Modeling and Control of Die-sinking EDM

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Abstract: - This paper explores the mathematical model for electrically discharged machining (EDM) machine. Conventionally, it is known that the machining efficiency of EDM is closed related to the so-called ignition delay time. Here, it can be observed that the ignition delay time is the RC time-constant of the circuit model of Die-sinking EDM. Then, it can be shown that ignition delay time is closed related to the gap resistance. According to electromagnetic theory, it can be shown that in the first and middle stages of machining process, the overall control system becomes a nonlinear Lur'e system. As for the final stage of machining process, the overall control system becomes a linear system.

For the Lur'e problem of Die-sinking EDM, we applied the SPR (Strict Positive Real) condition for the control of EDM. A differential-PID controller is proposed in the beginning and middle stage of machining process. In the final stage of machining process, A PID controller can applied to the final stage of machining process. An adaptive controller switch from Differential-PID (DPID) controller and PID controller is proposed in this paper. Experimental results will be verified. Especially, the machining efficiency can be enhanced up to 33% compared to the conventional optimal PID controller.

Key-words: Nonlinear Control System, Lur'e Problem, EDM, Differential-PID Controller

1. Introduction

Electro-discharge machining (EDM) is the machining method that voltage is applied through a dielectric medium between the tool electrode and work-piece. This method has a lot of engineering applications [1-5]. This mechanism can generate the electro-discharge phenomenon as the electrode and work-piece is positioned close enough and then, a section of the work-piece is melted and removed by the exothermic reaction accompanying by the electro-discharge. Machining is carried out by repeated melting and removal process; therefore, it is necessary to use a pulsating electrical discharge.

High precision, high quality machining is very important to the modern industry. The advantages of EDM can be described as follows: First, EDM can process any material that has electric conductivity. It can easily machine materials such as quenched steel and carbide mainly used for tools and difficult to machine by using cutting tools. Second, it is a non-contact machining method. Maintaining a gap between the electrode and the work-piece in order to ensure electro-discharge between them; therefore, machining can be done without applying pressure on the material. The, high precision machining can be done on curved surfaces, inclined surfaces and thin sheet materials that are difficult to drill. Third, it can achieve very high quality machining. It roughness of the machined surface Rmax can be achieved up to 0.1μm; However, the disadvantage of EDM is the speed of EDM machine is very slow. Therefore, there are lot of researchers propose many methods to increase the efficiency of EDM [5-15][26]. Some researcher already has shown that the efficiency is closed related to "Ignition Delay Time"[6][7]. To increase the efficiency of EDM, some researchers propose, the adaptive, PID, Fuzzy controllers respectively [5][6][9-13]. To increase the efficiency and protections, some researchers study different converter for EDM [14]. Also, some researchers propose AC power supply to the electrode and working pieces rather than the conventional DC power supply [16].

In this paper, a mathematical model for the EDM process is illustrated by electro-magnetic and circuit theory. This model considers the resistance and capacitance for the electrode gap by electromagnetic theory. It can be observed that the "Ignition Delay Time" is closed related to RC time constant for the circuit of EDM machine. It can be observed that as the electrode and work-piece are parallel and perfect flat surface, then the overall control system becomes a linear system. However, if the electrode and work-piece are not perfect parallel plate, the overall control system becomes a nonlinear Lur'e problem. It is well known that for a Lur'e problem, Circle criterion and Popov criterion can be applied to determine the stability of the control systems [14-17]. Also, it can be observed that the necessary condition for the loop transfer function in Lur'e problem to be SPR is that the relative degree of the linear transfer function must be less equal than one and no double poles on the imaginary axis. With this approach, a differential PID controller is proposed. The differential-PID (DPID) controller is a PID controller cascaded with a differential controller.

In the middle stage of machining process, according to electromagnetic theory, it can be also shown that the overall control system becomes a nonlinear system where the nonlinear element is in the sector $(0,1]$. According to the Circle Criterion, a suitable D-PID controller can stabilize the overall control system.

As for the final stage of machining process, according to electromagnetic theory, it can be also shown that the overall control system becomes a purely linear system, a suitable PID controller can ensured the overall system is globally asymptotically stable. Therefore, an adaptive controller switch from Differential-PID controller and PID controller is proposed in this paper. Experimental results for the EDM machine verify the results.

2 Ignition Delay Time

As shown in Fig.1, an EDM machine consists of a movable machining head in which the tool electrode is attached, a section containing a mechanism to position the material to be processed. Moreover, it also contains an electrical discharge circuit and an NC servo control circuit.

