_w}$ is an ($N \times p$) matrix of real numbers

$$\mathbf{F}_w = [F_w^T(\varphi_1);...;F_w^T(\varphi_k);...;F_w^T(\varphi_N)]^T \quad (32)$$

where k-th row $F_w^T(\varphi_k)$ is given by (25). The symbol $\mathbf{F}_v$ denotes an N -column real vector built from (26),

$$\mathbf{F}_v = [F_v(\varphi_1),...,F_v(\varphi_k),...,F_v(\varphi_N)]^T . \quad (33)$$

When only the restriction (u_T, y_T) of the pair (u, y) on the time interval T is available, any φ_i must have for its k derivative $\varphi_k^{(m)}(t), m = 1:n$ the same compact support T_k,

$$\mathrm{supp}\{\varphi_k^{(m)}(t)\} = T_i = [t_a^k, t_b^k] \subseteq T,\ \forall m = 1:n,\ k = 1:N \quad (34)$$

Below there are some simple testing functions $\varphi_k \in \Phi_n$,

$$\varphi_k(t) = \alpha_k \cdot \beta_k(t_a^k, t_b^k) \cdot \Psi_k(t, t_a^k, t_b^k) \quad (35)$$

$$\Psi(t, t_a^k, t_b^k) = \begin{cases} \sin^{n_k}[\pi\cdot(t - t_b^k)/(t_b^k - t_a^k)], \forall k, n_k \geq n \\ 0, \quad \forall t \in (-\infty, t_a^k] \cup [t_b^k, \infty) \end{cases} \quad (36)$$

where α_i is a scaling factor and β_i normalizes the area

$$\beta_k(t_a^k, t_b^k) = 1 / \int_{t_a^k}^{t_b^k} \Psi_k(t, t_a^k, t_b^k),\ \forall t_a^k < t_b^k . \quad (37)$$

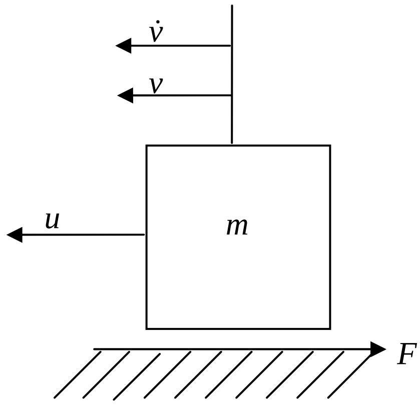
If $r = rank(\mathbf{F}_w) = p$, then a unique solution is obtained.

$$\hat{\theta} = (\mathbf{F}_w^T \cdot \mathbf{F}_w)^{-1} \cdot \mathbf{F}_w^T \cdot \mathbf{F}_v = \theta^* \quad (38)$$

4 Application for a Friction Mechanical System Identification Using the Modified of Friction LuGre Model

To combine the elements of GFS with the theory of identification based on distributions, as presented in [10], let us consider a simple mechanical system consisting of a mass m, moving on a surface with bristle. The system is excited by a force u, while the (unmeasurable) friction force f resists the motion (Fig.3). The model for this system is:

$$m\dot{v} = u - F \quad (39)$$



the zone of friction

Fig.3, A simple mechanical system with friction

where v and $\dot{v}$ represents the velocity, respectively the acceleration.

The friction force F is given of the modified LuGre model:

$$F = \sigma_0 z + \alpha v \quad (40)$$

where $\dot{z} = v - \dfrac{\sigma_0 |v|}{g(v)} z \quad (41)$

and $g(v) = F_s + (F_c - F_s)\dfrac{v}{v+1} \quad (42)$

with σ_0 known as dynamic friction parameters and is a stiffness parameter, respectively F_s, F_c and α known as static friction parameters, where F_c is the Coulomb friction, F_s is the static friction and α is a viscous friction coefficient, again z is the internal state of the modified friction model. We have did $\sigma_1 = 0$ of the original friction LuGre model and have replaced $g(v) = F_c + (F_s - F_c)\exp\left(-\left(\dfrac{v}{V_s}\right)\right)^2$ with

$$g(v) = F_s + (F_c - F_s)\frac{v}{v+1} .$$

From the equations above:

$$\begin{cases} m\dot{v} = u - F \\ F = \sigma_0 z + \alpha v \\ \dot{z} = \dfrac{-\sigma_0 |v|}{g(v)} z + v \end{cases} \quad (43)$$

