A Design Method of Narrow Band FIR Filters Based on Fluency Sampling Function of Quadratic Piecewise Polynomial

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Abstract: -In this paper, we propose a design method of the narrow band FIR filter in a Fluency signal space. As a result, we can make superior narrow band FIR filters than ordinary one. While an FIR filter has advantages that it is stable and can realize perfect linear phase, its transfer function can be of higher order according to desired accuracy. One of such difficult classes that indeed requires lots of multipliers is the narrow band filter. In proposed method, it is possible to design a narrow band FIR filter with multipliers much less than Remez method, a typical conventional method. In the first step of the proposed method, a pair of mother filters that are basic for the desired filter are defined. In the next step, scaled filters of the mother filters will be taken. Finally, we take an appropriate cascade connection of the scaled filters and make it the approximation model of the desired characteristics of narrow band FIR filter. By a numerical experiment, the proposed method is shown to be supeior to Remez method, as stated above.

Key–Words: -Fluency theory, Signal space, Dual space, Sampling function, Frequency scaling, Cascade connection, Remez algorithm

1 Introduction

Due to the information digitization in recent years, the importance of the circuits that perform digital signal processing is increasing. The conventional mathematical models for signal processing are based on Whittaker-Someya-Shannon's sampling theorem in the typical Hilbert space L^2, where L^2 is square integrable fucntion space. However, this space is too small because even sin and cos functions, which are analytic functions, does not belong to this space. So, it is inconvenience to design and analyze a system.

On the other hand, when we consider the Schwartz class S of infinitely many times differentiable functions that decay faster than any inverse polynomials as a signal subspace of L^2, the dual space S' of S is bigger than L^2. However, the dual space is very tempered because the space is metric space, that is, inner product is not defined in the space. Also, in view of real signals, conditions of S are too stringent.

One of authors has been studied modeling of real signals based on the Fluency information theory [14][15][16][17]. In the theory, the degree of smoothness of signals is taken as one of the main characteristics of signals, and signal spaces are classified into subspaces of L^2 by the smoothness of signals. We denote the Fluency signal space as D_m, which is function space of $(m-2)$-times differentiable functions that decay like with the order of inverse of $(m-2)$-th order polynomials. The signal space is included in conventional signal space L^2 and contains S. Also, the dual space D'_m of D_m is bigger than L^2 and

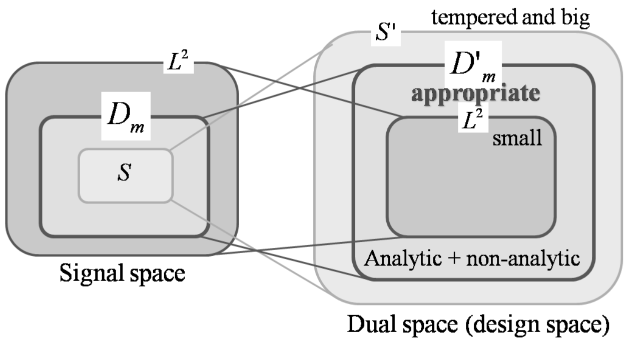


Figure 1: Signal spaces and dual spaces

smaller than S'. Therefore, D_m is appropriate signal space in view of both real signals and a system design, see Fig.1.

The conventional systems, for example, DA converter, filter, amplifier and so on, are designed based on the sinc function in L^2. They have many problems. In design of DA converters, it is impossible for conventional DA converters to reproduce the ultrasound components higher than about 22.05kHz from conventional CDs whose sampling rate are 44.1kHz because the analog signal reproduced from the digital signal of CD is strictly band-limited to frequency under 22.05kHz being based on the Shannon's sampling theorem. However, the DA converter designed by using a non-analytic sampling function of piecewise polynomial of degree 2 in D_3', named Fluency DAC, can reproduce the ultrasound components from the digital signal of CD. The Fluency Audio System implemented Fluency DAC has received many awards for audio systems and become the world standard. This is one of applications in D_3.

In this paper, we propose a design method of narrow band FIR filters with linear phase based on a sampling function in the dual space of the Fluency signal space D_3. As a result, we can make superior narrow band FIR filters than ordinary one.

Circuits that perform digital signal processing are implemented in the handheld units like cellular phone, audio player and PDA, as wel as in comparatively middle-scale systems such as PC and appliances. In those systems, digital filter is used in removing noise, adaptive signal processing and adjusting frequency components. Especially, FIR filters have a merit that it is stable for the bounded input. In addition, when the impulse response is symmetric, the FIR filter can perfectly realize the linear phase. The linear phase property is important in processing waveform, measuring and reproducing audio signal. However, there is a trade-off between the required frequency characteristics and circuit complexity [3][9]. In order to obtain a steeper characteristics, a higher order transfer function is necessary in general. For example, in the case of designing a band pass filter that has very narrow pass-band, the order of the transfer function become high, and a large number of multipliers are required according to the given accuracy. This fact causes a problem in dealing with multimedia information with mobile devices, that do not have sufficient computational power and memory. By using our proposed method, it is possible to design a narrow band FIR filter with smaller number of multipliers than conventional method.

This paper is organized as follows.

Section 2 reviews the Fluency information theory and give specifications of narrow band FIR filters.

