Research on Parameter Identification of Modified Friction LuGre Model Based Distributions Theory

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Abstract: - Based on distributions theory to continuous time systems with friction using the this paper presents a batch on-line method for the parameter identification of the modified LuGre friction model. Mainly it applies the results of the identification procedures based on distributions theory to continuous time systems with friction. There are defined the so called generalized friction dynamic systems (GFDS) as a closed loop structure around a smooth system with discontinuous feedback loops representing friction reaction vectors. Both GFDS with static friction models (SFM) and dynamic friction models (DFM), also the modified friction LuGre model is analyzed. The identification problem is formulated as a condition of vanishing the existence relation of the system. Then, this relation is represented by functionals using techniques from distribution theory based on testing function from a finite dimensional fundamental space. The advantage of representing information by distributions are pointed out when special evolutions as sliding mode, or limit cycle can appear. The proposed method is a batch on-line identification because identification results are obtained during the system evolution after some time intervals but not in any time moment. This method does not require the derivatives of measured signals for its implementation. Some experimental results are presented to illuminate its advantages and practical use. At last, the simulation results have shown effectiveness of the proposed method for friction parameter identification.

Keywords: - Identification; Distribution theory; Friction; the modified friction LuGre model.

1 Introduction

Motion in many mechanical, hydraulic or pneumatic systems is influenced by the so-called friction forces because of interactions with the environment or of the interaction between their components.

Friction is a complex phenomenon, not yet completely known, with many different physical causes, so it is a difficult task to model it. Such models contain some specific nonlinearity such as stiction, hysteretic, Stribeck effect, stick-slip, depending on velocity [1], [2], [3], [4]. These models depend on many parameters whose values can change during the system evolution or are influenced by some other causes as external temperature, quality of materials etc. In literature there are accepted a large variety of friction models as Coulomb friction model [1], LuGre model [24], Dahl model [5], [6], exponential model [7], bristle model [8], state variable model [9].

Ignoring friction in controlling such systems can lead to tracking errors, limit cycles, undesired stick-slip motion [2]. To avoid these difficulties, adaptive control strategies, named model-based friction compensation techniques [2], are recommended. Such adaptive strategies involve identification procedures of the controlled system, including identification of the model friction parameters.

Unfortunately friction models are nonlinear, involving a discontinuous dependence with respect to velocity. Because of this, many techniques, as identification based on time-discretized models fail to offer good results.

A survey of models, analysis tools and compensation methods for the control of machines with friction is presented in [11]. Furthermore, the application of classical identification methods for continuous time friction models requires the acceleration measurement that is not an easy task. A frequency domain approach to identification of mechanical systems with friction is developed in [12], which does not require the acceleration information but the procedure is available for periodic excitation input only.

Good results on continuous time system identification based on distribution theory are reported in [13], for linear systems, or in [14], for nonlinear systems. Because of their discontinuities, identification of systems with frictions is much more difficult. One
way is to perform continuous time domain identification transforming the system differential equations to an algebraic system that reveals the unknown parameters [15], [16], [17]. This can be done by using some modulating functions to generate functionals to avoid the direct computation of the input-output data derivatives [18], [19].

From computational point of view many advantages are obtained by using the classic methods based on orthogonal functions. The main disadvantages of them are the strong sensitivity to nonzero initial conditions and the fixed time interval for integrals.

A novel approach to identify the continuous-time system is the distribution-based approach, using deterministic distributions [13], or random distributions [20]. Through these approaches, derivatives are described in the sense of distribution theory and construct the input-output algebraic relationships using differential information produced in the distribution sense.

This paper extends the procedures of [13], [14], based on distributions, for parametric identification in continuous time systems with friction. By this method, it is possible to perform identification of these systems, processing only information on position and the sign of the velocity in any consistent transient response.

The proposed method is a batch on-line identification method because identification results are obtained during the system evolution after some time intervals but not in any time moment. Even if it is based on the input-output measurements only, the method is insensitive to the initial state of any transient.

The paper is organized as follows: After introduction in the first section, Section 2 is dedicated to the generalized friction dynamic systems GFDS, as presented in [10]. Section 3, presents the main steps of the continuous time system identification based on distributions. Section 4 illustrates application for a friction mechanical system identification using the modified friction LuGre model. Section 5 presents experimental results, again conclusions are resumed in Section 6.

