Applying Fuzzy Engineering Economics to Evaluate Project Investment Feasibility of Wind Generation

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Abstract— In this paper, a fuzzy engineering economic decision model is derived to evaluate the investment feasibility of wind generation project. A straightforward vertex parameters' fuzzy mathematics operation using the function principle is derived as an alternative to the traditional extension principle, and is applied to evaluate a number of different economic decision indexes. Compared to the extension principle, the function principle is simple to implement and is conceptually straightforward. Using Mellin transform, the geometric moments of the fuzzy economic indexes are established in order to determine their relative ranking as part of a decision-making process. The performance of the proposed fuzzy economic model is verified by considering their application to a practical wind generation project in Taiwan. These investigations not only confirm that the results of the fuzzy economic model is consistent with the conventional crisp model, but also demonstrate that the proposed model is more flexible, intelligent and computationally efficient compared to the extension principle fuzzy mathematics approach. The developed model represent readily implemented feasibility analysis tool for use in the arena of uncertain economic decision-making.

Index Terms—Function principle, Fuzzy mathematics, Mellin transform, Fuzzy ranking, Wind electricity, Decision-making.

1 Introduction

Since the Rio declaration from the Earth Summit in 1992, the energy supply and consumption relating to global warming has been focused on pursuing sustainable development and raising clean energy in recent years around the world.

In response to the global environmental issues, the Taiwan government formally established the National Council for Sustainable Development in 1997. In May 1998, a National Energy Conference mainly concluded with targets of formulating renewable energy development strategies and promotion measures. The National Energy Conference Action Plan and the New and Clean Energy Research Development Plan were completed in 1999, by which the renewable energy development potential assessment and other measures are reinforced. In 2003, the National Nuclear-Free Homeland Conference concluded, and combining with Kyoto Protocol to enhance use the Clean Energy.

The weight of renewable energy in Taiwan's electricity capacity was set up to 3%, 6500MW, in 2020. For this reason, energy strategies and policies for promoting renewable energy must be active in providing some environmental, financial and economic incentives.

Taiwan is a densely populated island with only limited natural resources. In response to the era of high oil prices and global trend of greenhouse gas emission reduction, promoting the development of renewable energy utilization is considered as a critical strategy internationally. Regarding the energy situation in Taiwan, 98% of energy supply is imported. Therefore, promoting renewable energy development can diversify the energy supply and increase the domestic energy proportion, as well as lead the development of local industry simultaneously, eventually to reach the goal of the three-wins of energy security, environmental protection, and economical development[1].

Generally, renewable energy includes wind energy, solar energy, biomass energy, hydro energy, and geothermal energy[2,3]. At present, Taiwan mainly places emphasis on wind power, solar photovoltaic and bio-fuel, and also promotes other renewable energies as an auxiliary means. Up to December of 2007, the total installed capacity of renewable power generation has reached 2,843 MW, which can approximately produce 7.65 billion kWh of electricity annually (roughly equivalent to the electricity generation of two sets of

Linko coal-fired power plant in 2006). With this amount of electricity from renewable energies, it can provide an annual electricity consumption of 1.91 million for households.

With abundant wind resources along the west coast and on offshore island, Taiwan has superior advantages in geographic location to develop wind energy. At present, Taiwan has completed 155 sets of wind turbines with total capacity of 281.6 MW, built by Taiwan Power Company and some private sectors. Assuming that 1 kW of the installed capacity averagely produces 2,700 kWh per year, wind power can totally generate 760 million kWh annually, providing enough electricity to 190 thousand households.

Moreover, projects under construction, being consented, and under planning are being added up, with a total installed capacity of 467.8 MW (equivalent to 230 sets of wind turbines). Besides onshore projects, the Executive Yuan has ratified the "The Program of First Stage of Offshore Wind Development" proposed by the Ministry of Economic Affairs, which targets on developing 300 MW of offshore wind power in the first stage, symbolizing that Taiwan's development of wind power has reached a new milestone.

Prior to adopting a project, potential investors must explore the soundness of the project by performing a feasibility study which investigates all aspects of the project, including its anticipated future financial and economic performance. The feasibility study mainly concerns the monetary aspects of the project and its financial rewards and profitability from the investors' perspectives. That is, an economic profitability model should be made available to potential investors to enable them to evaluate the benefit-costs of the project. In general, the greater the economic effectiveness of a project, the greater the degree of its acceptance by investor[4].

The cash flow models applied in many economic decision-making problems often involve an element of uncertainty. In the case of deficient data, decision-makers generally rely on an expert's knowledge of economic information when carrying out their economic modeling activities. Since the nature of this knowledge is typically vague as opposed to random, Dr. Zadeh introduced fuzzy set theory in 1965, which aimed to rationalize the uncertainty caused by vagueness or imprecision. The concept of fuzzy sets led to the definition of the fuzzy number by Nahmais[5], and Dubois and Prade[6]. This theory has been developed and successfully applied to numerous areas, such as control and decision making, engineering and medicine. Its application to economic analysis is natural due to the uncertainty inherent in many financial and investment decisions. However, practical applications of fuzzy number theory in the economic decision-making arena involve two laborious tasks, namely fuzzy mathematical operations and the comparison or ranking of the resultant complex fuzzy numbers.

