Intelligent Sea Transportation System Postoptimal Analysis

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Abstract: - A procedure for the ship dynamic positioning postoptimal analysis in Intelligent Sea Transportation System Optimization is proposed. The dynamic positioning control system design is based on the optimal constrained covariance control (OC^3). In that way some disadvantages of the classical optimal control technique are avoided. When the sensitivity of solutions to desired system performances is performed, then it can be shown that under particular circumstances, a slight change of desired system performances could significantly improve the optimal solution value. Namely, a slight relaxation of desired system position accuracy could result with significant energy savings. The presented numerical example illustrates some benefits of the proposed approach.

Key-Words: - Intelligent Transportation System, Dynamic Positioning, Mathematical Modelling, LQG control, Optimization problems, Postoptimal analysis, Robust control

1 Introduction

Dynamic Positioning of floating vessels is a technique for maintaining the position and heading of the vessel without the use of mooring system [1,2]. The basic forces and motions are presented in Fig. 1. Today this manoeuvre technique is very important for various logistics facility. A Mobile Offshore Base (MOB) Project as a large floating platform is well-known example. The Mobile Offshore Base is a Science & Technology Program conducted by US Office of Naval Research to advance technologies essential for establishing the feasibility of building a Mobile Offshore Base and to determine whether the MOB concept represents credible system for Naval and Marine forces. The envisioned system would consist of three to five interconnected modules and would accept cargo from conventional take-off and landing (CTOL) aircraft, as well as from container ships. It will be able to project these resources to the shore via landing craft.
Other key general characteristics of a MOB include:

1. Length up to 2 km and width of approximately 120 m
2. Low ocean wave-induced motion to support CTOL operation of cargo aircraft up to Sea State 6
3. High throughput, open-ocean ship to MOB, and MOB-to-cargo ship transfers through Sea State 3
4. Platform survivability in any incident storm (e.g., hurricane and typhoon)
5. Maintainability of 40 years between overhauls
6. Long-term station-keeping in deep water [3].

In a conventional floating vessel the forces required to overcome the effects of wind, waves and current are provided by the mooring system. The most significant limitation of that solution is the difficulty of mooring in deep water. In fact, at some water depth the multipoint mooring system is totally impractical. In a dynamically positioned vessel the forces are provided by thrust devices.
The main elements of a dynamic positioning system are the position reference system, the propulsion system and the control system. The position of the vessel can be measured using either an hydroacustic system (beacon), a taut-wire system, a micro-wave radio system, GPS or a combination of them. The deviation of the vessel heading is measured by a gyrocompass. The direction and magnitude of wind are measured by a wind sensor (anemometer). The propulsion system can be composed of various combinations of main engine, tunnel thrusters, steer able thrusters and cycloidal propellers. The control system receives signals of the position reference system and heading deviations, compares with ordered values and calculates the output commands for thrust magnitude and direction of thrust devices. In order to design an efficient DP control system using Kalman filtering approach, an accurate mathematical model of the vessel dynamics is required. The optimal control strategy for

the dynamic positioning design can be split into four distinct procedures, [4]:

a) Find the conditional mean estimate of LF state vector using the Kalman filter (filter problem);
b) Find the optimal feedback on the assumption that the conditional mean estimate of the LF state vector is the true system state (regulator problem);
c) The magnitude and direction of the wind are measured, converted to ship coordinate system, filtered and input into the wind feed-forward loop (feed-forward problem);
d) The thruster allocation algorithm calculates thruster output level and azimuth angle commands to an arbitrary combination of working propulsion plant (contribution problem), [5,6].

An essential problem in DP system design is removing the oscillatory components on the positioning measurement, that the propulsion system do not respond to the wave motions and, consequently, reduce energy loss and wear of the propulsion system. In existing presented DP investigations the controllers were determined on some different ways (using classical Linear Quadratic Gaussian (LQG) optimal control technique, [7], using the pole placement technique, [8], using characteristic locus design method, [9], etc. The classical Linear Quadratic Gaussian (LQG) optimal control approach is impractical in two reasons. First, there are not physical sense of weighting matrices Q and R, and second, the objectives are conflicting and no design exists which is best with respect to all objectives. Thus, a very difficult iterative design process must be used to determine the necessary optimal control performance criterion. The use of multiobjective LQG design has been rather limited, [10]. One reason for this may be the fact that multiobjective optimization is, in general, computationally demanding and the non-uniqueness of the controls. The pole placement technique and characteristic locus design method are inadequate, because they are not intended for stabilization's problems.