In Section 3, we describe our design method. In the proposed method, we first introduce a pair of mother filters that generates other intermediate filters used in constructing the desired filter. Usually, when one wants to design a filter, the one is to consider the filter characteristics such as filter length and coefficients, the width of pass-band and attenuation level, and whether or not the ripple is permitted. Our requirements here are flatness of pass-band and locality of support. As a solution to satisfy the conditions, we introduce the mother filters from the sampling function of piecewise polynomial of degree 2 that is derived by authors' previous research [14]. We then take scaled filters of the mother filters, as components of the desired filters. The pass-band of each scaled filter is narrowed by scaling frequency axis. Finally, the desired narrow band filter is realized by adding cascade connection of the scaled filters until the specified characteristics are satisfied. Since the scaled filter includes several 0 coefficients, the total multipliers can be reduced without changing the order of the desired transfer function. Also, the scaled filters have zeros and poles according to the value of the scaling. Therefore, by connecting the scaled filters cascade, we can make the width of pass-band narrow and the level of stop-band low. We thus form the desired characteristics.

In order to make the filter description be such that it is presenting the frequency characteristics of the sampling function in definitions of mother filters, we consider formulating the scaled filters appearing in the cascade connection through Dirac's delta function operating on the continuous Fluency sampling function. By the delta function argument, it turns out that the frequency scaling of the mother filters corresponds to the time scaling of the original Fluency function.

In Section 4, as an example to demonstrate the usefulness of the proposed method, we design a band-pass filter with very narrow band. As a result, the Parks-McClleran (Remez) algorithm that is a typical design method requires 829 multipliers, while the proposed method 95, just 1/9 of the conventional. The proposed method thus can realize narrow-band FIR filters with small number of multipliers.

In Section 5, we conclude this paper with some discussions.

2 Specifications

In this section, we will reviews the Fluency theory and set the specifications of the objective narrow band filter.

2.1 The Fluency theory

The Fluency information theory has been proposed by one of authors as a theory of modeling of real signals. In the theory, the degree of smoothness of signals is taken as one of the main characteristics of signals. Let m be the degree of smoothness of signals with $(m-2)$ times continuous differentiability, the Fluency information theory classifies signal spaces into subspaces based on the variable m attributed to signals. When $m = 1$, the subspace coincides with the Walsh signal space. When $m \geq 2$, the subspace is function space of piecewise polynomials of degree $(m - 1)$ having only $(m - 2)$ times continuous differentiability. When $m = \infty$, the signal space is similar to the band-limited signal space. We denote signal spaces characterized by parameter m as D_m. The signal space D_m is spanned by a basis of sampling function of piecewise polynomial that has been derived by one of authors. Moreover, the sampling funciton is a functional of D_m, that is, a element of the dual space D'_m of D_m.

By selecting appropriate signal spaces D_m according to the object, the Fluency analysis enables us to deal with signals flexibly and precisely. In the field of high-end audio and image processing, our new approaches based on the Fluency analysis have already been highly-acclaimed.

In this paper, we select the signal space with $m = 3$ and design narrow band FIR filters in the dual space D'_3 of D_3.

2.2 Narrow band FIR filter

In designing filters, we design filters whose -3dB bandwidth is very narrow and pass-band is flatter, transition-band is steeper, stop-band is a large of attenuation. Moreover, filters are linear phase. In Section 4, as an example, we design a narrow band FIR filter by using concrete numerical values.

3 Design Method

In this section, we describe the method to design the narrow band filter with specification given in section 2. Section 3.1 presents the basic concept of the proposed designing method. In section 3.2, we define a pair of mother filters based on a Fluency sampling function, and study their properties. Section 3.3 discusses the scaling operation on the mother filters. Section 3.4 explains how the desired filter is obtained by cascade connection of the scaled filters, through the Fourier transform argument. Section 3.5 gives the procedure to realize the desired filter.

3.1 Basic Concept: the Class One Can Reduce the Multipliers

First, we introduce a pair of mother filters with low-pass or high-pass characteristics from a Fluency sampling function of degree 2 in [14].

The Fluency sampling function used here has a low-pass characteristic and is derived from a class of piecewise polynomials of degree 2 satisfying maximally flat pass-band without ripple, linear phase, and compact support in time domain.

Second, we consider scaling the mother filters in frequency domain. The scaled filters involve several zeros in their coefficients. Because of this, filters composed of such scaled filters require less multipliers than those with the same order constructed by direct method.

Also, a scaled filter has its pass-band and zeros according to scale values. Hence, one can make the pass-band narrow and stop-band small, by connecting several scaled filters cascade. Based on this idea, we design the narrow band filters.

Now we consider when multipliers are reducible by the frequency scaling. As an example, let us consider FIR filters given by

$$G(z) = \sum_{k=0}^{8} g_k \, z^{-k}, \tag{1}$$

and

$$A(z) \triangleq \sum_{k=0}^{3} a_k \, z^{-k}, \tag{2}$$

and let $G(z)$ be constructed as

$$\begin{aligned} G(z) &= A(z) \cdot A(z^3) \\ &= (a_0+a_1z^{-1}+a_2z^{-2}) \cdot (a_0+a_1z^{-3}+a_2z^{-6}) \end{aligned} \tag{3}$$

the cascade connection of $A(z)$ and its scaled one $A(z^3)$. Then, the multipliers in the right hand side of (3) is 6, while those in (1) is 9. Since the scaled filters thus contain several zero coefficients, the multipliers can be reduced by the cascade connection of the scaled filters.