Conventional fuzzy mathematical operations using the extension principle[7-9] are applicable only to normalized fuzzy numbers. However, generalized fuzzy numbers (i.e. normalized and non-normalized fuzzy numbers) have the advantage that the *degrees of confidence* of a decision-makers' opinions can be represented by their *heights*[11]. Moreover, fuzzy mathematical operations using the extension principle change the membership function type of the fuzzy number following mathematical manipulation and involve complex and laborious mathematical operations. Accordingly, Chen proposed the *function principle*[10], which can be used to perform fuzzy mathematical operations on generalized fuzzy numbers. In [11-13], the authors pointed out that the fuzzy mathematical operations presented in [10] preserve the membership function type of the fuzzy number following mathematical manipulation and reduce the complexity and tediousness of the mathematical operations. Consequently, the present paper develops an easily implemented and conceptually straightforward *vertexes operation* using the function principle for application to fuzzy mathematics. The developed fuzzy mathematics operations are then applied to evaluate fuzzy economic indexes as part of a decision-making process. The proposed economic decision analysis method is more flexible and more intelligent than other methods since it takes the degree of confidence of the decision-makers' opinions into account.

Following the manipulation of fuzzy economic functions by fuzzy mathematics, the task of comparing or ranking the resultant fuzzy numbers can invoke a further problem since fuzzy numbers do not always yield a totally ordered set in the same way that crisp numbers do. Many authors have investigated the use of different fuzzy set ranking methods. These methods have been reviewed and compared by Chen and Hwang[17]. However, the majority of previous studies focused on the ranking of normalized fuzzy numbers, while relatively few considered the case of non-normalized fuzzy numbers[14-17]. In this paper, a *geometric moment* model is derived to rank generalized fuzzy numbers based on the probability measure of fuzzy events. The geometric moments of a fuzzy number comprise the *domain moments* and the *grade moments*.

The remainder of this paper is structured as follows: Section 2 introduces the fuzzy mathematics approach using the function principle, and discusses the ranking of the generalized fuzzy numbers using a geometric moment model. Section 3 develops fuzzy profitability model to assist project investors in evaluating the relative benefits of projects in an uncertain environment. Section 4 presents the application of the proposed fuzzy evaluation models to a practical case study. Finally, Section 5 presents the conclusions of the present study.

2 FUZZY MATHEMATICS AND RANKING

2.1 Fuzzy Number

Fuzzy set theory, a generalization of classical (crisp) set theory, was developed by Dr. Zadeh. Fuzziness describes sets that have no *sharp transition* from membership to non-membership. Let X be a collection of objects denoted generically by x.

Definition 1: A fuzzy set $\tilde{A}$ in X is a set of ordered pairs

$$\tilde{A} = \{ x, \mu_{\tilde{A}}(x) | x \in X \} \qquad (1)$$

Where $\mu_{\tilde{A}}(x)$ is the membership function of x in $\tilde{A}$, which maps X to the membership space M, $M \in [0,\alpha], \alpha \le 1$. If M is the closed interval [0,1], then $\tilde{A}$ is called a normal fuzzy set. Otherwise, is called non-normal fuzzy set.

Definition 2: A fuzzy set $\tilde{A}$ is convex if

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \ge min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)) \qquad (2)$$

for $x_1, x_2 \in X$ and $\lambda \in [0,1]$.

Definition 3: A fuzzy number is a convex fuzzy set on the real line R with a continuous, compactly supported, and convex membership function. Normality implies that $\forall x \in R, Max\{\mu_{\tilde{A}}(x)\} = 1$, means the maximum membership value of the fuzzy number in R is 1. Therefore, the non-normalized fuzzy number is $\forall x \in R, Max\{\mu_{\tilde{A}}(x)\} < 1$.

The most popular form of a fuzzy number is *L-R* representation, which developed by Dubois and Prade [8]. A *TrFN* fuzzy number is a particular form of fuzzy number in which the left-side and the right-side function are both straight-line segments, shown as in Eq.(3).

Quite often in interest rate, inflation rate and future cash amounts, the estimated values are usually accepted in the field of uncertain financial analysis. One usually employs educated guesses, based on expected values or other statistical techniques, to obtain future financial parameters. The *TrFN* and *TFN* can be appropriately used to state cases like *approximately between $12000 and $16000* and *about 5% annual interest rate* respectively, to capture the vagueness of those financial statements. *TrFN* and *TFN* are widely used in fuzzy financial analysis because of their simplicity in both representation and management levels as well as their intuitive interpretation.