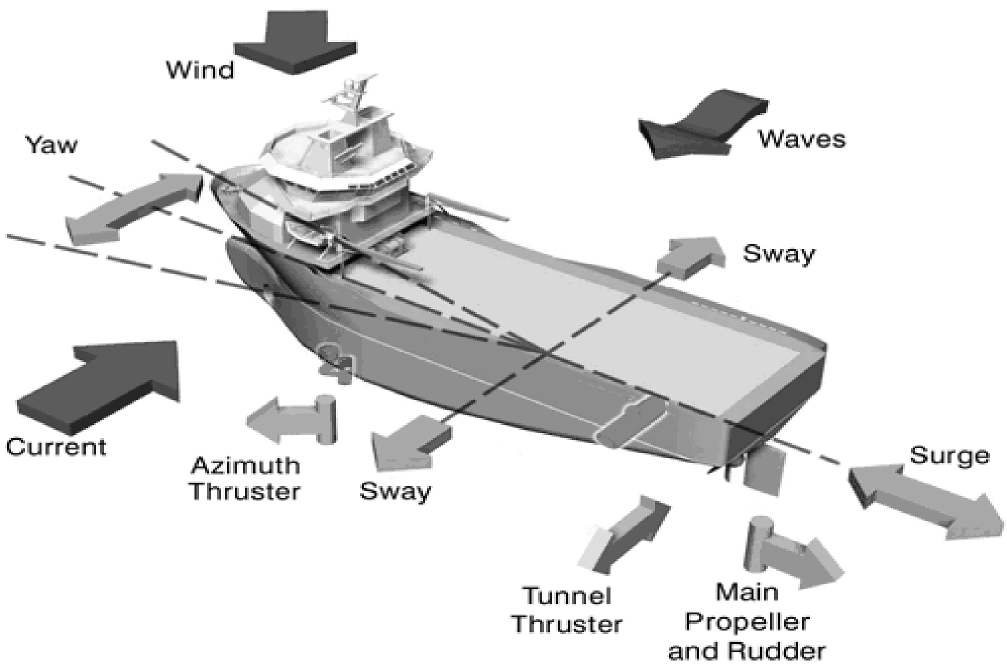

Fig. 1. Dynamic Positioning System

2 Mathematical Modeling

The mathematical model for the vessel dynamics is highly nonlinear and may be derived from theory and can be substantiated by model test, [11,12]. It is usual to assume that the vessel motions are the sum of the outputs from the low-frequency (LF) and high-frequency (HF) subsystem. The low-frequency subsystem is controllable and has an input from the thruster control signal u. The high-frequency subsystem is not controllable via the thrusters control input. Thus, the motions of the vessel are the superposition of the horizontal manoeuvring motion in a calm sea and the motions induced by the high-frequency wave exciting forces. The non-linear differential equations of vessel LF motions may be expressed in the following form:

$$\begin{aligned}(m-X_{\dot u})\dot u-(m-Y_{\dot v})rv&=X_A+X_H(u,v,r)\\(m-Y_{\dot v})\dot v+(m-X_{\dot u})ru&=Y_A+Y_H(u,v,r)\\(J_{zz}^2-N_{\dot r})\dot r&=N_A+N_H(u,v,r)\end{aligned} \quad (1)$$

where:

u - Surge velocity,
v - Sway velocity,
r - Yaw velocity,
X_A - Surge direction force,
Y_A - Sway direction force,
N_A - Turning moment on the vessel,
m - Mass of vessel,
J_{zz} - Moment of inertia,
$X_{\dot u}$, $Y_{\dot v}$, $N_{\dot r}$ - Added masses and inertia,
X_H, Y_H, N_H - Hydrodynamics forces and moment.

The low-frequency linear model is obtained by linearizing above equations of motions. The linearized equations are dependent upon nominal state (current, sea state, propulsion etc.) and it is reasonable to assume that the changes in velocity and position are small. In that way linear model very well describes dynamics of the vessel.
The high-frequency model is represented by colored noise model (color filter) which is obtained by approximation of sea spectrum in rational proper transfer-function model, [13]. It is assumed that. in the worst case, the high frequency motions of the vessel are not attenuated by vessel dynamics, [14]. The most representative mathematical models of sea waves is well-known Pierson-Moskowitz power spectral density function. It is given by the following non-rational expression:

$$S(\omega)=\frac{A}{\omega^5}\exp(-\frac{B}{\omega^4}) \quad (2)$$

where the values for A and B have been suggested by the ITTC (International Towing Tank Conference) as:

$A=0.0081*g^2$,
$B=3.109*h_s^{-2}$,

where:
hs - Significant wave height,
g - Gravitational constant (9.81 m/sec2),
ω - Angular frequency (rad/s).

This one-parameter spectrum is general enough to include many of the observations at oceans. It is also in agreement with theoretical prediction of high-frequency limit. The spectrum (2) for the sea states 5, 6 and 7 is shown in Fig 1.
The value of the resonant frequency can be calculated as:

$$\omega_0^4=\frac{4B}{5} \quad (3)$$

Unfortunately, the equations arising from such descriptions are difficult to manipulate in control theory. Following the linear theory, using spectral factorisation techniques, the sea spectrum can be modelled by transfer function representation (colour filter). There are at least two situations for modelling sea spectra by using transfer function methodology. First, it is necessary for digital simulation of the behaviour of floating vessels. The analysis of more complex systems on floating vessels and the design verification usually require simulation. These simulations use a computer to solve some algebraic and differential equations which model the system. Second, sea disturbances have to be defined adequately in the process of designing control systems using the modern optimal control and estimation theory. The traditional approach, using classical seakeeping methodology, is not suitable in this case. Remarkable progress has been made recently in the theory of stochastic process, not only in mathematics and physics, but also in communications, measuring and control engineering, as well as in economics and biology. Modern control and estimation theories are mostly based on linear models.

