An Approach for Design of Multi-element USBL Systems

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Abstract: - This paper presents an approach for design of multi-element ultra-short baseline (USBL) systems with the goal to improve the coordinate determination accuracy of the underwater objects. The object location estimation is performed on the base of the measurement of the distance to the object and the angular object position. It is supposed that the object is equipped with a transponder that receives the interrogation acoustical impulse and sends an acoustical impulse in reply. The idea of the design method is to increase of the number of receiving elements of the USBL antenna with a corresponding increase of the number of the receiving bases with the different spatial orientation. The arrangement of the array receiving elements is obtained by the rotations of the basic (three-element) receiving arrays around the longitudinal and lateral carrier axes (the case of the determination of the Cartesian coordinates of the object in the carrier coordinate system is considered). It is supposed that the control of the spatial orientation of the receiving USBL array is realized by means of measurement of its pitch and roll angles (in the carrier coordinate system). In the article the proposed method is applied for design of the nine-element USBL system. The special coordinate determination algorithm for the proposed USBL system is designed and tested. The simulation of the algorithm was realized with the assumption that the object can have an arbitrary location in the lower hemisphere and the receiving USBL antenna can have significant inclination. The coordinate determination accuracy of the proposed USBL system is evaluated.

Key-Words: - Ultra-short baseline (USBL) system, underwater object, transponder, carrier coordinate system, local coordinate system, pitch and roll angles.

1 Introduction

The methods and the systems for determining the position of underwater objects are constantly being developed with an intention to improve the reliability and accuracy of the object coordinate measurements. For many tasks in ocean engineering the accuracy of determination of the position of the object may be critical. Very often, the task of precise determination of the position of an underwater object, it is necessary to provide for real severe marine conditions where sea surface roughness and strong currents take place. In long distances the acoustical methods for position determination still remain the unique solution of the posed problem. In this paper we examine the design method for ultra-short baseline (USBL) systems where the location measurement of the underwater objects is based on the determination of the distance to the object and the object's angular position. The idea of the design method is to increase the number of receiving elements of the USBL array and use for the placement of receiving elements the natural rotations of the basic (three-element) receiving arrays around the longitudinal and lateral carrier axes. In addition to the design method the coordinate determination algorithm is proposed. The designed algorithm realizes the processing of multiple time delays obtained with the multi-element USBL array.

2 Problem Formulation

2.1 Location problem

In this article we will consider the position determination of an underwater object with USBL acoustic systems. The principle for measuring the object coordinates with USBL method is well known and is described in detail in [1][2]. Object position determination with this method is realized by means of the measuring of the distance to the object and its angular position relative to the measuring system location. During the last decades the improvements in accuracy and reliability of object position determination was the subject of investigation and development of the USBL systems [3-7]. To improve the USBL systems various special signal processing techniques were utilized. In particular the chirp signals and greater inter-element array separation [3] were used. Also the acoustic digital spread spectrum [4] and modulated Barker-coded signals [5] were applied. In [6] the USBL system with frequency-hopped pulses was investigated. The problem of instability of the position of the receiving antenna was studied in detail in [7]. In [8][9] the problem of low precision in coordinate determination for the case of the object found in the plane of receiving bases is studied.

In this paper the method to design a multi-element USBL is proposed. The objective of this design method is the further improvement of accuracy for the USBL systems (as well increasing the reliability of USBL systems) by means of increasing the number of receiving elements and the number of elemental USBL systems with different orientations in space. The designed algorithm processes multiple time delays in the output of the proposed multi-element USBL receiving array. Furthermore, in this paper we assume that the propagation medium is homogeneous and the multipath interference is absent.

2.2 Basic USBL System

The principle of operating of the USBL system is the following: the transmitter sends an interrogation acoustical impulse in the propagation medium where the object is located in an accessible distance. It is supposed that the object is equipped with a transponder that receives the interrogation impulse and sends an acoustical impulse in reply. The distance to the object is determined by the measurement of the values of propagation times of the interrogation impulse from the system and the transponder pulse response. The angular position of the object is determined by the measurement of the phase difference of the transponder pulse carrier frequency on the receiving array outputs. The minimum number of receiving elements for the USBL system for object coordinate determination is three [1].

To improve the reliability and accuracy of coordinate determination the number of elements of receiving antenna can be increased. This approach was initially proposed in [8] and studied in detail in [9] for the case of a five-element USBL system. The USBL system proposed in [8,9], has being designed by assembling of elemental (three-element) USBL arrays to single five-element USBL array.

We consider briefly the principle of coordinate determination for the three-element USBL system and then describe the case for designing of the multi-element USBL array.

Let $\Sigma=(0,x,y,z)$ be the carrier coordinate system (left-hand) with the origin in the point O in such a way that the x-coordinate axis coincides with the carrier longitudinal axis L-L' (the positive direction coincides with the direction of the straight arrowed line), the y-coordinate coincides with the carrier lateral axis B-B', and z-axis goes downwards. Now we can define the USBL array orientation in the introduced carrier coordinate system. Let the angle between the y-axis and base 1-2 (when it lies in horizontal plane) is 135° and the angle between the y-axis and the base 3-2 is 45°.

The geometry of the receiving antenna with the carrier longitudinal and lateral axes is presented in Fig.1. We also define a local coordinate system for this elemental (three-element) USBL system. Let $\Sigma_{123}=(0,x_{123},y_{123},z_{123})$ be the local coordinate system for the considered USBL system (with antenna elements 1,2,3).

In Fig.1 we specify the angles (α, β and γ) that define the position of the underwater object located in point P (firstly the object position is defined in the $\Sigma_{123}=(0,x_{123},y_{123},z_{123})$ coordinate system). We also assume that the USBL system is equipped with special unit to measure the pitch and roll angles of the receiving antenna. Let angles ξ and ζ be pitch and roll angles of the receiving USBL antenna (in the figure these angles show the rotations of the receiving three-element USBL array relative to the carrier lateral B-B' axis and the carrier longitudinal L-L' axis).

Let that the interrogation impulse has been sent and the reply impulse is being received by antenna. The distance to the object is defined by measuring the propagation times of the interrogation and reply pulses.

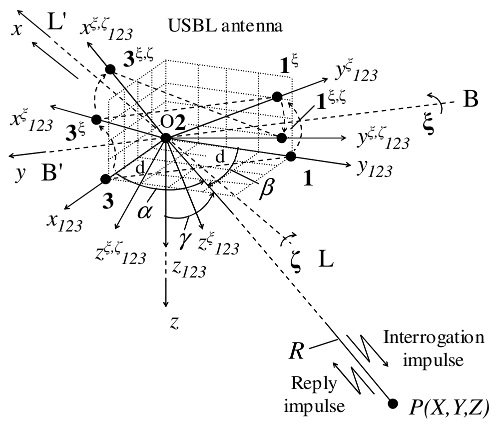

Fig.1. Geometry of the three-element USBL receiving array and the carrier longitudinal and lateral axes.

