The Combined Decision Making Technology based on the Statistical and Fuzzy Analysis and its Application in Forecast's Modeling

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Abstract: - In the present work the combined Decision Making Technology based on the Statistical and Fuzzy Analysis is offered. This is a novel technology, which is based on the use of fuzzy-statistical methods. The technology involves two stages of decision-making. While considering the same object, the first three methods make a decision independently. These methods are – the Statistical Method of Fuzzy Grades' Analysis, the Fuzzy Discrimination Analysis and the Case Based Reasoning. Each makes a decision with its own approach to a problem and uses statistical and expert data. These statistical data are the existing historical cases of correctly made decisions with exhibited activities. At the second stage, the fourth method – an Expertons method – is used to make a final decision. The method works with the expert data only and chooses the most believable decision from those offered by the first three methods.

Proposed Decision Making Technology is applied to a specific forecasting problem – the decision-making on the possibility of earthquake occurrence. The present work describes the modifications of the "classical" variants of fuzzy-statistical methods, which became necessary to solve the specific forecasting problem and, also, introduces the Decision Support System, which was developed completely based on the proposed combined technology. The article provides an example clearly illustrating the work of the developed system.

Key-Words: - Decision Support System, Fuzzy Relative Frequencies, Membership Functions, most believable Decision, the similarity Measure between two Cases, Experton, Possibility Distribution.

1 Introduction

As is well known, in the tasks of decision-making the deterministic or probabilistic approaches are traditional. However, for complex enough object, its description in traditional mathematical terms, likewise, development of its exact mathematical model becomes impossible. The description of such objects is impossible without introduction of fuzzy representations. Many authors clearly support the use of the fuzzy sets theory and soft computing methods to expand human ability in making optimal decisions involving uncertainty ([2-6,8,13,15-22,25,26] and so on). In particular, the use of fuzzy sets theory is considered to be effective enough to build decision-making support systems, because, often, the development of such systems is based on expert knowledge and representations.

Thus, for complex by nature object it is expedient to develop a fuzzy model of the object. Moreover, developing several fuzzy models makes it possible to reflect expert knowledge of various properties of researched object. Purposeful development of the model implies to overlook minor details in conformity with the final purpose. It, naturally, gives non-precise model. Therefore, simultaneous consideration of a number of models allows for creating the best, more complete representation of a problem. Such an approach is offered in the presented work [9], where three fuzzy models describing researched object from the different points of view are built. Models are based on the following fuzzy-statistical methods: Statistical method of Fuzzy Grades' Analysis [4,10,29], Fuzzy Discrimination Analysis [15,18,19] and Case Based Reasoning [14,23]. For the final decision-making, i.e. for a choice of the most optimal decision, an Expertons method [5,6,12] is applied.

For the object of the decision-making containing fuzziness in the definition, with known set of activities and in view of the investigated fuzzy methods the model of Decision Support System (DSS) is developed. The model is entirely based on the offered combined technology of decision-making.

DSS is tested for a specific target of the forecast – decision-making regarding the possibility of earthquake occurrence. Some geophysical activities of an atmosphere are taken as the forecasting factors. To solve the problem mentioned above, the software package was created, which is the forecast making system (algorithms of representation the fuzzy information in a computer, algorithms of creation and use of the knowledge base, and also algorithms of all fuzzy-statistical methods applied for making decision are developed and implemented).

At the end of paper the example of a forecast is given, which represents a result of DSS work.

2 Fuzzy Decisions Making Methods

Methods of decision-making are characterized by various approaches. For each method the information is received from general database, which contains primary historical data with exhibited activities and correctly made decisions.
Let's consider each of them.

2.1 Statistical Method of Fuzzy Grades' Analysis

According to a statistical method of fuzzy grades' analysis (hereinafter referred to as statistics of fuzzy grades) the object of decision making (forecasting) is described by the forecast value. Area of the forecast value is divided into forecasting grades (classes). For each class the numerical interval is put in conformity. Corresponding membership functions are defined. Definition of the membership functions includes a human factor, since an expert has a subjective viewpoint on a degree of belonging of the given forecasting object to the forecasting classes [24, 28]. The mentioned classes are fuzzy, therefore supports of membership functions are intersected.

The forecasting value depends on the certain parameters, or of forecasting factors (activities). Each of activities, in turn, is divided into classes. The number of activities, their classes and the range of their numerical intervals can be selected arbitrarily.
Let's introduce some designations: Forecasting grades – $M_1, M_2, ..., M_\ell$; Corresponding membership function – $\mu_1, \mu_2, \dots, \mu_\ell$; Activities – $A_1, A_2, \dots, A_m$; Classes of activities – $a_{k1}, a_{k2}, \dots, a_{kr}$; $k = \overline{1, m}$; $A_k = \bigcup_{j=1}^{r} a_{kj}$.