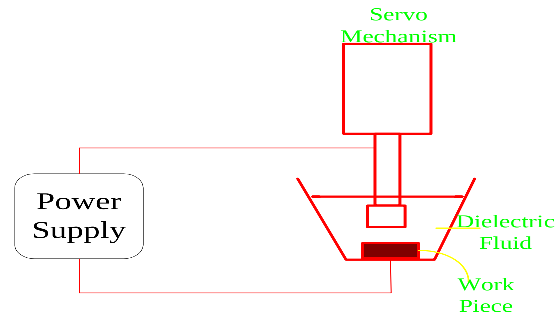

Fig. 1. The Feedback Loop for EDM

The control system of an EDM machine can be shown as Fig.1 [1,2,15]. In Fig.1, a DC power supply supplies the voltage V_{dc} is around 80-120 V. In addition, a servomechanism controls the position of electrode. As the gap between the electrode and work piece below certain distance (about 10-90 um), the electro-discharge will happen. In the meantime, as seen in Fig.2, the gap voltage V_g initially maintained the DC voltage V_{dc}, then at the time t_D, the gap voltage V_g drops to a very low voltage (in general 25 V(20-30V depend on the materials of electrode and work piece)). Here, the time, t_D, as seen in Fig.2, is called ignition delay time. The servo controller tries to keep the ignition delay time t_D around certain value 20~120 μs. If the ignition delay time t_D can be kept as a constant means the electro-discharge is normal and happened, which can be shown in Fig.2; otherwise, it is abnormal such that the EDM is in "open" or "short" condition.

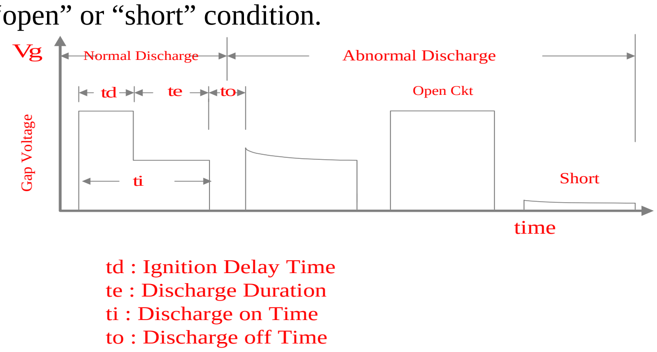

Fig. 2.Gap Voltage Waveform for EDM

As seen in Fig.2, if the ignition delay time t_D is too long, shown in Fig.2, this means the circuit is "open" and if the ignition delay time is too short, this means the circuit is a "short" circuit. Both cases are abnormal and electro-discharge is not happened. Therefore, it is very important keep the ignition delay time t_D **to be** a constant. The error of the ignition delay time should be kept as small as possible. When the discharge is happened, the time t_E as seen in Fig.2 and Fig.3, is called "Discharge Duration", which is the time duration such that the discharge is happened. In general, the discharge duration t_E is closed related the roughness of the work-piece can be controlled by the hardware such as a timer. As the discharge happens, it continues the process until "discharge duration time" t_E and then disconnected the power supply. This stops of the discharge. t_o (Discharge off Time) is the time that EDM disconnected by the power supply. Discharge duration time t_E and Discharge off Time t_o can be controlled by the hardware such as timer. To maintain same quality of roughness for work-piece, the "discharge duration time" t_E should kept as a constant, which is called the equal energy control as seen in Fig.3 [1,2, 15]. However, the most important time index for EDM is the ignition delay time t_D since it determines whether electro-discharge is happened or not**.** If this time t_D is kept as a constant, then the efficiency can be enhanced.

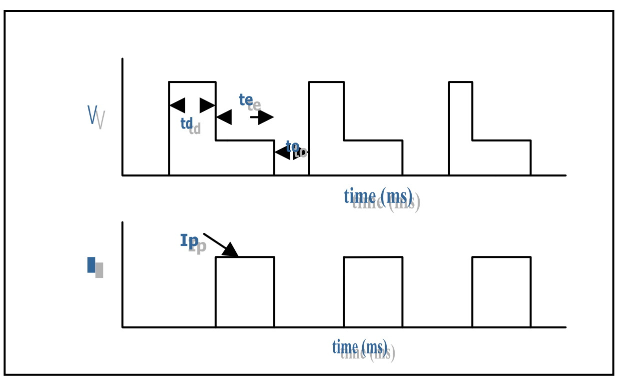

Fig. 3. The Current and Gap Voltage Waveform for Equal Energy Discharge

The duty factor D is defined as follows:

$$D = \frac{t_E}{t_D + t_E + t_o} \quad (1)$$

The meaning of duty factor D is the average percentage of overall time such that the discharge is happened.