Time delays on receiving elements define the object's angular position. The time delays τ_{12} and τ_{32} at the outputs of the receiving elements of the base 1-2 and the base 3-2 (it is supposed that $R>>d$) can be expressed in the following way:

$$\tau_{12} = \frac{d\cos\beta}{c}, \quad \tau_{32} = \frac{d\cos\alpha}{c}, \tag{1}$$

where c is the speed of the sound in the water. With d/c defined as τ_d we can write the direction cosines $cos\alpha$ and $cos\beta$:

$$\cos\alpha = \tau_{32}/\tau_d, \quad \cos\beta = \tau_{12}/\tau_d . \tag{2}$$

The third direction cosine is:

$$\cos\gamma = \sqrt{1-\left(\tau_{32}/\tau_d\right)^2-\left(\tau_{12}/\tau_d\right)^2} . \tag{3}$$

If we know the incline distance R to the object (distance R is defined by measuring the propagation times of the interrogation and reply pulses) Cartesian coordinates of the point P in the $\Sigma_{123}=(0,x_{123},y_{123},z_{123})$ coordinate system define as:

$$X_{123} = R\cos\alpha, \ Y_{123} = R\cos\beta, \ Z_{123} = R\cos\gamma. \tag{4}$$

To obtain the coordinates of point P in the carrier coordinate system (coordinates of the point P in the $\Sigma=(0,x,y,z)$ coordinate system, see Fig.1) it is necessary realize the corresponding transformation of obtained coordinates X_{123}, Y_{123}, Z_{123}.

Let that the pitch and roll rotations of the receiving antenna take place. The pitch and roll rotations on the pitch angle ξ and on the roll angle ζ of the elemental three-element USBL antenna are shown in Fig.1. After the first rotation on pitch angle ξ relative to B-B' axis the receiving elements are displacing to the points 1^{ξ} and 3^{ξ} respectively. After second rotation on the angle ζ relatively L-L' axis the receiving elements are displacing to the points $1^{\xi,\zeta}$ and $3^{\xi,\zeta}$ respectively (see Fig.1). With the rotations of receiving bases the corresponding transformations of coordinate systems from $\Sigma_{123}=(0,x_{123},y_{123},z_{123})$ to $\Sigma^{\xi}_{123}=(0,x^{\xi}_{123},y^{\xi}_{123},z^{\xi}_{123})$ and to $\Sigma^{\xi,\zeta}_{123}=(0,x^{\xi,\zeta}_{123}, y^{\xi,\zeta}_{123}, z^{\xi,\zeta}_{123})$ are have taken place.

Calculation expressions for the case of the pitch and roll of the three element receiving antenna with introduced orientation relative to the carrier have been obtained in [8][9]. So we describe the calculation procedure here very briefly. If we introduce vectors: $\boldsymbol{p}^{\xi,\zeta}_{123}=[X^{\xi,\zeta}_{123}, Y^{\xi,\zeta}_{123}, Z^{\xi,\zeta}_{123}]^T$ and $\boldsymbol{p}^{\xi}_{123}=[X^{\xi}_{123}, Y^{\xi}_{123}, Z^{\xi}_{123}]^T$ (vector $\boldsymbol{p}^{\xi,\zeta}_{123}$ represents the coordinates of object in $\Sigma^{\xi,\zeta}_{123}$ coordinate system and vector $\boldsymbol{p}^{\xi}_{123}$ represents the coordinates of object in Σ^{ξ}_{123} coordinate system) and the transformation matrix $\boldsymbol{B}=\boldsymbol{B}[\zeta,,\eta^{\xi}_{123}(\xi),\chi^{\xi}_{123}(\xi),\nu^{\xi}_{123}(\xi)]$ with direction cosines $\eta^{\xi}_{123}=cos(x^{\xi}_{123},L)$, $\chi^{\xi}_{123}=cos(y^{\xi}_{123},L)$ y $\nu^{\xi}_{123}.=cos(z^{\xi}_{123},L)$ (matrix $\boldsymbol{B}$ transforms vector $\boldsymbol{p}^{\xi}_{123}$ to vector $\boldsymbol{p}^{\xi,\zeta}_{123}$) we can write the equation:

$$\boldsymbol{p}^{\xi}_{123} = \boldsymbol{B}^{-1}\boldsymbol{p}^{\varsigma}_{123} . \tag{5}$$

If we introduce vector $\boldsymbol{p}_{123}=[X_{123}, Y_{123}, Z_{123}]^T$ (vector $\boldsymbol{p}^{\xi}_{123}$ represents the coordinates of the object in Σ_{123} coordinate system) and the transformation matrix $\boldsymbol{A}= \boldsymbol{A}[\xi, \ \eta_{123}, \ \chi_{123}, \ \nu_{123}]$ with direction cosines $\eta_{123}=cos(x_{123} ,B)$, $\chi_{123}=cos(y_{123} ,B)$, $\nu_{123} =cos(z_{123} ,B)$ (matrix $\boldsymbol{A}$ transforms vector $\boldsymbol{p}_{123}$ to vector $\boldsymbol{p}^{\xi}_{123}$) we can write the equation:

$$\boldsymbol{p}_{123} = \boldsymbol{A}^{-1}\boldsymbol{p}^{\xi}_{123} . \tag{6}$$

These two transformations can be combining in the equation:

$$\boldsymbol{p}_{123} = \boldsymbol{A}^{-1}\boldsymbol{B}^{-1}\boldsymbol{p}^{\xi\varsigma}_{123} . \tag{7}$$

In order to obtain the coordinates of the object in a carrier coordinates system $\Sigma=(0,x,y,z)$ it is necessary to make one more rotation of the coordinate system Σ_{123} around the axis z on the angle of 135° (see Fig.1). For the Σ coordinate system, we have the following direction cosines for the z-axis: $cos(x, z)=0$, $cos(y,z)=0$, $cos(z, z)=1$. If we introduce vector $\boldsymbol{p}=[X, Y, Z]^T$ (vector $\boldsymbol{p}$ represents the coordinates of the object in Σ coordinate system) and the transformation matrix $\boldsymbol{C}$ (matrix $\boldsymbol{C}$ transforms vector $\boldsymbol{p}$ to vector $\boldsymbol{p}_{123}$) the final equation to find vector $\boldsymbol{p}$ be the next:

$$\boldsymbol{p} = \boldsymbol{C}^{-1}\boldsymbol{A}^{-1}\boldsymbol{B}^{-1}\boldsymbol{p}^{\xi\varsigma}_{123} . \tag{8}$$

2.3 Five-element USBL system

Articles [8][9] noted the problem of the low precision coordinate determination for the cases when the controlled object is found in planes of receiving bases of elemental USBL systems. To resolve this problem a five-element USBL system with different spatial orientation of receiving bases was proposed. First, one more element (element number 4) was added to the antenna array in the horizontal plane. This element is added in such a way that a square is formed (the sides of the square form the receiving bases of the antenna). As result we have four basic three-element USBL arrays located in a horizontal plane. The fifth element (element number 5) is placed underneath the four-element USBL antenna plane exactly underneath its geometric center, in such a way that the four obtained inclined bases would be the same size as the size of the horizontal receiving bases. In this case, the USBL system obtains two additional three-element basic USBL arrays placed in two orthogonal vertical planes. Finally, we have the five-element USBL array formed with the six basic three-element USBL arrays with the six different orientations: $USBL_{123}$, $USBL_{234}$, $USBL_{341}$, $USBL_{412}$, $USBL_{153}$ and $USBL_{254}$. (see Fig.2).

For this five-element USBL system a special algorithm had been designed and investigated and significant coordinate determination accuracy improvement was obtained.