Further, let us define selective frequencies n_{kj}^{i}, that represent the frequencies of j class of A_k activity occurring in i forecasting grade. The values of n_{kj}^{i} are calculated from the initial data received as a result of observations and measurements [29]. n_{kj}^{i} and μ_i numbers are used to define the fuzzy selective frequencies, the fuzzy relative frequencies and the weights of each interval of the activity in accordance to the known formulas [4]:

$$\tilde{n}_{kj}^{m} = \sum_i \mu_i^m \cdot n_{kj}^i \ , \ \tilde{f}_{kj}^{m} = \tilde{n}_{kj}^{m} \Big/ \sum_i \tilde{n}_{kj}^{i} \ ,$$
$$w_{kj} = \sum_i \tilde{n}_{kj}^{i} \Big/ \sum_j \sum_i \tilde{n}_{kj}^{i} \ , \qquad (1)$$

where μ_i^m is average value of membership function when the forecasting value from i forecasting interval belongs to m forecasting grade.

After that, it becomes possible to make a decision for the certain sample of forecasting factors. For this purpose, we need to define fuzzy weights of each activity according to its interval, and then to carry out multifactor linear synthesis of fuzzy weights and fuzzy relative frequencies. As a result of multifactor linear synthesis we receive the generalized decision (weighed vector of the possible decisions) [29]:

$$\vec{D}_\alpha = \vec{w}_\alpha \cdot \tilde{f}_\alpha \ . \qquad (2)$$

At last, in order to receive the unique decision it is necessary to use an additional principle. For example, it is possible to use a principle of a maximum of possibilities. The final decision will be [16]:

$$D_{Class}^{(\alpha)} = \max_i (D_\alpha(i)) \ , \qquad (3)$$

where $D_\alpha(i)$ is i component of a vector $\vec{D}_\alpha$.

2.2 Fuzzy Discrimination Analysis

Discrimination analysis is better approach for modeling intellectual activity of the expert during decision-making [15, 18].

The essence of the discrimination analysis comprises the following: from the information in general database the frequency distribution table $\{ f_{ij} \}$ is built, where f_{ij} is the relative frequency of activity A_i accompanying decision D_j:

$$f_{ij} = m_{ij} / N \ , \qquad (4)$$

where m_{ij} is number of those correctly accepted decisions D_j for which A_i activity was exhibited, and N is the general number of cases.

The frequency distribution table, which is the basis of the numerical-tabular knowledge base, contains the primary information for two other tables called positive discrimination table { p_{ij} } and negative discrimination table { n_{ij} }, which are calculated as follows:

$$p_{ij} = \frac{1}{C_D - 1} \sum_{\substack{k \in D \\ k \neq j}} \chi_{Large-ratio}\left(\frac{f_{ij}}{f_{ik}}\right),$$

$$n_{ij} = \frac{1}{C_D - 1} \sum_{\substack{k \in D \\ k \neq j}} \chi_{Large-ratio}\left(\frac{f_{ik}}{f_{ij}}\right). \quad (5)$$

Here, $p_{ij}, n_{ij} \in [0;1]$; $C_D = Card\ D$ denote cardinality of set of decisions; *Large-ratio* denote the fuzzy subset R_0^+ with membership function: $\chi_{Large-ratio} : R_0^+ \to [0,1]$, which puts relations (real numbers) f_{ij} / f_{ik} into the interval [0,1].

The heuristic interpretation of positive and negative discrimination is the following: p_{ij} represents an accumulated belief that A_i is more characteristic for decision D_j than for other decisions, and n_{ij} represents an accumulated belief that A_i is more characteristic for decision non-D_j than for others.

The generalized decision is represented as a fuzzy subset of the set of possible decisions with the following membership function:

$$\delta(D_j) = \frac{1}{2}\left(\chi_{Large}(\pi_j) + \chi_{Small}(\nu_j)\right), \quad j \in D, \quad (6)$$

where

$$\pi_j = \frac{1}{C_{A'}} \sum_{i=1}^{r} p'_{ij}, \quad \nu_j = \frac{1}{C_{A'}} \sum_{i=1}^{r} n'_{ij}. \quad (7)$$

Here π_j and ν_j represent the averages of positive and negative discrimination measures, respectively, for decision D_j; The fuzzy sets *Large* and *Small* have characteristic membership functions: $\chi_{Large}, \chi_{Small} : [0,1] \to [0,1]$, where χ_{Large} is monotonic increasing, and χ_{Small} – monotonic decreasing in its argument; p'_{ij} and n'_{ij} are elements of matrixes { p'_{ij} } and { n'_{ij} } corresponding to a particular set of activities $A' = \{A'_1, \ldots A'_r\}$. These matrixes are produced by selecting from { p_{ij} } and { n_{ij} } only those rows which correspond to A'; $C_{A'} = CardA' = Card\{A'_1,.., A'_r\} = r$.

To make the final "classic" decision an additional defuzzification principle is needed [19]. For example, the decision can be made according to the maximum of function $\delta(D_j)$:

$$\delta^{Class} = \max_{j \in D} \delta(D_j),$$

i.e. the decision j with maximum value in { δ_j } can be recognized as a most believable decision.