2 Generalized Friction Dynamic Systems

As presented in [12], a generalized dynamic friction system (GFDS) is a system characterized by the state equation of the form

\[ \dot{x} = f(x, u, r_i), \]  

where \( x \) is the state vector and \( u \) is the input vector. The vectors \( r_i \) are called friction reaction vectors.

They depend on \( x \) and \( u \) through a specific operator \( \Psi_i \), called friction operator,

\[ r_i = \Psi_i(x, u), \quad i = 1 : p \]  

There are two categories of friction models: static friction models (SFM) and dynamic friction models (DFM). For SFM, we deal with only in this paper, the operator (2) is a non-dynamic mapping with a specific structure as follows. For any \( i = 1 : p \) there are two functions \( v_i = v_i(x, u) \), which determines the so called generalized velocity vector \( v_i \), and \( a_i = a_i(x, u) \), expressing the so called active component of the velocity vector \( v_i \). In SFM, the non dynamic mapping (2) can be expressed as a function of \( v_i \), and \( a_i \) only, as depicted in Figure 2,

\[ r_i = F_i(x, u), \quad i = 1 : p \]  

Fig. 1. The feedback structure of a GDFS with SFM.

Inspired from mechanics [11], the function \( \rho_i \) is explicitly defined for \( v_i = 0 \) and for \( v_i \neq 0 \). As a result, two components of the friction reaction vectors \( r_i, i = 1 : p \) can be defined: static friction reaction \( r_i^s \) and cinematic friction reaction \( r_i^c \), where, particularly,

\[ r_i^s = \rho_i(v_i, a_i), v_i = 0; \quad r_i^c = 0, v_i \neq 0 \]  

\[ r_i^c = \rho_i^c(v_i, a_i), v_i \neq 0; \quad r_i^c = 0, v_i = 0 \]  

The adjective static and dynamic, for the friction reaction vectors \( r_i, i = 1 : p \), must be understood with respect to the velocity vector \( v_i \) only. Also, for a vector \( v_i \in \mathbb{R}^m, i = 1 : p \), it is defined the function \( sgn(v_i) \) as \( sgn(v_i) = v_i / \|v_i\| \), where \( \|v_i\| \) is the Euclidian norm of \( \mathbb{R}^m \). In this norm the function \( sgn(v_i) \) is a discontinuous function in the point \( v_i = 0 \). It is observed that

\[ sgn(v_i) = sgn(\|v_i\|) = 1, \quad v_i \neq 0, \quad sgn(\|0\|) = 0 \]  

ISSN: 1109-2777 979 Issue 8, Volume 8, August 2009
If $m_i = 1$, $v_i$ is a scalar variable, then (12) can be presented by using inequalities. Because of (3) and (4), the system state evolution $x(t)$ is characterized by a status of two values, related to each friction reaction vectors $r_i$, $i = 1 : p$

1. Evolution inside a surface characterized by zero value of the velocity vector $v_i$, $x(t) \in S_i$, where $S_i = \{ x \in X, v_i = 0 \}$.

2. Evolution with nonzero value of the velocity vector $v_i$, that means outside the surface $S_i$, $x(t) \notin S_i$.

Outside the surface $S_i$, $r_i$ is a vector opposite to $v_i \neq 0$ but inside the surface $S_i$, $r_i$ is a vector opposite to $a_i$. There is a closed subset $S^0_i(u) \subseteq S_i$, called sticky area (SA), which keeps the system state inside. This means

$$\frac{d}{dt}v_i(x(t)) = \frac{d}{dt}v_i(x(t))^T \cdot \mathbf{R}(t) = 0, \forall x \in S^0_i(u).$$

(6)

Inside the SA $r_i = -a_i$. Because the input $u$ can change the SA position the system can be forced to be out of $S^0_i(u)$, crossing its border. For any admissible $u$, the function $r_i = f_i(x,u)$ is continuous with respect to $\forall x \in S^0_i(u)$. Because of this, when the system state $x(t)$ arrives on or leaves out $S^0_i(u)$ the friction reaction $r_i(t)$ is a continuous time function. Condition c, is called the smooth sticky condition (SSC). However, when $x(t) \in S_i \setminus S^0_i(u)$, $r_i(t)$ has a discontinuity and $\frac{d}{dt}v_i(x(t)) \neq 0$. In this case $x(t)$ passes from one side to other of $S_i \setminus S^0_i(u)$, as a switching mode or as a sliding mode. For example, expressions as (7) and (8) of (3) and (4) respectively, satisfy the above conditions, where by $a_i$ it must understand $a_i = a_i(x,u)$,

$$r_i^c = \gamma(\beta(\varphi, a_i)) = -a_i \cdot \left| v_i \right|, \{ 1 - sgn(\beta) \}$$