At the same time the detailed analysis of the algorithm simulation results represented in [9] shows that some imperfections of the modernized USBL system still remain. Thus the algorithm simulation for some orientations of the receiving antenna relative to the object shows significant fluctuations of coordinate determination errors [9]. The frequency of the time delay counter (160MHz) of the system looks unjustifiably high. The proposed in [9] algorithm also has threshold level election sensitivity.

In the present paper the additional increase of the number of the receiving elements is proposed (as a result the new receiving array has a larger number of elemental USBL systems with different spatial orientations). In the paper the method of design of a nine-element USBL array is considered. The method is also allows us to realize the further increase of the number of the receiving elements for the USBL systems. Also the significant improvement of the coordinate determination algorithm is realized. For example, in new algorithm the Z-coordinate sign determination is realized without utilizing any threshold value. In new multi-element USBL system the frequency of the time delay counter was significantly reduced.

3 Problem Solution

3.1 Nine-element USBL system

The proposed nine-element USBL array is shown in Fig.3. To demonstrate the development of the USBL array we will use the five-element array as a reference (see Fig.2). Let that the 1, 2, 3 elements of the three-element array and of the five-element array have the same location in the carrier coordinate system (see Figs.1,2). In this case the rotation around the lateral axis *L-L'* coincides with the rotation to pitch angle ξ and the rotation around the longitudinal axis *B-B'* coincides with the rotation to roll angle ζ. The additional new receiving elements (receiving elements with numbers 6, 7, 8, 9) can be formed by means of the rotation – first, of the three-element $USBL_{123}$ array ($USBL_{123}$ array is formed with the receiving elements 1,2,3) around the B-B' axis and then by means of the rotations of other three-element arrays located in a horizontal plane (the $USBL_{234}$ array with the receiving elements 2,3,4; the $USBL_{341}$ array with the receiving elements 3,4,1; and the USB_{412} array with the receiving elements 4,1,2).

We explain in detail only how the $USBL_{163}$ array was formed. The element 2 rotates around the axis B-B' on angle of 45° and moves to point 6 (6 is also the number of the new receiving element) whereas elements 1 and 3 are fixed. Rotation direction coincides with the curved arrow that indicates the positive direction of pitch angle ξ. The plane of location of the new $USBL_{163}$ array is shown in Fig.3 with hatching. The other new three-element USBL arrays (the $USBL_{274}$, the $USBL_{381}$ and the $USBL_{492}$) are obtained in the same way (see Fig.3). The obtained locus of points (receiving elements 6,7,8,9) forms the spherical surface of designed nine-element USBL array.

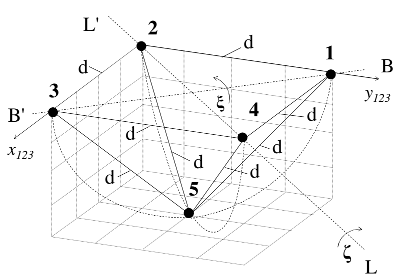

Fig.2. Five-element USBL array

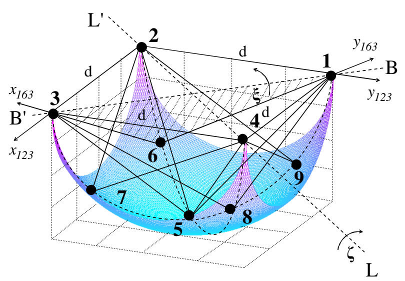

Fig.3. Nine-element USBL array

3.2 Calculation expressions

In the case of the nine-element array the USBL system consists of ten elemental tree-element USBL systems with ten different orientations. These elemental USBL systems are: $USBL_{123}$, $USBL_{234}$, $USBL_{341}$, $USBL_{412}$, $USBL_{153}$, $USBL_{254}$, $USBL_{163}$, $USBL_{274}$, $USBL_{381}$, and $USBL_{492}$. We can divide the three-element USBL systems in three groups: horizontal USBL systems ($USBL_{123}$, $USBL_{234}$, $USBL_{341}$, $USBL_{412}$,), vertical USBL systems ($USBL_{153}$, $USBL_{254}$) and inclined USBL systems ($USBL_{163}$, $USBL_{274}$, $USBL_{381}$, and $USBL_{492}$).

The coordinates of the object are determined individually in each elemental USBL system. The horizontal $USBL_{234}$, $USBL_{341}$ and $USBL_{412}$ systems differ from the examined earlier $USBL_{123}$ system in their own values of the pitch and roll angles and in their own angles of rotation of each USBL antenna around the z-axis.

The coordinate determination for the vertical $USBL_{153}$ system is almost the same as for the $USBL_{123}$ system, the difference is that one additional step is required to reduce the $USBL_{153}$ system coordinates to a horizontal plane (by rotation the $USBL_{153}$ system on a 90° angle). We have to do the same with coordinates obtained with the $USBL_{254}$ system.

The inclined $USBL_{163}$ system differs from the $USBL_{123}$ only in its additional inclination on an angle of 45° (see Fig.3). We have the same for the $USBL_{274}$ and $USBL_{234}$ systems (for $USBL_{341}$ and systems $USBL_{381}$; and for the $USBL_{412}$ and $USBL_{492}$ systems). So for $USBL_{163}$, $USBL_{274}$, $USBL_{381}$ and $USBL_{492}$ systems we have to accomplish the additional step (to realize the additional rotation on an angle of 45° for each system).

To carry out these additional rotations for non horizontal USBL arrays we introduce for each system the transformation matrix $\boldsymbol{D}$ (for example, for the $USBL_{163}$ system matrix $\boldsymbol{D}_{163}$ transforms vector $\boldsymbol{p}_{123}$ to vector $\boldsymbol{p}_{163}$). In order to distinguish the results of the measured coordinates by different basic USBL systems we introduce the following designations for the measured vectors: $\boldsymbol{p}_{\mathrm{USBL}_{123}}$, $\boldsymbol{p}_{\mathrm{USBL}_{234}}$, $\boldsymbol{p}_{\mathrm{USBL}_{341}}$, $\boldsymbol{p}_{\mathrm{USBL}_{412}}$, $\boldsymbol{p}_{\mathrm{USBL}_{153}}$, $\boldsymbol{p}_{\mathrm{USBL}_{254}}$, $\boldsymbol{p}_{\mathrm{USBL}_{163}}$, $\boldsymbol{p}_{\mathrm{USBL}_{274}}$, $\boldsymbol{p}_{\mathrm{USBL}_{381}}$, $\boldsymbol{p}_{\mathrm{USBL}_{492}}$.

We also introduce the corresponding indexes for the transformation matrixes for each particular three-element USBL system.