When a desired transfer function $G(z)$ is given, with scaled filters $A(z^{k_n})$ (i.e. frequency $f \mapsto k_n f$) of a basic filter $A(z) = a_0 + \cdots + a_{\mu-1} z^{-(\mu-1)}$ of μ multipliers, by

$$G(z) = \prod_{n=1}^{N} A(z^{k_n}), \tag{4}$$

μN multipliers are necessary. We can write this down to the direct form as

$$G(z) = g_0 + g_1 z^{-1} + \cdots + g_d z^{-d},$$

where $d = (\mu - 1)\sum_{n=1}^{N} k_n$. Necessary number of multipliers to realize this is $1 + (\mu - 1)\sum k_n$ in general. Therefore, as a sufficient condition, if

$$\mu N \leq (\mu - 1)\sum_{n=1}^{N} k_n,$$

i.e.

$$\frac{1}{N}\sum_{n=1}^{N} k_n \geq \frac{\mu}{\mu - 1},$$

then the multipliers to construct the desired filter is less than those by direct form.

3.2 Definition of Mother Filters from a Fluency Sampling Function

In this subsection, we first present a sampling function of piecewise polynomial of degree 2, named Fluency sampling function, in the signal space D_3 and state its properties. We then define a pair of mothers filters based on the Fluency sampling function and study their properties.

The Fluency sampling function of degree 2 has been derived in [14]. It is a non-regular function having singular points at which it is only one time differentiable. The Fluency function is given by

$$\psi_0(t) = -\frac{h}{2}\phi\left(t + \frac{h}{2}\right) + 2h\phi(t) - \frac{h}{2}\phi\left(t - \frac{h}{2}\right), \tag{5}$$

where

$$\phi(t) \triangleq \int_{-\infty}^{\infty} \left\{\frac{\sin(\pi f h)}{\pi f h}\right\}^3 e^{j2\pi f t} df. \tag{6}$$

Here $h > 0$ is a real and is a sampling interval when one reconstruct signals using ψ_0. The Fluency sampling function ψ_0 is only one time differentiable at every $\frac{1}{2}h$ point. The Fluency function ψ_0 have the following properties as well:

$$\psi_0(t) = 0, \quad \text{for } |t| \geq 2h \tag{7}$$

$$\psi_0(t) = \psi_0(-t) \tag{8}$$

$$\psi_0(kh) = \delta_{k,0}, \quad k = 0, \pm 1, \cdots \tag{9}$$

Equations (7), (8) and (9) indicate that ψ_0 is compact support, even and has value zero at every sampling points other than origin, respectively. The symbol $\delta_{k,0}$ in (9) is the Kronecker's δ. Denoting the frequency characteristics of ψ_0 by Ψ_0, we can write

$$\Psi_0(f) = h\{2 - \cos(\pi f h)\}\left\{\frac{\sin(\pi f h)}{\pi f h}\right\}^3. \tag{10}$$

This Ψ_0 possesses maximal flatness of 3rd order and linear phase. The reason why we define our mother filter from this ψ_0 is as follows:

1. Since the function is compact support, the number of filter coefficients is finite.
2. Since the function is symmetric, the filter is a linear phase.
3. Since it has maximally flat characteristic, no ripple is caused in the pass band by connecting the filters defined from the function.

We define a pair of mother filters, an elementary component of the desired filter as follows:

Definition 1 (Mother Filters) *We define the filters $L_0(z)$ and $H_0(z)$ with coefficients given by $\psi_0(k \cdot h/2)$, for $k \cdot h/2 \subset supp(\psi_0) = [-2h, 2h]$:*

$$L_0(z) \triangleq \sum_{k=-3}^{3} a_k z^{-k} \tag{11}$$

and

$$H_0(z) \triangleq L_0(-z), \tag{12}$$

where

$$a_k = \psi\left(\frac{h}{2}k\right), \qquad \psi(t) = \frac{1}{2}\psi_0(t). \tag{13}$$

Let $L_0(f)$ and $H_0(f)$ denote the frequency characteristics of the mother filters $L_0(z)$ and $H_0(z)$, respectively. Since a_k is symmetric by (8), they can be written as

$$L_0(f) = e^{-j6\pi f\tau}\left(a_0 + 2\sum_{k=1}^{3} a_k \cos(2\pi k f \tau)\right), \tag{14}$$

$$H_0(f) = L_0(f + 0.5 f_s). \tag{15}$$

Here $\tau > 0$ correspnds to the sampling interval of the signal input to the filter and $f_s = 1/\tau$ is the sampling frequency. Since the coefficients of $L_0(f)$ and $H_0(f)$ are symmetric, the two filters are linear phase. Also, since every second coefficients of the filters from the central ($k = 0$) are 0, the two filters turns out to be half-band filters. Further, $L_0(f)$ and $H_0(f)$ have the maximal flatness property [2][12].

Lemma 2 (Maximal Flatness) *L_0 and H_0 have the maximal flatness of third order:*

$$L_0(z)|_{z=1} = H_0(z)|_{z=-1} = 1\ , \quad (16)$$

$$\left.\frac{d^k L_0(z)}{dz^k}\right|_{z=1} = \left.\frac{d^k H_0(z)}{dz^k}\right|_{z=-1} = 0 \quad (17)$$
$$(k = 1, 2, 3)\ ,$$

$$\left.\frac{d^n L_0(z)}{dz^n}\right|_{z=-1} = \left.\frac{d^n H_0(z)}{dz^n}\right|_{z=1} = 0 \quad (18)$$
$$(n = 0, 1, 2, 3).$$

Lemma 3 (Half-Band Filter Pair) *L_0 and H_0 are mutually a half-band filter pair, i.e.*

$$\left|L_0(f) + H_0(f)\right| \equiv 1, \qquad \textit{for all} \quad f \in [0, \tfrac{f_s}{2}].$$

Lemma 4 (Monotonicity) *L_0 is monotonically decreasing on $f \in [0, \frac{f_s}{2}]$.*