In [10], Chen proposed a generalized trapezoidal fuzzy number (*TrFN*) $\tilde{A}$ as $\tilde{A} = (a,b,c,d;h)$ with the membership function and grade function shown in Eqs.(3) and (4), respectively. The membership degree h, where $0 \le h \le 1$, represents the *confidence level* (or *height*) of the fuzzy number, and can be considered to indicate the degree of confidence of the decision-makers' opinions. Meanwhile, the vertexes a, b, c and d are real numbers denoting *the smallest possible value*, *the most promising interval value* and *the largest possible value* of a fuzzy event, respectively. The *support* of $\tilde{A}$ is the crisp set that contains all the elements have a nonzero membership grade in $\tilde{A}$, i.e. the interval [a,d] [9,19].

$$\mu_{\tilde{A}}(x) = \begin{cases} \mu_{\tilde{A}_L}(x) = h(x-a)/(b-a) & a \le x \le b \\ h & b \le x \le c \\ \mu_{\tilde{A}_R}(x) = h(d-x)/(d-c) & c \le x \le d \end{cases} \qquad (3)$$

The *membership function* given in Eq.(3) represents the mapping of any given value of x to its corresponding grade of membership, α. Meanwhile, the *grade function* expressed by Eq.(4) is an inverse mapping of any given α to its corresponding x value. In the case where b equals c, the *TrFN* becomes a triangular fuzzy number (*TFN*), $\tilde{A} = (a,b,d;h)$.

$$x = \begin{cases} \nu_{\tilde{A}_L}(\alpha) = \mu_{\tilde{A}_L}^{-1} = a + (b-a)\alpha / h & 0 \le \alpha \le h \\ [b,c] & \alpha = h \\ \nu_{\tilde{A}_R}(\alpha) = \mu_{\tilde{A}_R}^{-1} = d - (d-c)\alpha / h & 0 \le \alpha \le h \end{cases} \qquad (4)$$

2.2 Fuzzy Mathematics using Function Principle

Under the extension principle (*E/P*), the multiplication of two fuzzy numbers, each with a trapezoidal membership function, results in a fuzzy number with a two-sided parabolic drum-like shape membership function. In other words, the result of fuzzy multiplication is a complicated fuzzy number.

The function principle is defined as follows [10]: let g be a mapping from n-dimension real numbers R^n into a real line R, f_g be a corresponding mapping from an *n*-dimension fuzzy number into a fuzzy number, and $\tilde{A}_i = (a_i, b_i, c_i, d_i; h_i), i = 1,2,\cdots n$ be *n* fuzzy numbers, each belonging to the trapezoidal family F^1. The fuzzy number $\tilde{B}$ in *R* induced from the *n* fuzzy numbers through function *g* is:

$$f_g(\tilde{A}_1, \tilde{A}_2, \cdots \tilde{A}_n) = \tilde{B} = (a, b, c, d; h) \quad (5)$$

where: $h = min\{h_1, h_2, \cdots h_n\}$,

$$A_{i,s} = min\{x \mid \mu_{\tilde{A}_{i,L}}(x) \geq h\} = \nu_{\tilde{A}_{i,L}}(h) = a_i + (b_i - a_i)\frac{h}{h_i}, i = 1,2,\cdots,n,$$

$$A_{i,t} = max\{x \mid \mu_{\tilde{A}_{i,R}}(x) \geq h\} = \nu_{\tilde{A}_{i,R}}(h) = d_i - (d_i - c_i)\frac{h}{h_i}, i = 1,2,\cdots,n,$$

$$T = \{g(x_1, x_2, \cdots, x_n) \mid x_i = a_i \text{ or } d_i, i = 1,2,\cdots,n\},$$

$$T_1 = \{g(x_1, x_2, \cdots, x_n) \mid x_i = A_{i,s} \text{ or } A_{i,t}, i = 1,2,\cdots,n\},$$

$$a = min\,T, \quad b = min\,T_1, \quad c = max\,T_1, \quad d = max\,T,$$

and $min\,T \leq min\,T_1$, $max\,T_1 \leq max\,T$.

It should be noted that the *F/P* fuzzy mathematics is approached partly from the *E/P* fuzzy mathematics. The *F/P* operation linearizes the complicated non-linear membership functions given by the *E/P* operation which eases the calculation without introducing a significant error. Besides, the *F/P* operation also can be used to operate in case of non-normal fuzzy numbers, while the *E/P* operation can be used to operate normal fuzzy numbers only. The four basic mathematical operations between two positive *TrFNs*, $\tilde{A}_1$ and $\tilde{A}_2$, can be derived as follows:

(1) Fuzzy addition and multiplication:

$$\tilde{A}_1 \bullet \tilde{A}_2 = (a_1, b_1, c_1, d_1; h_1) \bullet (a_2, b_2, c_2, d_2; h_2) = (a_1 \bullet a_2, A_{1,s} \bullet A_{2,s}, A_{1,t} \bullet A_{2,t}, d_1 \bullet d_2, min(h_2, h_1)) \quad (6)$$

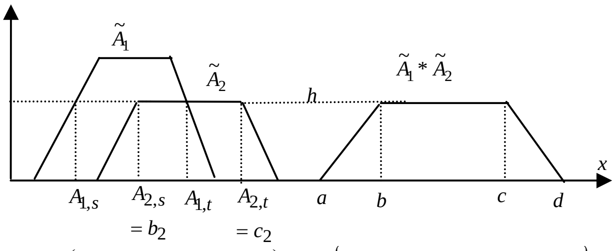
where $\bullet$ denotes addition or multiplication operations, respectively.