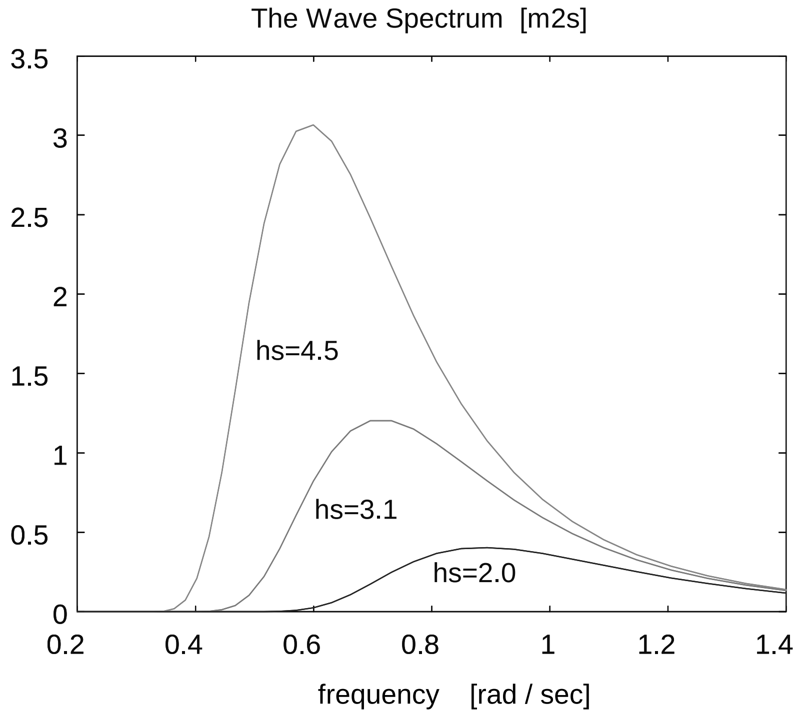

Fig 1. Pierson-Moskowitz spectrum

Important step in identification of colored filter transfer function is determination of its structure (order). Consequently, there exists a compromise problem between approximation quality (accuracy) and simplicity (complexity) of a filter. The majority of research efforts in the area of sea spectrum modelling suggest the following structure:

$$F(s) = \frac{Ks^2}{s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4} \tag{4}$$

It represents agreeable compromise solution for real range of sea states. The maximum slope of (4) in low-frequency range is defined by the numerator (40 dB/dec), while the maximum slope in high-frequency range is determined by difference of degrees of numerator and denominator (-40 dB/dec).

More complex spectrum shapes indicate the more complex transfer function structure.

The global state equations of the vessel may be derived in the usual form as follows:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{Ax}(t) + \mathbf{Bu}(t) + \mathbf{Gw}(t) \\ \mathbf{y}(t) &= \mathbf{Cx}(t) + \mathbf{v}(t) \end{aligned} \tag{5}$$

where **x** is the n-dimensional state vector, **u** is the p-dimensional input vector, **y** is the r-dimensional output vector, and **w** and **v** are Gaussian white noise with zero mean and covariance matrices $\mathbf{R}_w$ and $\mathbf{R}_v$, respectively. The state vector may be partitioned into low- and high frequency sections to obtain the following combined equations:

$$\begin{aligned} \begin{bmatrix} \dot{x}_l \\ \dot{x}_h \end{bmatrix} &= \begin{bmatrix} A_l & 0 \\ 0 & A_h \end{bmatrix} \begin{bmatrix} x_l \\ x_h \end{bmatrix} + \begin{bmatrix} B_l \\ 0 \end{bmatrix} u + \begin{bmatrix} D_l & 0 \\ 0 & D_h \end{bmatrix} \begin{bmatrix} w_l \\ w_h \end{bmatrix} \\ z &= \begin{bmatrix} C_l & C_h \end{bmatrix} \begin{bmatrix} x_l \\ x_h \end{bmatrix} + v \end{aligned} \tag{6}$$

The first partition in (6) corresponds to the subsystem (A_l, B_l, C_l) associated with the low-frequency vessel motions. This includes the effect of steady wind disturbances; current forces and second order (drift) wave forces on the vessel. The second subsystem (A_h, B_h, C_h) represents the high-frequency ship motions which are due to the first-order (oscillatory) wave forces. The assumption that the linear models may be employed will be validated by simulation based on the nonlinear model of the vessel.

3 General Problem Formulations

Consider the continuous linear system described by (5). The required performances of the control system are given in the form of inequality constraints:

$$\mathrm{diag}(\mathbf{D_x}) \le \mathbf{d_0} \tag{7}$$

where D_X is the state covariance matrix of closed loop system and d_0 are desired upper limits for diagonal elements of D_X.