Proposition 5 (Estimation around $f = 0$ and $f = f_s/2$) *L_0 satisfies the following evaluations on the neighborhood of $f = f_s/2$ and $f = 0$, respectively*:

$$|L_0(f)| \ \le\ \frac{3}{16}\left(\frac{\pi(f - f_s/2)}{f_s/2}\right)^4, \qquad \textit{for} \left|f - \frac{f_s}{2}\right| \le \frac{\sqrt{3}\, f_s/2}{\pi} \quad (19)$$

and

$$|L_0(f)| \ \ge\ 1 - \frac{3}{16}\left(\frac{2\pi f}{f_s}\right)^4, \qquad \textit{for}\ |f| \le \frac{\sqrt{3}\, f_s/2}{\pi}\ . \quad (20)$$

3.3 Introducing scaled filters

Next, we define scaled filters based on mother filters $L_0(z)$ and $H_0(z)$ as follows:

Definition 6 (Scaled Filters) *We define scaled filters as follows:*

$$L_p(z) \ \triangleq\ L_0\left(z^{p+1}\right) \quad \textit{and} \quad (21)$$
$$H_q(z) \ \triangleq\ L_q(-z), \quad (22)$$

where p and q are non-negative intergers.

The frequency responses of $L_p(z)$ and $H_q(z)$ defined in Eqs. (21) and (22) can be described as follows:

$$L_p(f) = L_0((p+1)f) \quad \text{and} \quad (23)$$
$$H_q(f) = L_q(f + 0.5 f_s) \quad (24)$$

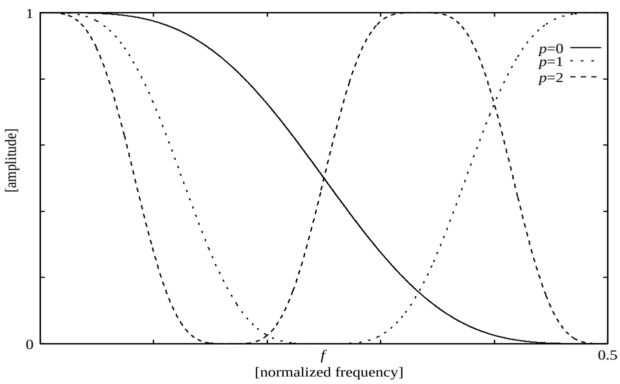


Figure 2: Frequency characteristics of $L_p(f)$

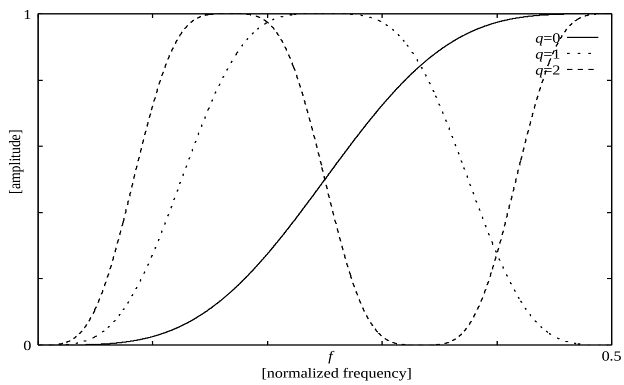


Figure 3: Frequency characteristics of $H_q(f)$

Figures 2 and 3 show examples of frequency responses of $L_p(z)$ and $H_q(z)$ for $p = 0, 1, 2$, respectively. From these figures, $L_p(z)$ and $H_q(z)$ are low-pass and high-pass filter, respectively.

Zeros and pass-band central points of $L_p(z)$ and $H_q(z)$ are as follows:

The center frequencies of the pass-band of $L_p(f)$ are $f_{c,k} = \frac{2k}{2(p+1)} f_s \quad (k = 0, 1, \cdots, \lfloor \frac{p+1}{2} \rfloor)$, and the zeros of $L_p(f)$ are $f_{z,l} = \frac{2l+1}{2(p+1)} f_s \quad (l = 0, 1, \cdots, \lfloor \frac{p}{2} \rfloor)$ on the interval $[0, \frac{f_s}{2}]$. On the other hand, the center frequencies of the pass-band of $H_p(f)$ are $f_{z,l}$ and the zeros of $H_p(f)$ are $f_{c,k}$:

$$L_p(f_{c,k}) = H_p(f_{z,l}) = 1 \quad (25)$$
$$L_p(f_{z,l}) = H_p(f_{c,k}) = 0 \quad (26)$$
$$k = 0, 1, \ \cdots\ , \left\lfloor \tfrac{p+1}{2} \right\rfloor; \quad l = 0, 1, \cdots, \left\lfloor \tfrac{p}{2} \right\rfloor$$

Here $\lfloor K \rfloor$ is the maximum integer which is smaller than or equal to K.

In this subsection, we have defined the scaled fliters necessary to design the desired filter and showed their properties.

3.4 Principle of the Narrowband Filter Construction

This subsection describes the principle of constructing the narrowband filter based on cascade connection of the scaled filters. We will clarify the relation between the characteristics of the digital filter as a discrete system and that of Fluency sampling function as a continuous system.