2.3 Case Based Reasoning

To receive a correct new result of decision-making or forecasting, among the existing known cases analogues to a newly introduced case are searched and the same decision which was correct for analogues is accepted [1,14,23]. Therefore this method is often named a method of "the nearest neighbour". The method has the following advantages:

- It is closest to real processes of decision-making by experts, i.e. the problem is processed by means of comparison with known similar situations;
- Automation of the association between historical and new knowledge is simple enough, in this case the numerical-tabular knowledge base is used;
- It is possible to give the best explanation and a substantiation of the decision on the basis of consideration of the previous cases, etc.

Measure of similarity between a new case and other cases stored in the general base include two stages:

a) The distance between two i activities of two cases is calculated according to the formulae:

$$DV_i = \min(CB_i, ND_i), \quad i = \overline{1, n}, \quad (8)$$

where n is number of all activities; CB_i is the value of i activity of the existing case; ND_i is the value of i activity of a new case; DV_i is the distance value between two i activities.

b) The similarity measure between two cases is calculated as follows:

Let SV_j be the similarity value between the new case and the j case existing in the general database. SV_j can be calculated as follows:

$$SV_j = \frac{1}{n} \sum_{i=1}^{n} w_i \cdot DV_i, \quad j = \overline{1, k}, \quad i = \overline{1, n}, \quad (9)$$

where k is the number of precedents; $\vec{w}$ represents a vector of weights w_i component of which indicates the importance of the i activity for decision making. $w_i \in [0, 1]$ and $w_i = 0$ means that i activity is not important, $w_i = 1$ means that i activity is absolutely important, $0 < w_i < 1$ indicates the importance degree of i activity. Determination of weights often can be based on the experience of experts.

The final decision could be derived according to the following: find the r which satisfies to a condition

$$SV_r = \max(SV_j)\,,\; j = \overline{1,k}\,. \qquad (10)$$

Experts provide some threshold for similarity measure in advance. If the degree of similarity SV_r is equal or greater than this threshold, conclusion for examined case is the same as that of the precedent. Otherwise, conclusion for the new case cannot arrive at the same conclusion as that of the precedent. Then it is necessary to infer decision using other precedents or to change approaches for this query case.

2.4 Expertons Method

An experton is the generalized notion of a probable random fuzzy event when the probability of a random event of each α-cut is replaced by confidence intervals. These intervals in their turn are statistically defined by the group of experts. The concept of the expertons theory can be briefly described as follows [5,6,12].

Let E be a finite or infinite set of certain objects, factors and so on. The group of r experts is requested to express their subjective opinion regarding each element from E in the form of a confidence interval

$$\forall P \in E : \left[a_*^j(P), a_j^*(P) \right] \subset [0,1],$$

where the symbol $\subset$ denotes an inclusion and j the order number of an expert. We consider the statistics when to each element $P \in E$ we assign both the lower and the upper bound of confidence intervals. The cumulative distribution law $F_*(\alpha, P)$ is constructed on the basis of $a_*^j(P)$, and $F^*(\alpha, P)$ on the basis of $a_j^*(P)$. Thus we obtain

$$\forall P \in E, \forall \alpha \in [0,1] : \widetilde{A}(P) = \left[F_*(\alpha, P), F^*(\alpha, P) \right],$$

where $\widetilde{A}$ denotes an experton.

The following properties of an experton are obvious:

$$\forall P \in E\,,\; \forall \alpha, \alpha' \in [0,1] :\; (\alpha < \alpha') \Rightarrow$$
$$\left(\left[F_*(\alpha', P), F^*(\alpha', P) \right] \subset_i \left[F_*(\alpha, P), F^*(\alpha, P) \right] \right),$$

where $\subset_i$ denotes an interval inclusion,

i.e. $(\alpha < \alpha') \Rightarrow \left(\left[F_*(\alpha', P) \geq F_*(\alpha, P) \right] \text{ and } \left[F^*(\alpha', P) \leq F^*(\alpha, P) \right] \right)$.

3 Fuzzy-statistical Methods for Concrete Decision Making Task

Offered technology applied in concrete forecasting task - decision-making regarding the possibility of earthquake occurrence. As the factors-precursors some geophysical activities of an atmosphere are taken. The forecasting object - earthquake - is described by great number of activities. Initial data comprises the earthquakes' statistics in the Dusheti Region of Georgia and was received from national Center of Seismic Monitoring.

Below are described the modifications of fuzzy-statistical methods necessary to solve the specific problem of decision-making regarding the possibility of earthquake occurrence.

a) *Statistics of Fuzzy Grades:*

The "classical" variant of a method became a subject to modification in order to satisfy the condition of a great number of activities. For this case method has been generalized. With this purpose, the author introduced a concept of a measure of possibility which is used to obtain a generalized decision [11]:

$$\overrightarrow{Poss_\alpha} = \vec{D}_\alpha \Big/ \max_j (D_\alpha(j))\,, \qquad (11)$$

where $D_\alpha(j)$ is j component of a vector $\vec{D}_\alpha$.

To receive the unique decision, now, it is possible to use a principle of a maximum of possibilities. Then the final decision will be:

$$D_{Class}^{(\alpha)} = \max_i (Poss_\alpha(i))\,, \qquad (12)$$

where $Poss_\alpha(i)$ is i component of a vector $\overrightarrow{Poss}_\alpha$.