The cost function (price) is given in the form:

$$J = \mathrm{trace}(\mathbf{RD_u}) \tag{8}$$

where D_u is the control input covariance matrix of closed loop system and R is weighting matrix. D_X and D_u are defined as:

$$\begin{aligned} \mathbf{D_x} &= E\left[\mathbf{x}(t)\mathbf{x}(t)^T\right] \\ \mathbf{D_u} &= E\left[\mathbf{u}(t)\mathbf{u}(t)^T\right] \end{aligned} \tag{9}$$

The optimal regulator has the form:

$$\begin{aligned} \dot{\hat{\mathbf{x}}} &= \mathbf{A}\hat{\mathbf{x}} + \mathbf{Bu} + \mathbf{K_f}(\mathbf{y} - \mathbf{C}\hat{\mathbf{x}}) \\ \mathbf{u} &= -\mathbf{K_r}\hat{\mathbf{x}} \end{aligned} \tag{10}$$

where $\hat{x}$ is optimal estimation of x, K_f is Kalman filter gain and K_r is full-order static controller. The solution of the Kalman filter equation gives the stationary state estimation error covariance matrix:

$$\mathbf{P}_{\tilde{\mathbf{x}}} = E[\mathbf{x}(t) - \hat{\mathbf{x}}(t)][\mathbf{x}(t) - \hat{\mathbf{x}}(t)]^T \tag{11}$$

The estimation error is uncorrelated with any estimate of the state [15, 16].

The required performances are transformed in the new form of inequality constraints:

$$\mathrm{diag}(\mathbf{D}_{\hat{\mathbf{x}}} + \mathbf{P}_{\tilde{\mathbf{x}}}) \le \mathbf{d_0} \tag{12}$$

In the next procedure step, we define weighting matrix Q as:

$$\mathbf{Q} = \mathbf{XX}^T \tag{13}$$

where X is an arbitrary matrix and Q is always symmetric and positive semi-definite matrix. The terms of X are variables in the considered optimization problem. In that way, a local minimizer is defined as the well-known LQR problem:

$$\mathbf{K_r} = \mathbf{LQR}(\mathbf{A}, \mathbf{B}, \mathbf{Q}, \mathbf{R}) \tag{14}$$

Since innovations signal is a white noise process with mean zero and covariance R_v which is independent of x the stationary covariance matrices of the estimate and the input can be computed from:

$$\begin{aligned} (\mathbf{A} - \mathbf{BK_r})\mathbf{D}_{\hat{\mathbf{x}}} + \mathbf{D}_{\hat{\mathbf{x}}}(\mathbf{A} - \mathbf{BK_r})^{\mathbf{T}} + \mathbf{K_f R_v K_f^T} &= \mathbf{0} \\ \mathbf{D_u} &= \mathbf{K_r D_{\hat{x}} K_r^T} \end{aligned} \tag{15}$$

In this way, the algorithm is based on solving a sequence of standard linear quadratic control problems. This can be done with the algorithm shown in Fig 2.

Without proof, it is clear that the suggested procedure holds the original convexity of the LQR problem. There is no problem to supply the SQP algorithm with the analytically defined gradients of the cost function (8) and constraints (7). Some characteristics of dynamic positioning control design are described in [4].

There is a notable difference between the standard LQG procedure and constrained LQG. The standard LQG solution is not dependent of the characteristics of the disturbances (w(t) and v(t)). The constrained LQG procedure takes into account both the process disturbances and the estimation solution.

