A new index for monitoring the duration test

Niola V., Quaremba G., Avagliano V.

Department of Mechanical Engineering for Energetics
University of Naples "Federico II"
Via Claudio 21, 80125, Napoli, ITALY
vincenzo.niola@unina.it http://niola.dime.unina.it/

Abstract – The aim of this work is to improve the algorithm proposed for monitoring the reliability test (duration test). In order to improve this task, a new index was developed for studying the Side Bands. In fact each frequency of the spectrum has a specific meaning as well as the Side Bands. They appear when an anomaly occurs (in particular, in this work when an anomaly occurs in the gearbox under study). Several diagrams and considerations when an unexpected event occurs are presented. In the first part it is described the typical frequencies due to the gearbox vibration. In the second part it is illustrated the concept of harmonics, side bands and their meaning. In the third part a new index for monitoring the reliability test is presented. Such an index was designed in order to answer to many questions before the crack exceeds. In fact it gives the opportunity of checking the various aspects of the spectrum of the accelerometer signal. In particular it gives information about the order exceeding the target, so we can find earlier the cause of damage. An alarm starts if a damage on a gear wheel is detected.

Key-Words: - Assembly-Phase-Passage Frequency, Tooth Repeat Frequency, Harmonics, Side Bands.

1 Introduction

During some gearbox reliability trials performed in laboratory, we found out many fundamental characteristics on the spectrum computed from an accelerometer signal.

These aspects carefully investigated led the research for the gear transmission monitoring to a further improvement.

As it is showed in the following pages, a new index for monitoring was proposed, which can be used in parallel with the following one (see [1] for details)

$$Index = \Delta\%(Area\_cur / Area\_ref).$$

It gives more strength to the monitoring assessment because by means of its more specificity and sensitivity we can stop earlier the duration test.

The idea of designing a new index to monitor the gearbox reliability test followed from the study of typical frequencies of gearbox. First of all, the main question are:

a) what are the frequencies of interest?
b) what is their meaning?
c) where do other frequencies (Side Bands) appear when an unexpected event occurs?

In the recent bibliography, Side Bands are symptomatic of several types of damage. Our attention is focused on the study of this kind of frequency (Side Bands), appearing near the main frequency under study.

So, we need an index for monitoring the trials which gives information about each singular frequency. We studied this index and we called it Index_SideBand.

By using the index proposed in [1], we monitor all the range of frequencies. No information about a singular order we obtain. Using such an Index_SideBand we are able to monitor other features of the spectrum and more information about each vibration of singular components of the gearbox could be obtained.

2 Gearbox vibrational frequencies

The vibrations of gear transmission system are characterized by certain "typical" frequencies (signatures) that can be analytically determined before starting the monitoring.

Usually, the spectrum of vibrations of a gear system is dominated by peaks belonging to typical frequencies, but in many cases, it shows other frequencies that don't coincide with typical ones. However, this irregularity is not true; in fact these frequencies can be made in relation to the typical ones.

Below we can find the definitions of four frequencies that usually occur as dominant peaks in the gearbox vibrations spectrum. The formulas for the calculation of these frequencies are valid both for a cylindrical gear with straight-toothed and for those with helical teeth, and for the wheel crown gear. As we show, before using the following formulas to calculate these frequencies, it is necessary to know the number of teeth and the speed of rotation of all the gears under testing.

2.1 – Frequency of rotation

Let's consider a typical pair of wheels, where Z_1 is the number of teeth of drive wheel, n_1 its rotation speed (rpm), Z_2 the number of teeth of the driven wheel and n_2 its speed of rotation (rpm). We define, for both the wheels, the frequency of rotation

$$f_1 = n_1/60 \quad (1)$$

for the drive wheel, and

$$f_2 = n_2/60 \quad (2)$$

for driven wheel.

2.2 – Geared Frequency

It is well known that, during the rotation, teeth are subjected to collision, sliding and bending. These phenomena are repeated every time a pair of teeth comes in contact, and therefore they appear as a vibration of which the fundamental frequency, characteristic of the pair of wheels, is named gear mesh frequency (GMF).

Using the same symbolism used before, the gear mesh frequency is given by [2,3]

$$GMF = Z_1 \cdot n_1/60 = Z_2 \cdot n_2/60 \quad (3).$$

The second equivalence concerns the fact that for a pair of gears, the gear ratio is given by

$$\varepsilon = n_1/n_2 = Z_2/Z_1 \quad (4)$$

and therefore,

$$Z_1 \cdot n_1 = Z_2 \cdot n_2 \quad (5)$$

and from this formula we can infer immediately the equivalence (3). From the definition of rotational speed, it is possible to rewrite the expression of GMF

$$GMF = Z_1 \cdot f_1 = Z_2 \cdot f_2 \quad (6).$$

Gear mesh frequency and its harmonics are the dominant components in the vibration spectrum as well as the noise of any gear transmission system [2,4].

2.3 – Assembly-Phase-Passage frequency

This frequency could occur in the gearbox vibration spectrum when the gears are disassembled (*e.g.*, for a detection or a repair) and then reassembled.

For that reason the contact of teeth is never the same.

Before showing the formula to calculate this frequency, it is necessary to introduce the concept of Phase Assembly [2,3].

Let's take into consideration a typical pair of wheels, and let's suppose to numerate all teeth of both the wheels starting from the real point of contact, as it is showed in Fig. 1.