Let Δ_τ be the sequence of the impulses defined as

$$\Delta_\tau(t) \triangleq \sum_{k=-\infty}^{\infty} \delta(t - k\tau) \tag{27}$$

where $\delta(t)$ is the Dirac's delta function, τ is the interval between two successive impulses. The Fourier transform $\hat{\Delta}_\tau$ of Δ_τ is given by [13]

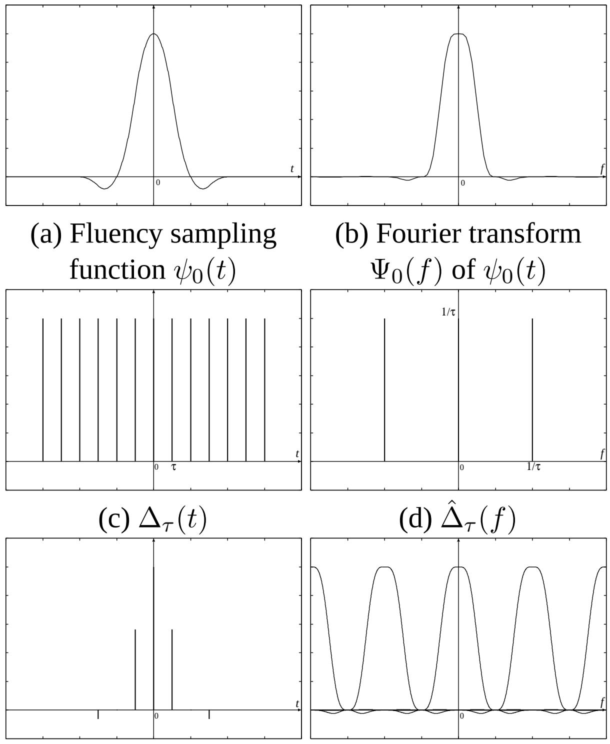
$$\hat{\Delta}_\tau(f) = \frac{1}{\tau}\sum_{k=-\infty}^{\infty} \delta\left(f - \frac{k}{\tau}\right) = \frac{1}{\tau}\Delta_{1/\tau}(f). \tag{28}$$

Using Δ_τ and the Fluency sampling function ψ_0, we can rewrite $L_0(f)$

$$\begin{aligned} L_0(f) &= \sum_{k=-3}^{3} a_k e^{-j2\pi f k\tau} \\ &= \int_{-\infty}^{\infty} \{\psi_0(t)\Delta_\tau(t)\}\, e^{-j2\pi f t} dt \\ &= \Psi_0 * \frac{1}{\tau}\Delta_{1/\tau}(f) \qquad (29) \\ &= \frac{1}{\tau}\sum_{k=-\infty}^{\infty} \Psi_0\left(f - \frac{k}{\tau}\right), \qquad (30) \end{aligned}$$

where Ψ_0 is the Fourier transform of ψ_0 and $\tau = h/2$. The meaning of Eq.(30) is illustrated in Fig.4. Figures 4(a) and 4(b) show the waveform of the Fluency sampling function and its Fourier transform, respectively. Figures 4(c) and 4(d) show the waveform of Δ_τ and its Fourier tansform, respectively. The sampled values of the Fluency sampling function can be represented by the product of Δ_τ and ψ_0. Therefore the Fourier transform of the sampled values is given by the convolution of $(1/\tau)\Delta_{1/\tau}$ and Ψ_0. As a result, the frequency characteristics of $L_0(f)$ are shown in Fig.4(f). Similarly, $L_p(f)$ can be written as

$$\begin{aligned} L_p(f) = L_0\left((p+1)f\right) &= \sum_{k=-3}^{3} a_k e^{-j2\pi(p+1)fk\tau} \\ &= \int_{-\infty}^{\infty} \left[\psi_0\left(\frac{t}{p+1}\right)\Delta_{(p+1)\tau}(t)\right] e^{-j2\pi f t} dt \\ &= \Psi_0((p+1)\,(\cdot)) * \frac{1}{\tau}\Delta_{\frac{1}{(p+1)\tau}}(f) \\ &= \frac{1}{\tau}\sum_{k=-\infty}^{\infty} \Psi_0\left((p+1)f - \frac{k}{\tau}\right) \qquad (31) \end{aligned}$$



(a) Fluency sampling function $\psi_0(t)$ (b) Fourier transform $\Psi_0(f)$ of $\psi_0(t)$

(c) $\Delta_\tau(t)$ (d) $\hat{\Delta}_\tau(f)$

(e) (a)×(c) (f) (b)∗(d)

Figure 4: Summary of Eq.(30)

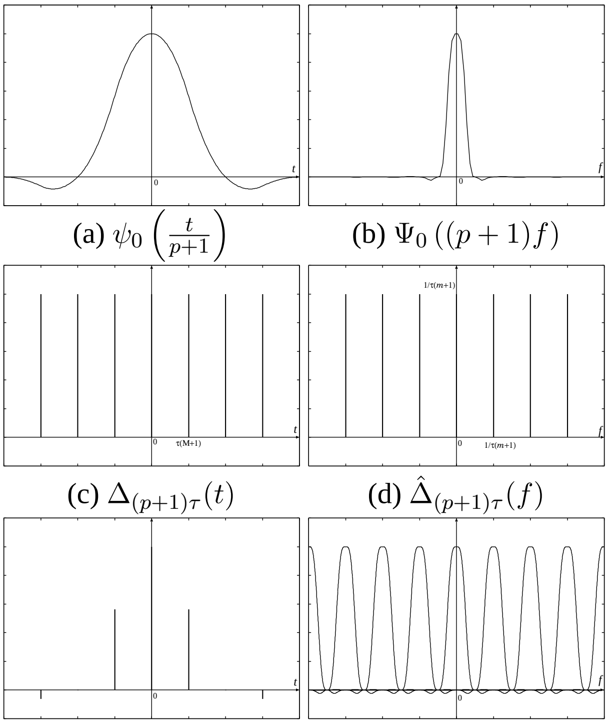
Figure 5 illustrates Eq.(31) for $p = 1$. In case of $p = 1$, while the time axis expands to $(p + 1)$ times, the frequency axis reduces $1/(p + 1)$. Eq.(31) gives the description of the digital filter $L_p(f)$ by using Fourier transform of the continuous Fluency sampling function.