Belonging to a certain class of forecasting is determined through application of a so-called membership functions. Definition of membership functions is based on intellectual activity of experts. Since membership functions are defined with experts' subjective viewpoint, they can be of any kind [7]. "Right" definition of membership functions is the basic guarantee of the method's success. The present work offers a model of membership function developed for a concrete case of the forecast. It represent a new modification of Zadeh's model (see formulas (15)).

b) *Discrimination Analysis:*

By analogy to a *Statistics of Fuzzy Grades* each activity is divided into classes. Since initial data is only objective, it helps to calculate relative frequencies f_{ij}. The quantity of classes and range of their numerical intervals are selected on the basis of expert data.

$\chi_{Large-ratio}$, χ_{Large} and χ_{Small} membership functions are defined according to their properties and to the initial data:

$$\chi_{Large-ratio}\left(f_{ij}/f_{ik}\right)=k\cdot\left(f_{ij}/f_{ik}\right),$$

where coefficient k is found in accordance with the primary data;

χ_{Large} define as $\chi_{Large}(\pi_j)=\pi_j$;

χ_{Small} - as $\chi_{Small}(\nu_j)=-\nu_j+1$.

c) *Case Based Reasoning:*

Here also each activity is divided into classes. It allows for preparing initial data for the method on a stage of fuzzification. The quantity of classes and their numerical intervals coincide with those, which are definitions in *Discrimination Analysis*. Since classes of activities are fuzzy subsets, the distance between two i-th activities of investigated case and precedent is calculated as follows [17]:

$$DV_i = 1-\left|CB_i - ND_i\right|. \quad (13)$$

As activities are the known values, while calculating the similarity measure between two cases, all activities should be considered as absolutely important for decision making. I.e. all $w_i = 1$. Calculations are made under the formulae:

$$SV_j = \sum_{i=1}^{2n} DV_i \Big/ 2n. \quad (14)$$

Recall that n is the number of activities.

d) *Expertons Method*:

If during decision-making in the fuzzy system more than one method is applied, their composition is necessary to receive of a unique fuzzy subset of values. To ensure such a composition it is possible to apply an Expertons method.

Let E be a fuzzy subset of decisions and include k elements:

$$E=\{\delta_1,\delta_2,...,\delta_k\}.$$

Using joint interval estimations of experts for each decision, the expertons method will allow to find the unique most believable decision as

$$\delta^{Class}=\max(\delta_j),\ j=\overline{1,k},$$

where j is number of decision.

4 Decision Making Example

For a specific example of earthquake forecasting we consider the following geophysical atmosphere data to be factor-precursors:
A_0 - Value of intensity of the electric fields (volt/m); A_1 - Temperature of air (in degrees of Celsius); A_2 - Temperature of ground (in degrees of Celsius); A_3 - Atmospheric pressure (in mb); A_4 - Absolute humidity (elasticity water pair in mb); A_5 - Relative humidity (in %); A_6 - The general overcast (in points); A_7 - The bottom overcast (in points); A_8 - Speed of a wind (in m/s). Values of factors were measured during the day in three hour interval.

Decision-making process consists of the following:

The object of the forecasting – earthquake – is described by means of a linguistic variable with the following values: "noise", "moderate earthquake", "strong earthquake" [27] and is characterized by numerical value of magnitude (*M*).

At $0 \le M \le 3$ "noise" is observed; at $3 < M < 5$ "moderate earthquake" is observed; at $5 \le M \le 8$ – "strong earthquake" is observed. Let us designate the defined forecast classes (intervals of earthquake intensity) as M_0, M_1 and M_2.

To illustrate the work of the offered technology we consider 7 known cases of earthquakes for each intensity interval.

a) *Statistics of Fuzzy Grades*:

Let's define the corresponding membership functions. The model of membership function, applied to the given method, is:

$$\mu_0(M)=\begin{cases}\dfrac{1}{1+(\alpha_1 M)^2}, & 0\le M\le 3;\\ 0, & M>3\end{cases}$$

$$\mu_1(M)=\begin{cases}0, & M<4.4;\\ \dfrac{1}{1+(\alpha_2(M-4.9))^2}, & 4.4\le M\le 5.4;\\ 0, & M>5.4\end{cases} \quad (15)$$

$$\mu_2(M)=\begin{cases}0, & M<4.4;\\ \dfrac{1}{1+(\alpha_3(M-8))^2}, & 4.4\le M\le 8;\\ 1, & M>8\end{cases}$$

Coefficients α_1, α_2 and α_3 are chosen empirically in accordance with the available data and experts' recommendations. In our case $\alpha_1=0.15$, $\alpha_2=4.99$ and $\alpha_3=0.5$.