(7)

$$r_i^c = \gamma(\beta(\varphi, a_i)) = -a_i \cdot \left| v_i + c_i \right| - \beta(\left| v_i \right|) \cdot \left( 1 - sgn(\beta) \right)$$

(8)

As it can be observed, the cinematic reaction $r_i^c$ is a sum of three components, $r_i^c = r_i^{cc} + r_i^{cv} + r_i^{cs}$ expressing respectively Coulomb friction, viscous friction and the so called Striebeck effect, [4], [11].

$$r_i^c = r_i^{cc} + r_i^{cv} + r_i^{cs}.$$ (9)

For $m_i = 1$, all $r_i, a_i, v_i$ are scalar variables so the static reaction (7), $r_i^s$, is illustrated in Fig.2a and the cinematic reaction (8), $r_i^c$, in Fig.2b.

A friction reaction vector $r_i$, as above defined, has a sticky characteristic which means there is a subset $S^0_i(u) \subseteq S_i$, called sticky set (SS), such $\mathbf{R}(t) = d/dt \{ v_i(x(t)) \} = 0, \forall x(t) \in S^0_i(u(t)) \subseteq S_i$ (10)

The position of SS depends on input vector $u$. When the system state $x(t)$ approaches $S^0_i(u)$, generated by a vector $r_i$, it remains inside of that SS till the input $u(t)$ changes the position of $S^0_i(u)$, forcing $x(t)$ to be outside of it. Substituting (4) into (1) and denoting

$$f(x,u) = f(x,u,F_i(x,u),...,F_i(x,u),...,F_i(x,u))$$ (11)

the GDFS takes the compact form

$$\mathbf{R} = \mathbf{f}(x,u), x(t_0) = x_0, t \geq t_0.$$ (12)

This is a differential system with a discontinuous function on right side so for its analytical description, special mathematical approaches are necessary. For example approaches describing the solution in the Charatheodory sense [4], using the Filippov approach or differential inclusions and differential inequalities. However, for the identification it is supposed a solution exist for (28) and are available as measurements the input variable $u$ and the output variable $y$ where

$$y = h(x,u).$$ (13)

### 3 Continuous Time System

#### Identification Based on Distributions

This section presents the main results on continuous time system identification based on distribution, as have been presented in [11]. Let $\Phi_\Phi$ be the fundamental space from distribution theory [17] of the real testing functions, $\varphi: i \rightarrow i, t \rightarrow \varphi(t)$, having continuous derivatives at least up to the order $n$, with compact support $T$ for any of the above derivative. The linear space $\Phi_\phi$ is organized as a topological space considering a specific norm [17]. A
distribution is a linear, continuous real functional on $\Phi_{a}$, $F : \Phi_{a} \rightarrow 1$, $\varphi \rightarrow F(\varphi) \in 1$.

Let $q : 1 \rightarrow 1$, $t \rightarrow q(t)$ be a function that admits a Riemann integral on any compact interval $T$ from $1$. Using this function, a unique distribution $F_{q} : \Phi_{a} \rightarrow 1$, $\varphi \rightarrow F_{q}(\varphi) \in 1$ can be build by the relation

$$F_{q}(\varphi) = \int q(t) \varphi(t) dt, \forall \varphi \in \Phi_{a}.$$  

Considering, at least, $q \in C^{0}(1)$, the following important equivalence take place [18],

$$F_{q}(\varphi) = 0, \forall \varphi \in \Phi_{a}, \Leftrightarrow q(t) = 0, \forall t \in 1 \quad (14)$$

The m-order derivative of a distribution is a new distribution, $F_{q}^{(m)} \in \Phi_{a}$ uniquely defined by the relations,

$$F_{q}^{(m)}(\varphi) = (-1)^{m} F_{q}(\varphi^{(m)}), \forall \varphi \in \Phi_{a} \quad (15)$$