Hence, we could confirm that proposed digital filters hold properties of the Fluency sampling function.

With this method, the narrowband filter can be constructed by cascade connection of the scaled filters and the mother filters. For example,

$$\begin{aligned} &G(f) \\ &= \prod_{p=0}^{P} L_p(f) = L_0(f)\cdot L_1(f)\cdots L_P(f) \\ &= \prod_{p=0}^{P}\left[\int_{-\infty}^{\infty}\left\{\psi_0\left(\frac{t}{p+1}\right)\Delta_{(p+1)\tau}(t)\right\} e^{-j2\pi f t}dt\right] \\ &= \prod_{p=0}^{P}\left[\frac{1}{\tau}\sum_{k=-\infty}^{\infty}\Psi_0\Big((p+1)f - \frac{k}{\tau}\Big)\right]. \end{aligned}$$

Figure 6 illustrates the performance of cascade connection of scaled filters. We see that the the more the value of p and the number of scaled filters to be connected be, the more bandwidth of the filter becomes narrow and the stop-band level attenuates.



(a) $\psi_0\left(\frac{t}{p+1}\right)$ (b) $\Psi_0\left((p+1)f\right)$

(c) $\Delta_{(p+1)\tau}(t)$ (d) $\hat{\Delta}_{(p+1)\tau}(f)$

(e) (a)×(c) (f) (b)∗(d)

Figure 5: Illustration of Eq.(31) ($p = 1$)

Based on the principle, the narrowband filter is constructed.

3.5 Estimation of Stop- and Pass-band Attenuation

In this subsection, we indicate that how we can estimate the attenuation in the process of adding the cascade connection of scaled filters. It is roughly estimated by using the evaluations in Proposition 5. We will explain it with the example of $|L_0(f)L_1(f)|$ for stop- and pass-band.

For stop-band, by (19), we have

$$|L_0(f)\cdot L_1(f)|$$

$$\leq \frac{3}{16}\left[\frac{\pi(f-\frac{f_s}{2})}{f_s/2}\right]^4\cdot\frac{3}{16}\left[\frac{\pi(2f-\frac{f_s}{2})}{f_s/2}\right]^4$$

$$= \left(\frac{3}{16}\right)^2\left(\frac{\pi}{f_s/2}\right)^8\left[2(f-\frac{f_s}{2})(f-\frac{1}{2}\cdot\frac{f_s}{2})\right]^4$$

$$\leq \frac{9}{16}\left(\frac{\pi}{4}\right)^8 = -21.78 \text{ [dB]},$$

on $[\frac{f_s}{4},\ \frac{f_s}{2}]$. Here we have used that $y=(f-\frac{f_s}{2})(f-\frac{1}{2}\cdot\frac{f_s}{2})$ takes its minimum $(\frac{1}{4}\,\frac{f_s}{2})^2$ at $f=\frac{1+\frac{1}{2}}{2}\,\frac{f_s}{2}=\frac{3}{4}\,\frac{f_s}{2}$, since the derivative of y is

$$y' = 2\Big(f-\frac{1+\frac{1}{2}}{2}\,\frac{f_s}{2}\Big)\ .$$

Since the attenuation on $[\frac{f_s}{4},\ \frac{f_s}{2}]$ is not adequate, one may take those L_p or H_q that place a zero around $f=\frac{3}{4}\cdot\frac{f_s}{2}$ (for example).

Similarly, $|L_1\cdot L_2|$ has its estimation of stop-band level on $[\frac{1}{3}\,\frac{f_s}{2},\ \frac{1}{2}\,\frac{f_s}{2}]$, as

$$|L_1(f)\cdot L_2(f)| \ \leq\ \left(\frac{1}{12}\right)^2\left(\frac{\pi}{4}\right)^8 = -60.0 \text{ [dB]}.$$

For pass-band, we have at $f=f_s/12$ for example,

$$\big|L_0(f)L_1(f)\big|$$

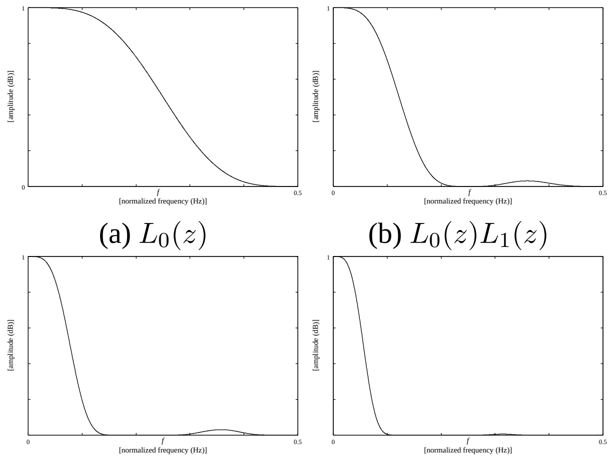
$$\geq \left[1-\frac{3}{16}\left\{\frac{\pi(f-\frac{f_s}{2})}{f_s/2}\right\}^4\right]\cdot\left[1-\frac{3}{16}\left\{\frac{\pi(2f-\frac{f_s}{2})}{f_s/2}\right\}^4\right]$$

$$= \left[1-\frac{3}{16}\Big(\frac{\pi}{6}\Big)^4\right]\cdot\left[1-\frac{3}{16}\Big(\frac{\pi}{3}\Big)^4\right]$$

$$= -2.34\text{[dB]},$$

by (20). In order to attenuate siderobes, we may wonder whether multiplying L_0 one more time, i.e. taking $L_0^2L_1$ is possible or not. It turns out possible by

$$\big|L_0^2(f)L_1(f)\big| \ \geq\ -2.47\text{[dB]}.$$



(a) $L_0(z)$ (b) $L_0(z)L_1(z)$

(c) $L_0(z)L_1(z)L_2(z)$ (d) $L_0(z)L_1(z)L_2(z)L_3(z)$

Figure 6: Connecting scaled filters

3.6 Design Procedure

This subsection explains the procedure for designing the narrowband filter. The target filter is the one that satisfies the specification given in Section 2, and it is designed by the following procedure. For simplicity, the sampling frequency is assumed to be unity below.