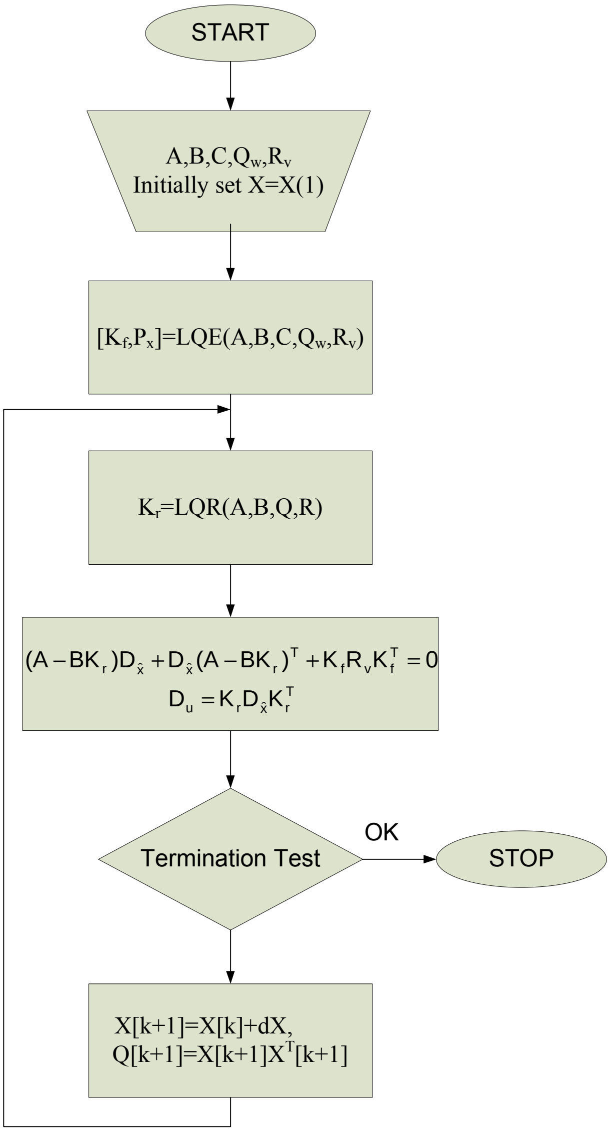

Fig. 2. OC^3 Algorithm

4 Control System Design

The linearized low-frequency equations for a vessel may be assumed to be independent of the sea state and known. The high-frequency model depends upon the sea state, but may be assumed constant for given weather conditions. The covariance of the white noise signal feeding this high-frequency block is then fixed by the assumed sea spectrum Thus, the only unknown quantities which are required before the Kalman filter may be specified are the low-frequency process noise covariance and the measurement noise covariance. An estimate of the power spectral density of w_l can be based upon the Davenport wind gust spectrum, [5]. The measurement noise covariance may be obtained from the manufacturer for the particular position measurement system. The high-frequency model is represented by colored noise model (color filter) which is obtained by approximation of sea spectrum in rational proper transfer-function model and its state space presentation. The signal w_h is unit white noise uncorrelated with others. The extended Kalman filtering technique was first applied to dynamic positioning systems by [7]. It was assumed that the high frequency motions were purely oscillatory and could be modeled by a second order sinusoidal oscillator with variable center frequency. Some disadvantages of this approach are discussed [9]. It is used a fourth order wave model in the specification of high frequency motions. However, the dominant wave frequency varies with weather conditions and corresponding Kalman filter gain must therefore be switched for different operating conditions. The structure of proposed optimal estimation algorithm is presented in Fig. 3..

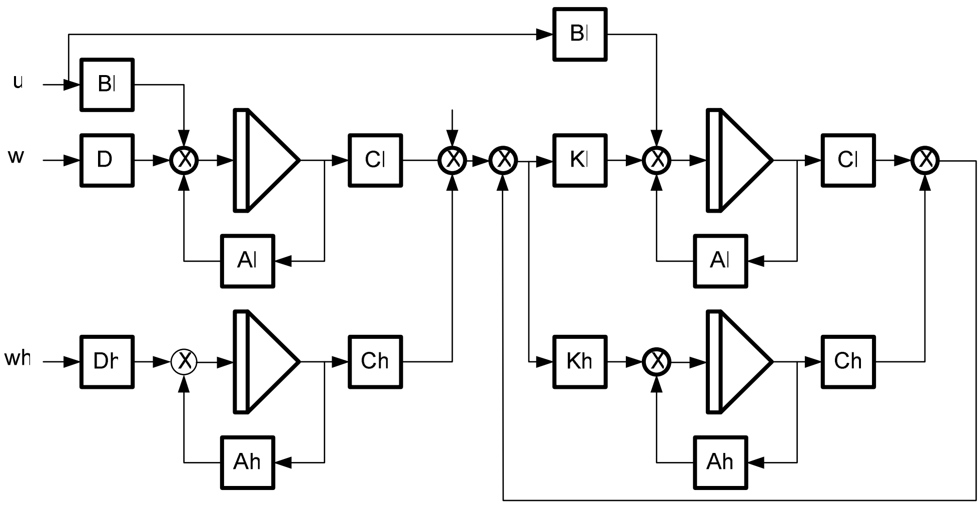

Fig. 3. Optimal estimation algorithm

Theoretically, the problem of constrained LQG control design can be solved using linear LQG theory by iterative change of matrices Q and R in the traditional performance index which is given as:

$$J(u) = E\left\{ \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \left[\mathbf{x}^T(t)\mathbf{Q}\mathbf{x}(t) + \mathbf{u}^T(t)\mathbf{R}\mathbf{u}(t) \right] dt \right\} \quad (16)$$

That approach is impractical, because there is not physical (cause and effect) sense of weighting matrices Q and R (free design parameters). Thus, a very difficult iterative design process must be used to determine the necessary limitations on controlled variables and optimal control performance criterion. The constrained LQG design procedure overcomes these problems, because in process control the reduction of the variances of a controlled variable makes it possible to design a control system which satisfies tighter specification limits, [10, 17]. In the case of dynamic positioning control design, the cost function is proposed in the form (8) and process constraints are in the form (7).