Also, the discharged current I_P is shown in Fig.3, the average current I has the relation with the discharged current I_P

$$I = DI_P \quad (2)$$

Considering Cu (copper electrode) in the anode (+) , working piece is steel (in the cathode) and according to the formula[3][4], the cutting speed (metal removal rate) W can be written as

$$W = 0.097 \cdot I_p^{1.5} \cdot D \quad (3)$$

, the surface roughness $R_{\max}$ can be expressed by

$$R_{\max} = 1.6 \cdot I_p^{0.43} \cdot t_E^{0.38} \quad (4)$$

the electrode wear ε can be expressed as

$$\varepsilon \approx 1.5 \cdot \frac{I_p^{1.74}}{t_E^{1.35}} \quad (5)$$

In addition, considering the other cases[3][4], the cutting speed W and surface roughness $R_{\max}$ have very similar formula of Eq.(3)-Eq.(5).

3 The Circuit Model for EDM

The circuit model for EDM can be shown in Fig.4(a). In Fig.4(a), the capacitor C is the filter's capacitor for the rectifier of the power supply respectively.

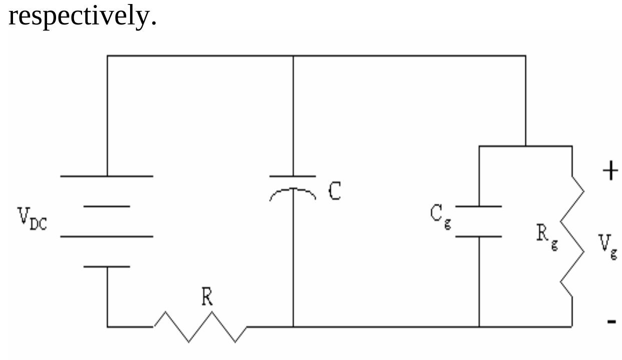

Fig. 4(a). The Equivalent CKT for EDM

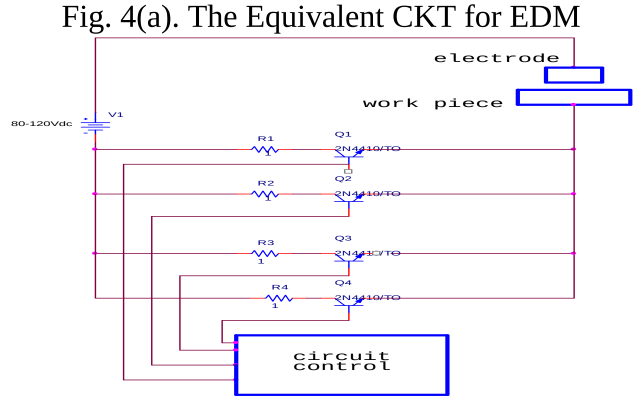

Fig. 4(b). Transistor Controlled EDM

To reduce the voltage ripple or harmonics, the filter's capacitor of DC power supply, C, in general, is large enough. (note that some EDM machine add a additional capacitor in parallel with the electrode and work-piece) Also, in Fig.4 and Fig.5, C_g and R_g are denoted as the gap capacitance and resistance between the electrode and work-piece. As the electro-discharge (ignition) is happened, the gap voltage V_g is around $R_g/(R_g+R)V_{dc}$, as shown in Fig.2 and Fig.6. In literature [1, 2, 14], as the discharge is happened, the gap voltage V_g is dropped around 28 V(In EDM machine, V_g is about 20-30 V and depends on the materials of electrode and work-piece) .

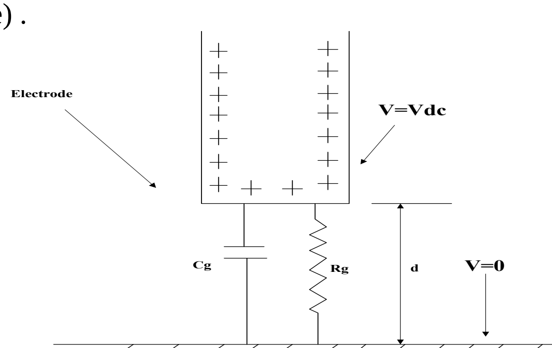

Fig. 5. The Capacitance and Resistance of Gap Electrode for EDM

Also, the discharge current I_P can be expressed by

$$I_P = \frac{V_{DC} - V_g}{R} \tag{6}$$

In Eq.(6), it can be observed that the discharge current I_P can be controlled by the resistance R. Then, there are EDM machine, as shown in Fig. 4(b), the resistors $R_1, R_2, R_3 \bullet\bullet\bullet R_n$ are in series with power transistors (For example : IGBT, Power BJT or Power MOS) . The switch ("on and off") of the power transistors can control the discharge current I_P