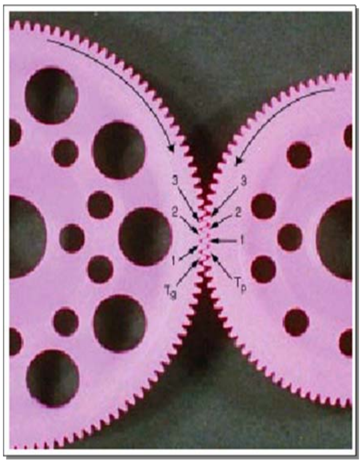

Fig. 1 - Teeth numbering for a generic pair of wheels

Fig. 2 shows the sequence of teeth involved during their contact. Each wheel has 6 teeth involved.

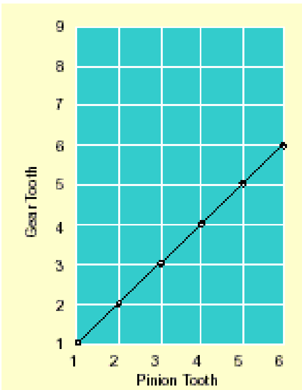

Fig. 2 - Sequence of contact between the teeth Z_1=Z_2=6

During the rotation each tooth of the wheel 1 (pinion) comes in contact with a single tooth of the wheel 2 (gear). In this case, the teeth showing the same number develop during their contact, a complementary wear so that it compensate the imperfections due to manufacturing and/or assembling operation. If after a certain working period these two wheels are disassembled and then reassembled without taking the right sequence of the numbered teeth into consideration, each tooth would no longer be in contact with its "complementary" and it would bring out a changing in the vibration. We say that this pair of wheels has 6 Assembly Phases (AP).

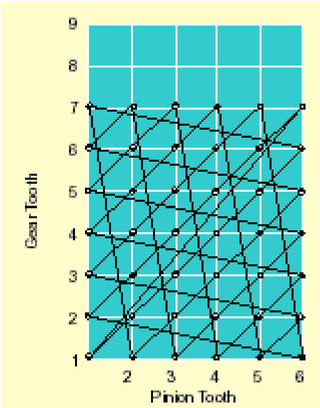

Fig. 3 - Sequence of contact between the teeth $Z_1=6$, $Z_2=7$

The Fig. 3 shows the sequence for two gears having respectively 6 and 7 teeth. In this case the situation is completely different. After the 7th rotation, all the teeth of the pinion are in contact with all the teeth of the wheel; all the teeth of both gears wear in the same way, and therefore, even if two gears have to be disassembled and then reassembled in a different way, there will be no change in vibration. This pair of gears has a single AP.
In the following other two examples of the contact between the teeth are showed.

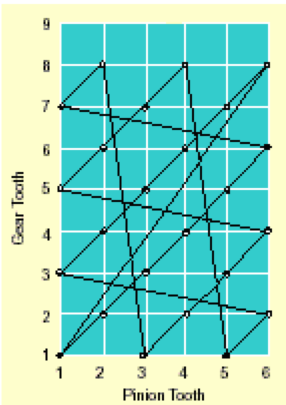

Fig. 4– Sequence of contact between the teeth $Z_1=6$, $Z_2=8$

In Fig.4 it is illustrated the sequence of contact for $Z_1=6$, $Z_2=8$. Each tooth of the pinion is in contact with 4 teeth of the gear, while each tooth of the gear is in contact with 3 teeth of the pinion.
Looking at the sequence of contact, the same pair of teeth comes into contact every 4 revolutions of the pinion (3 revolutions of the gear). For this pair of gears the teeth 1-3-5 of the pinion come into contact only with the teeth 1-3-5-7 of the gear, while the teeth 2-4-6 of the pinion come into contact only with 2-4-6-8 of the gear, and no other combination of contact occurs. For that reason, each of these two series of teeth have a different wear. This pair of gears has 2 AP.

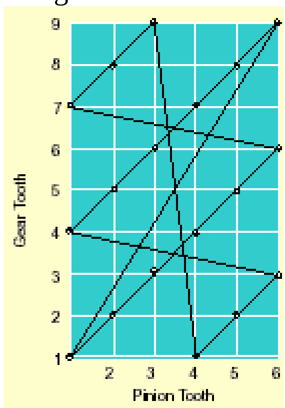

Fig.5 – Sequence of contact between the teeth $Z_1=6$, $Z_2=9$

Fig.5 shows the sequence for $Z_1=6$, $Z_2=9$. In this case, each tooth of the pinion comes into contact with 3 teeth of the gear, while each tooth of the gear is in contact with 2 teeth of the pinion. From this sequence, we note that a pair of teeth is in contact every 3 revolutions of the pinion (2 revolutions of the gear).
The teeth 1-4 of the pinion are in contact only with the teeth 1-4-7 of the gear; the teeth 2-5 of the pinion only with 2-5-8 of the gear; the teeth 3-6 of the pinion only with the teeth 3-6-9 of the gear. This pair of gears has 3 AP.
Therefore, the number of AP (N_{AP}) of a pair of gearwheels can be considered as the number of "types of wear" (wear patterns) developed by the contact of teeth of the pair of wheels. Therefore the type of wear depends on the combinations of contact between the teeth. Numerically speaking, N_{AP} is the "highest common factor" of the number of teeth for two wheels tested [2,3].
The "Assembly-frequency Phase-Passage", is given by

$$f_{AP} = GMF / N_{AP} \quad (7)$$

where GMF is gear mesh frequency (Hz), N_{AP} is the number of Assembly Phases for pair of wheels tested. Note that if $N_{AP}=1$, f_{AP} coincides with the gear mesh frequency.