When $q \in C^{m}(1)$, then

$$F_{q}^{(m)}(\varphi) = F_{q_{m}}(\varphi) = \int q^{(m)}(t) \varphi(t) dt$$

that means the k-order derivative of a distribution generated by a function $q \in C^{m}(1)$ equals to the distribution generated by $q^{(m)}$, the k-order time derivative of the function $q$. If $q \in C^{m}(1)$, from (13), (16) one can write, $orall \varphi \in \Phi_{a}$

$$F_{q}^{(m)}(\varphi) = \int q^{(m)}(t) \varphi(t) dt = (-1)^{m} \int q(t) \varphi^{(m)}(t) dt \quad (16)$$

Let us consider a dynamical continuous time system expressed by a differential operator,

$$q_{0}(u_{i}, y) = F_{i}(u_{i}, y, \theta) \quad (17)$$

whose expression depends on a vector of parameters

$$\theta = \theta_{1}, ..., \theta_{p}.$$

It represents a family of models with a given structure in constant parameters. A special case is the model (17) expressing a linear relation in the parameters

$$q_{0}(u_{i}, y) = F_{C}(u_{i}, y, \theta) = \sum_{i=1}^{p} w_{i} \theta - v = w^{T} \theta - v \quad (19)$$

where $w_{i}$ and $v$ represent a sum of the derivatives of some known, possible nonlinear, functions $\psi_{i}^{j}, \psi_{0}^{j}$, with respect to the input and output variables,

$$w_{i} = \sum_{j=1}^{P_{i}} [\psi_{i}^{j}(u_{i}, y)]^{(n_{i})}, i = 1 : p$$

$$v = \sum_{j=1}^{N_{0}} [\psi_{0}^{j}(u_{i}, y)]^{(n_{0})}, \quad (20)$$

where parameters $P_{i}, n_{i}, P_{0}, n_{0}$ are given integer numbers. In [11] the existence and uniqueness conditions for a problem of distribution based continuous time system identification are presented. Suppose that it is possible to record the functions $(u, y)$ in an the time interval $T \subset 1$, called observation time interval or just time window. The restriction of the functions $(u, y)$ to the time interval $T$ is denoted by $(u_{i}, y_{i})$ respectively. If no confusion would appear, then we may drop the subscript $T$.

An identification problem means to determine the parameter $\theta = \hat{\theta}$, given the priori information on the model structure $F_{C}, (17)$, and a set of observed input-output pairs $(u_{i}, y_{i})$, $\hat{\theta} = [u_{i}, y_{i}, F_{C}]$ in a such a way that,

$$q_{0}(u_{i}, y_{i}) = 0, \forall t \in 1 \quad (22)$$

This condition involves,

$$q_{0}(u_{i}, y_{i}) = 0, \forall t \in 1, \forall (u, y) \in \Omega \times \Gamma \quad (23)$$

for any input-output pair $(u, y)$ observed to that system.

Let us consider two families of regular distributions, $F_{w}, i = 1 : p$, and $F_{w}(\varphi)$ created based on the functions (20), (21),

$$F_{w}(\varphi) = \sum_{j=1}^{P_{i}} \int R [\psi_{i}^{j}(t)]^{(v_{j})} \varphi(t) dt =$$

$$= \sum_{j=1}^{P_{i}} (-1)^{v_{j}} \int R [\psi_{i}^{j}(t)]^{(v_{j})} \varphi(t) dt \quad (24)$$

which determines the row vector,

$$F_{w}(\varphi) = [F_{w_{1}}(\varphi), ..., F_{w_{p}}(\varphi)]_{1} \quad (25)$$

Any input-output pair $(u, y)$ observed from the system (27) is described by a pair of regular distribution $(F_{u}, F_{y})$ for any $\varphi \in \Phi_{a}$, [11]. The solution of the system (17) parameter identification can be represented now by distributions. For example, the regular distribution generated by the continuous function $q_{0}(u_{i}, y)$ from (17), is related to the parameter vector $\theta$, $\forall \varphi \in \Phi_{a}$ as

$$F_{w} \varphi(\varphi) = F_{w_{0}}(u_{i}, y) \varphi(\varphi) = \sum_{j=1}^{P_{i}} F_{w_{i}}(\varphi) \theta_{j} - F_{y}(\varphi) =$$

$$= F_{w_{0}}(\varphi) \theta - F_{y}(\varphi) \quad (27)$$

If a triple $(u_{i}, y_{i}, \theta)_{*}$ is a realization of the model (17), then the identity (39) takes place,