(2) Fuzzy subtraction and division:

$$\tilde{A}_1 \bullet \tilde{A}_2 = (a_1, b_1, c_1, d_1; h_1) \bullet (a_2, b_2, c_2, d_2; h_2) = (a_1 \bullet d_2, A_{1,s} \bullet A_{2,t}, A_{1,t} \bullet A_{2,s}, d_1 \bullet a_2, min(h_2, h_1)) \quad (7)$$

where $\bullet$ denotes subtraction or division operations, respectively.

The multiplication of $\tilde{A}_1$ and $\tilde{A}_2$ is shown in Fig.1 It can be seen that fuzzy mathematical operations performed by the function principle do not change the membership function type of the fuzzy number following mathematical operation. Furthermore, this approach reduces the difficulty and laboriousness of the mathematical operations. A similar finding was also reported in [11-13].



$T = \{a_1 a_2, a_1 d_2, d_1 a_2, d_1 d_2\}$, $T_1 = \{A_{1,s} b_2, A_{1,s} c_2, A_{1,t} b_2, A_{1,t} c_2\}$

$a = \min T = a_1 a_2$, $b = \min T_1 = A_{1,s} b_2$, $c = \max T_1 = A_{1,t} c_2$, $d = \max T =$

Fig.1. Multiplication of two *TrFNs*

2.3 Fuzzy Ranking

This paper develops a geometric moment fuzzy ranking algorithm, based on the probability measure of fuzzy events, for the ranking of generalized fuzzy numbers. The probabilistic method is one of two previously published fuzzy ranking methods[9,19-23]. In an earlier study[24,25], the current author suggested using the Mellin transform to perform the fuzzy ranking of normalized fuzzy numbers. The proportional probability density function was adopted due to its computationally straightforward nature and conceptual consistence.

The proportional probability density function (*pdf*) corresponding to the membership function of a fuzzy number, $\mu(x)$, is $p(x) = h_p \mu_{\tilde{A}}(x)$, where h_p denotes the conversion constant which ensure that the area under the continuous probability density function is equal to 1.

Operational calculus techniques are particularly useful when analyzing probabilistic models as part of a decision-making process. In the probabilistic modeling context, it is often possible to reduce complex operations involving differentiation and integration to simple *algebraic manipulations* in the transform domain.

The Mellin transform is a useful tool for studying the distributions of certain combinations of random variables; particularly those concerned with the random variables associated with products and quotients. The Mellin transform $M_x(s)$ of a function *f(x)*, where *x* is positive, is defined in [32,33] as:

$$M_x(s) = \int_0^\infty x^{s-1} f(x)dx \quad 0 < x < \infty \quad (8)$$

The moments of a distribution represent the expected values of the power of a random variable with a $f(x)$ distribution. In general, the p^{th} moment of a random variable, *X*, about a real number, *c*, is defined as:

$$M_p(x) = E[(X-c)^p] = \int_X (x-c)^p f(x)dx \quad (9)$$

The moments of interest in economic analyses are those about the origin ($c=0$) and those about the mean ($c=\mu$), typically for p =1,2,3 and 4. If the p^{th} moments about the origin and the mean are denoted by $E[X^p]$ and m_p respectively, then:

$$m_p = E[(X-\mu)^p] = \int_X (x-\mu)^p f(x)dx \quad (10)$$

The first moment about the origin represents the *mean* of the distribution $\mu = E[X]$, while the second moment about the mean represents the *variance* σ^2. Meanwhile, the *skew* and the *kurtosis* of the distribution are denoted by m_3 and m_4, respectively. Comparing Eq. (8) with Eq. (9) shows that $M_x(s)$ is a special case of $M_p(x)$, where $c=0$ and $p=s-1$. In other words, if *f(x)* is viewed as a *pdf*, the Mellin transform $M_x(s) = E[X^{s-1}]$ provides the means of establishing a series of moments of the distribution. Comparing the first two moments of a distribution using the Mellin transform, allows the mean and variance to be expressed as Eqs.(11) and (12), respectively. Computing $M_x(s)$ at *s*=2 and 3 in Eqs.(7) and (8), gives the domain mean and domain variance of the *TrFN* and *TFN*, respectively.

$$\mu = E[X] = M_x(2) \quad (11)$$

$$\sigma^2 = m_2 = Var[X] = M_x(3) - (M_x(2))^2 \quad (12)$$

From the grade functions of the fuzzy number represented in Eq.(4), the *grade mean* μ_g and the *grade variance* Var_g are expressed by Eqs.(13) and (14), respectively. The grade mean and grade variance of the *TrFN* $\tilde{A} = (a,b,c,d;h)$ and the *TFN* $\tilde{A} = (a,b,d;h)$ are derived in Eqs.(9) and (10) , respectively. The geometric mean μ and geometric variance Var of a fuzzy number are then defined by Eqs.(13) and (14), respectively.

$$\mu_g = \int_0^h \alpha(\nu_{\tilde{A}_R}(\alpha) - \nu_{\tilde{A}_L}(\alpha))d\alpha \quad (13)$$

$$Var_g = \int_0^h \alpha^2(\nu_{\tilde{A}_R}(\alpha) - \nu_{\tilde{A}_{L1}}(\alpha))d\alpha - (\mu_g)^2 \quad (14)$$