After introducing these designations the calculation expressions for the horizontal USBL systems will be written as follows:

$$\begin{aligned}
\boldsymbol{p}_{\mathrm{USBL}_{123}} &= \boldsymbol{C}^{-1}_{123}\boldsymbol{A}^{-1}_{123}\boldsymbol{B}^{-1}_{123}\boldsymbol{p}^{\xi\varsigma}_{123}\,;\\
\boldsymbol{p}_{\mathrm{USBL}_{234}} &= \boldsymbol{C}^{-1}_{234}\boldsymbol{A}^{-1}_{234}\boldsymbol{B}^{-1}_{234}\boldsymbol{p}^{\xi\varsigma}_{234}\,;\\
\boldsymbol{p}_{\mathrm{USBL}_{341}} &= \boldsymbol{C}^{-1}_{341}\boldsymbol{A}^{-1}_{341}\boldsymbol{B}^{-1}_{341}\boldsymbol{p}^{\xi\varsigma}_{341}\,;\\
\boldsymbol{p}_{\mathrm{USBL}_{412}} &= \boldsymbol{C}^{-1}_{412}\boldsymbol{A}^{-1}_{412}\boldsymbol{B}^{-1}_{412}\boldsymbol{p}^{\xi\varsigma}_{412}\,;
\end{aligned} \tag{9}$$

where the vector $\boldsymbol{p}_{\mathrm{USBL}_{123}}$ is the vector that represents the object coordinates (in the carrier coordinate system) obtained with the $USBL_{123}$ system, the vector $\boldsymbol{p}_{\mathrm{USBL}_{234}}$ is the vector obtained with the with the $USBL_{234}$ system and so on.

The calculation expressions for coordinate vectors obtained with the vertical USBL systems are the next:

$$\begin{aligned}
\boldsymbol{p}_{\mathrm{USBL}_{153}} &= \boldsymbol{C}^{-1}_{153}\boldsymbol{D}^{-1}_{153}\boldsymbol{A}^{-1}_{153}\boldsymbol{B}^{-1}_{153}\boldsymbol{p}^{\xi\varsigma}_{153}\,;\\
\boldsymbol{p}_{\mathrm{USBL}_{254}} &= \boldsymbol{C}^{-1}_{254}\boldsymbol{D}^{-1}_{254}\boldsymbol{A}^{-1}_{254}\boldsymbol{B}^{-1}_{254}\boldsymbol{p}^{\xi\varsigma}_{254}\,.
\end{aligned} \tag{10}$$

The calculation expressions for coordinate vectors obtained with the inclined USBL systems are written in the following way:

$$\begin{aligned}
\boldsymbol{p}_{\mathrm{USBL}_{163}} &= \boldsymbol{C}^{-1}_{163}\boldsymbol{D}^{-1}_{163}\boldsymbol{A}^{-1}_{163}\boldsymbol{B}^{-1}_{163}\boldsymbol{p}^{\xi\varsigma}_{163}\,;\\
\boldsymbol{p}_{\mathrm{USBL}_{274}} &= \boldsymbol{C}^{-1}_{274}\boldsymbol{D}^{-1}_{274}\boldsymbol{A}^{-1}_{274}\boldsymbol{B}^{-1}_{274}\boldsymbol{p}^{\xi\varsigma}_{274}\,;\\
\boldsymbol{p}_{\mathrm{USBL}_{381}} &= \boldsymbol{C}^{-1}_{381}\boldsymbol{D}^{-1}_{381}\boldsymbol{A}^{-1}_{381}\boldsymbol{B}^{-1}_{381}\boldsymbol{p}^{\xi\varsigma}_{381}\,;\\
\boldsymbol{p}_{\mathrm{USBL}_{492}} &= \boldsymbol{C}^{-1}_{492}\boldsymbol{D}^{-1}_{492}\boldsymbol{A}^{-1}_{492}\boldsymbol{B}^{-1}_{492}\boldsymbol{p}^{\xi\varsigma}_{492}\,.
\end{aligned} \tag{11}$$

3.3 Algorithm description

It is assumed that the measured values are: ξ and ζ – pitch and roll angles of the receiving nine-element antenna (see Fig.3); t – interrogation pulse and response pulse separation; τ_{12}, τ_{32}, τ_{23}, τ_{43}, τ_{34}, τ_{14}, τ_{41}, τ_{21}, τ_{15}, τ_{35}, τ_{25}, τ_{45}, τ_{16}, τ_{36}, τ_{27}, τ_{47}, τ_{38}, τ_{18}, τ_{49}, τ_{29}, - time delays for receiving bases of the corresponding $USBL_{123}$, $USBL_{234}$, $USBL_{341}$, $USBL_{412}$, $USBL_{153}$, $USBL_{452}$, $USBL_{163}$, $USBL_{274}$, $USBL_{381}$, and $USBL_{492}$ systems (twenty time delays are measured, to provide the positive values of time delays the second-indexed outputs are inverted).

The time delays can be measured by the standard digital method (with the infilling of the interval corresponding to time delay with the high-frequency impulses). The time delays on each base can be expressed in number of impulses as follows:

$n_{ij}=f_c(\tau_{ij}+T/2)$, where f_c is the frequency of the time delay counter, τ_{ij} is the time delay between the receiving elements of the USBL array (j is the index of the common receiving element), T is the period of the transponder pulse carrier frequency.

First the vector $\boldsymbol{p}^{\xi,\zeta}_{153}=[X^{\xi,\zeta}_{153},Y^{\xi,\zeta}_{153},Z^{\xi,\zeta}_{153}]^T$) is calculated and the sign of the $Z^{\xi,\zeta}_{153}$ coordinate is determined. The coordinates $X^{\xi,\zeta}_{153},Y^{\xi,\zeta}_{153},Z^{\xi,\zeta}_{153}$ are calculated in accordance with the calculation formulas for Cartesian coordinates in the $\Sigma^{\xi,\zeta}_{153}=(0,x^{\xi,\zeta}_{153}, y^{\xi,\zeta}_{153}, z^{\xi,\zeta}_{153})$ coordinate system.

The sign of the $Z^{\xi,\zeta}_{153}$ coordinate is defined by utilizing the time delay values obtained for the $USBL_{452}$ system (values τ_{25}, τ_{45}). If $\tau_{45}>\tau_{25}$ the sign of the $Z^{\xi,\zeta}_{153}$ coordinate is assumed to be negative. If $\tau_{45}\leq \tau_{25}$ is assumed the $Z^{\xi,\zeta}_{153}$ coordinate is positive. With the obtained values of the vector $\boldsymbol{p}^{\xi,\zeta}_{153}=[X^{\xi,\zeta}_{153},Y^{\xi,\zeta}_{153}, Z^{\xi,\zeta}_{153}]^T$ the values of the time delays τ'_{25} and τ'_{45} are calculated for the $USBL_{452}$ system. The modules of the differences $\delta1= |\tau'_{25}-\tau_{25}|$ and $\delta2=|\tau'_{45}-\tau_{45}|$ are then calculated (the values τ_{25} and τ_{45} are obtained through measurement; and the values of τ'_{25} and τ'_{45} are calculated). Then the value $\Delta1=(\delta1^2+\delta2^2)^{0.5}$ is calculated. The same procedure is repeated with the opposite sign of $Z^{\xi,\zeta}_{153}$ coordinate (with calculation of the corresponding values of τ'_{25} and τ'_{45}, the differences $\delta1= |\tau'_{25}-\tau_{25}|$ and $\delta2=|\tau'_{45}-\tau_{45}|$ and the corresponding value $\Delta2$). If $\Delta1>\Delta2$ the latter sign is assumed as correct. Otherwise the initial sign value of $Z^{\xi,\zeta}_{153}$ coordinate is assumed as correct.