It is well-known that dynamic positioning control system must respond to the LF vessel motion only (thruster modulation will be minimized). The control strategies u(t) is given by:

$$u(t) = -K_r \hat{x}_l(t) \quad (17)$$

The stationary covariance matrices of the LF state and the control input can be computed by:

$$\begin{aligned} D_{xl} &= D_{\hat{x}l} + P_{xl} \\ D_u &= K_r D_{\hat{x}l} K_r^{\,T} \end{aligned} \quad (18)$$

where $D_{\hat{x}l}$ is stationary covariance matrix of the estimate. We can write the estimate dynamics as:

$$\begin{bmatrix} \dot{\hat{x}}_l \\ \dot{\hat{x}}_h \end{bmatrix} = \begin{bmatrix} A_l - B_l L & 0 \\ 0 & A_h \end{bmatrix} \begin{bmatrix} \hat{x}_l \\ \hat{x}_h \end{bmatrix} + \begin{bmatrix} K_l \\ K_h \end{bmatrix} \tilde{y}$$
$$\tilde{y}(t) = C\tilde{x}(t) + v(t) \quad (19)$$

where K_l and K_h are Kalman LF and HF filter gains, respectively. Since $\tilde{y}$ (t) is a white noise process with mean zero and covariance R which is independent of $\hat{x}$.

The stationary covariance matrix of the estimate can be computed using Lyapunov equation:

$$(A_l - B_l K_r) D_{\hat{x}l} + D_{\hat{x}l} (A_l - B_l K_r)^T + K_l R K_l^T = 0 \quad (20)$$

The stationary covariance matrix of the control input is computed from:

$$D_u = K_r D_{\hat{x}l} K_r^{\,T} \quad (21)$$

There is a notable difference between the standard LQG procedure and constrained LQG. The standard LQG solution is not dependent of the characteristics of the disturbances (w(t) and v(t)).

5 Postoptimal Analysis

When the optimization procedure is finished, the sensitivity of the solutions to desired system performances, model inaccuracies and other initial conditions have to be analyzed. This analysis is known as postoptimal analysis [20, 21, 22]. When the sensitivity of solutions to desired system performances is of our concern, then it can be shown that under particular circumstances, a slight change of desired system performances could significantly improve the optimal solution value. Namely, a slight relaxation of desired system position accuracy could result with significant energy savings [23]. As part of postoptimal analysis, the possibilities of price-performance (cost-effectiveness) improvements can be tested. The optimization problem can be given in the form by:

$$\begin{aligned} &\min_x f^0(x) \\ &f^i(x) \le \theta_i \qquad i = 1,\dots,p \end{aligned} \quad (22)$$

where:

$f^0(\mathbf{x})$ - cost (price) function,
$f^i(\mathbf{x})$ - constraint function,
θ_i - constraint (performance) value.

The above optimization is easy to explain. The cost function represents the price of realization (such as energy consumption), while the constraint function represents the desired technical performances of our system (such as desired position accuracy). The corresponding augmented Lagrange function is:

$$L(\mathbf{x}, \lambda, \theta) = f^0(\mathbf{x}) + \sum_{i=1}^{p} \lambda_i (f^i(\mathbf{x}) - \theta_i) \quad (23)$$

Assuming that Slater's condition [22] is valid for some point $\mathbf{x}^*$ and Θ^*, then:

$$\frac{\partial f^0(\mathbf{x}^*, \theta^*)}{\partial \theta_i^*} = -\lambda_i^* \quad (24)$$

Equation (24) can be interpreted as the shadow price. This term is often used in economics when the optimal solutions are sought. It gives the relation for the sensitivity of solution to small change of constrained value (22). A small (or zero) value of Lagrange multiplier indicates that a slight change in this constraint does not have influence on the cost function. On the other hand, a large value of Lagrange multiplier indicates that the corresponding optimal value of the cost function is more susceptible to changes in this constraint. The shadow price is important for the following reasons:

a) To identify which constraints might be the most beneficially changed, and to initiate these changes as a fundamental means to improve the solution.
b) To react appropriately when external circumstances create opportunities or threats to change the constraints, [21].

In the case of OC^3 design, equation (13) expresses the cost sensitivity related to the slight change of control system accuracy performances. However, sometimes the normed equation is preferred [23], and is given by:

$$s_i = \frac{\dfrac{\partial f^0(\mathbf{x}^*,\theta^*)}{f^0(\mathbf{x}^*,\theta^*)}}{\dfrac{\partial \theta_i^*}{\theta_i^*}} = -\lambda_i^* \frac{\theta_i^*}{f^0(\mathbf{x}^*,\theta^*)} \qquad (25)$$

Here the relative change of the cost function optimal value and constrained values are used.

6 Example

The proposed method of a postoptimal analysis is applied to the dynamic positioning of the floating vessel, given in [8]. Only the sway motion is analyzed. The LF subsystem of mathematical model is given by:

$$A = \begin{bmatrix} -0.0546 & 0 & 0.5435 \\ 1 & 0 & 0 \\ 0 & 0 & -1.55 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1.55 \end{bmatrix} \qquad G = \begin{bmatrix} 0.5435 \\ 0 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

$$Q_w = 5.1984 * 10^{-6} \qquad R_v = 10^{-5}$$

In this demonstrative example only low-frequency part of the system is analysed. The response of free floating vessel to random disturbance is presented in Fig. 4.