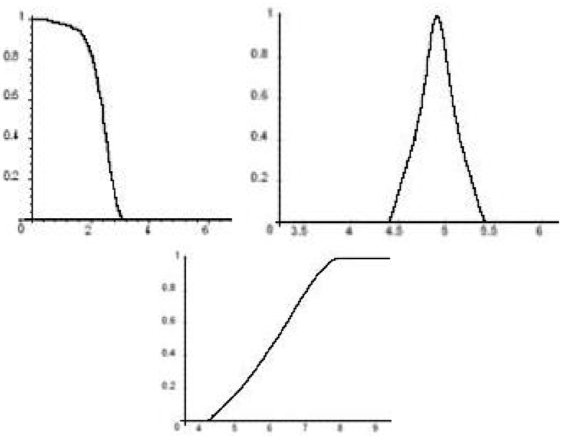
Since forecasting classes are presented in the form of intervals, it is necessary to average membership functions on these intervals. Let μ_i^j be an average value of μ_j considering the intersection of a support of i forecasting fuzzy class and supp μ_j. Then:

$$\mu_0(M) = \frac{0.93968}{0 < M \le 3} + \frac{0}{M > 3};$$

$$\mu_1(M) = \frac{0}{0 < M < 4.4} + \frac{0.55192}{4.4 \le M \le 5} + \frac{0.3641}{5 < M \le 5.4};$$

$$\mu_2(M) = \frac{0.26968}{4.4 \le M < 5} + \frac{0.6552}{5 \le M \le 8} + \frac{1}{M > 8}.$$

Corresponding graphs of membership functions are:



Each of the factors-precursors is divided into three classes (sub factors). Intervals of the classes are fuzzy sets. Their boundaries are chosen empirically in accordance with the available data and estimations by the experts.

Selective frequencies n_{kj}^i for each interval of intensity and each class of each activity are calculated. According to our initial data, they have a the following values:

Activity		n_{kj}^0	n_{kj}^1	n_{kj}^2
A_0	a_{00}	2	0	1
	a_{01}	5	5	4
	a_{02}	0	2	2
A_1	a_{10}	3	3	2
	a_{11}	1	1	4
	a_{12}	3	3	1
A_2	a_{20}	3	2	3
	a_{21}	2	2	3
	a_{22}	2	3	1
A_3	a_{30}	1	2	3
	a_{31}	3	5	2
	a_{32}	3	0	2
A_4	a_{40}	3	2	1
	a_{41}	2	4	4
	a_{42}	2	1	2
A_5	a_{50}	2	2	1
	a_{51}	4	3	5
	a_{52}	1	2	1
A_6	a_{60}	4	2	3
	a_{61}	2	4	2
	a_{62}	1	1	2
A_7	a_{70}	2	3	2
	a_{71}	4	4	4
	a_{72}	1	0	1
A_8	a_{80}	1	4	4
	a_{81}	5	2	2
	a_{82}	1	1	1

The $\tilde{n}_{kj}^i$ fuzzy selective frequencies, $\tilde{f}_{kj}^i$ fuzzy relative frequencies and w_{kj} fuzzy weights of the forecasting factors are calculated on the basis of formulae (1). Now all the data necessary for decision-making exists and the numerical-tabular knowledge base is built:

Activity		w_{kj}	$\tilde{f}_{kj}^0$	$\tilde{f}_{kj}^1$	$\tilde{f}_{kj}^2$
A_0	a_{00}	0.1489	0.6484	0.1256	0.2260
	a_{01}	0.6619	0.3647	0.3272	0.3081
	a_{02}	0.1892	0	0.4976	0.5024
A_1	a_{10}	0.3762	0.3850	0.3256	0.2894
	a_{11}	0.2999	0.1609	0.34402	0.4951
	a_{12}	0.3238	0.4472	0.3205	0.2323
A_2	a_{20}	0.3864	0.3749	0.2920	0.3331
	a_{21}	0.3381	0.2856	0.3337	0.3807
	a_{22}	0.2756	0.3504	0.3766	0.2730
A_3	a_{30}	0.2898	0.1666	0.3893	0.4440
	a_{31}	0.4606	0.3144	0.3890	0.2966
	a_{32}	0.2496	0.5803	0.1499	0.2698
A_4	a_{40}	0.2816	0.5143	0.2678	0.2179
	a_{41}	0.4749	0.2033	0.3964	0.4003
	a_{42}	0.2435	0.3965	0.2701	0.3334
A_5	a_{50}	0.2333	0.4138	0.3232	0.2630
	a_{51}	0.5816	0.3320	0.3071	0.3609
	a_{52}	0.1851	0.2609	0.4075	0.3316
A_6	a_{60}	0.4346	0.4443	0.2596	0.2961
	a_{61}	0.3701	0.2609	0.4075	0.3316
	a_{62}	0.1952	0.2473	0.3369	0.4158
A_7	a_{70}	0.3279	0.2944	0.3735	0.3321
	a_{71}	0.5714	0.3379	0.3294	0.3326
	a_{72}	0.1006	0.4797	0.1859	0.3345
A_8	a_{80}	0.4266	0.1132	0.4413	0.4455
	a_{81}	0.4305	0.5607	0.2186	0.2207
	a_{82}	0.1429	0.3379	0.3294	0.3326

Assume, we need to study a new case and values describing its activities are:

5.125, 9.675, 8.875, 913.98, 8.12, 69,

3.25, 2.625, 6.125.