$$F_{w_{0}}(\varphi) = F_{w_{0}}(u_{i}, y_{i}) \varphi(\varphi) = 0, \forall \varphi \in \Phi_{a}$$

and vice versa, if an input-output pair $(u_{i}, y_{i})$ of the family of models (27), with unknown parameter $\theta$, generates a distribution
\[ F_{\psi_i}(\varphi) = F_{\psi_i/(\varpi, \gamma_i)}(\varphi) = \sum_{i=1}^{p} F_{\psi_i}(\varphi) \cdot \theta_i - F_i(\varphi) \] (29)

which satisfies
\[ F_{\psi}(\varphi) = F_{\psi/(\pi, \gamma)}(\varphi) = 0, \forall \varphi \in \Phi_n \implies \theta = \theta^* \] (30)

As \( \theta \) has \( p \) components it is enough a chose (utilize) a finite number \( N \geq p \) of fundamental function \( \phi_i, i=1:N \) and to build an algebraic equation,
\[ F_w \cdot \theta = F_v \] (31)

where \( F_w \) is an \((N \times p)\) matrix of real numbers
\[ F_w = [F_w(\varphi_1); \ldots; F_w(\varphi_k); \ldots; F_w(\varphi_n)]^T \] (32)

where k-th row \( F_w^T(\varphi_k) \) is given by (25). The symbol \( \Phi \) denotes an \( N \)-column real vector built from (26),
\[ F_v = [F_v(\varphi_1), \ldots, F_v(\varphi_k), \ldots, F_v(\varphi_n)]^T. \] (33)

When only the restriction \((u_t, y_t)\) of the pair \((u, y)\) on the time interval \( T \) is available, any \( \varphi \) must have for its k derivative \( \varphi_k^{(m)}(t), m=1:n \) the same compact support \( T_k \),
\[ \text{supp}(\varphi_k(t)) = T_k = \left[ t_k^s, t_k^b \right] \subseteq T, \forall m=1:n, k = 1:N \] (34)

Below there are some simple testing functions \( \varphi_k \in \Phi_n \),
\[ \varphi_k(t) = \alpha_k \cdot \beta_k(t_k^s, t_k^b) \cdot \Psi_k(t, t_k^s, t_k^b) \] (35)

\[ \Psi(t, t_k^s, t_k^b) = \begin{cases} \sin \alpha_k \left[ \frac{\pi}{2} \cdot \frac{t - t_k^s}{t_k^b - t_k^s} \right], & \forall k, n_k \geq n \\ 0, & \forall t \in (-\infty, t_k^s] \cup [t_k^b, \infty) \end{cases} \] (36)

where \( \alpha_k \) is a scaling factor and \( \beta_k \) normalizes the area
\[ \beta_k(t_k^s, t_k^b) = 1 / \int_{t_k^s}^{t_k^b} \Psi_k(t, t_k^s, t_k^b) \mathrm{d}t, \quad \forall t_k^s < t_k^b. \] (37)

If \( r = \text{rank}(F_w) = p \), then a unique solution is obtained.
\[ \hat{\theta} = (F_w^T \cdot F_w)^{-1} \cdot F_w^T \cdot F_v = \theta^* \] (38)

4 Application for a Friction Mechanical System Identification Using the Modified of Friction LuGre Model

To combine the elements of GFS with the theory of identification based on distributions, as presented in [10], let us consider a simple mechanical system consisting of a mass \( m \), moving on a surface with bristle. The system is excited by a force \( u \), while the (unmeasurable) friction force \( f \) resists the motion (Fig.3). The model for this system is:
\[ m \ddot{\varphi} = u - F \] (39)

\[ \varphi = \begin{cases} m \ddot{\varphi} = u - F \\ F = \sigma_0 \varphi + \alpha v \\ \dot{z} = \frac{-\sigma_0 \varphi}{g(v)} - v + z \end{cases} \] (43)

will obtain the state of equations:
\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{1}{m} \left( u - \sigma_0 x_3 - \alpha x_2 \right) \\
\dot{x}_3 &= \frac{-\sigma_0 x_2}{F_i + (F_c - F_i) x_2} x_3 + x_2 \\
\dot{x}_4 &= \frac{-\sigma_0 x_2}{F_i + (F_c - F_i) x_2} x_3 + x_2
\end{align*}
\]  

(44)

where \( x_1 = x \) is position, \( x_2 = v \) represents velocity, again \( x_3 = z \) represents the displacement of bristles which is unmeasureable. Therefore, the state \( x \) of (1) has three components, \( x_i = x, \ x_2 = v \) and \( x_3 = z \). Because \( z \) is the internal state of the friction model, which is unmeasurable, must get to an independent relation of \( z \), for can to identify by way of distributions the parameters of our model. For to get to so of relation, will proceeding in next mode.