Then the vector $\boldsymbol{p}^{\xi,\zeta}_{254}=[X^{\xi,\zeta}_{254}, Y^{\xi,\zeta}_{254},Z^{\xi,\zeta}_{254}]^T$ is calculated and the sign of the $Z^{\xi,\zeta}_{254}$ coordinate is determined. To define the sign of the $Z^{\xi,\zeta}_{254}$ coordinate the procedure is described above is applied to the vector $\boldsymbol{p}^{\xi,\zeta}_{254}=[X^{\xi,\zeta}_{254}, Y^{\xi,\zeta}_{254},Z^{\xi,\zeta}_{254}]^T$. The difference is that in this time the measured time delays τ_{15} and τ_{35} are utilized to compare with the calculated values τ'_{15} and τ'_{35} for the different signs of the $Z^{\xi,\zeta}_{254}$ coordinate.

The important part of the algorithm is solving the problem of the low precision of coordinate determination in cases when an object is found in the plane (or near to the plane) of the receiving bases. This problem was investigated in detail in [9] and it was found that the data obtained with some elemental USBL system can be utilized in calculation if the latitude angle to object is more than 10°. Otherwise (if the latitude angle to the object relative to plane of the measuring antenna is equal or less than 10°) the calculated values are discarded and do not participate in the calculation of the final means of the coordinates of the object.

Through the next stage of the algorithm is the calculation of the values of the angles between the planes of the measuring three-element antennas and the directions to the object (latitude angles to object). For that the coordinate vectors in the spherical coordinate can be expressed as follows:

$$\boldsymbol{q}^{\xi,\varsigma}_{153} = (R_{153},\psi^{\xi,\varsigma}_{153},\varphi^{\xi,\varsigma}_{153})^T;$$
$$\boldsymbol{q}^{\xi,\varsigma}_{254} = (R_{254},\psi^{\xi,\varsigma}_{254},\varphi^{\xi,\varsigma}_{254})^T, \quad (12)$$

where $\psi^{\xi,\zeta}_{153}$, $\varphi^{\xi,\zeta}_{153}$ are the polar and azimuth angles in the $USBL_{153}$ spherical coordinate system and $\psi^{\xi,\zeta}_{254}$, $\varphi^{\xi,\zeta}_{254}$ are the polar and azimuth angles in the $USBL_{254}$ spherical coordinate system. The values of polar angles ($\psi^{\xi,\zeta}_{153}$ and $\psi^{\xi,\zeta}_{254}$) for each USBL system define the decision to utilize or no utilize this system in the calculation of coordinate means. So if the values of the corresponding polar angles of both systems are found outside [80°, 100°] diapason, the values of both systems are utilized. If the value of the polar angle of one of the USBL systems lies outside the diapason [80°, 100°] and the value of the polar angle of the other system belongs to the [80°, 100°] then the value of the first system is utilized and the value obtained from another system is not taken into account. If the values of the polar angles of both systems are found within [80°, 100°] diapason, the coordinates obtained with the both vertical USBL system are discarded. The case when the values of the polar angles for the both vertical USBL systems lie within [80°, 100°] diapason corresponds to the location of the object in the narrow cone under the USBL array and are measured with good accuracy with the horizontal and inclined elemental USBL arrays. The final step in the coordinate determination of this part of the algorithm is the calculation (if it is necessary) of the Cartesian coordinates of the object in the coordinate system of the carrier (calculation of the vectors $\boldsymbol{p}_{USBL_{153}}$ and $\boldsymbol{p}_{USBL_{254}}$). These calculations for the $USBL_{153}$ and $USBL_{254}$ systems are carried out according to the formulas (10).

In the second stage of the algorithm the object coordinates are calculated using the time delays obtained by the horizontal measuring systems ($USBL_{123}$, $USBL_{234}$, $USBL_{341}$, $USBL_{412}$). It is also supposed that the receiving USBL array can have pitch and roll inclinations. We consider the $USBL_{123}$ system in order to describe this procedure (for the other horizontal USBL systems this part of the algorithm functions in the same way).

First we calculate the coordinates of the object based on the delays measured with the $USBL_{123}$ system ($\boldsymbol{p}^{\xi,\zeta}_{123}=[X^{\xi,\zeta}_{123}, Y^{\xi,\zeta}_{123}, Z^{\xi,\zeta}_{123}]^T$) and then the sign of the $Z^{\xi,\zeta}_{123}$ coordinate is determined. The sign of the $Z^{\xi,\zeta}_{123}$ coordinate is obtained using the values of time delays τ_{15}, τ_{35} of the $USBL_{153}$ system and the values of the time delays τ_{25}, τ_{45} of the $USBL_{254}$ system. For the horizontal USBL systems we initially assume that the value of the Z coordinate (for the $USBL_{123}$ system it is the $Z^{\xi,\zeta}{}_{123}$ coordinate) is positive. Just as in the first stage of the algorithm the values of the components of the vector $\boldsymbol{p}^{\xi,\zeta}{}_{123}=[X^{\xi,\zeta}{}_{123},, Y^{\xi,\zeta}{}_{123}, Z^{\xi,\zeta}{}_{123}]^T$ allow us to calculate the values of the delays τ'_{15} and τ'_{35} for the $USBL_{153}$ system and the values of delays τ'_{25} and τ'_{45} for the $USBL_{254}$ system. Next, we calculate the absolute values of the following differences: $\delta1=|\tau'_{15}-\tau_{15}|$, $\delta2=|\tau'_{35}-\tau_{35}|$, $\delta3=|\tau'_{25}-\tau_{25}|$ and $\delta4=|\tau'_{45}-\tau_{45}|$ (the values τ_{15}, τ_{35}, τ_{25} and τ_{45} are obtained through the measurement). Now we calculate the geometric mean $\Delta1$ of the $\delta1$, $\delta2$, $\delta3$ and $\delta4$ ($\Delta1=(\delta1^2+\delta2^2+\delta3^2+\delta4^2)^{0.5}$). The same procedure is repeated with opposite sign designation for the coordinate $Z^{\xi,\zeta}{}_{123}$. The geometric mean in this case will be $\Delta2$. The coordinates corresponding to a less geometric mean are taken as correct. The same procedure is applied to the $USBL_{234}$, $USBL_{341}$, $USBL_{412}$ systems.

The next stage of the algorithm is solving the problem of the low precision of coordinate determination in cases when an object is found in the plane (or near to the plane) of the receiving horizontal bases. For that we calculate the spherical coordinates:

$$\boldsymbol{q}^{\xi,\varsigma}_{123} = (R_{123}, \psi^{\xi,\varsigma}_{123}, \varphi^{\xi,\varsigma}_{123})^T \,;\; \boldsymbol{q}^{\xi,\varsigma}_{234} = (R_{234}, \psi^{\xi,\varsigma}_{234}, \varphi^{\xi,\varsigma}_{234})^T \,;$$
$$\boldsymbol{q}^{\xi,\varsigma}_{341} = (R_{341}, \psi^{\xi,\varsigma}_{341}, \varphi^{\xi,\varsigma}_{341})^T \,;\; \boldsymbol{q}^{\xi,\varsigma}_{412} = (R_{412}, \psi^{\xi,\varsigma}_{412}, \varphi^{\xi,\varsigma}_{412})^T \,; \quad (13)$$

If the values of the polar angle of some USBL systems lie within the diapason [80°, 100°], the calculated values are discarded. If the values of the polar angle of the analyzed USBL systems lay outside of the diapason [80°, 100°] the corresponding Cartesian coordinates of these systems are considering as reliable and the values of the object coordinates ($\boldsymbol{p}_{\mathrm{USBL}_{123}}, \boldsymbol{p}_{\mathrm{USBL}_{234}}, \boldsymbol{p}_{\mathrm{USBL}_{341}}$ and $\boldsymbol{p}_{\mathrm{USBL}_{412}}$) in the carrier coordinate system are calculated according to the formulas (9).