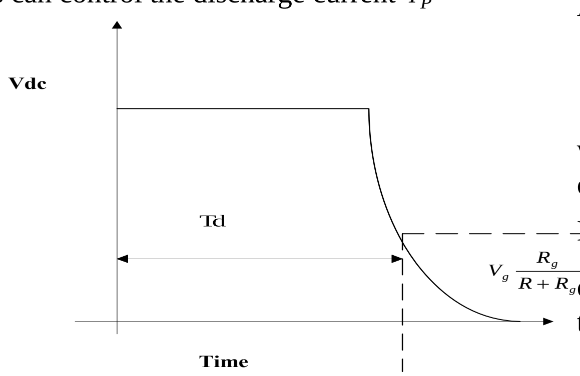

Fig. 6. The Ignition Delay Time for EDM

Moreover, when the discharge is happened, the discharged duration t_E can be easily controlled by the hardware. From Eq.(3), it can be observed that the cutting speed W is closed related to duty ratio D *and* Discharge current I_P . As the electro-discharge phenomenon (ignition) is stably happened, t_o (Discharge off Time) is the time of EDM disconnected by the DC power supply and easily controlled by the hardware such as a timer. If the electro-discharge(ignition) cannot be stably happened, from Eq.(3), Discharge off Time t_o is increased**.** **Then** the cutting speed W will be decreased very seriously**.** Therefore to improve the cutting speed, it is very important for the EDM machines that the servo control system try to stabilize the phenomenon of electro-discharge.

In this sequel, we try to explore the overall servo-control model for electro-discharge in EDM. We find that if the electrode and work-piece are two parallel plates, the overall control system is a purely linear system. (For die-sinking EDM, this case is adapt for final furnishing). In general, for die-sinking EDM, the overall control system is not a linear system. Let's state this as follows:

According to the circuit theory, the ignition delay time t_D

$$t_d \cong 2.2\tau \cong 2.2R_g(C_g + C) \tag{7}$$

where t_D is the ignition delay time; C_g and R_g are some nonlinear function of the gap distance d . Note that the capacitor C also includes the capacitor of rectifier of DC power supply.
Also, according to electro-magnetic theory, we have

$$R_g C_g = \frac{\varepsilon}{\sigma} \cong 0 \tag{8}$$

where ε is the permeability and σ is the resistance coefficient of the dielectric liquid oil respectively. Note, in general, for dielectric fluid $\frac{\varepsilon}{\sigma} \cong 1\ ns$ (when compared to ignition delay time $\approx 100\,us$), this term can be neglected.

If the capacitance C is large enough, from Eq.(7) we have

$$t_d = 2.2R_gC + 2.2\frac{\varepsilon}{\sigma} \approx 2.2R_gC \tag{9}$$

From Eq.(9), it can be observed that the ignition delay time t_D is determined by the gap resistance R_g. The gap resistance R_g is some nonlinear function of the gap distance d Then, let's consider the gap resistance R_g of the following cases:

3.1Parallel Plate

If the shape of work-piece and electrode is assumed perfect parallel plate, as seen in Fig.7, then we have

$$R_g = \frac{d}{\sigma A}, C_g = \frac{\varepsilon A}{d} \tag{10}$$

where A is area for the parallel plate , d is the gap distance and C_g is the gap capactance.

From Eq.(10) and Eq.(9), it can be observed that the ignition delay time t_D is a linear function of the gap distance d .

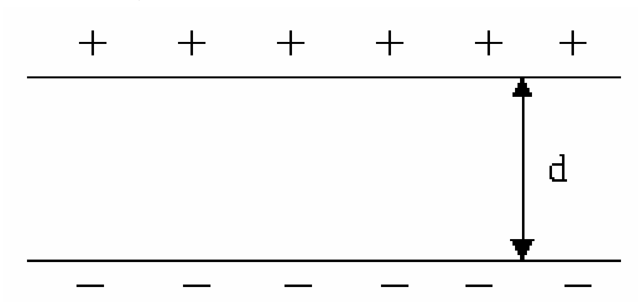

Fig. 7. Infinite Parallel Plate

3.2 General Cases of Electrode and Work-piece:

For the general cases of electrode and work-piece as seen in Fig.5, the gap resistance R_g can be obtained by solving the following Laplace equation

$$\nabla^2 V = 0 \tag{11}$$

where $\boldsymbol{V}$ is the voltage potential.

The boundary conditions can be described as follows:

$$\frac{\partial E}{\partial n} = \frac{\rho_s}{\sigma} \tag{12}$$

where ρ_s is the surface charge density and $E = \nabla V$. In addition, at the surface boundary of electrode, $V = V_{dc}$.

The total charge Q can be written as

$$Q = \iint_S \rho_s ds$$

where S is the surface of electrode or infinite working piece.
Then, we have

$$C_g = \frac{Q}{V_{dc}} \tag{13}$$

where C_g is the gap capacitance.