The relations (39) and (40) can be rewritten so:

\[
F = u - m\ddot{x} \\
\sigma_0 x = F - \alpha v
\]  

(45)

from result:

\[
\sigma_0 z = -m\dot{v} - \alpha \dot{v} + u
\]  

(46)

Will multiply the relation (41) with \( \sigma_0 \) and will have:

\[
\sigma_0 \dot{z} = \sigma_0 v - \frac{\sigma_0 \dot{v}}{F_i (v + 1)} (v + 1) \sigma_0 z
\]  

(47)

We will derive the relation (45) and will obtain:

\[
m\dot{v} = \ddot{u} - \sigma_0 \dot{z} - \alpha \dot{v}
\]  

(49)

In continuance, will replace the relation (47) in the relation (48), again the relation which will obtain, will introduce in (49) and will result the independent relation of \( z \) (50):

\[
mF_i \ddot{v} + \sigma_0 F_i \dddot{v} + \alpha mF_i \dddot{v} + \sigma_0 \dddot{v} + m\sigma_0 \left( \dddot{v} + \dddot{v} \right) + \sigma_0 \alpha \left( v \dot{v}^2 + \dddot{v} \right) + \sigma_0 \alpha \left( \dddot{v} + \dddot{v} \right) + \sigma_0 \alpha \left( \dddot{v} + \dddot{v} \right) = F_i \dddot{u}
\]  

(50)

Relation (51) is expressed as the operator (19), where \( p = 10 \),

\[
w_1 = \dddot{v} ; \quad w_2 = \dddot{v} ; \quad w_3 = \dddot{v} ; \quad w_4 = \dddot{v} ; \quad w_5 = \dddot{v} ; \quad w_6 = \dddot{v} + \dddot{v} ; \quad w_7 = \dddot{v} + \dddot{v} ; \quad w_8 = \dddot{v} ; \quad w_9 = -\dddot{v} ; \quad w_{10} = -\left( \dddot{v} + \dddot{v} \right) ; \quad \nu = \dddot{v}
\]  

(53)

except a set of points of a zero measure.

The distribution image (29) of this differential operator, evaluated for a testing function \( \varphi_k \) on the time interval \( T_k = \left[ t^k_a, t^k_b \right] \subseteq T \), contains the elements given by (24), (25), (26) of the form

\[
F_{w_1} (\varphi_k) = \int_{t^k_a}^{t^k_b} w_1 (\varphi_k) \dd t
\]  

where,

\[
F_{w_1} (\varphi_k) = -\int_{t^k_a}^{t^k_b} \dd \dot{v} (\varphi_1) \dd t
\]  

(54)

\[
F_{w_1} (\varphi_1) = \int_{t^k_a}^{t^k_b} \dd \dot{v} (\varphi_1) \dd t
\]  

(55)

\[
F_{w_1} (\varphi_2) = \int_{t^k_a}^{t^k_b} \dd \dot{v} (\varphi_2) \dd t
\]  

(56)

\[
F_{w_1} (\varphi_3) = \int_{t^k_a}^{t^k_b} \dd \dot{v} (\varphi_3) \dd t
\]  

(57)

\[
F_{w_1} (\varphi_4) = \int_{t^k_a}^{t^k_b} \dd \dot{v} (\varphi_4) \dd t
\]  

(58)

\[
F_{w_1} (\varphi_5) = \int_{t^k_a}^{t^k_b} \dd \dot{v} (\varphi_5) \dd t
\]  

(59)

\[
F_{w_1} (\varphi_6) = \int_{t^k_a}^{t^k_b} \dd \dot{v} (\varphi_6) \dd t
\]  

(60)

\[
F_{w_1} (\varphi_7) = \int_{t^k_a}^{t^k_b} \dd \dot{v} (\varphi_7) \dd t
\]  

(61)

\[
F_{w_1} (\varphi_8) = \int_{t^k_a}^{t^k_b} \dd \dot{v} (\varphi_8) \dd t
\]  