Since the proposed method is limited to the band-pass filter design, the class of scaled filters used in the cascade connection is predetermined. The center frequency for the targeted filter is $f_c = \frac{1}{4}$, and so must be the filters connected cascade.

The scaled filter satisfying this condition can be determined by Eq.(25). From equation (25) we have

$$\frac{2k}{2(p+1)} = f_c = \frac{1}{4},$$

and hence $p = 4k - 1$. Since $p, k \geq 0$ we have

$$p = 4k + 3, \qquad k = 0, 1, \cdots . \tag{32}$$

Similarly, we conclude that

$$q = 4l + 1, \qquad l = 0, 1, \cdots , \tag{33}$$

because $H_q(f)$ has a center frequency located at $f = \frac{2l+1}{2(q+1)}$. Therefore, in order to design the narrowband filter the following scaled filters may be used

$$L_{4k+3}(z), \qquad k = 0, 1, \ldots, \tag{34}$$

$$H_{4l+1}(z), \qquad l = 0, 1, \ldots. \tag{35}$$

In the following, we summarize the procedure for designing the narrowband filter.

step 1. Initialization
Select a scaled filter such that one of whose pass-band is closest to the target filter. If we denote the -3dB bandwidth of $L_I(z), H_I(z)$ by f_I, then

$$f_I = \frac{f_0}{I+1}. \tag{36}$$

According to Eqs.(34), (35) and (36), the initial construction $X_0(z)$ is given by $L_{4M+3}(z)$ or H_{4M+1}, where M is a natural number.

step 2. Filter Addition
The scaled filters determined by Eqs.(34) and (35) are connected cascade. If in Step 1 we initialize the filter by $X_0(z) = H_{4M+1}(z)$, then we set $P = Q = M - 1$; otherwise if $X_0(z) = L_{4M+3}(z)$, then we set $P = M - 1$, $Q = M$. Using $X_0(z)$ and parameters P, Q, we consider the filter

$$X(z) = X_0(z) \prod_{k=0}^{P} L_{4k+3}^{\alpha_k}(z) \prod_{l=0}^{Q} H_{4l+1}^{\beta_l}(z) \tag{37}$$

where α_k and β_l are the numbers of filters $L_{4k+3}(z)$ and $H_{4k+1}(z)$ respectively, used in cascade connection. It is assumed that $\alpha_k = 0$ and $\beta_l = 0$ for any k, l as initialization. Next, α_k and β_l are increased one by one until it satisfies the specification for X(z).

step 3. Computation of Frequency Spectrum
When the filter satisfying the specification is obtained by Step 2, the coefficient is quantized with the given number of bits. The frequency characteristics of the filter with resulting coefficient is computed. If the frequency spectrum satisfies the target specification, the algorithm terminates, otherwise, α_k and β_l are increased, and return to Step 2.

4 Comparison with a Conventional Method

In this section, we compare the proposed method with Parks-McClellan method (Remez algorithm), a typical conventional methods.

In designing filters, an adequate number of filter coefficients are required according to the accuracy of desired characteristics in general. Therefore, as the practical problem, the filter length increases significantly for the severe characteristics such as steep attenuation and narrow bandwidth. As an example, we design the following filter whose -3dB bandwidth is very narrow with a small number of multipliers compared with conventional methods.

(Specifications for the objective filter)

- Sampling frequency: 2.4[GHz]
- Central frequency: 600[MHz]
- -3dB Pass band: 592[MHz]–608[MHz]
- Stop band: 0[MHz]–584[MHz] and 616[MHz]–1.2[GHz]
- Stop band level: -80[dB]
- quantization bit number: 18[bit]
- Phase condition: linear phase

This sampling frequency 2.4[GHz] is set in relation to ISM (Industrial, Scientific and Medical) band. There are several frequency bands in ISM that are assigned to each of the applications. The 2.4[GHz] band is used for microwave, digital cordless phone, wireless LAN, Bluetooth, etc. Because the ISM band is offered free-license, the use of the band is increasing.

Table 1 shows the numbers of multipliers used in each filter. The ideal narrowband filter designed by Remez algorithm requires 1707 multipliers, and after quantization, 829 multipliers. In contrast, the filter designed using our method requires only 95 multipliers, $1/9$ of the conventional. By this, it turns out that our method can design a filter satisfying the specified very narrow band filter with much less multipliers than conventional algorithm.

Table 1: Number of multipliers

method	Ideal	Quantized
Conventional	1707	829
Proposed	95	95

Figures 7 and 8 show the frequency responses of the filters designed in accordance with the specification described above. Figure 7 shows the frequency response of the narrowband filter designed by Remez algorithm. The theoretical frequency response is shown in Fig.7(a). The frequency response of the filter with coefficients quantized to 18 bits is shown in Fig.7(b). Figure 8 shows the frequency response of the narrowband filter designed by proposed method, where (a) is ideal and (b) quantized to 18 bits. We can see that the quantized filter designed using the proposed method satisfies the specification required in section 2 with slight deficiency around the boundaries between stopbands and transition bands.