The gap resistance R_g can be obtained by the following relation [22]

$$R_g C_g = \frac{\varepsilon}{\sigma} \tag{14}$$

The calculations of R_g in Eq.(14) is very lengthy, which is omitted here. Some relevant results can be found in the literatures [23][24].

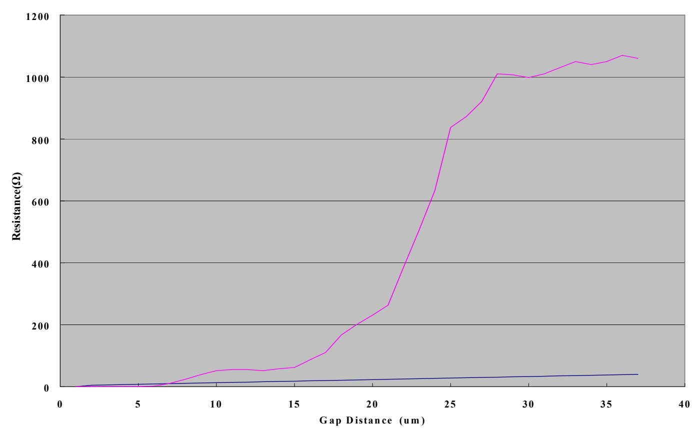

Fig. 8.The Actual Measurement for the Gap Resistance of EDM

Fig.8 shows the actual measurement of the gap resistance R_g for EDM. As seen in Fig.8, it can be observed that as the gap distance varies from 0 to 40 um, the gap resistance becomes from 0 Ω to 1.2k Ω and suddenly increases to 1k as the gap distance is around 25 um. From Fig.8, it can be observed that the gap resistance R_g is some nonlinear function of gap distance Expressed by

$$R_g = R(d) \tag{15}$$

which is in the sector $\left(0, \frac{\pi}{2}\right)$, where d is the gap distance, which is in the beginning of machining process for the die-sinking EDM as seen in Fig. 9(a).

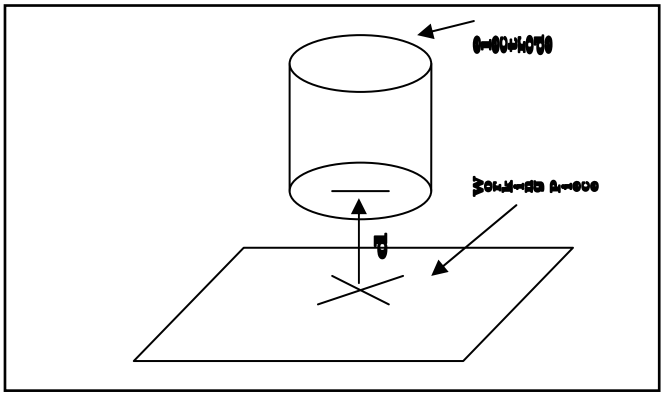

Fig.9(a) The beginning of machining process of die-sinking EDM

4 The Control System for EDM

In this section, we will address the control system of the EDM process. It can be shown that the overall control system can be represented by a transformed Lur'e problem. As shown in Fig.9(b), the control output is the ignition delay time t_D . In general, the ignition delay time is selected as **20~100μs.** Fig.10 shows the block diagram of the servo control of the ignition delay time t_D for EDM machine. From Eq.(10), it can be observed that if the work-piece and electrode are perfect parallel plate, the servo control system becomes a perfect linear system. However, if the work-piece and electrode is the general case of Fig.9(a), the servo system becomes a nonlinear Lur'e problem shown in Fig.10. In the Lur'e problem, the nonlinear function is due to the gap resistance R_g , which is some nonlinear function of the gap distance d. From Eq.(3), it can be observed that the ignition delay time t_D is closed related to the gap resistance.

In the sequeal, let's consider the general case of electrode and working piece and let's state as follows:

4.1 At the beginning of machining process of die sinking EDM

At the beginning of machining process of die-sinking EDM, the gap resistance R_g is shown in Eq.(15), which in general is a nonlinear function in the sector $\left(0, \frac{\pi}{2}\right)$. Also, in Fig.10, the plant transfer function $G_p(s)$ represents the transfer function of servo motor.

If the servo motor is a DC motor, then the plant transfer function $G_p(s)$ becomes

$$G_p(s) = \frac{k}{s(s\tau_e + 1)(s\tau_m + 1)} \qquad \textbf{(16)}$$

where τ_e , τ_m are electrical time constant and mechanical time constant respectively and k is the torque constant for the DC servo motor.