(62)

\[
F_{w_1} (\varphi_9) = \int_{t^k_a}^{t^k_b} \dd \dot{v} (\varphi_9) \dd t
\]  

(63)

\[
F_{w_1} (\varphi_{10}) = \int_{t^k_a}^{t^k_b} \dd \dot{v} (\varphi_{10}) \dd t
\]  

(64)

For the evaluation of these integrals only input-output pairs \( (v, u) \), respectively \( (\dddot{v}, u) \) and the module of \( v \) are necessary. Otherwise also the speed \( v \) and the acceleration \( \dddot{v} \) will be measured, for to measure acceleration will use an accelerator. Integrals are utilized to build the system (31), (32), (33), whose solution is (38).
5 Experimental Results

To implement distribution based identification methods, an experimental platform (DBI) has been developed. It allows creating testing functions with settable parameters, automatically to create and solve the system (31). The input-output data for identification are obtained from an external source (a data file) or internally by simulation.

Many examples and types of friction systems have been implemented for identification but, in this paper, only one example is analysed, based on the application presented in section 4.

In first case, the measured signals, as indicated in Fig.4., are generated by a step input \( u(t) = I(t) \) passed through function of transfer \( \frac{0.1s}{0.01s^3 + 0.01s + 1} \) with initial state \( x(0) = [0 0 0] \) again \( m, \sigma_0, F_s, \alpha, F_i \) are parameters necessary for identification. Ten testing functions \( \varphi_k \) on \( T_k \), as (35), with \( n_k = 10 \) and \( T_1 = [1.1, 1.4]; T_2 = [1.4, 1.8]; T_3 = [1.8, 2.2]; T_4 = [2.2, 2.6]; T_5 = [2.6, 3]; T_6 = [3, 3.4]; T_7 = [3.4, 3.8]; T_8 = [3.8, 4.2]; T_9 = [4.2, 4.6]; T_{10} = [4.6, 5] \) are utilized.

![Fig.4. Measured variables for the system with the modified friction LuGre model and initial state \( x(0) = [0 0 0] \)](image)

The matrices \( F_w \) and \( F_i \), respectively are:

\[
\begin{bmatrix}
0.184 & 0.000 & -0.001 & -0.038 & -0.000 & 0.000 & -0.000 \\
-0.051 & -0.013 & 0.000 \\
-0.019 & -0.000 & -0.001 & 0.000 & -0.087 & 0.006 & 0.000 \\
-0.006 & 0.000 & 0.000 \\
0.000 & 0.044 & 0.001 & -0.000 & -0.002 & -0.000 & 0.000 \\
-0.000 & 0.007 & -0.000 \\
-0.000 & -0.000 & 0.000 & -0.000 & -0.000 & -0.001 & 0.000 \\
0.000 & -0.000 & 0.000 \\
0.000 & 0.000 & 0.004 & 0.000 & -0.000 & -0.000 & -0.000 \\
0.000 & -0.000 & 0.001 \\
-0.000 & -0.000 & 0.000 & -0.000 & -0.000 & -0.000 & 0.002 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
0.000 & 0.000 & -0.000 \\
0.000 & 0.000 & 0.000 & 0.002 & 0.000 & -0.000 & 0.000 \\
-0.000 & 0.000 & -0.000 \\
-0.000 & 0.007 & 0.000 & 0.000 & 0.002 & -0.000 & 0.000 \\
-0.000 & -0.000 & 0.000 & 0.000 & -0.000 & -0.000 & 0.000 \\
-0.000 & 0.000 & -0.000 & 0.000 \\
-0.000 & -0.000 & 0.000 & -0.000 & -0.000 & 0.000 & 0.000 \\
-0.000 & -0.000 & 0.000 \\
0.000 & 0.004 & 0.000 & 0.000 & 0.002 & -0.000 & 0.000 \\
\end{bmatrix}
\]

The real and identified parameter values are respectively:

\[
m : 0.2000000000000 \quad 0.1999919109469 \\
\sigma_0 : 300.0000000000000 \quad 299.99937408757 \\
F_s : 0.3300000000000 \quad 0.3299672364812 \\
\alpha : 0.0300000000000 \quad 0.0300235552826 \\
F_i : 0.2900000000000 \quad 0.2899840840260 \\
\]

and the conditioning number of \( F_w \) is:

\[
\text{cond}(F_w) = 2.011849741648770e+006.
\]