The Mellin transforms of the *TrFN* $\tilde{A} = (a,b,c,d;h)$ and the *TFN* $\tilde{A} = (a,b,d;h)$ are derived in Eqs.(15) and (16) , respectively. Computing $M_x(s)$ at *s*=2 and 3, gives the mean and variance of the *TrFN* and *TFN*, respectively. These mean and variance of a *TrFN* and a *TFN* are dependent only on the *domain vertexes*, i.e. they are independent of the grade (height). Accordingly, they are generally referred to as the *domain mean* μ_d and the *domain variance* Var_d, respectively.

$$M_x(s) = \frac{h_p}{s(s+1)}\left(\frac{(d^{s+1}-c^{s+1})}{(d-c)} - \frac{(b^{s+1}-a^{s+1})}{(b-a)}\right),$$
$$h_p = \frac{2}{(d+c)-(b+a)} \quad (15)$$

$$M_x(s) = \frac{h_p}{s(s+1)}\left(\frac{d(d^s-b^s)}{(d-b)} - \frac{a(b^s-a^s)}{(b-a)}\right),$$
$$h_p = \frac{2}{(d-a)} \quad (16)$$

The grade mean and grade variance of the *TrFN* $\tilde{A} = (a,b,c,d;h)$ and the *TFN* $\tilde{A} = (a,b,d;h)$ are derived in Eqs.(17) and (18) , respectively. It is found that the grade mean and the grade variance are both functions of the height and vertexes of the fuzzy number.

$$\mu_g = \left(\frac{d+2c-2b-a}{6}\right)h^2 ,$$
$$Var_g = \left(\frac{d+3c-3b-a}{12}\right)h^3 - (\mu_g)^2 \quad (17)$$

$$\mu_g = \left(\frac{d-a}{6}\right)h^2 ,$$
$$Var_g = \left(\frac{d-a}{12}\right)h^3 - (\mu_g)^2 \quad (18)$$

The geometric mean μ and geometric variance Var of a fuzzy number are defined by Eqs.(19) and (20), respectively.

$$\mu = \sqrt{\mu_d^2 + \mu_g^2} \quad (19)$$

$$Var = \sigma^2 = \sqrt{Var_d^2 + Var_g^2} \quad (20)$$

Fig.2 presents a flow chart describing the proposed ranking process for fuzzy numbers. Initially, the fuzzy numbers are converted to their equivalent *pdfs*. Eqs.(19) and (20) are then used to calculate their geometric means and geometric variances. Fuzzy numbers which share the same geometric mean value are ranked using Rule 1, while the remaining fuzzy numbers are ranked using Rule 2. These two rules are summarized as follows: *Rule 1*: a fuzzy number with a lower geometric variance is ranked above fuzzy numbers whose geometric variances are higher. *Rule 2:* a fuzzy number with a superior geometric mean is ranked above fuzzy numbers having inferior geometric means. Note that when performing a least-cost analysis, a smaller geometric mean cost is superior to higher geometric mean costs. Conversely, in a cost-benefit analysis, a higher geometric mean benefit is superior to lower geometric mean benefits.

3 FUZZY ECONOMIC FEASIBILITY MODEL

The cash flow models applied in economic decision-making problems relating to project evaluation frequently involve an element of uncertainty. Previous researchers, including Kaufmann and Gupta [26] and Ward [27], conducted fuzzy discounted cash flow analyses in which either the periodic cash flow or the discount rate was specified as a fuzzy number. Furthermore, Buckley [28], Chiu and Park [29] and Kahraman *et al.* [30] addressed problems in which both the periodic cash flow and the discount rate were expressed as fuzzy numbers.

These studies also developed various economic equivalence formulae for use in rudimentary economic calculations. However, these models have only limited application in the economic decision-making arena since they consider only a single payment, or at best, a few payments, when deriving their economic indexes. However, in real-world applications, the periodic cash flow may be subject to occasional uncertain variations. Accordingly, the present study adopts a parameter, *d*, to represent the inflation rate. This parameter is specified in the form of a fuzzy number and is used to reflect an uncertain geometric series of cash flows. At the planning stage, a decision-maker is seldom in possession of all the information required to make an accurate assessment of the initial investment $\tilde{I}$, and the annual cash flow-in (or out) $\tilde{A}$. Therefore, it is appropriate to specify the initial investment, the periodic cash flow, the inflation rate and the interest rate as *TrFNs*.