According to initial data and selected classes of activities the following set of the classes of activities corresponds to the above given set of activities:

$$a_{01}, a_{12}, a_{21}, a_{31}, a_{41}, a_{51}, a_{60}, a_{70}, a_{82}$$

From the knowledge base we select a vector of fuzzy weights

$$\vec{w} = (w_{01}, w_{12}, w_{21}, w_{31}, w_{41}, w_{51}, w_{60}, w_{70}, w_{82})$$

and matrix $\tilde{f}$ of fuzzy relative frequencies, where to each component w_{kj} of a vector $\vec{w}$ corresponds a row $\tilde{f}^0_{kj}, \tilde{f}^1_{kj}, \tilde{f}^2_{kj}$ of a matrix $\tilde{f}$.

In our case

$\vec{w} = (0.6619,\ 0.3238,\ 0.3381,\ 0.4606,\ 0.4749,\ 0.5816,\ 0.4346,\ 0.3279,\ 0.1429)$,

a matrix of fuzzy relative frequencies is:

$$\tilde{f} = \begin{pmatrix} 0.3647 & 0.3272 & 0.3081 \\ 0.4472 & 0.3205 & 0.2323 \\ 0.2856 & 0.3337 & 0.3807 \\ 0.3144 & 0.3890 & 0.2966 \\ 0.2033 & 0.3964 & 0.4003 \\ 0.3320 & 0.3071 & 0.3609 \\ 0.4443 & 0.2596 & 0.2961 \\ 0.2944 & 0.3735 & 0.3321 \\ 0.3379 & 0.3294 & 0.3326 \end{pmatrix}.$$

According to formulae (2) we receive the weighed vector of possible decisions

$$\vec{D}_\alpha = (1.25522,\ 1.26162,\ 1.22950)$$

with the corresponding measure of possibility (see (11))

$$\overrightarrow{Poss} = (0.99493,\ 1,\ 0.97454).$$

And, finally, we receive the forecast (see (12)):

$$D_{Class} = 1\ (\Rightarrow M_1 \equiv \text{"moderate earthquake"}).$$

b) *Discrimination Analysis:*

In this method each of the activities (factor-precursors) is divided into two classes: A_{k1}, A_{k2}; $k = \overline{1,9}$; $A_k = \bigcup_{j=1}^{2} a_{kj}$. Relative frequencies of each class of each activity are calculated. On the basis of these calculations (made by rules of type "If-Then") the first table of the knowledge base $\{f_{ij}\}$ (see (4)) is received. In accordance with the historical data in our general database, the table looks like:

Activity		f_{ij}		
A_0	a_{00}	3	2	4
	a_{01}	4	5	3
A_1	a_{10}	4	4	6
	a_{11}	3	3	1
A_2	a_{20}	5	4	6
	a_{21}	2	3	1
A_3	a_{30}	2	6	5
	a_{31}	5	1	2
A_4	a_{40}	5	6	5
	a_{41}	2	1	2
A_5	a_{50}	3	5	5
	a_{51}	4	2	2
A_6	a_{60}	4	2	3
	a_{61}	3	5	4
A_7	a_{70}	2	3	1
	a_{71}	5	4	6
A_8	a_{80}	3	4	5
	a_{81}	4	3	2

Then under formulas (5) positive $\{p_{ij}\}$ and negative $\{n_{ij}\}$ discrimination tables are built. Coefficient $k = 1/3.75$ in $\chi_{Large-ratio}$ membership function.

	p_{ij}			n_{ij}		
a_{00}	0.3	0.1556	0.4444	0.2667	0.4667	0.1667
a_{01}	0.2844	0.3889	0.18	0.2667	0.1867	0.4
a_{10}	0.2222	0.2222	0.4	0.3333	0.3333	0.1778
a_{11}	0.5333	0.5333	0.0889	0.1778	0.1778	0.8
a_{20}	0.2778	0.1956	0.36	0.2667	0.3667	0.2
a_{21}	0.3556	0.6	0.1111	0.2667	0.1333	0.6667
a_{30}	0.0978	0.56	0.4444	0.7333	0.1556	0.2133
a_{31}	1	0.0933	0.32	0.08	0.9333	0.4
a_{40}	0.2444	0.32	0.2444	0.2933	0.2222	0.2933
a_{41}	0.4	0.1333	0.4	0.2	0.5333	0.2
a_{50}	0.16	0.3556	0.3556	0.4444	0.2133	0.2133
a_{51}	0.5333	0.2	0.2	0.1333	0.4	0.4
a_{60}	0.4444	0.1556	0.3	0.1667	0.4667	0.2667
a_{61}	0.18	0.3889	0.2844	0.4	0.1867	0.2667
a_{70}	0.3556	0.6	0.1111	0.2667	0.1333	0.6667
a_{71}	0.2778	0.1956	0.36	0.2667	0.3667	0.2
a_{80}	0.18	0.2844	0.3889	0.4	0.2667	0.1867
a_{81}	0.4444	0.3	0.1556	0.1667	0.2667	0.4667

These tables represent knowledge base of the Discrimination Analysis method.

Let's make the forecast for the same set of activities:

5.125, 9.675, 8.875, 913.98, 8.12, 69, 3.25, 2.625, 6.125.