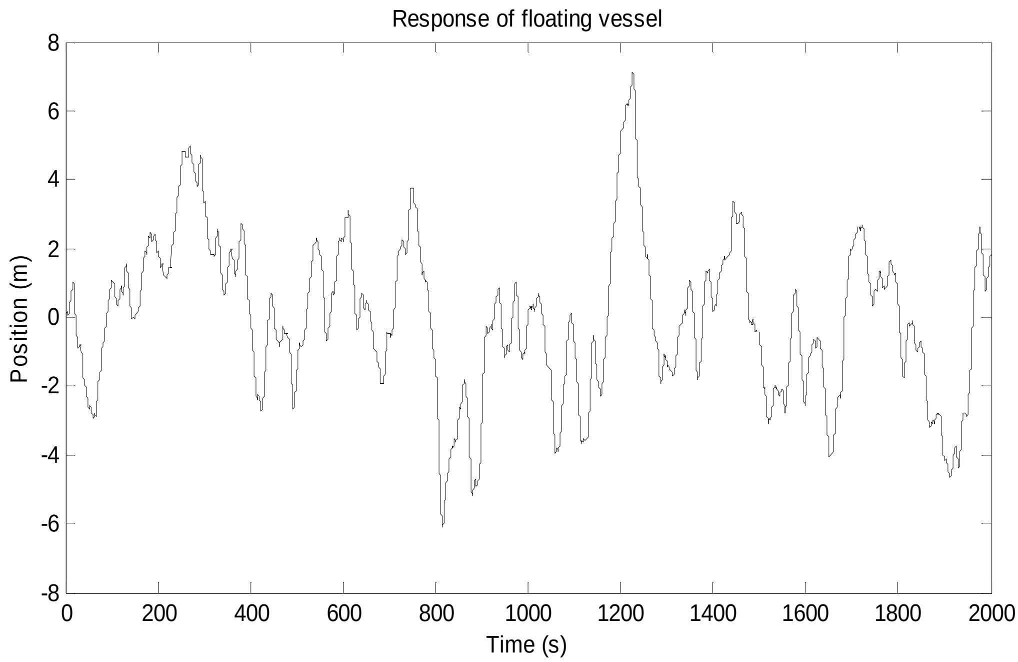

Fig. 4. Response of free floating vessel to random disturbance

Kalman filter gain matrix (**K**$_f$) and the corresponding error covariance matrix for LF subsystem ($\mathbf{P}_{\tilde{x}}$) are:

$$K_f = \begin{bmatrix} 0.3464 \\ 0.8324 \\ 0 \end{bmatrix}$$

$$P_{\tilde{x}} = \begin{bmatrix} 0.3073 & 0.3464 & 0 \\ 0.3464 & 0.8324 & 0 \\ 0 & 0 & 0 \end{bmatrix} * 10^{-5}$$

The solution of the Kalman filter equation gives the stationary state estimation error covariance matrix in the form (11).
The estimation error is uncorrelated with any estimate of the state. The required performances are transformed in the new form of inequality constraints (22).

The response of controlled floating vessel to random disturbance is presented in Fig. 4.