2.3 – Tooth Repeat frequency

This frequency (Hunting-Tooth frequency [2]) usually occurs when each of the two wheels has a defect localized on a tooth [2,3,5]. In this case, the pair of damaged teeth comes into contact one more time, with a frequency equal to [2,3]

$$f_{TR} = (GMF \cdot N_{AP}) / (Z_1 \cdot Z_2) \quad (8)$$

where:

- GMF is the gear mesh frequency (Hz)
- N_{AP} is the number of Assembly Phases
- Z_1 and Z_2 represent the number of teeth of the drive and driven wheel respectively.

Once more, the value of the frequency under consideration depends on N_{AP}.

3 The gearbox spectrum: harmonics and side bands

It was already observed that in the gearbox spectrum several peaks appear with frequencies which are different from the typical ones. These frequencies are "unknown" and are often referable to one or more typical frequencies because:

- they are the multiple of a characteristic frequency (in this case they are called typical harmonic frequency)
- or they are the sum or the difference of the typical frequencies.

The rotation frequency for each wheel usually occurs when there is a residual imbalance, which causes a vibration and therefore harmonic component around the rotation frequency [4,6].
Usually, the residual imbalance (static and/or dynamic) is always present, more or less, in all types of rotor. Static imbalance appears as a radial vibration characterized by a frequency equal to N/60 (where N is the speed of rotation of the rotor expressed in revolutions per minute) and then, in the spectrum, it appears as a harmonic component at this frequency [4]. However, it is well known that the non-linearity of some components (*e.g.*, lubricant film, bearings), may change the feedback of the system to generate components at higher frequencies [6]. The amplitude of vibration, due to the imbalance, growths with the square of the rotation speed.
Usually, in the gearbox vibrational spectrum, there are also the harmonics belonging to the frequencies of rotation. In particular, a wide second harmonic of the rotation frequency is usually taken as a "signal" due to the shaft damage [2].
This points out the situations in which the shafts of a pair of gears are not perfectly parallel, or those in which the axes of the shafts of two generic machines are not perfectly coincident, in this case there are two types of problems: parallel and angular. Usually such situations are generated by errors made during the assembling process.

The peaks due to GMF and their harmonics are usually dominant in the spectrum. In some cases, in the neighbourhood of GMF, may appear peaks due to, for example, a small eccentricity of the drive/driven wheel. The peak frequencies are

$$GMF \pm f_1 \text{ or } GMF \pm f_2 \quad (9)$$

where f_1 and f_2 are the frequencies of drive and driven wheel respectively.
The presence of these frequencies is due to the fact that the eccentricity modulates the amplitude of the forces acting during the mesh [2,6,7].
Localized defects on the teeth, such as crack or broken part determine the appearance of several peaks in the neighbourhood of the GMF and its harmonics, due to changing of deflection of the tooth during the mesh. This happens once for each rotation of the damaged wheel. This change could be seen as an amplitude modulation while the frequency modulation is the frequency of rotation of the damaged wheel. Therefore, the distance from the side of the peak of the GMF can identify the modulated frequency, and then the wheel generating the defect [7,8].
In [7] (Chapter 16), talking about peak side, it is stated that:

- for localized defects on teeth such as crack or wear, the side peaks are smaller than the gear mesh frequency and they cover a very wide range of frequencies
- for distributed defects such as wear or errors in working-process, the peaks side are more extended and are more clustered around the gear mesh frequency and its harmonics.

In a gearbox, each wheel can be a source of vibration and therefore of side peaks [9-13], in order to measure, with low error, their distance from the GMF, it is necessary that the spectra have, at least, the same resolution equal to the lower rotation frequency. However, even this kind of resolution might not be enough, to differ from neighbourhood of GMF, because of its very low value. This is a negative point, because of a loss of information if two gears show a well localized damage [4]. In this case, if it is impossible a further improvement of the resolution, it is fundamental to carry out an analysis based on the techniques of the close-up.
The frequencies f_{AP} and f_{TR} could appear as peaks side of the GMF, where, respectively, the pair of wheels is disassembled and reassembled in a

different sequence and contemporary both the wheels are damaged.
It should be remarked that if both wheels have the number of teeth such as N_{AP} = 1 their f_{AP}=GMF, and therefore the Assembly-Phase-frequency Passage is no more distinguishable from the GMF.
The Tooth-Repeat frequency, however, because of its very low value, could hardly be distinguished, especially when N_{AP} = 1.

4 The Side Bands Index

The idea to improve a new index for monitoring the duration test of a gearbox was born starting from the following considerations. First of all we thought to monitor the most important frequency only(or their orders [1]). But it should be a very expensive system. A new index has been proposed to enforce the previous index proposed in [1].
For each order the Side Bands index is the ratio between the "actual spectrum" and the “target spectrum”. This set index was stated doesn’t exceed the threshold level of 3.
So this index, takes on account how the instantaneous spectrum is related to the target spectrum: the index should ideally be close to "1" for each order.
As it is well known in the literature, and as we reported before, the presence of side-band and the changing of the spectrum, due to typical frequencies different from the typical ones of gearbox, indicate the presence of defects.
In these cases the Index_SideBand is greater than 1.
Moreover, referring to the first threshold [1]

$$\Delta\%(Area\_cur / Area\_ref) \quad (10)$$

meaningful changes of some orders could not stop the test (*e.g.*, the order of bearing which has no weight on the overall spectrum). In these cases, if the Index_SideBand exceeds 3 we stop the test and we also have an indication on which order is increased.
For this type of monitoring is not required the use of additional sensors and this is a great additional benefit.
So, on the algorithm proposed in [1], we thought to modify the STEP 5 only, in which the Index_SideBand must be controlled.
So the process is modified as in Fig. 6, where Index_SB stands for Index_Side_Band.