In the third stage of the algorithm the object coordinates are calculated using the time delays obtained by the inclined measuring systems ($USBL_{163}$, $USBL_{274}$, $USBL_{381}$ and $USBL_{492}$).

We consider the $USBL_{163}$ system in order to describe this procedure (for the other inclined USBL systems this part of the algorithm functions in the same way).

First we calculate the coordinates of the object based on the delays measured with the $USBL_{163}$ system ($\boldsymbol{p}^{\xi,\zeta}{}_{163}=[X^{\xi,\zeta}{}_{163}, Y^{\xi,\zeta}{}_{163}, Z^{\xi,\zeta}{}_{163}]^T$) and then the sign of the $Z^{\xi,\zeta}{}_{163}$ coordinate is determined. The sign of the $Z^{\xi,\zeta}{}_{163}$ coordinate just as in the case of the horizontal systems is obtained using the values of time delays τ_{15}, τ_{35} of the $USBL_{153}$ system and the values of the time delays τ_{25}, τ_{45} of the $USBL_{254}$ system. For the inclined USBL systems we initially also assume that the value of the Z coordinate (for the $USBL_{163}$ system it is the $Z^{\xi,\zeta}{}_{163}$ coordinate) is positive. Just as in the first and the second stages of the algorithm the values of the components of the vector $\boldsymbol{p}^{\xi,\zeta}{}_{163}=[X^{\xi,\zeta}{}_{163},, Y^{\xi,\zeta}{}_{163}, Z^{\xi,\zeta}{}_{163}]^T$ allow us to calculate the values of the delays τ'_{15} and τ'_{35} for the $USBL_{153}$ system and the values of delays τ'_{25} and τ'_{45} for the $USBL_{254}$ system. Next, we calculate the absolute values of the following differences: $\delta1=|\tau'_{15}-\tau_{15}|$, $\delta2=|\tau'_{35}-\tau_{35}|$, $\delta3=|\tau'_{25}-\tau_{25}|$ and $\delta4=|\tau'_{45}-\tau_{45}|$ (the values τ_{15}, τ_{35}, τ_{25} and τ_{45} are obtained through the measurement). Now we calculate the geometric mean $\Delta1$ of the $\delta1$, $\delta2$, $\delta3$ and $\delta4$. The same procedure is repeated with opposite sign designation for the coordinate $Z^{\xi,\zeta}{}_{163}$. The geometric mean in this case will be $\Delta2$. The coordinates corresponding to a less geometric mean are taken as correct. The same procedure is applied to the $USBL_{274}$, $USBL_{381}$ and $USBL_{492}$ systems.

The next stage of the algorithm is the calculation of the spherical coordinates of the object for the elemental inclined USBL systems (for solving the accuracy problem when the object is found in the plane or near the plane of some inclined array). The coordinate vectors in the spherical coordinate can be expressed as follows:

$$\boldsymbol{q}^{\xi,\varsigma}_{163} = (R_{163}, \psi^{\xi,\varsigma}_{163}, \varphi^{\xi,\varsigma}_{163})^T \,;\; \boldsymbol{q}^{\xi,\varsigma}_{274} = (R_{274}, \psi^{\xi,\varsigma}_{274}, \varphi^{\xi,\varsigma}_{274})^T \,;$$
$$\boldsymbol{q}^{\xi,\varsigma}_{381} = (R_{381}, \psi^{\xi,\varsigma}_{381}, \varphi^{\xi,\varsigma}_{381})^T \,;\; \boldsymbol{q}^{\xi,\varsigma}_{492} = (R_{492}, \psi^{\xi,\varsigma}_{492}, \varphi^{\xi,\varsigma}_{492})^T . \quad (14)$$

If the values of the polar angle of some USBL systems lie within the diapason [80°, 100°], the calculated values are discarded. If the values of the polar angle of the analyzed USBL systems lay outside of the diapason [80°, 100°] the corresponding Cartesian coordinates of these systems are considering as reliable and the values of the object coordinates ($\boldsymbol{p}_{\mathrm{USBL}_{163}}, \boldsymbol{p}_{\mathrm{USBL}_{274}}, \boldsymbol{p}_{\mathrm{USBL}_{381}}, \boldsymbol{p}_{\mathrm{USBL}_{492}}$) in the carrier coordinate system are calculated according to the formulas (11).

The last step of the algorithm implies the calculation of the means of the object coordinates in the carrier coordinate system with reliable data obtained by the elemental USBL systems: $\boldsymbol{p}=(X,Y,Z)^T$ = mean ($\boldsymbol{p}_{\mathrm{USBL}_{153}}, \boldsymbol{p}_{\mathrm{USBL}_{254}}, \boldsymbol{p}_{\mathrm{USBL}_{123}}, \boldsymbol{p}_{\mathrm{USBL}_{234}}, \boldsymbol{p}_{\mathrm{USBL}_{341}}, \boldsymbol{p}_{\mathrm{USBL}_{412}}, \boldsymbol{p}_{\mathrm{USBL}_{163}}, \boldsymbol{p}_{\mathrm{USBL}_{274}}, \boldsymbol{p}_{\mathrm{USBL}_{381}}, \boldsymbol{p}_{\mathrm{USBL}_{492}}$).

3.4 Simulation results

For the algorithm simulation a special computer program was designed. During the simulation of the algorithm, it was assumed that the distance to the object, and the pitch and roll angles were being measured precisely. We assume that the measurement of the time delays is provided by utilizing the binary counters and the signal-to-noise ratio (SNR) on the inputs of receiving elements and signal reception conditions allow us to measure the time delays without errors. It is also supposed that the accuracy of measurement of the time delays is limited by the clock drive frequency of the time delay counters. The different values of the horizontal distance to the object, the depth of the object, the azimuth angle, and the pitch and roll angles were utilized for modeling the difficult conditions to measure the object coordinates with high accuracy (cases when an object is found in the plane of the horizontal receiving bases or near to the plane of the horizontal receiving bases). The computer simulation of algorithm will estimate the instrumental precision of the USBL system.

Let X, Y, Z be the true values of the coordinates of the object in the carrier coordinate system $\Sigma=(0,x,y,z)$. Let R be the true incline distance to the object. Let X_{USBL}, Y_{USBL}, Z_{USBL} be the values of the coordinates obtained by applying of the developed algorithm (the coordinates of the object are calculated utilizing the expression (9), (10) and (11) for $USBL_{153}$, $USBL_{254}$ $USBL_{123}$, $USBL_{234}$, $USBL_{341}$, $USBL_{412}$, $USBL_{163}$, $USBL_{274}$, $USBL_{381}$, and $USBL_{492}$ systems). The values of the true errors of determination of the coordinates are: $\Delta X=X_{USBL}-X$; $\Delta Y= Y_{USBL}-Y$, $\Delta Z= Z_{USBL}-Z$. The values of the relative true errors of the coordinates are: $\Delta X/R$; $\Delta Y/R$; $\Delta Z/R$. In process of the simulation the azimuth angle φ is changing clockwise (if looking down on the horizontal plane, see Figs.1,3) from 0° to 360° in the *(x,y)* coordinate plane (zero reading is coincided with x-axis of the $\Sigma=(0,x,y,z)$ carrier coordinate system, see Fig.1). The other parameters of algorithm simulation have following values: the speed of the sound in the water c=1500m/s; the size of receiving bases d=0.056m; transponder pulse carrier frequency f=11KHz (operating frequency of USBL system); the frequency of the time delay counter f_C=25MHz.