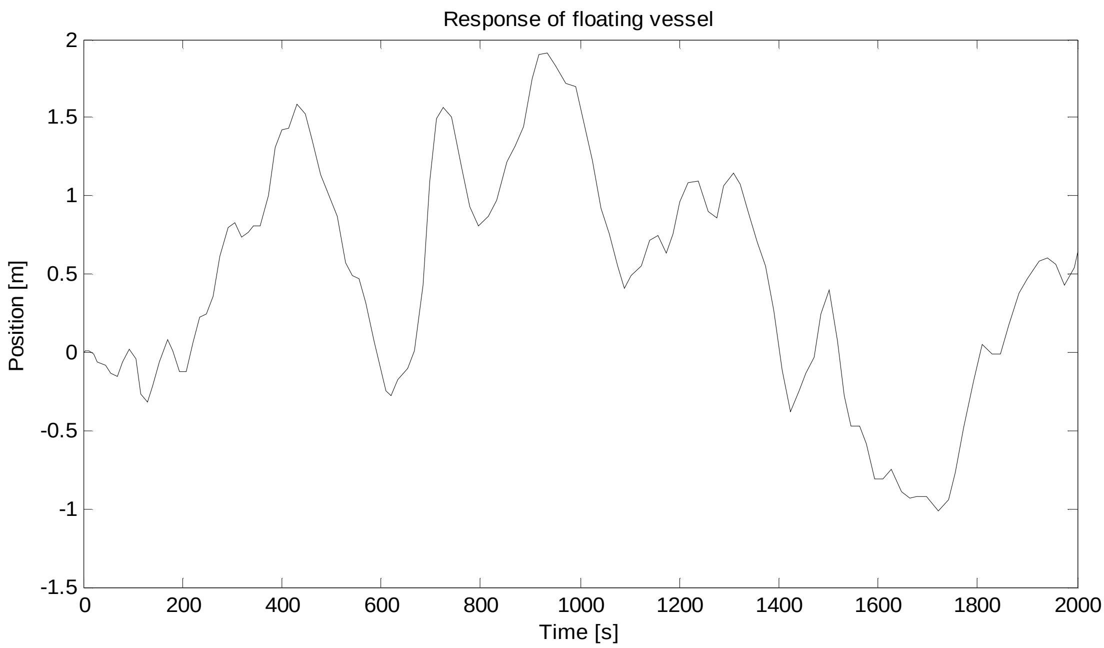

Fig. 4. Response of controlled floating vessel to random disturbance

The cost function in the postoptimal analysis was chosen to represent the total energy consumption, while the constraint function represents the positioning error. The results of the postoptimal analysis are given in Fig. 6. It can be seen from Fig.6. that the shadow price parameter s_i is approximately one until the positioning error dispersion becomes 2 meters. After that the shadow price value steeply rises. The interpretation of this example from the economic aspect is that there is the price to be paid if we insist to have the positioning accuracy better than 2 meters. Subsequent techno-economical analysis must establish justification for accuracy improvement below 2 meters.

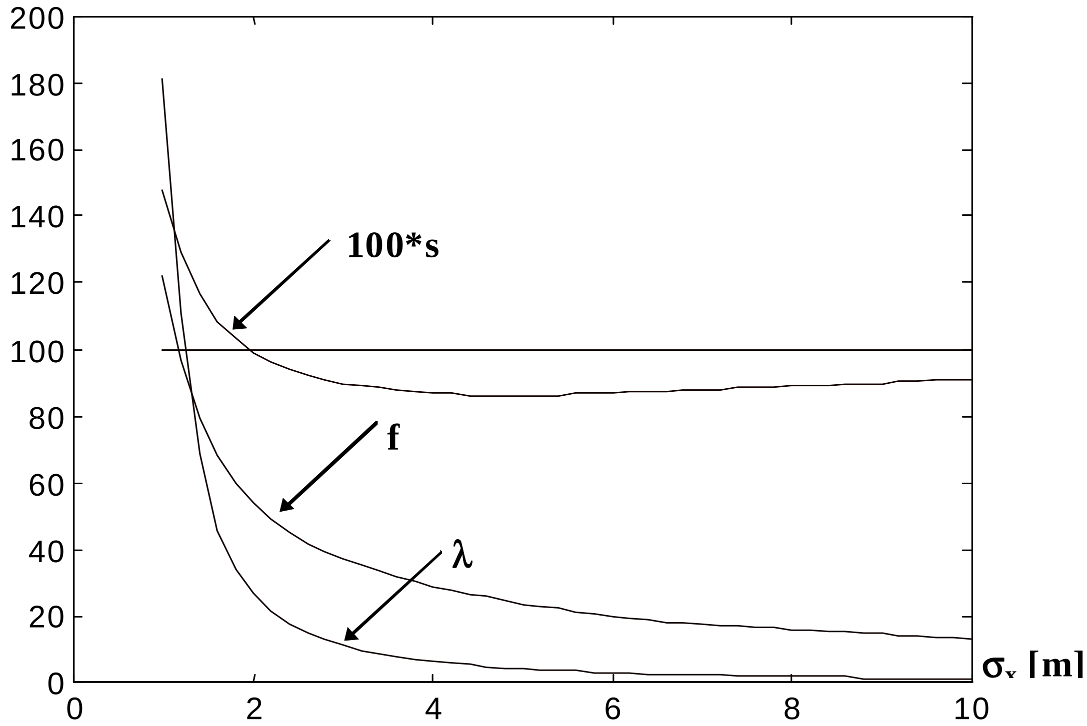

Fig. 5. Shadow price function

7 Conclusion

The main motivation for use Optimal Constrained Covariance Control Theory (OC3) is that many real control systems have performance requirements naturally stated in the terms of the root-mean-square (RMS) values. These requirements are usually given in the form of inequality constraints. The optimal control problem is characterized by compromises and tradeoffs, with performance requirements and magnitude of the input energy. For example, the objective of a dynamic positioning system is to maintain the position and heading of a vessel at reference values with acceptable accuracy. The design of the systems involves a compromise between the accuracy of holding a position and the need to suppress excessive thruster response.

When the sensitivity of solutions to desired system performances is of our concern, then it can be shown that under particular circumstances, a slight change of desired system performances could significantly improve the optimal solution value. Namely, a slight relaxation of desired system position accuracy could result with significant energy savings.

The proposed method provides a means for the analysis of desired performance, set by the designer, for the intelligent control system, according to the total cost (energy consumption), [24]. Sometimes it can be concluded that a slight relaxation of desired accuracy specifications (if technically sound) can result in significant total energy savings. Future research should investigate interdependence between parameters of shadow price and robustness of the nonlinear control system [25]. A preliminary analysis shows that some form of interdependence exists, because with significant growth of the shadow price, the robustness of the control system deteriorates.

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