will obtain the state of equations:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{1}{m}\left(u - \sigma_0 x_3 - \alpha x_2\right) \\ \dot{x}_3 = \frac{-\sigma_0 |x_2|}{F_s + (F_c - F_s)\cdot \frac{x_2}{x_2+1}} x_3 + x_2 \end{cases} \quad (44)$$

where $x_1 = x$ is position, $x_2 = v$ represents velocity, again $x_3 = z$ represents the displacement of bristles which is unmeasureable. Therefore, the state x of (1) has three components, $x_1 = x$, $x_2 = v$ and $x_3 = z$. Because z is the internal state of the friction model, which is unmeasureable, must to get to a independent relation of z, for can to identify by way of distributions the parameters of our model. For to get to so of relation, will proceeding in next mode.

The relations (39) and (40) can be rewritten so:

$$F = u - m\dot{v} \quad (45)$$

$$\sigma_0 z = F - \alpha v \quad (46)$$

from result:

$$\sigma_0 z = -m\dot{v} - \alpha v + u \quad (47)$$

Will multiply the relation (41) with σ_0 and will have:

$$\sigma_0 \dot{z} = \sigma_0 v - \frac{\sigma_0 |v|(v+1)}{F_s(v+1) + (F_c - F_s)v}\sigma_0 z \quad (48)$$

We will derive the relation (45) and will obtain:

$$m\ddot{v} = \dot{u} - \sigma_0 \dot{z} - \alpha \dot{v} \quad (49)$$

In continuance, will replace the relation (47) in the relation (48), again the relation which will obtain, wiil introduce in (49) and will result the independent relation of z (50):

$$mF_s\ddot{v} + \alpha F_s\dot{v} + \sigma_0 F_s v + mF_c v\ddot{v} + \alpha F_c v\dot{v} + m\sigma_0\left(|v|\dot{v} + |v|\dot{v}v\right) +$$
$$+\sigma_0\alpha\left(|v|v^2 + |v|v\right) + \sigma_0 F_c v^2 - F_c v\dot{u} - \sigma_0\left(|v|u + |v|vu\right) = F_s\dot{u}$$

relation, can be rewritten through divide with F_s so (51):

$$m\ddot{v} + \alpha\dot{v} + \sigma_0 v + \frac{mF_c v\ddot{v}}{F_s} + \frac{\alpha F_c v\dot{v}}{F_s} + \frac{m\sigma_0\left(|v|\dot{v} + |v|\dot{v}v\right)}{F_s} +$$
$$+\frac{\sigma_0\alpha\left(|v|v^2 + |v|v\right)}{F_s} + \frac{\sigma_0 F_c v^2}{F_s} - \frac{F_c v\dot{u}}{F_s} - \frac{\sigma_0\left(|v|u + |v|vu\right)}{F_s} = \dot{u}$$

Will note wth $\theta_1 = m$; $\theta_2 = \alpha$; $\theta_3 = \sigma_0$; $\theta_4 = \frac{mF_c}{F_s}$;

$\theta_5 = \frac{\alpha F_c}{F_s}$; $\theta_6 = \frac{m\sigma_0}{F_s}$; $\theta_7 = \frac{\sigma_0\alpha}{F_s}$; $\theta_8 = \frac{\sigma_0 F_c}{F_s}$;

$\theta_9 = \frac{F_c}{F_s}$ and $\theta_{10} = \frac{\sigma_0}{F_s}$ (52)

the parameters which will be identified, again the relation (51) is expressed as the operator (19), where $p = 10$,

$w_1 = \ddot{v}$; $w_2 = \dot{v}$; $w_3 = v$; $w_4 = v\ddot{v}$; $w_5 = v\dot{v}$;
$w_6 = |v|\dot{v} + |v|\dot{v}v$; $w_7 = |v|v^2 + |v|v$; $w_8 = v^2$;
$w_9 = -v\dot{u}$; $w_{10} = -\left(|v|u + |v|vu\right)$; $v = \dot{u}$ (53)

except a set of points of a zero measure.