To the set above corresponds the set of the classes of activities:

$$a_{00}, a_{11}, a_{20}, a_{30}, a_{40}, a_{50}, a_{60}, a_{70}, a_{81}.$$

Out of tables $\{p_{ij}\}$ and $\{n_{ij}\}$ only those rows which correspond to $\{A_{kj}\}$ are selected and new $\{p'_{ij}\}$ and $\{n'_{ij}\}$ tables are produced:

p'_{ij}			n'_{ij}		
0.3	0.1556	0.4444	0.2667	0.4667	0.1667
0.5333	0.5333	0.0889	0.1778	0.1778	0.8
0.2778	0.1956	0.36	0.2667	0.3667	0.2
0.0978	0.56	0.4444	0.7333	0.1556	0.2133
0.2444	0.32	0.2444	0.2933	0.2222	0.2933
0.16	0.3556	0.3556	0.4444	0.2133	0.2133
0.4444	0.1556	0.3	0.1667	0.4667	0.2667
0.3556	0.6	0.1111	0.2667	0.1333	0.6667
0.4444	0.3	0.1556	0.1667	0.2667	0.4667

The subsequent calculations are received under formulas (6) and (7):

Forecast classes	π_j	ν_j	δ_j
M_0	0.317531	0.309136	0.504198
M_1	0.352839	0.274321	0.539259
M_2	0.278272	0.365185	0.456543

The final, "classical" decision it is received as $\delta^{Class} = \max_j \delta_j$. In our case

$$\delta^{Class} = 0.539259.$$

This maximal value corresponds to an interval of intensity "moderate earthquake". That also will be the forecast.

c) *Case Based Reasoning:*

Values of ativities, which describe known cases (precedents) as "noise", "moderate" and "strong" are selected from a general database. Each activity for each case is divided into two classes. Then by rules of type "If-Then", we build the table of the knowledge base where for each class of each activity we have values 0 or 1. We have 1, if value of activity of a precedent belongs to a class of activity. Otherwise we have 0:

A_0		A_1		A_2		A_3		A_4		A_5		A_6		A_7		A_8	
a_{00}	a_{01}	a_{10}	a_{11}	a_{20}	a_{21}	a_{30}	a_{31}	a_{40}	a_{41}	a_{50}	a_{51}	a_{60}	a_{61}	a_{70}	a_{71}	a_{80}	a_{81}
M_0																	
0	1	1	0	1	0	0	1	1	0	0	1	0	1	0	1	1	0
1	0	1	0	1	0	0	1	1	0	0	1	1	0	1	0	0	1
1	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	1	1	0	1	0	1	0	1	0	1	0	0	1	0	1
1	0	1	0	1	0	0	1	1	0	0	1	0	1	0	1	1	0
0	1	1	0	1	0	0	1	1	0	1	0	1	0	0	1	0	1
0	1	0	1	0	1	1	0	0	1	1	0	1	0	1	0	1	0
M_1																	
0	1	1	0	1	0	1	0	1	0	1	0	0	1	0	1	1	0
0	1	0	1	0	1	1	0	0	1	0	1	1	0	1	0	1	0
0	1	0	1	0	1	1	0	1	0	1	0	1	0	1	0	0	1
0	1	1	0	1	0	0	1	1	0	1	0	0	1	1	0	0	1
0	1	0	1	0	1	1	0	1	0	1	0	0	1	0	1	0	1
1	0	1	0	1	0	1	0	1	0	0	1	0	1	0	1	1	0
1	0	1	0	1	0	1	0	1	0	1	0	0	1	0	1	1	0
M_2																	
1	0	1	0	1	0	0	1	1	0	0	1	0	1	0	1	0	1
0	1	1	0	1	0	1	0	1	0	1	0	0	1	0	1	1	0
0	1	1	0	1	0	1	0	1	0	1	0	1	0	0	1	1	0
0	1	0	1	0	1	1	0	0	1	1	0	1	0	0	1	1	0
1	0	1	0	1	0	0	1	0	1	0	1	0	1	0	1	1	0
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	0	1
1	0	1	0	1	0	1	0	1	0	1	0	0	1	0	1	1	0

Let's make the forecast for a set of activities:

5.125, 9.675, 8.875, 913.98, 8.12, 69, 3.25, 2.625, 6.125.

At the predetermined boundaries of classes to the set above corresponds the following set

1 0 0 1 1 0 1 0 1 0 1 0 1 0 1 0 0 1

The distance between two i activities of investigated case and precedent is calculated according formulae (13). The similarity measure between two cases is calculated according formulae (14).

After necessary calculations

	SV_j		
	M_0	M_1	M_2
1	0.2222	0.4444	0.4444
2	0.6667	0.4444	0.4444
3	0.3333	0.7778	0.5556
4	0.7778	0.5556	0.4444
5	0.3333	0.5556	0.2222
6	0. 5556	0.4444	0.8889
7	0. 5556	0.5556	0.5556
max	*0.7778*	*0.7778*	*0.8889*

we receive set of measures of the maximal similarity on forecasting classes:

$$(0.7778,\ 0.7778,\ 0.8889).$$

The final decision is found according formulae (10). The forecast is - "strong earthquake".

d) *Expertons Method*:

Methods give an ambiguous forecast. This is a precondition to apply Expertons method.