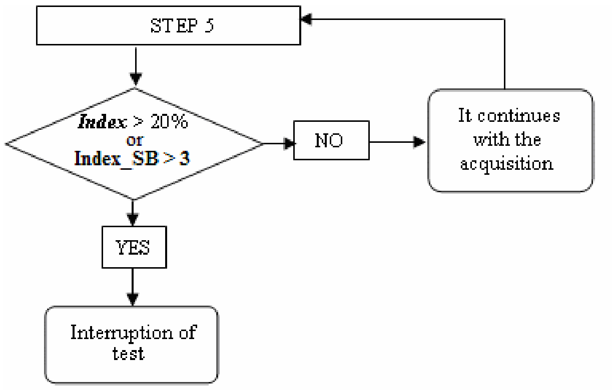

Fig. 6 - How the algorithm stops the test

Using this method for monitoring, the energy content of the whole spectrum does not exceed the threshold. In addition an order doesn’t overthrow the other threshold.
Note that if an order growths significantly, but not the first index in [1] then the Index_SideBand should be employed in order to solve this kind of problem.
By using two indexes, as illustrated in Fig.6, we improve the algorithm proposed in [1] and we get many important information about the occurrence of an unexpected event.
In fact, in this case we control, with the first index, the area of the spectrum. So we control the hole energetic content of the spectrum. But we lose information about the order damaged.
By using Index_Side_Band we evaluate for each order the ratio between the Target Spectrum and the actual spectrum. So if an order exceeds the threshold level set at 3, we know an important information, in fact we know, what is the order that overthrows the target. Consequently we know on which component of gearbox we must investigate in order to understand earlier what is the cause of the damage inside the gearbox.
This technique of monitoring gives an advantage for inspecting inside the gearbox eventually before a crack.
In the next paragraph it will be showed how the above index should be used for stopping earlier the test.

5 Experiments and discussion

Taking now the same test showed in [1], we can see that the duration test is stopped to the 13% of the total test. Conversely, by means of the index proposed in [1] the performance was less good, in fact the test-run was stopped at 16% of the total test.
The diagrams reported below, obviously, show only a part of the test results. They are selected to best

underline the duration test performed with this technique.
We analyze the 6th cycle (in particular, two seconds: the instants corresponding to 40s and 80s). Below we illustrate the test cycle before the condition of alarm occurs. Finally it will be shown the 13th cycle where the test-run was stopped.

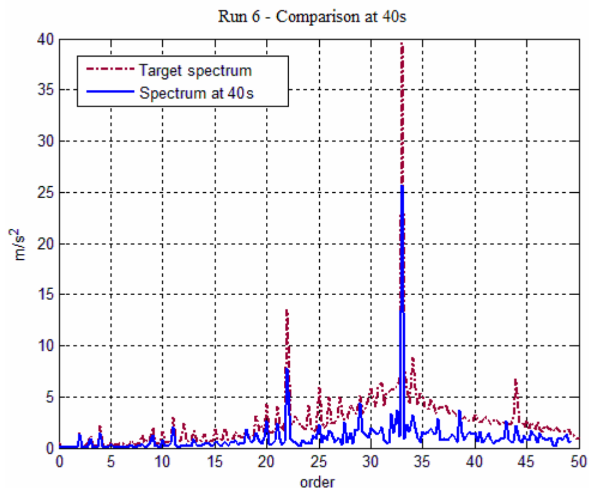

Fig. 7 – Run 6, comparison at 40s

In the Fig. 7 it is represented the target spectrum and the actual spectrum (at 40s). To check the effectiveness of the Index_SideBand, it must be done the ratio between these two spectra as it is shown in the Fig. 8

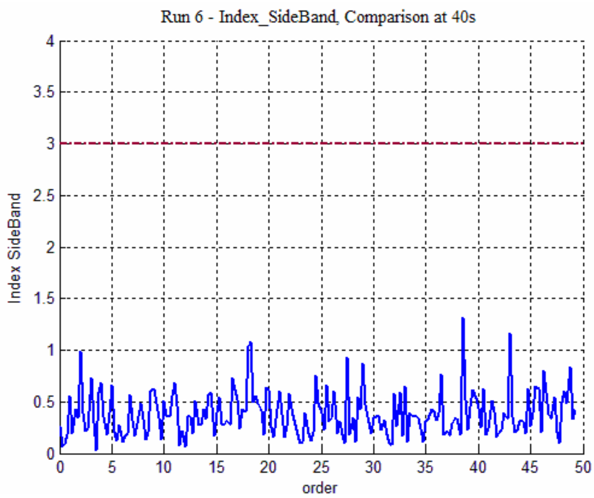

Fig. 8 – Run 6, Index_SideBand at 40s

It's clear that the Index_SideBand doesn't overthrow the threshold level, so the test-run can go on. In Fig. 9 it is shown the spectrum at 80s.