It is known that for a Lur'e problem, the stability can be determined by Popov and Circle criterions. On the other hand, if linear transfer function is SPR (Strict Positive Real) then the overall system is globally asymptotically stable. To ensure the loop transfer function $G(s) = G_c(s)G_p(s)$ to be SPR, where $G_c(s)$ is the controller, a necessary condition is that the relative degree should be less than 1. However, the relative degree of plant transfer function in Eq.16 is *3*. This relative degree's requirement of overall transfer function requires the controller's transfer function $G_c(s) = N(s)/D(s)$ should own relative degree 2; i.e.

$$\partial N(s) - \partial D(s) \geq 2 \qquad (17)$$

In addition, with conventional PID controller, the loop transfer function becomes

$$G(s) = \frac{k(k_I + k_p s + k_d s^2)}{s^2(s\tau_e + 1)(s\tau_m + 1)} \qquad (18)$$

where k_p, k_I, k_d are P, I, D coefficients for PID controller.

In Eq.(18), it can be observed that the loop transfer function $G(s)$ cannot to be SPR, for all $k_p, k_I, k_D \in R$ since $G(s)$ in Eq.(18) has double roots of zero, which is on the imaginary axis [17].

To satisfy the requirements of relative degree of Eq.(16) and the elimination of double roots of zero, a differential-PID controller is proposed in this paper. The differential-PID(DPID) controller is a PID controller cascaded with the differential controller can be described as follows:

$$G_c(s) = s(k_p + \frac{k_I}{s} + k_d s) \qquad (19)$$

Not that the controller in Eq.(19), which not only makes the relative degree of loop transfer function $G(s)$ become one but it also eliminates the double roots of zero.

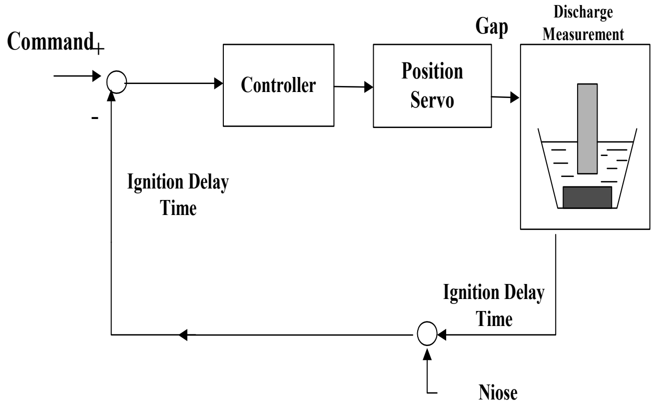

Fig. 9(b) The Block Diagram for EDM

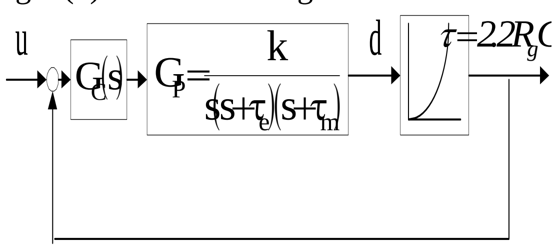

Fig.10. The Overall Controlled System for EDM (A Transformed Lur'e Problem)

Then we have the following results:
To ensure SPR condition of loop transfer function $G(s)$, $\mathrm{Re}(G(j\omega)) > 0, \forall \omega$, the PID coefficients of the proposed differential-PID controller must satisfy

$$k_d > \frac{\tau_e \tau_m}{\tau_e + \tau_m} k_p ,\ k_p > (\tau_e + \tau_m) k_I \qquad (20)$$

The derivation of Eq.(19) is listed in **Appendix A**.
Note that in general $\tau_m >> \tau_e$, Eq.(19) can be approximately rewritten as

$$k_d > \tau_e k_p ,\ k_p > \tau_m k_I \qquad (21)$$

On the other hand, some drivers of DC motor are designed to have a feedback-loop inside the current loop. This type of DC servo motor is called torque-mode driver. For torque-mode DC motor drivers, similar conclusions can be drawn. The conclusion is that a PID controller cannot make the loop transfer function $G(s) = G_c(s) G_p(s)$ to be **SPR.** Also, the proposed differential-PID (DPID) controller can make the loop transfer function SPR. The derivation of this conclusion is shown in **Appendix A**.

4.2 In the middle stage of machining process of die sinking EDM

Consider that in the middle stage of machining process of die sinking EDM of Fig.9(b) and let's assume that the gap distance **d** and gap lateral distance d_1 are small enough when compare the size of electrode, as seen in Fig.10 such that the surface of electrode and working pieces can be approximately as a flat surface. .

Then, from Eq.(10), the gap capacitance C_g can be expressed by

$$C_g = \frac{\varepsilon}{A_l d_1} + \frac{\varepsilon}{A_b d} \qquad (22)$$

where A_l is the area of later surface and A_b is the area of bottom surface for the electrode respectively. In addition, d_1 is the lateral distance and d is the gap distance of bottom surface as seen in Fig.9(b). Eq(22) consider two capacitors are in parallel. The first term of the gap capacitance C_g is due to the lateral surface and the second term is due to the bottom surface.