For the second case with same input but with \( x(0) = [1 \ 1 \ 1] \) and \( T_1 = [1.1, 1.3]; T_2 = [1.3, 1.6]; T_3 = [1.6, 1.9]; T_4 = [1.9, 2.2]; T_5 = [2.2, 2.5]; T_6 = [2.5, 2.8]; T_7 = [2.8, 3.1]; T_8 = [3.1, 3.4]; T_9 = [3.4, 3.7]; T_{10} = [3.7, 4] \), the matrices \( F_w \) and \( F_i \) are:

\[
\begin{bmatrix}
0.095 & -0.005 & -0.001 & -0.021 & 0.001 & -0.000 & -0.000 \\
-0.012 & 0.000 & 0.000 & 0.000 & 0.053 & -0.003 & -0.000 \\
-0.006 & -0.041 & 0.001 & 0.000 & -0.003 & 0.000 & 0.000 \\
-0.000 & 0.016 & -0.000 \\
-0.000 & -0.000 & 0.000 & -0.000 & -0.000 & -0.000 & -0.000 \\
-0.000 & 0.000 & -0.000 & 0.000 & -0.000 & 0.000 \\
0.000 & 0.000 & 0.020 & -0.000 & -0.000 & 0.000 & 0.000 \\
-0.000 & -0.000 & 0.017 \\
-0.000 & 0.000 & -0.000 & -0.000 & -0.000 & 0.000 & 0.000 \\
-0.000 & -0.000 & 0.000 \\
0.000 & 0.004 & 0.000 & 0.000 & 0.000 & -0.000 & 0.000 \\
\end{bmatrix}
\]
The measured variables for this case are illustrated in Fig. 5.

The third case refers to the same conditions as in the second example but considering errors in the measurement of both input and output. A zoom of these measurements containing error is shown in Fig. 6. Also the real and identified cinematic friction characteristic is presented in Fig. 7.

Using these noise contaminated measurements, the matrix \( F_w \) is still well conditioned, \( \text{cond}(F_w) = 2.438921172332657e+005 \), with the results:

\[
\begin{align*}
m : & \quad 0.2000000000000 \quad 0.1988765213456 \\
\sigma_0 : & \quad 300.00000000000 \quad 298.39993561175 \\
F_r : & \quad 0.3300000000000 \quad 0.3201031566981 \\
\alpha : & \quad 0.0300000000000 \quad 0.0289971466911 \\
F_s : & \quad 0.2900000000000 \quad 0.2999897673226
\end{align*}
\]

again the matrices \( F_w \) and \( F_r \) takes next values:

\[
\begin{align*}
0.089 & \quad -0.001 \quad -0.004 \quad -0.012 \quad 0.007 \quad -0.002 \quad -0.001 \\
-0.082 & \quad 0.002 \quad 0.001 \\
-0.020 & \quad 0.001 \quad 0.001 \quad 0.001 \quad 0.045 \quad -0.009 \quad -0.001 \\
-0.009 & \quad 0.001 \quad -0.001 \\
-0.001 & \quad -0.034 \quad 0.006 \quad 0.001 \quad -0.009 \quad 0.001 \quad 0.001 \\
0.001 & \quad 0.023 \quad -0.001 \\
-0.001 & \quad -0.001 \quad 0.001 \quad -0.001 \quad -0.001 \quad -0.001 \quad -0.001 \\
0.001 & \quad 0.001 \quad -0.001 \\
-0.001 & \quad 0.015 \quad -0.001 \quad -0.001 \quad -0.001 \quad -0.001 \\
0.001 & \quad -0.001 \quad -0.001 \\
0.001 & \quad 0.001 \quad -0.001 \\
0.034 & \quad 0.001 \quad -0.001 \\
-0.001 & \quad 0.001 \quad -0.001 \\
-0.001 & \quad -0.001 \\
0.001 & \quad 0.012 \\
0.001 & \quad 0.001 \\
0.001 & \quad 0.007 \\
0.001 & \quad -0.001 \\
0.001 & \quad 0.001
\end{align*}
\]
6 Conclusions

The above results illustrates the advantages of distribution based identification for systems with discontinuities on the right side. Description by functionals allows to enlarge the area of systems to which identification procedures can be applied. This paper is development of a paper “Identification of systems with friction via distributions”.

ACKNOWLEDGEMENTS

This work was supported by the National University Research Council - CNCSIS, Romania, under the research projects ID 786, 2007 (PNCDI II)

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