Table 2 shows the charasteristics of each filter. The characteristics are bandwidth, and ideal and quntized stop-band levels. It is clear that bandwidth of the filter designed by Remez algorithm fails below the specified bandwidth. Also, the quantization makes the characteristics much poor, exceeding the stop band level, -80dB, at several frequencies. On the other hand, our method satisfies the specification of bandwidth, and the stop-band level is kept under -80dB.

5 Conclusion

In this paper, we proposed a design method of narrow band FIR filters with linear phase in the dual space of the Fluency signal space D_3. As a result, we could make superior narrow band FIR filters than ordinary one.

Our proposed method is described as follows: First we defined low-pass and high-pass mother filters from quadratic piecewise polynomial Fluency sampling function with maximally flat pass-band characteristic and local support in the Fluency signal space D_3. These mother filters inherit maximally flat pass-band characteristic and linear phase property. Next we introduced scaled filters from the mother filters by scaling. Then we obtained the narrowband filter by cascade connection of the scaled filters.

We show efficiency of our proposed method with the example of designing a very narrowband filter; the filter requires only 95 multipliers, while the filter designed by Remez algorithm requires 829 multipliers.

We are applying the narrowband filter to sound quality improvement of mobile terminal music players. We will report the achivement in further papers.

Acknowlegement

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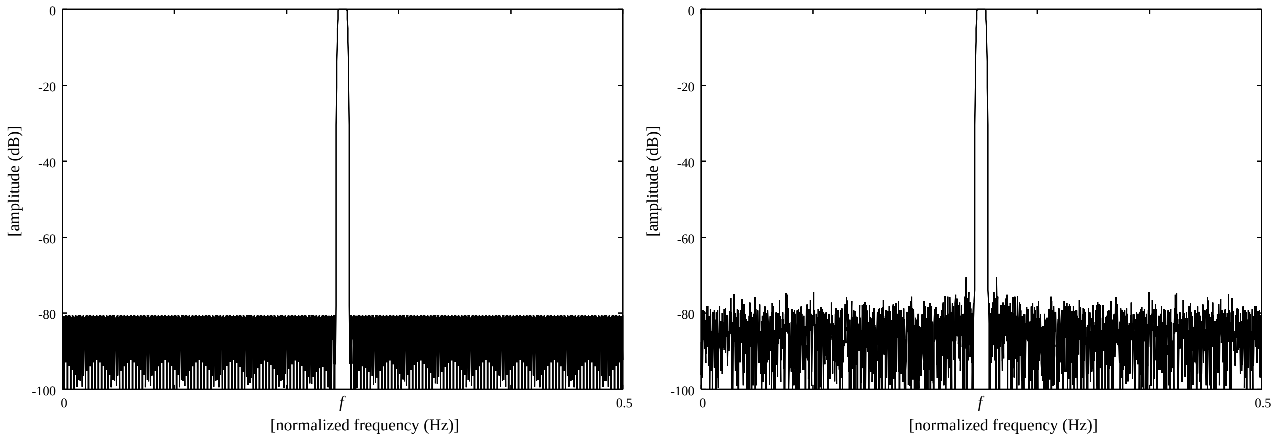
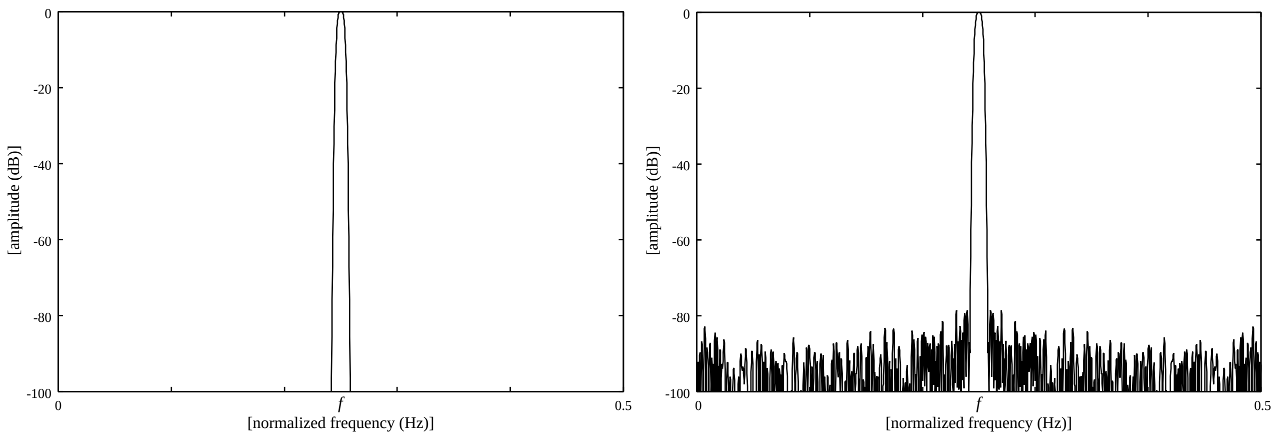


(a) Ideal (b) Quantized to 18 bit

Figure 7: Frequency characteristics of the filter designed by Remez method



(a) Ideal (b) Quantized to 18 bit

Figure 8: Frequency characteristics of the filter designed by proposed method

Table 2: Filter Characteristics by Conventional and Proposed Methods

	-3dB bandwidth	stopband (ideal)	stopband (quantized)
Conventional	9.72MHz	under -80dB	around -80dB
Proposed	7.44MHz	under -100dB	under -80dB

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