From Eq.(21) and Eq.(14), we have the gap resistance can be expressed by

$$R_g = \beta \frac{d}{\lambda + d} \qquad (23)$$

where $\beta = \dfrac{A_1 d_1}{\sigma}$ and $\lambda = \dfrac{A_1 d_1}{A_b}$ are constant.

In Eq.(22), it can be observed that from Eq.(9) and Eq.(23), the overall control system is still a nonlinear system, however, the nonlinear element $R_g = R(d)$ in Eq.(23) is in the sector $(0, \beta)$ also, it can be observed that with Circle Criterion, suitable choice of PID controller of Eq.(18) can stabilize the overall control system.

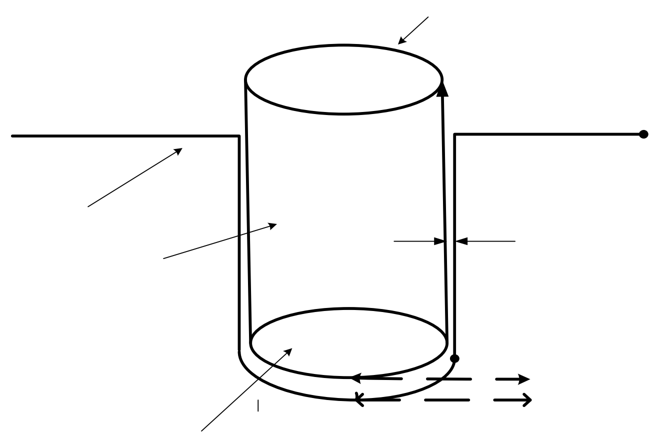

Fig.11 In the middle and final stage of machining process of die-sinking EDM

4.3 In the final stage of machining process of die sinking EDM

Consider the final stage of machining process of die-sinking EDM, the gap distance d is small enough when compare to λ; then we have

$$R_g \approx \frac{\beta d}{\lambda} \tag{24}$$

From Eq.(24), it can be observed that gap resistance R_g in Eq.(24) is a linear function for gap distance $\boldsymbol{d}$; Then form Eq.(9) and Eq.(24), the overall system is a linear system. Similarly, suitable chose of PID controller of Eq.(18) can make the overall control system globally asymptotically stable.

5 Experiment Results

In this section, we will investigate the experimental results to verify the theoretical predictions for EDM. In the experiment, the first controller is a PID controller with a torque-mode DC motor drive; however, the coefficients k_p, k_I, k_D already has been optimized. Secondly, the proposed controller is an adaptive controller which switchs from differential-PID(DPID) controller to PID controller. Also, the coefficients Kp, Ki, Kd are also selected to satisfy Eq.29 and optimized. Then according to Bilinear Transformation $s = \frac{2}{T}\frac{z-1}{z+1}$, we can have the digital controller where the sampling time T=2 ms.

The experimental results of Ignition Delay Time are shown in the following Table where the controlled Ignition Delay Time is set up to 100 us. In Table 1, it can be observed that the servo error of ignition delay time already has been reduced. For the original controller, the average error for five-time experiments is about 68.07 us. Also, for the proposed adaptive controller, the average error for five-time experiments is about 22.05 us. The controlled results are shown in Fig.11, Fig.12 respectively.

Experiment Times	1	2	3	4	5	Average Error of t_D **(us)**
Optimal PID Controller	68.73	59.04	77.46	54.99	77.1	68.07
Differential PID Controller	23.94	21.72	29.53	11.4	23.67	22.05

Table 1: The control error of Ignition Delay Time **Td**

Fig.11 and 12 show the real-time control of the error of ignition delay time for the optimal PID (with Gain scheduling) controller and the proposed differential-PID controller respectively.

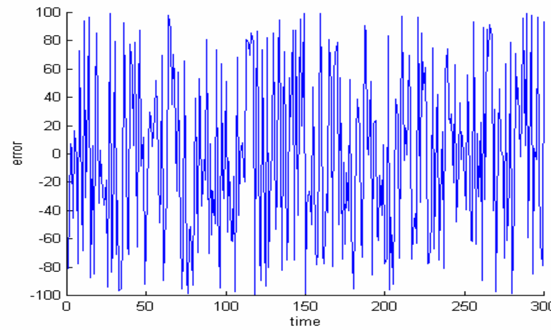

Fig. 12. The Control Error of Ignition Delayed Time for Optimal PID Controller --(time unit 2ms)

Fig.13 shows the efficiency of EDM for the original controller and the proposed controller. It can be observed that the efficiency already has been increased 33% for the first stage. For machining the depth of 12.7 mm, the optimized PID controller is about 60 min. However, for the proposed controller, it takes about 41 min. It is really increase the efficiency at the beginning of machining process of die sinking EDM. As to the middle and final stages of machining process of die sinking EDM. The controller becomes the conventional PID controller. Fig.14 is the photo of EDM.