The results of designed algorithm simulation are shown in Figs.4-8. First we will consider the case when the receiving antenna does not have any inclination and the object is located in the horizontal distance of 100 meters and the relative depth is 5 meters. The angular position of the object (azimuth angle φ) is changing with the step of 1°. The results of the algorithm simulation for the examining case (R=100m; Z=5 m; ξ=0°; ζ= 0°) are shown in Fig.4. In the absence of the pitch and roll the modulus of the latitude angle to the object (in graphs this angle is designated as $|\psi^{\xi,\zeta}_{1234}$ -90°|) should be invariable and the value of the latitude angle is approximately 2.86°. It means that the horizontal USBL systems are not participated in the calculation of coordinate means and the maximum number of systems that are taken into account in this case is 6 (N=6). In the relatively lengthy diapasons of azimuth angle changing, the number of utilized elemental USBL is 6 (see Fig.4). In other azimuth angle diapasons, the number of utilized USBL systems varies from 3 to 5.

We will consider in detail the behavior of the function $N=N(\varphi)$ only in the diapason of changing of the angle φ from 0° to 90°. In the other diapasons of φ ([90°-180°], [180°-270°], [270°-360°]) the behavior of the function $N=N(\varphi)$ will be analogous. In the azimuth angle range from 0° to 10° the number of the utilized USBL system is 3 (N=3) and the systems that are utilized in the calculation of the coordinate means are: $USBL_{153}$, $USBL_{163}$ and $USBL_{381}$ (see Figs.3,4). For the φ=11° the number of used systems is 4 (N=4, utilized systems are: $USBL_{153}$, $USBL_{163}$, $USBL_{381}$ and $USBL_{452}$). The system $USBL_{452}$ is utilized in the calculation of coordinate means because the azimuth angle φ is more than 10°. For the $USBL_{452}$ system (the case: ζ= 0°) the azimuth angle φ also can be interpreted as the altitude angle to the object relative to the plane of the

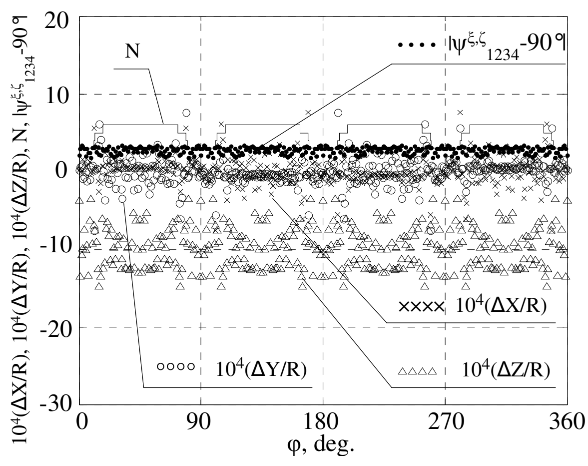

Fig.4. Variation of the relative errors of the object coordinates, number of the elemental USBL systems N that are used for calculation of coordinate means and modulus of the latitude angle to the transponder (object) relative to the $USBL_{1234}$ plane; R=100m; Z=5 m; ξ=0°; ζ= 0°.

$USBL_{452}$ three-element array. With the further increasing of the φ the altitude angle to object relative to the $USBL_{274}$ array became more then 10° and this system ($USBL_{274}$ system) is also begun to utilize for calculation of the coordinate means (on the graph of N this is the case when the azimuth angle φ belongs to the interval [12°-17°]). Thus, if the angle φ belongs to the interval [12°-17°] and in the calculation of the coordinate means are utilized next USBL systems (N=5): $USBL_{153}$, $USBL_{452}$, $USBL_{163}$, $USBL_{274}$ and $USBL_{492}$. In the interval [18°-72°] both vertical ($USBL_{153}$ and $USBL_{452}$) systems and all inclined ($USBL_{163}$, $USBL_{274}$, $USBL_{381}$ and $USBL_{492}$) systems are utilized. In diapason [73°-78°] the 5 systems (N=5) are utilized: $USBL_{153}$, $USBL_{452}$, $USBL_{274}$, $USBL_{381}$ and $USBL_{492}$. In difference from the diapason [12°-17°] in the interval [73°-78°] the $USBL_{492}$ system is utilized instead of the $USBL_{163}$ system. For the angle φ=79° the following four (N=4) systems are used: $USBL_{153}$, $USBL_{452}$, $USBL_{274}$ and $USBL_{492}$. In the diapason [80°-90°] the next three (N=3) systems are utilized: $USBL_{452}$, $USBL_{274}$ and $USBL_{492}$. The behavior of the function $N=N(\varphi)$ is repeated in the other three 90°-sectors.

The utilization of the different elemental USBL arrays is defined according to algorithm described above. The relative errors of all three object coordinates ($\Delta X/R$, $\Delta Y/R$, $\Delta Z/R$) have not exceeded the threshold of 0.2% of inclined distance to the object. The behavior of the relative errors is also similar within each 90°-sector. The greatest errors take place for the Z-coordinate of the object. The relative errors of Z-coordinate are reached the values of 0.15% whereas the errors of X- and Y-coordinates not exceeded the values of 0.1%.

Let that the USBL array has some inclination (pitch and roll angles are not zeros). The simulation results for these cases are shown in Figs.5-8.

From Fig.5 (R=100m; Z=15m; ξ=5°; ζ= –6°) it is seen that the relative errors ($\Delta X/R$, $\Delta Y/R$, $\Delta Z/R$) of all three coordinates have not exceeded the threshold of 0.2% of inclined distance to the object. The values of the relative errors $\Delta X/R$ and $\Delta Y/R$ are less than 0.1% in all examined diapason of φ. The number of the utilizing elemental systems (N) is varying from 3 to 10. From the graphs it is seen that if the $|\psi^{\xi,\zeta}_{1234}$ - 90°$|\leq$10° the number of the utilized system is less or equal to 6. If the $|\psi^{\xi,\zeta}_{1234}$ -90°$|>$10° the number of the utilized systems (N) is more than 6 and the exact number is defined by the values of altitude angle for the other three-element array planes.

The graphs in Fig.6 illustrate the variation of relative errors of object coordinates for the case when R=100m; Z=40m; ξ= –20°; ζ=10°. The relative location of the measuring system and the object (with predetermined spatial orientation of receiving antenna) defines the case of significant inclination of receiving antenna and when the object can be found in the plane of horizontal receiving bases of USBL system. It is seen that the relative errors do not exceed the threshold of 0.15% of incline distance in all diapason of changing azimuth angle φ. Also it is seen that when the modulus of altitude angle $|\psi^{\xi,\zeta}_{1234}$ -90°$|$ is more than 10° the number of the utilized three-element systems is found in interval between 7 and 10 (7≤N≤10).