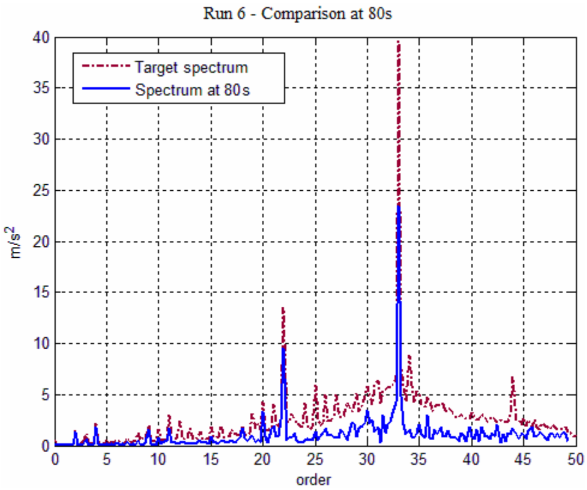

Fig. 9 - Run 6, comparison at 80s

In order to see the trend of the Index_SideBand, in Fig. 10 it is represented the ratio between these last two spectra.

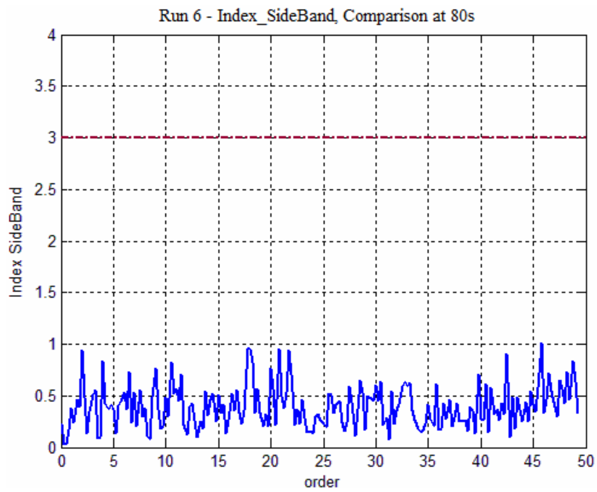

Fig. 10 – Run 6, Index_SideBand at 80s

It follows that the test can go on.
In the next picture we illustrate the 7th RUN: a comparison between the target spectrum and the actual spectrum at 40s.

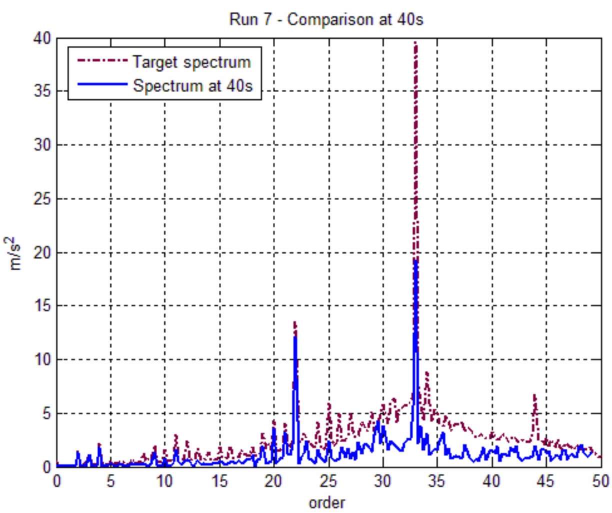

Fig. 11 – Run 7, comparison at 40s

Fig. 12 illustrates the behaviour of Index_SideBand. There aren't orders exceeding the threshold set to 3. Conversely, the index shows a trend to 1. It indicates a normal working condition of the gearbox. Then the reliability test can go on.

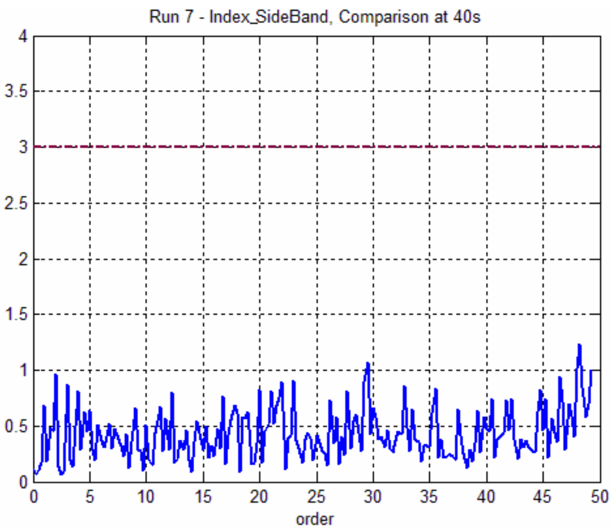

Fig. 12 – Run 7, Index_SideBand at 40s

In the Fig. 13 we have illustrated another comparison for 7th test cycle. In particular it's showed the instant 80s. From this picture it is clear that there aren't condition to stop the test, and so it can go on.

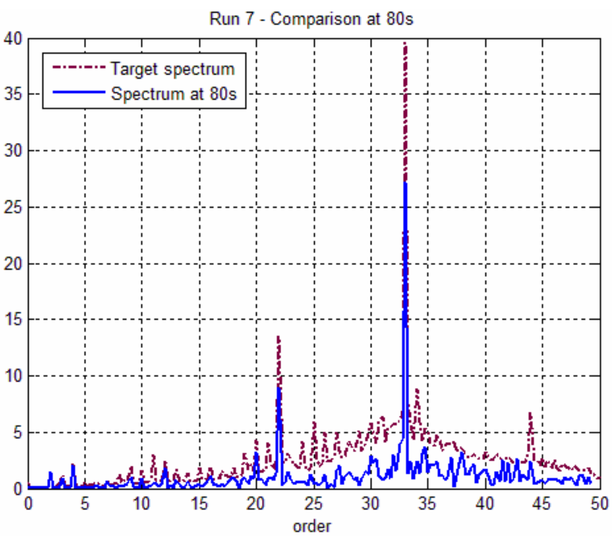

Fig. 13 – Run 7, comparison at 80s

The Fig. 14 illustrates the trend of the Index_SideBand at 80s. It shows all the orders under the fixed threshold. So the reliability test must not be stopped.
We remarked that we have only chosen two instants of the test cycle to illustrate the results obtained with the proposed technique. For that reason for all the other instants of the test, the index shows always the same trend.
This condition assures us that the gearbox works in normal condition and that there aren't anomalies. Then we continue with the reliability test.