The distribution image (29) of this differential operator, evaluated for a testing function φ_k on the time interval $T_k = \left[t_a{}^k, t_b{}^k\right] \subseteq T$, contains the elements given by (24), (25), (26) of the form $F_{w_i}(\varphi_k) = \int_{t_a^k}^{t_b^k} w_i(t)\cdot\varphi_k(t)dt$ where,

$$F_{w_1}(\varphi_k) = -\int_{t_a^k}^{t_b^k} \dot{v}(t)\cdot\dot{\varphi}_k(t)dt \quad F_{w_2}(\varphi_k) = \int_{t_a^k}^{t_b^k} \dot{v}(t)\cdot\varphi_k(t)dt$$

$$F_{w_3}(\varphi_k) = \int_{t_a^k}^{t_b^k} v(t)\cdot\varphi_k(t)dt$$

$$F_{w_4}(\varphi_k) = -\int_{t_a^k}^{t_b^k} \dot{v}(t)\cdot v(t)\cdot\dot{\varphi}_k(t)dt - \int_{t_a^k}^{t_b^k} \dot{v}^2(t)\cdot\varphi_k(t)dt$$

$$F_{w_5}(\varphi_k) = \int_{t_a^k}^{t_b^k} v(t)\cdot\dot{v}(t)\cdot\varphi_k(t)dt$$

$$F_{w_6}(\varphi_k) = \int_{t_a^k}^{t_b^k} |v(t)|\cdot\dot{v}(t)\cdot\varphi_k(t)dt + \int_{t_a^k}^{t_b^k} |v(t)|\cdot\dot{v}(t)\cdot v(t)\cdot\varphi_k(t)dt$$

$$F_{w_7}(\varphi_k) = \int_{t_a^k}^{t_b^k} |v(t)|\cdot v^2(t)\cdot\varphi_k(t)dt + \int_{t_a^k}^{t_b^k} |v(t)|\cdot v(t)\cdot\varphi_k(t)dt$$

$$F_{w_8}(\varphi_k) = \int_{t_a^k}^{t_b^k} v^2(t)\cdot\varphi_k(t)dt$$

$$F_{w_9}(\varphi_k) = \int_{t_a^k}^{t_b^k} \dot{v}(t)\cdot u(t)\cdot\varphi_k(t)dt + \int_{t_a^k}^{t_b^k} v(t)\cdot u(t)\cdot\dot{\varphi}_k(t)dt$$

$$F_{w_{10}}(\varphi_k) = -\int_{t_a^k}^{t_b^k} |v(t)|\cdot u(t)\cdot\varphi_k(t)dt - \int_{t_a^k}^{t_b^k} |v(t)|\cdot v(t)\cdot u(t)\cdot\varphi_k(t)dt$$

$$F_v(\varphi_k) = -\int_{t_a^k}^{t_b^k} u(t)\cdot\dot{\varphi}_k(t)dt.$$

For the evaluation of these integrals only input-output pairs (v,u), respectively ($\dot{v},u$) and the module of v are necessary. Otherwise also the speed v and the acceleration $\dot{v}$ will be measured, for to measure acceleration will use an accelerator. Integrals are utilized to build the system (31), (32), (33), whose solution is (38).

5 Experimental Results

To implement distribution based identification methods, an experimental platform (DBI) has been developed. It allows creating testing functions with settable parameters, automatically to create and solve the system (31). The input-output data for identification are obtained from an external source (a data file) or internally by simulation.

Many examples and types of friction systems have been implemented for identification but, in this paper, only one example is analysed, based on the application presented in section 4.

In first case, the measured signals, as indicated in Fig.4., are generated by a step input $u(t) = \mathbf{1}(t)$ passed through function of transfer $\dfrac{0.1s}{0.01s^2 + 0.01s + 1}$ with initial state $x(0) = [0 \;\; 0 \;\; 0]$ again m, σ_0, F_c, α, F_s are parameters necessary for identification. Ten testing functions φ_k on T_k, as (35), with n_k=10 and T_1=[1,1.4]; T_2=[1.4,1.8]; T_3=[1.8,2.2]; T_4=[2.2,2.6]; T_5=[2.6,3]; T_6=[3,3.4]; T_7=[3.4,3.8]; T_8=[3.8,4.2]; T_9=[4.2,4.6]; T_{10}=[4.6,5] are utilized.