Let 3 experts give the interval estimations concerning reliability of acceptance of each of decisions: δ_0 - possibly "noise" will be observed; δ_1 - possibly "moderate earthquake" will be observed; δ_2 - possibly "strong earthquake" will be observed. The interval estimations of experts are given in the table:

Expert	Method	δ_0	δ_1	δ_2
1	1	[0.2, 0.3]	[0.4, 0.6]	[0.6, 0.7]
	2	[0.5, 0.6]	[0.6, 0.8]	[0.5, 0.6]
	3	[0.1, 0.7]	1	[0.7, 0.8]
2	1	[0.3, 0.4]	1	1
	2	0.6	[0.7, 1]	[0.6, 0.8]
	3	[0.8, 1]	[0.6, 1]	[0.2, 0.3]
3	1	[0.4, 0.8]	0.6	[0, 0.1]
	2	[0.4, 0.5]	[0.3, 0.7]	1
	3	[0.2, 0.4]	[0.2, 0.9]	[0.2, 0.9]

Here 1 designates method of Statistics of Fuzzy Grades; 2 - Discrimination Analysis; 3 - Case Based Reasoning.

For each of the possible decisions δ_i, $i = 0,1,2$ we calculate two statistics on set of the levels $\{0, 0.1, 0.2, \ldots, 0.9, 1\}$: one for the lower boundary of an interval and the other for the upper boundary. Then we obtain the following table which is an experton:

Level	δ_0	δ_1	δ_2
0	[1, 1]	[1, 1]	[1, 1]
0.1	[1, 1]	[1, 1]	[0.9, 1]
0.2	[0.9, 1]	[1, 1]	[0.9 , 0.9]
0.3	[0.7, 1]	[0.9, 1]	[0.7, 0.9]
0.4	[0.6, 0.9]	[0.8, 1]	[0.7, 0.8]
0.5	[0.3, 0.7]	[0.7, 1]	[0.7, 0.8]
0.6	[0.2, 0.6]	[0.7, 1]	[0.6, 0.8]
0.7	[0.1, 0.3]	[0.3, 0.8]	[0.3, 0.7]
0.8	[0.1, 0.2]	[0.2, 0.7]	[0.2, 0.6]
0.9	[0, 0.1]	[0.2, 0.6]	[0.2, 0.3]
1	[0, 0.1]	[0.2, 0.4]	[0.2, 0.2]

Further, we will transform the experton as follows:

(a). we will calculate averaged experton by taking a mean arithmetic value of the boundaries of each interval:

Level	δ_0	δ_1	δ_2
0	1	1	1
0.1	1	1	0.95
0.2	0.95	1	0.9
0.3	0.85	0.95	0.8
0.4	0.75	0.9	0.75
0.5	0.5	0.85	0.75
0.6	0.4	0.85	0.7
0.7	0.2	0.55	0.5
0.8	0.15	0.45	0.4
0.9	0.05	0.4	0.25
1	0.05	0.3	0.2

(b). averaged experton is reduced to a possibility distribution on decisions set $\{\delta_i\}$, $i = 0,1,2$ by taking mean value of all levels;

(c). if necessary, we search a nonfuzzy set, the closest to the fuzzy one.

After the transformation (b) we will obtain the possibility distribution on decisions set:

δ_0	δ_1	δ_2
0.536364	0.7500	0.654545

The principle of a maximum is applied to acquire the unique decision:

$$\delta(\delta_i) = \max_i \delta_i, \; i = 0,1,2 .$$

In our case, in conformity with the common opinion of the experts, the experton gives preference to the decision δ_1, i.e. final forecast is - possibly "moderate earthquake" will be observed.

The described result conforms with the statistical data: values of forecasting factors in the given example correspond to a real data for November, 20-th, 1981 when in $18^{\underline{00}}$ there was an earthquake with magnitude 4.6 (according to our classification – " moderate earthquake").

5 Conclusion

The article proposed decision making technology combining fuzzy-statistical methods. As shown, three methods make a decision independently with different approaches. During the decision-making these methods are applied simultaneously and their composition is necessary to receive the unique decision.Thus, we use the fourth fuzzy method.

By means of the offered technology the DSS was developed, which does the forecast of earthquake. The illustrated example is the result of this system's work. Being applied to the specific earthquake task, the technology proved approximately 70% accuracy.

It is the satisfactory result, if taking into account the fact that the geophysical activities of an atmosphere are not the main factor-precursors of earthquakes.

The advantage of the technology is that the small amount of the initial data suffices to receive the forecast. Increasing the number of known cases in the general database does not improve the forecast much.

To tell more about DSS, it is based on numerical-tabular knowledge base and is implemented by means of Web-programming and client-server technology. The DSS utilizes MySQL DBMS resources.

The DSS is a general use system, since it is possible to use the created software in various research fields (for example, medical diagnostics, weather forecast, forecasting of flooding, etc.).

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