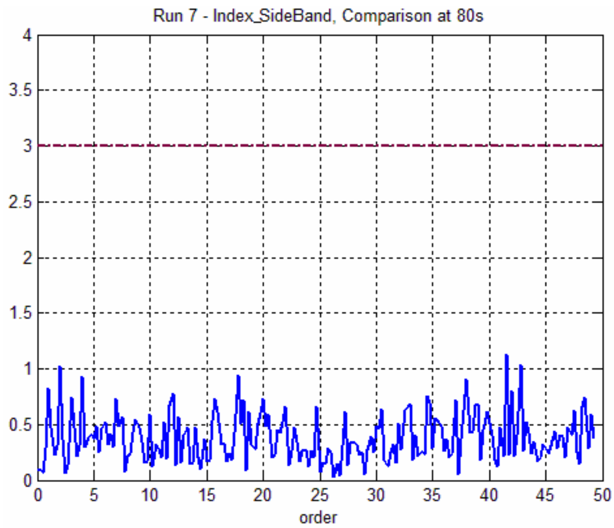

Fig. 14 – Run 7, Index_SideBand at 80s

An unexpected event occurs during the RUN 13. In fact the comparison between the target spectrum and the actual spectrum shows the value of the order 1 exceeding the threshold (Fig. 15). So the test must be stopped.

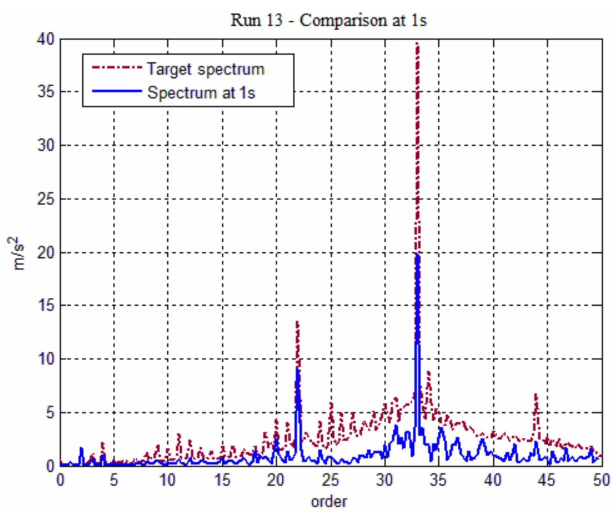

Fig. 15 – Run 13, comparison at 1s

At this moment the order 1 of the actual spectrum exceeds the amplitude of the same order of the target spectrum. It's quite clear looking at the Fig. 15, but it is more clear looking at the Fig. 16 in which the Index_SideBand and the threshold imposed to stop the duration test are reported.

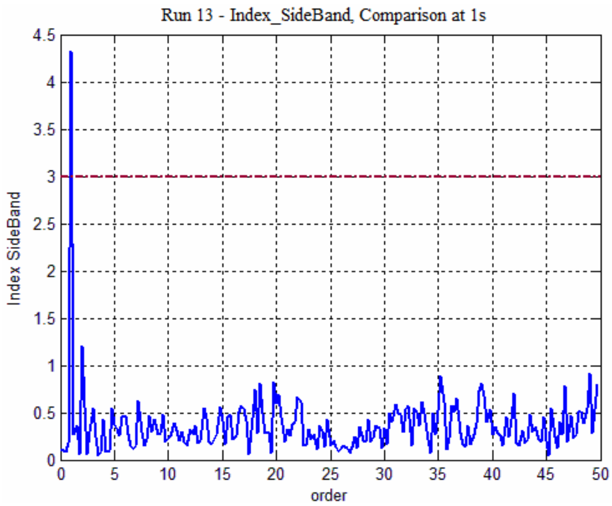

Fig. 16 – Run 13, Index_SideBand at 1s

So looking at the last diagram the duration test must be stopped at the 13th cycle, with all benefits.
A close-up on the order 1 clears what happens (Fig. 17).

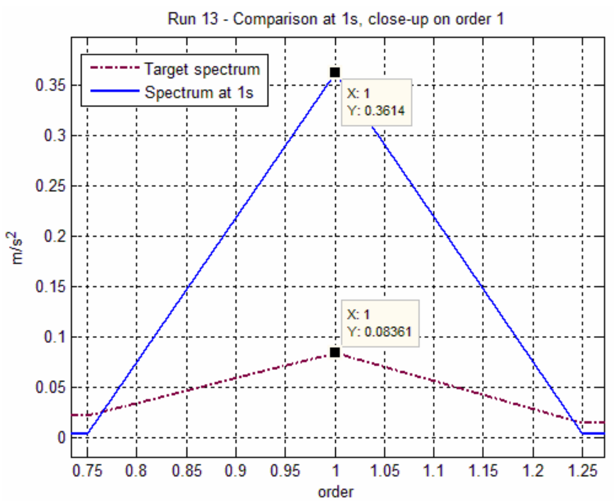

Fig. 17 – Run 13, comparison at 1s, close-up on order 1

It's quite clear that the ratio between the order 1 of the spectrum exceeds the threshold level of 3 times.
The diagram reported in Fig. 18 shows that the threshold overcoming is permanent and not temporary.
This condition is more clear in the Fig. 19 where the diagram illustrates a close-up on the order 1 of the target spectrum with reference to the actual one.
It's clear that the value of the actual spectrum at the 1st order exceeds the target spectrum more than 4 times.