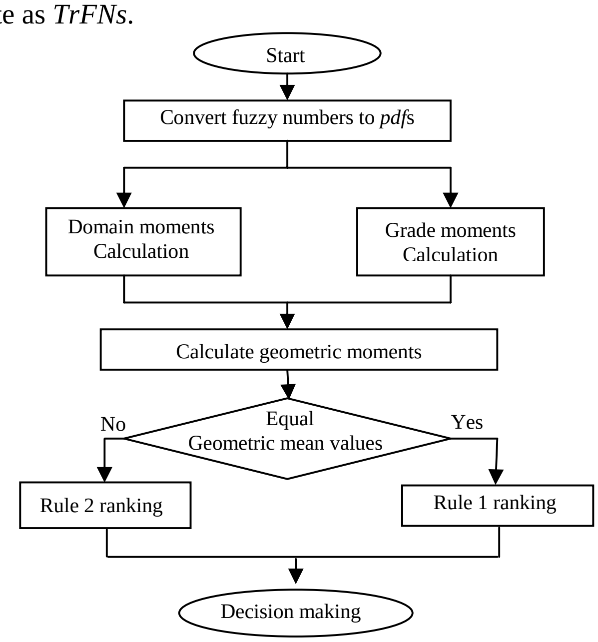


Fig.2. Flow chart of fuzzy number ranking process

In evaluating certain projects, investors may take the cash flow-out to be the initial capital investment *I*, and consider the cash flow-in to be the annual net profit, A_t, which is calculated as the difference between the annual production revenue and the annual operating cost. The present study develops three fuzzy cost-benefit evaluation models, i.e. net present value (*NPV*), pay back year (*PBY*) and benefit/cost ratio (*BCR*), to assess the profitability of projects. Although the internal rate of return indicator is commonly used in conventional crisp cost/benefit analysis, it has been noted by previous researchers that this index is not applicable to the fuzzy case [16, 29].

For the inflation-free interest rate *r* and the inflation rate *d*, the geometric series present value factor *GPVF*, is given by Eq.(21).

$$GPVF = \frac{1-((1+d)/(1+r))^n}{(r-d)} \tag{21}$$

It should be noted that *r* is the inflation-free interest rate (or the so-called real interest rate), which is given by the difference between the market interest rate and the inflation rate, *d*, when either the real interest rate or the inflation rate is relatively small [31]. Generally speaking, the interest rate, *r*, is higher than the inflation rate, *d*. Although the inflation rate seldom exceeds the interest rate, this *hyperinflation* situation can sometimes arise, for example in countries where political instability, government overspending, international trade balance weaknesses, etc. are present.

The *NPV*, *PBY* and *BCR* measures are expressed in Eqs. (22), (23) and (24), respectively. Meanwhile, In the fuzzy case, the initial investment $\tilde{I}$, the annual cash flow-out $\tilde{A}$, the fuzzy inflation rate $\tilde{d}\%$, and the fuzzy interest rate $\tilde{r}\%$, should all be denoted as *TrFN*, i.e. $(I_1,I_2,I_3,I_4;h_I)$, $(A_1,A_2,A_3,A_4;h_A)$, $(d_1,d_2,d_3,d_4;h_d)$, $(r_1,r_2,r_3,r_4;h_r)$, respectively. Let $h_\beta = min(h_I,h_A,h_d,h_r)$. The corresponding fuzzy models can be derived and represented in Eqs.(25), (26), and (27), respectively.

$$NPV = -I + A * GPVF \tag{22}$$

$$PBY = \frac{ln(1-(r-d)I_0/A)}{ln((1+d)/(1+r))} \tag{23}$$

$$BCR = \frac{\sum_{t=1}^{n} \frac{A(1+d)^{t-1}}{(1+r)^t}}{I} = \frac{A}{I}\left(\sum_{t=1}^{n} \frac{(1+d)^{t-1}}{(1+r)^t}\right) \tag{24}$$

$$NPV = (NPV_1, NPV_2, NPV_3, NPV_4; h_\beta) \tag{25}$$

where:

$$NPV_1 = -I_4 + A_1 * gpvf_1 , \quad NPV_1 = -I_t + A_s * gpvf_2 ,$$
$$NPV_1 = -I_s + A_t * gpvf_3 , \quad NPV_1 = -I_1 + A_4 * gpvf_4$$

$$PBY = (PBY_1 ,PBY_2 ,PBY_3 ,PBY_4 ;h_\beta) \quad (26)$$

Where:

$$PBY_1 = \frac{ln(1 - (r_1 - d_4)(I_1 / A_4))}{ln((1 + d_4)/(1 + r_1))},$$
$$PBY_2 = \frac{ln(1 - (r_s - d_t)(I_s / A_t))}{ln((1 + d_t)/(1 + r_s))},$$
$$PBY_3 = \frac{ln(1 - (r_t - d_s)(I_t / A_s))}{ln((1 + d_s)/(1 + r_t))},$$
$$PBY_4 = \frac{ln(1 - (r_4 - d_1)(I_4 / A_1))}{ln((1 + d_1)/(1 + r_4))}$$

$$BCR = (BCR_1 ,BCR_2 ,BCR_3 ,BCR_4 ;h_\beta) \quad (27)$$

where:

$$BCR_1 = (A_1 / I_4)* gpvf_1 , \quad BCR_2 = (A_s / I_t)* gpvf_2 ,$$
$$BCR_3 = (A_t / I_s)* gpvf_3 , \quad BCR_4 = (A_4 / I_1)* gpvf_4 .$$

When seeking to establish the least-cost solution, the two most commonly applied discounting methods are the present value of cost (*PVC*) method and the equivalent uniform annual cost (*EUAC*) method. The present value of the costs illustrated in Fig. 3 can be derived from Eq.(28), while the geometric series present value factor *GPVF*, is given by Eq.(29).