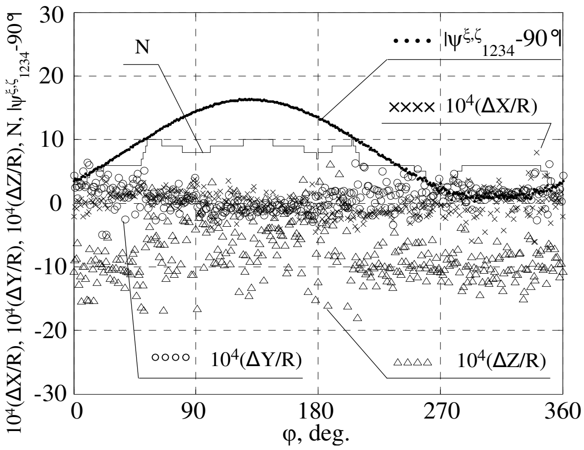

Fig.5. Variation of the relative errors of the object coordinates, number of the elemental USBL systems N that are used for calculation of coordinate means and modulus of the latitude angle to the transponder (object) relatively the $USBL_{1234}$ plane; R=100m; Z=15 m; ξ=5°; ζ= -6°.

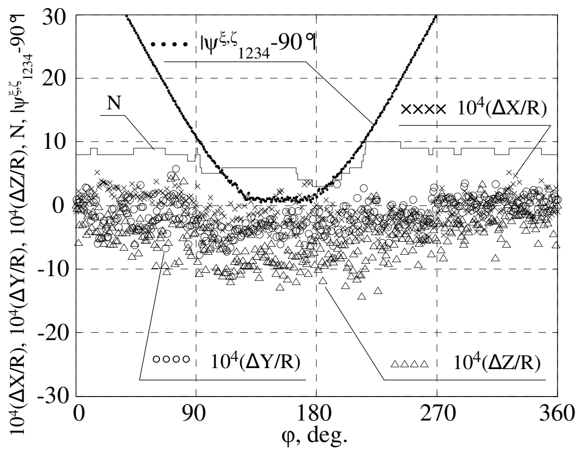

Fig.6. Variation of the relative errors of the object coordinates, number of the elemental USBL systems N that are used for calculation of coordinate means and modulus of the latitude angle to the transponder (object) relatively the $USBL_{1234}$ plane; R=100m; Z=40 m; ξ= -20°; ζ= 10°.

The results of the calculation of the relative errors for the significant inclination angles of the receiving antenna are shown in Fig.7 and Fig.8. In Fig.7 the pitch and roll angles have values: $\xi = -30°$; $\zeta = 20°$; in the Fig.8 the pitch and roll angles are: $\xi = -40°$; $\zeta = -40°$. From the presented curves we can note that as before the relative errors do not exceed the threshold of 0.15% of incline distance in all diapason of changing azimuth angle φ. In the case of the big depth (Z=150m, see Fig.8) the values of the relative errors do not exceed the threshold of 0.12% of incline distance. It is necessary to notice that with the increasing of the relative depth (Z) the values of the relative errors $\Delta Z/R$ are decreasing (the mean of the relative errors is displaced from the negative values to zero, see Figs.4-8). The number of utilizing USBL systems as before depends on the latitude angle modulus of the horizontal arrays and on the latitude angles of the other systems. If $|\psi^{\xi,\zeta}_{1234} - 90°| \leq 10°$ the number of the utilized systems is varies from 3 to 6, if $|\psi^{\xi,\zeta}_{1234} - 90°| > 10°$ the number of the utilized systems is varies from 7 to 10.

The designed algorithm has been examined for various relative locations of the measuring system and object (object location in the lower hemisphere was considered, the maximum horizontal distance was assumed to be 100m). The values of pitch and roll angles ξ and ζ are assumed to be in the range from $-40°$ to $+40°$. The computer simulation demonstrated the reliable operation of the designed algorithm for all tested angular antenna positions and verified locations of object. The results of the calculation of the errors of the determination of the coordinates of the object show that the relative true errors of the coordinates have values less than 0.2% of the slant distance to the object. It is also necessary to mention, that in a wide range of distances, depths, pitch and roll angles the values of true relative errors were less than 0.1%.

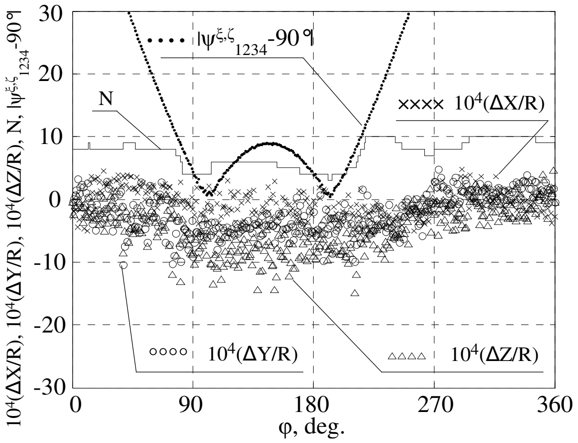

Fig.7. Variation of the relative errors of the object coordinates, number of the elemental USBL systems N that are used for calculation of coordinate means and modulus of the latitude angle to the transponder (object) relatively the $USBL_{1234}$ plane; R=100m; Z=50 m; $\xi = -30°$; $\zeta = 20°$.

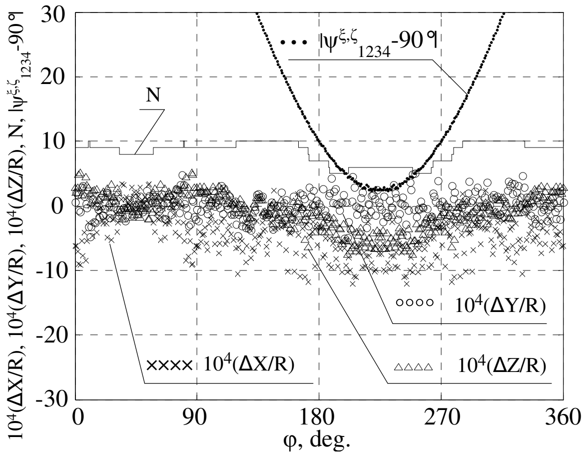

Fig.8. Variation of the relative errors of the object coordinates, number of the elemental USBL systems N that are used for calculation of coordinate means and modulus of the latitude angle to the transponder (object) relatively the $USBL_{1234}$ plane; R=100m; Z=150 m; $\xi = -40°$; $\zeta = -40°$.

4 Conclusion

In this article we have investigated the approach for design of a multi-element USBL system. A case for the design of a nine-element USBL system is presented. The paper focused on the problem of improving the precision of coordinate determination in conditions when the location of the object in the lower hemisphere is arbitrary and the receiving antenna can have significant inclinations. In this article this problem has been solved by means of increasing the number of elemental three-element USBL arrays and exploiting their different spatial orientations. The proposed algorithm allows us to accomplish the selection of reliable elemental USBL arrays utilizing the analysis of the values of latitude angles to the object for each elemental array. The presented algorithm is a significantly modified and improved version of the algorithm designed for the five-element USBL system [9]. In particular the logical part of algorithm has been notably modified and the algorithm threshold dependence of the Z-coordinate sign determination was eliminated.

Algorithm simulation was realized for different USBL systems and object mutual positions in wide range of pitch and roll angles of the receiving array (pitch and roll angles ξ and ζ are assumed to be in the range from -40° to +40°). For all tested angular receiving array positions and object locations the designed algorithm showed reliable operation. The accuracy of coordinate determination for the proposed nine-element USBL system can be evaluated as the 0.2% of slant distance to the object.

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