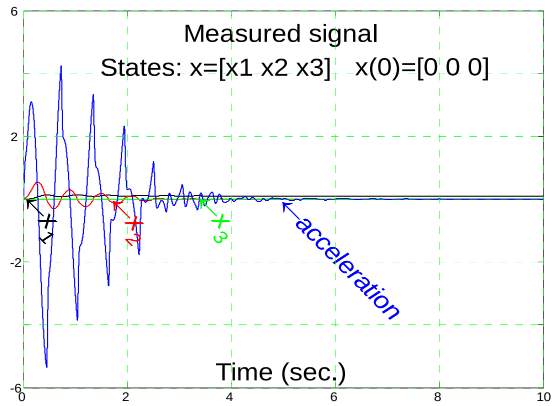


Fig.4, Measured variables for the system with the modified friction LuGre model and initial state x(0)=[0 0 0]

The matrices F_w and F_v, respectively are:

0.184 0.000 -0.001 -0.038 -0.000 0.000 -0.000
-0.051 -0.013 0.000
-0.019 -0.000 -0.001 0.000 -0.087 0.006 0.000
-0.006 0.000 0.000
0.000 0.044 0.001 -0.000 -0.002 -0.000 0.000
-0.000 0.007 -0.000
-0.000 -0.000 0.000 -0.000 -0.000 -0.001 0.000
0.000 -0.000 0.000
0.000 0.000 0.004 0.000 -0.000 -0.000 -0.000
0.000 -0.000 0.001
-0.000 -0.000 -0.000 0.000 -0.000 -0.000 0.002
0.000 0.000 -0.000
0.000 0.000 0.000 0.002 0.000 -0.000 0.000
-0.000 0.000 -0.000
0.000 0.007 0.000 0.000 0.002 -0.000 0.000
-0.000 -0.000 0.000
0.000 0.000 0.000 -0.000 -0.000 0.000 0.000
-0.000 0.000 -0.000
0.000 0.000 -0.000 -0.000 0.000 -0.000 0.000
0.000 -0.000 0.000

0.035
-0.032
0.011
0.009
-0.018
0.012
-0.001
-0.006
0.008
-0.004

The real and identified parameter values are respectively:

m: 0.2000000000000 0.1999919109469
σ_0: 300.00000000000 299.99937408757
F_c: 0.3300000000000 0.3299672364812
α: 0.0300000000000 0.0300235552826
F_s: 0.2900000000000 0.2899840840260

and the conditioning number of F_w is:

cond(F_w)=2.011849741648770e+006.

For the second case with same input but with x(0)=[1 1 1] and T_1=[1,1.3]; T_2=[1.3,1.6]; T_3=[1.6,1.9]; T_4=[1.9,2.2]; T_5=[2.2,2.5]; T_6=[2,5.2.8]; T_7=[2.8,3.1]; T_8=[3.1,3.4]; T_9=[3.4,3.7]; T_{10}=[3.7,4], the matrices F_w and F_v are:

0.095 -0.005 -0.001 -0.021 0.001 -0.000 -0.000
-0.091 0.004 0.000
-0.012 0.000 0.000 0.000 0.053 -0.003 -0.000
-0.006 0.000 -0.000
-0.000 -0.041 0.001 0.000 -0.003 0.000 0.000
0.000 0.016 -0.000
-0.000 -0.000 0.000 -0.000 -0.000 -0.000 -0.000
0.000 0.000 -0.000
-0.000 0.000 0.020 -0.000 -0.000 -0.000 0.000
-0.000 -0.000 -0.019
0.000 0.000 -0.000 0.000 0.000 0.000 0.017
0.000 -0.000 -0.000
-0.000 0.000 -0.000 -0.009 -0.000 0.000 -0.000
-0.000 -0.000 0.000
0.000 0.004 0.000 0.000 0.002 -0.000 0.000

0.000 0.000 0.000
0.000 -0.000 0.000 -0.000 0.000 0.000 -0.000
-0.000 0.000 -0.000
0.000 0.000 -0.000 -0.000 0.000 -0.000 0.000
0.000 -0.000 -0.000

0.017
-0.011
0.006
-0.002
-0.000
0.003
-0.004
0.005
-0.006
0.006

with cond(F_w)=2.056712069444360e+005.

again the identification results are:

m : 0.2000000000000 0.1999999236194
σ_0 : 300.00000000000 299.99993551165
F_c : 0.3300000000000 0.3300031565991
α : 0.0300000000000 0.0299974467911
F_s : 0.2900000000000 0.2899997173826

The measured variables for this case are illustrated in Fig.5.