$$PVC = I + \sum_{t=1}^{n} \frac{A_t}{(1 + r)^t} = I + \sum_{t=1}^{n} \frac{A(1 + d)^{t-1}}{(1 + r)^t} = I + A * GPVF \quad (28)$$

where:
$$GPVF = \frac{1 - ((1 + d)/(1 + r))^n}{(r - d)} \quad (29)$$

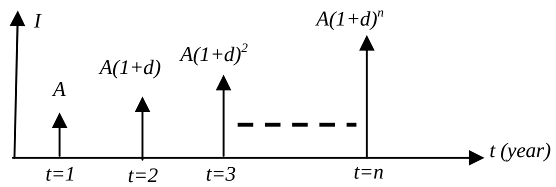


Fig.3. Cash flow of a least cost program

It should be noted that *r* is the inflation-free interest rate (or the so-called real interest rate), which is given by the difference between the market interest rate and the inflation rate, *d*, when either the real interest rate or the inflation rate is relatively small [28]. Generally speaking, the interest rate, *r*, is higher than the inflation rate, *d*. Although the inflation rate seldom exceeds the interest rate, this *hyperinflation* situation can sometimes arise, for example in countries where political instability, government overspending, international trade balance weaknesses, etc. are present.

The *EUAC* is given by the summation of the uniform annualized investment cost and the uniform annualized geometric-gradient annual operating cost, and is expressed Eq.(30), where *CRP* and *GSAF* represent the *capital recovery factor* and *geometric-series annuity factor*, expressed as Eqs.(31) and (32), respectively.

$$EUAC = I * CRF + A * GSAF \quad (30)$$

$$CRF = \frac{r(1 + r)^n}{(1 + r)^n - 1} \quad (31)$$

$$GSAF = \frac{r[(1 + r)^n - (1 + d)^n]}{(r - d)[(1 + r)^n - 1]} \quad (32)$$

In the fuzzy case, the initial investment $\tilde{I}$, the annual cash flow-out $\tilde{A}$, the fuzzy inflation rate $\tilde{d}\%$, and the fuzzy interest rate $\tilde{r}\%$, should all be denoted as *TrFN*, i.e. $(I_1,I_2,I_3,I_4;h_I)$, $(A_1,A_2,A_3,A_4;h_A)$, $(d_1,d_2,d_3,d_4;h_d)$, $(r_1,r_2,r_3,r_4;h_r)$, respectively. Let $h_\alpha = \min(h_d, h_r)$ and $h_\beta = \min(h_I, h_A, h_d, h_r)$. The fuzzy *GPVF*, *CRF*, *and GSAF* are given in Eqs.(33)-(35), respectively.

$$GPVF = (gpvf_1 , gpvf_2 , gpvf_3 , gpvf_4 ; h_\alpha) \quad (33)$$

where:

$$gpvf_1 = \frac{1 - ((1 + d_1)/(1 + r_4))^n}{(r_4 - d_1)}, \quad gpvf_2 = \frac{1 - ((1 + d_s)/(1 + r_t))^n}{(r_t - d_s)},$$
$$gpvf_3 = \frac{1 - ((1 + d_t)/(1 + r_s))^n}{(r_s - d_t)}, \quad gpvf_4 = \frac{1 - ((1 + d_4)/(1 + r_1))^n}{(r_1 - d_4)}$$

$$CRF = (crf_1, crf_2, crf_3, crf_4; h_\alpha) \quad (34)$$

where:
$$crf_1 = \frac{r_1(1 + r_1)^n}{(1 + r_1)^n - 1}, \quad crf_2 = \frac{r_s(1 + r_s)^n}{(1 + r_s)^n - 1},$$
$$crf_3 = \frac{r_t(1 + r_t)^n}{(1 + r_t)^n - 1}, \quad crf_4 = \frac{r_4(1 + r_4)^n}{(1 + r_4)^n - 1}$$

$$GSAF = (gsaf_1, gsaf_2, gsaf_3, gsaf_4; h_\alpha) \quad (35)$$

Where:

$$gsaf_1 = \frac{r_4[(1 + r_4)^n - (1 + d_1)^n]}{(r_4 - d_1)[(1 + r_4)^n - 1]}, \quad gpvf_2 = \frac{r_t[(1 + r_t)^n - (1 + d_s)^n]}{(r_t - d_s)[(1 + r_t)^n - 1]},$$
$$gsaf_3 = \frac{r_s[(1 + r_s)^n - (1 + d_t)^n]}{(r_s - d_t)[(1 + r_s)^n - 1]}, \quad gpvf_4 = \frac{r_1[(1 + r_1)^n - (1 + d_4)^n]}{(r_1 - d_4)[(1 + r_1)^n - 1]},$$

The resultant fuzzy *PVC* and *EUAC* measures are then derived in Eqs.(36) and (37), respectively, i.e.

$$PVC = (PVC_1, PVC_2, PVC_3, PVC_4; h_\beta) \quad (36)$$

where:
$$PVC_1 = I_1 + A_1 * gpvf_1, \quad PVC_2 = I_s + A_s * gpvf_2,$$
$$PVC_3 = I_t + A_t * gpvf_3, \quad PVC_4 = I_4 + A_4 * gpvf_4$$

$$EUAC = (EUAC_1, EUAC_2, EUAC_3, EUAC_4; h_\beta) \quad (37)$$

where:

$$EUAC_1 = I_1 * crf_1 + A_1 * gsaf_1, \quad EUAC_2 = I_s * crf_2 + A_s * gsaf_2$$

$$EUAC_3 = I_t * crf_3 + A_t * gsaf_3, \quad EUAC_4 = I_4 * crf_4 + A_4 * gsaf_4$$

4 CASE STUDY

The fuzzy economic decision-making procedures are briefly described. Firstly, the estimated input parameters, such as interest rate, inflation rate, investment, and operating revenue and/or cost, which are needed in economic index calculation, should be provided by the expert in form of fuzzy numbers. The fuzzy economic decision indexes are then calculated according to the models developed in Section 3. The fuzzy economic decision is made finally according to the relative ranking of the resultant fuzzy economic indexes, which is performed following the process described in Fig.2.