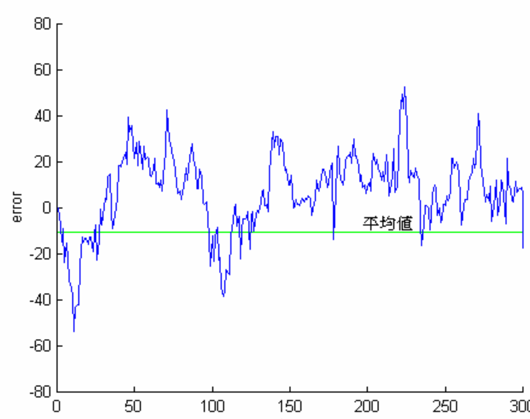

Fig. 13. The Control Error of Ignition Delayed

Time for Proposed Adaptive Controller --(time unit 2ms)

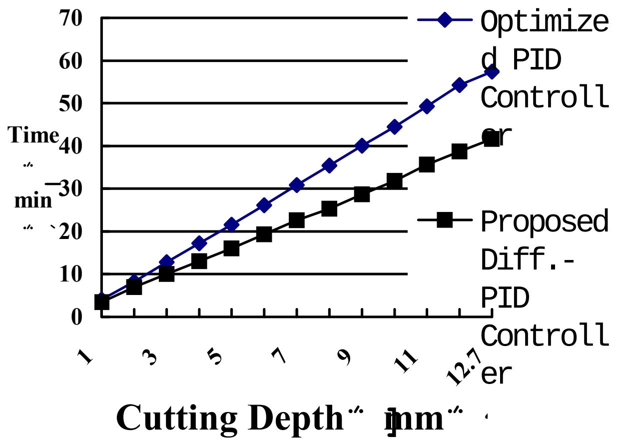

Fig. 14. The Controlled Results of for Proposed differential PID(DPID) Controller

Fig.15.The EDM Machine

6 Conclusions

This paper explores the circuit model of Die-sinking EDM. We have shown that the machining efficiency of EDM is closed related to gap resistance. According to electromagnetic theory, it can be shown that in the first and middle stages of machining process, the overall control system becomes a nonlinear Lur'e problem. As for the final stage of machining process, the overall control system becomes a linear system.

For the Lur'e problem of Die-sinking EDM, and the SPR (Strict Positive Real) condition can be applied to the control of EDM. A differential-PID(DPID) controller is proposed in the beginning and middle stage. In the final stage of machining process, A PID controller can applied to the final stage of machining process. Therefore an adaptive controller switch from Differential-PID controller and PID controller is proposed in this paper. Experimental results already has been verify the results. The machining efficiency already has been enhanced up to 30%.

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7 Appendix A

From Eq.(9) and Eq.(11), the overall transfer function can be written as

$$G(s) = \frac{k(k_I + k_p s + k_d s^2)}{s(s\tau_e + 1)(s\tau_m + 1)} \qquad (25)$$

With partial fractional expansion of Eq.(25), we have

$$G(s) = \frac{kk_I}{s} + \frac{[(k_p - (\tau_e + \tau_m)k_I) + (k_{dd} - \tau_e\tau_m k_I)s]k}{(s\tau_e + 1)(s\tau_m + 1)} \qquad (26)$$

From Eq.(26), with $\mathrm{Re}\,G(j\omega) = G(j\omega) + G(-j\omega) > 0$ and substituting $s = j\omega$ into Eq.(26), we can have the results of Eq.(20)

For a torque-mode DC motor driver, if we neglected the dynamics of current feedback loop, the plant transfer function can be simplified as

$$G_p(s) = \frac{k}{s(Js + D)} \qquad (27)$$

where J is the moment of inertia and D is the damping coefficient.

If the controller $G_c(s)$ is a PID controller, then the loop transfer function $G(s) = G_c(s)G_p(s)$ can be written as

$$G(s) = \frac{k(k_I + k_p s + k_d s^2)}{s^2(Js + D)} \qquad (28)$$

It can be observed that in Eq.(28), the loop transfer function $G(s)$ cannot to be SPR $\forall k_p, k_I, k_D \in R$ since $G(s)$ has double roots of zero, which is on the imaginary axis [15]. Even though $G(s)$ has relative degree 1, the SPR condition cannot be satisfied. Also, a differential PID controller can eliminate the double roots of zero. Also, for SPR requirements, similar conclusions can be obtained as

$$k_p > k_d\left(\frac{D}{J}\right) + k_I\left(\frac{J}{D}\right),\ \textit{and}\ k_I > 0 \qquad (29)$$