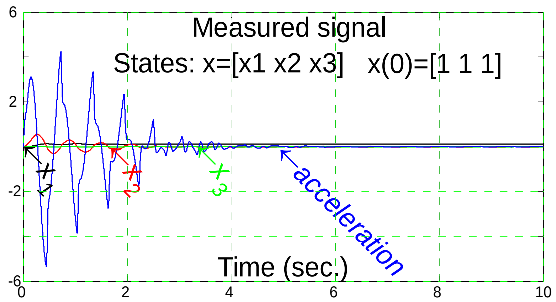


Fig.5, Measured variables for the system with the modified friction LuGre model and initial state x(0)=[1 1 1]

The third case refers to the same conditions as in the second example but considering errors in the measurement of both input and output. A zoom of these measurements containing error is shown in Fig.6.

Also the real and identified cinematic friction characteristic is presented in Fig.7.

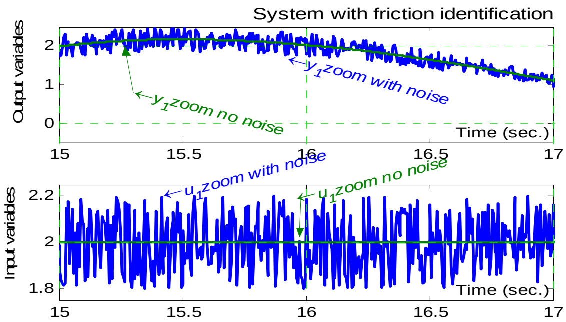


Fig.6, A zoom representation of measurements containing error for the friction system and initial state x(0)=[1 1 1]

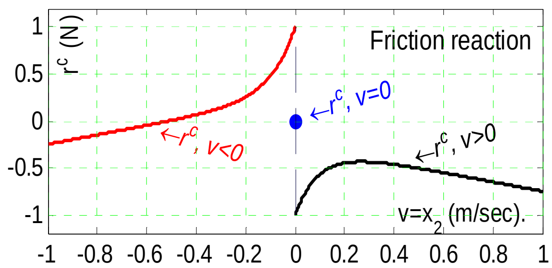


Fig.7, The real and identified cinematic friction reaction r^c.

Using these noise contamined measurements, the matrix F_w is still well conditioned,
cond(F_w)=2.438921172332657e+005.
with the results:

m : 0.2000000000000 0.1988765213456
σ_0 : 300.00000000000 298.39993561175
F_c : 0.3300000000000 0.3201031566981
α : 0.0300000000000 0.0289971466911
F_s : 0.2900000000000 0.2999897673226

again the matrices F_w and F_v takes next values:

0.089 -0.001 -0.004 -0.012 0.007 -0.002 -0.001
-0.082 0.002 0.001
-0.020 0.001 0.001 0.001 0.045 -0.009 -0.001
-0.009 0.001 -0.001
-0.001 -0.034 0.006 0.001 -0.009 0.001 0.001
0.001 0.023 -0.001
-0.001 -0.001 0.001 -0.001 -0.001 -0.001 -0.001
0.001 0.001 -0.001
-0.001 0.001 0.015 -0.001 -0.001 -0.001
0.001 -0.001 -0.001 -0.023
0.001 0.001 -0.001 0.001 0.001 0.001
0.034 0.001 -0.001 -0.001
-0.001 0.001 -0.001 -0.019 -0.001 0.001 0.001
-0.001 -0.001 0.001
0.001 0.012 0.001 0.001 0.007 -0.001 0.001

0.001 0.001 0.001
0.001 -0.001 0.001 -0.001 0.001 0.001 -0.001
-0.001 0.001 -0.001
0.001 0.001 -0.001 -0.001 0.001 -0.001 0.001
0.001 -0.001 -0.001

0.031
-0.039
0.017
0.010
-0.023
0.019
-0.009
-0.001
0.012
-0.005

6 Conclusions

The above results illustrates the advantages of distribution based identification for systems with discontinuities on the right side. Description by functionals allows to enlarge the area of systems to which identification procedures can be applied. This paper is development of a paper “Identification of systems with friction via distributions”.

ACKNOWLEDGEMENTS

This work was supported by the National University Research Council - CNCSIS, Romania, under the research projects ID 786, 2007 (PNCDI II)

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