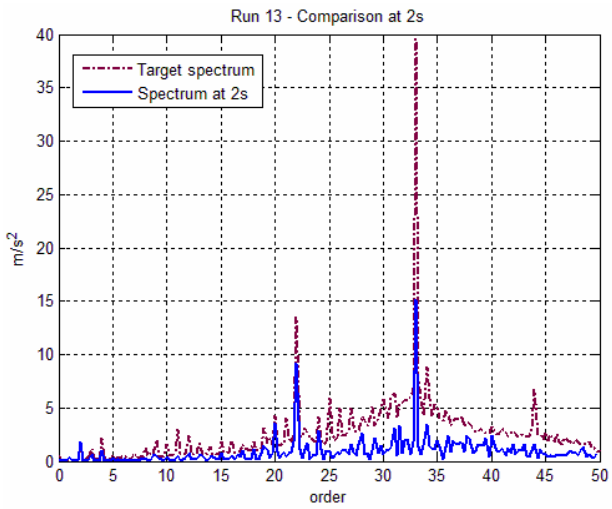

Fig. 18 – Run 13, comparison at 2s

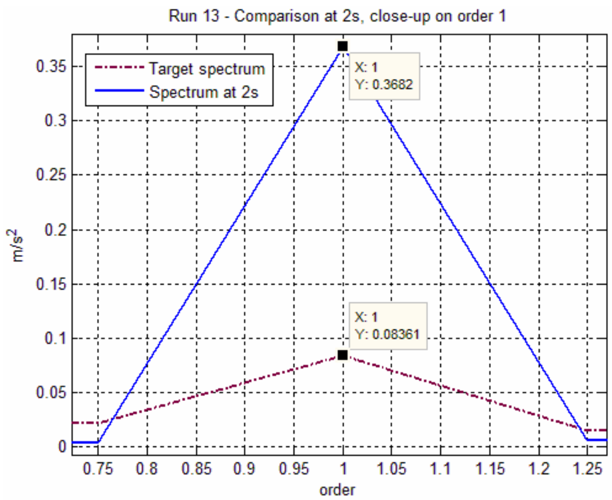

Fig. 19 – Run 13, comparison at 2s, close-up on order 1

To better observe what's happen at 2s, in Fig.20 it is illustrated the trend of the Index_SideBand.

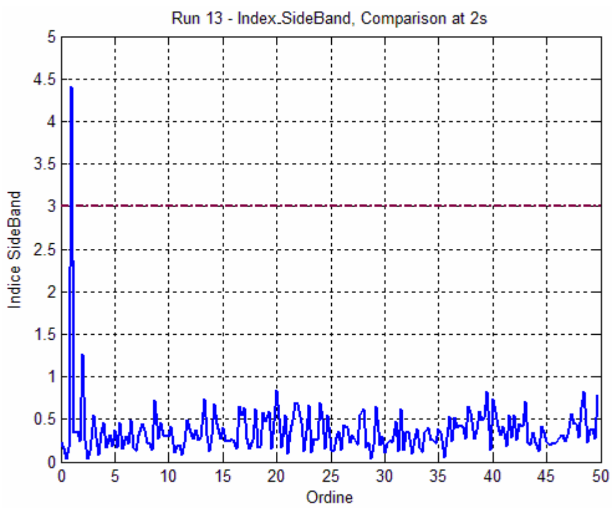

Fig. 20 – Run 13, Index_SideBand at 2s

Before concluding we want to remark that at Run 13 of the reliability test, the Index_Area doesn't overthrow the threshold: so it's clear the opportunity earlier stop of the test for obtaining a precise information about the order exceeding the threshold.
This advantage is clear during the inspection of the inside gearbox Fig 21.

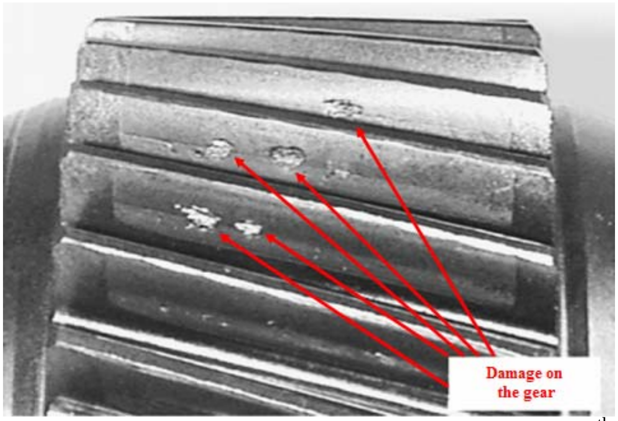

Fig.21 – Inspection of the inside gearbox at the 13th cycle test

Fig. 21 shows the presence of "*pitting*" on the gears. It is symptomatic of damage on the same gear.
The inspection following the RUN-stop confirms the importance of the information allowed by the proposed new index (Index_SideBand).