The developed fuzzy decision models were used to evaluate a wind electricity generation in Taiwan. The project was built by four Vestas 660kW wind asynchronous generators with totally approximate 90 million NT$ (M$) initial investment and estimated 8000*MWh* annual electricity generation. Assume the annual operation and maintenance cost approximately 1.5% of the initial investment.

Table I presents the corresponding trapezoidal fuzzy investments and profit determined by the experts' opinions, and the calculated cost-benefit measures of this alternative. The experts assume the trapezoidal fuzzy yearly interest rate, *r*, and the fuzzy inflation rate, *d*, to be (5,5.5,6.5,7;1.0)% and (1,1.5,2.5,3;1.0)% in *TrFN* form, respectively. Additionally, the plant life is assumed to be 20 years. The resultant measures indicate that the proposed wind generation system has a 7.57 mean *PBY*, 110.6M$ mean *NPV*, and 2.29 mean *BCR*.

It should be noted that the conventional crisp economic decision represents one particular case of fuzzy economic evaluation. A specific crisp economic index can be established by setting all the fuzzy rates, costs and revenues to their most promising values. For example, when conducting the crisp (i.e. certain) evaluation of the wind generation system discussed above, the interest rate, inflation rate, investment cost and operating costs can be taken from Table I to be 6%, 2%, 90 M$ and 15M$, respectively. Subsequently, using Eqs.(22)-(24), the crisp *NPV*, *PBY* and *BCR* values are calculated to be 111.25M$, 7.13 year and 2.24, respectively. It can be seen that these values are located within the most promising interval values of the fuzzy *NPV*, fuzzy *PBY* and fuzzy *BCR*, respectively, in Table I.

TABLE I

BENEFIT/COST ANALYSIS OF WIND GENERATION PROGRAM

Initial investment(M$)		(87.5,90.0,92.5,95.0;0.8)
Annual profit(M$)		(14.0,14.5,15.0,15.5;1.0)
***NPV* method(M$)**		
	Fuzzy NPV	(64.46,82.07,135.04,159.95;0.8)
	NPV mean	**110.63**
***BCR* method(year)**		
	Fuzzy BCR	(6.23,6.74,8.18,9.06;0.8)
	BCR mean	**7.57**
***PBY* method**		
	Fuzzy PBY	(1.68,1.88,2.51,2.83;0.8)
	PBY mean	**2.29**

Note: Units of profit/cost expressed in *Millions of* NT$, otherwise *PBY* in years and *BCR* is dimensionless

A possibility analysis can be performed by setting a specific confidence level in the fuzzy economic models in order to obtain a possible economic value range. For the case of the wind generation system shown in Table I, if two specific confidence levels of 0.6, and 0.4 are selected, the possible *NPV* range, *PBY* range and *BCR* range are then calculated to be [77.74,141.27]M$, [6.61,8.40] year and [1.83,2.59], respectively. It should be noted that a fuzzier (larger interval) economic index is obtained as the lower confidence level is adopted.

5 CONCLUSIONS

This study has derived fuzzy profitability models which enable project investors to perform an economic evaluation of investment alternatives. The proposed economic decision analysis method is more flexible and more intelligent than other methods since it takes the degree of confidence of the decision-makers' opinions into consideration. The cost-benefit analysis is performed using the *NPV*, *PBY* and *BCR* indexes. The geometric moments of the resultant fuzzy indexes are derived in order to determine the relative ranking of the fuzzy economic indexes to support the decision-making process. In a cost-benefit analysis, a higher geometric mean benefit represents a better solution than one with a lower geometric mean benefit. Meanwhile, a computer simulation is performed to explore the main uncertainties typically encountered in this analysis. The results show that the fuzziness of the decision indexes is not significantly influenced by the change in the values of the investment and the annual cost (benefit). However, it is strongly influenced by the values of interest rate *r* and inflation rate *d* due to the presence of the n^{th} power of *r* and *d* within the economic decision indexes. The simulation also shows that a fuzzier economic index is obtained as the lower confidence level is adopted. It is found that regardless of whether the least-cost analysis method or the cost-benefit analysis method is applied, all of the economic measures suggest the same result, and hence any one of the economic decision indexes can be chosen for decision-making purposes.

The performance of the proposed fuzzy economic model is verified by considering their application to a practical wind electricity project in Taiwan. It has been demonstrated that the results generated using the proposed fuzzy models are consistent with those provided by the conventional crisp models. The results of this present study have confirmed that the proposed methods provide readily implemented possibility analysis tools for use in the arena of financial uncertain decision-making.

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