6 Conclusion

The new index proposed in this work allows new performance. In fact, the algorithm is designed for detecting earlier the gearbox damage.
The Index_SideBand monitoring of each order gives us information about each of these orders. So it helps to find the origin of damage. In fact, as we showed in the above paragraphs, each event is showed like frequency. Observing the frequencies which overthrow the threshold, we can detect the wheel to which frequency corresponds.
Note that test is also stopped if one order exceeds the threshold.
The Index_Area leads to stop the duration test only if the total contribution, given by the area of spectrum, overthrows the threshold level.
The proposed algorithm, using two indexes, offers a better control of the test and gives more information about the presence of an unexpected event that can cause a serious damage to the gearbox. We have a complete control on the whole process of gearbox reliability test. By using Index_Area we control the aspects regarding the increasing of all the orders on the spectrum, in particular we control if the energy content of the actual spectrum with respect to the target one. By using Index_SideBand we control each order, so we look for an order exceeding the fixed threshold. It is a punctual information, in fact this index shows 2 important aspects. The first one is the improvement of the gearbox monitoring reliability test. The second one is the indication which on order gives problems. If an unexpected event occurs the index gives us information about the exact order exceeding the threshold imposed.
By using this technique of monitoring we are sure to stop the gearbox reliability test if there is a problem inside the gearbox under testing and by contemporary we have information on the order exceeding the threshold level. Consequently can start the phase of inspection considering that an order indicates a particular gear inside the gearbox. So it is more easy to look for the problem which caused a damage on a gear.
We want to remark, the importance of the inspecting phase. In fact if we consider that the reliability test could be performed on all the gearboxes before their commercialization, it is quite evident the advantage for the industries which can perform the reliability test during their production. If the gearbox hasn't problem, the algorithm doesn't stop the test and we are sure of the commercialization of the gearbox (due to the positive response of the duration test). If during the test an unexpected problem occurs, the technique of monitoring point out this aspect and give information on which component can be the cause of damage, involving, of course, a great time saving.
The proposed index is a logical evolution of the research developed during the last ten years near the Mechanical Dept. of University of Naples, Federico II. It started by applying to the signals the Fourier and Wavelet processing for noise reduction [14-17].
One of the most important step in the next future is the integration between the wavelet and chaos theory in order to better process the noised vibrational signals acquired both through accelerometers and thermography.

References

[1] V. Niola, G. Quaremba, V. Avagliano, *"Vibration monitoring of gear transmission"* in press.

[2] G. F. Lang, *"S&V Geometry 101"*, Sound & Vibration, March 1999.

[3] J. G. Winterton, "*Component identification of gear-generated spectra*", Machinery Diagnostic Services, Bently Nevada Corporation, 1991.

[4] Hewlett-Packard, "*Dynamic Signal Analyzer Applications. Effective Machinery Maintenance Using Vibration Analisys*", Application Note 243-1, 1983.

[5] A. V. Barkov, N. A. Barkova, "*Diagnostic of Gearing and Geared Couplings using Spectrum Methods*", Proc. of the 20th Annual Meeting of the Vibration Institute, Saint Louis, Missouri, USA 1996.

[6] R. B. Randall, "*State of the art in monitoring rotating machinery – Part I*", Sound & Vibration, March 2004.

[7] C. M. Harris, "*Shock and Vibration Handbook*", 4th ed., McGraw-Hill, 1995.

[8] K.Wang "*Phase information at tooth mesh frequency for gear crack diagnosis*", Proc. 2nd *IEEE* Conference on Industrial Electronics and Application, 2007.

[9] E.B. Halim, S.L. Shah, M.J. Zuo and M.A.A. S. Choudhury, "*Fault Detection of Gearbox from Vibration Signals using Time-Frequency Domain Averaging*", Proc. Of the 2006 American Control Conference, Minneapolis, Minnesota, USA, June 14-16, 2006, pp. 4430-4435.

[10] P.D. Mc Fadden, "*A revised model for the extraction of periodic waveforms by time domain averaging, Mechanical Systems and Signal Processing*", vol. 1(1), pp. 83-95, 1987.

[11] F.K. Choy, D.H. Mugler and J. Zhou, "*Damage Identification of a Gear Transmission Using Vibration Signatures*", ASME Trans. J. Mechanical Design, Vol. 125 , pp. 394-403, 2003.

[12] G. Dalpiaz, A. Rivola, R. Rubini, "*Dynamic modelling of gear systems for condition monitoring and diagnostics*", Proc. of the Congress on Technical Diagnostics, KDT'96, 09/17-20/1996, Gdanks, Poland, 2, 185-192.

[13] G. Dalpiaz, A. Rivola, R. Rubini, "*Gear fault monitoring: comparison of vibration analysis techniques*", Proc. of the 3rd Int. Conf. on Acoustical and Vibratory Surveillance Methods and Diagnostic Techniques, 13-15/10/1998, Senlis, France, Ed. Société Française des Mécaniciens, Courbevole, 1, 263-272.

[14] V. Niola, G. Quaremba, A. Forcelli, "*Gear Noise Detection. An Integrated Method using Fourier and Wavelet Approach*", 6th Int. Conf. on SYSTEM SCIENCE and SIMULATION in ENGINEERING, Venice, 11/20-23/2007.

[15] I. Daubechies, "*Ten Lectures on Wavelets*", SIAM, 1992.

[16] W. Härdle, G. Kerkyacharian, D. Picard and A. Tsybakov, "*Lecture Notes in Statistics - Wavelets, Approximation, and Statistical Applications*", Springer, 1998.

[17] A. Antoniadis, G. Oppenheim, "*Lecture notes in Statistics. - Wavelets and Statistics*", Springer, 1995.