A Hybrid Grey & ANFIS Approach to Bullwhip Effect in Supply Chain Networks

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Abstract: Demand forecasting and decision making processes are among the key activities which directly affect the performance of successful supply chain networks. The variability of the demand information between the stages of the supply chains and the increase in this variability as the demand data moves upstream from the customer to consequent stages of the supply chain networks is called Bullwhip Effect. As demand pattern varies due to the field of activity and architecture of supply chains networks, determining the appropriate forecasting and order decision model for system interested in is complicated. This paper analyzes the response of bullwhip effect to a hybrid grey GM (1, 1) forecasting and ANFIS based order decision model under demand with relatively medium variation in a two stage supply chain network simulation.

Key-Words: Forecasting, Supply chain network, Bullwhip effect, ANFIS, Grey GM (1, 1).

1 Introduction
Supply chain networks (ScNs) are multi stage complex dynamical systems consist of various involved organizations performing different processes and activities in each and consequent stages which are connected through upstream and downstream linkages to produce value in the form of products and services [1, 2].

Bethinking of the definition exposes that the performance of a successful ScN system directly depends on accurate and appropriate demand information, as this vital data influences all decision making processes of ScN. The information flow in ScN consists of cumulative data about costs parameters, production activities, inventory systems and levels, logistic activities and many other related complex processes. But basically; in addition to the architecture, system performance of successful ScNs directly depends on accurate, constant, on time and appropriate demand information flow through the stages of the system grounding on the decision making process and estimated values obtained from the selected forecasting activities and decision making processes performed in each stage.

The variability of the demand information between the stages of ScN and the increase in this variability as the demand data moves upstream from the customer to the consequent stages is called Bullwhip Effect (BE).

This phenomenon triggers several system defects which directly influence total performance of ScNs, such as
undesirable excess inventory levels, inefficient labor force, cost increases, overload errors in production activities and etc. [2, 3, 4].

Just like many other ScN related research topics, BE also has a long research history. The first academic research on this topic grounds on to the pioneer empirical work of Forrester [5, 6] in which with a simple ScN simulation consists of retailer, wholesaler, distributor and factory stages, discovered the existence of BE as ‘demand amplification’ in ScN systems and emphasized on the decision making process in each stage of ScN. After arguing about the possible causes, he concluded that the decision making process and time delays in each phase of ScN and the factory capabilities could be the main reasons of the demand amplification, as the amount of the demand accrual rate amplifies not only by taking account the real demand increases but also potential future increases from the retailer to the factory (upstream through the chain). Sterman used an experimental four-stage ScN role-playing simulation that simulates the beer distribution in ScN which successfully depicts the notion of system dynamics (i.e., Beer Distribution Game) and became world wide teaching tool showing the behavior, concept and structure of ScNs [7, 8]. The model was so simple but despite to its simplicity, it successfully showed the impact of the decision process in each echelon on the demand variability. Lee et al. [9, 10, 11, 12]; focusing on the operational causes of the problem and proving the existence by documentary evidences provided from several companies from different sectors (such as their well-known case P&G), declared four major causes and triggers of BE as demand forecast updating, rationing game, order batching and price fluctuations. Chen et al. [13, 14, 15, 16, 17] studied the effects of forecasting, lead times and information sharing on BE which is quantified as a ratio of demand variances of two consequent stages of simple ScN system. They showed order variances in the upstream stage will be amplified if upstream stage demand decisions are renewed systematically using the monitored values of predecessor stage orders periodically. And also concluded that, even though the demand data is available for all stages (i.e. centralized demand information) in addition to unique the forecasting technique and inventory system in each stage through whole chain, BE will exist. Gavirneni et al. [18], Cachon et al. [19], Dejonckheere et al. [20], Sohn et al. [21], Saeed [22] and Sucky [23] are same of the other researchers who investigated the effects of forecasting and information flow on this phenomenon.

As concluded in many past researches, two major minimization tools for handling this undesirable phenomenon are i.) appropriate and accurate demand forecasting and, ii.) demand/production decision processes. Although motivation for the problem is specific, finding the adequate forecasting model for the demand pattern in many ScNs is snarl [3, 4]. For the dynamic and chaotic systems like ScN, where information is relatively few and the rate of uncertainties are considerably high fuzzy logic(FL) introduced by Zadeh [24] and grey system theory introduced by Deng [25, 26] best fits for application due to the uncertain and vagueness nature of the prediction [27, 28, 29, 30]. But building proper fuzzy rules and determining appropriate membership functions for the parameters of forecasting and decision activities according to the system that will be analyzed is; unfortunately, not simple. This paper analyses the response of BE to a hybrid system consists of grey GM (1, 1) (GrGM) demand forecasting in addition to neuro-fuzzy (more specifically adaptive neuro-fuzzy inference systems; ANFIS) decision making process in a two stage ScN under demand data with relatively medium variation (\( \mu = 50, \sigma_c = 8.685 \)).

The following sections of the paper are organized as follow. In section 2, grey system theory, GrGM, forecasting model and neuro-fuzzy systems (NFS) are explained. In section 3 ScN simulation model is introduced. In sections 4 the application of discussed forecasting and decision models on ScN is analyzed and research findings together are illustrated. And finally in section 5 conclusions are presented.

## 2 GrGM, NFS

### 2.1 GrGM

Grey system theory introduced by Deng [25, 26] is pretty much similar to FL in nature thought it’s completely crisp. The theory also; like FL, is comparatively new. The grey system theory can simply be summarized as a methodology that concerns with the systems comprising uncertainties and lack of sufficient amount of information (like most ScNs); in which, the term ‘grey’ indicates the system information that lays between the clearly and certainly known ones (the white part) and the unknown ones which contains any knowledge of the system structure (the black part); so that grey systems include partially known and partially unknown characteristics [31].

![Fig.3. Grey system theory](image_url)
From this point of view, grey system theory similar to FL, which does not enforce to set a clear-cut boundary between the decision variables and alternatives. Again like FL, it has successfully been applied to many fields including medical, engineering, control and military problems [25, 26, 32].

Differently from statistical forecasting methods which usually needs large past periods data sets to have better regularity of variables of random process, grey theory uses accumulated generating operation (AGO) to obtain regularity via which noise is reduced by converting ambiguous original time series data to a monotonically increased series [33].

The importance of AGO in grey system theory arise from its capability of turning unimproved stochastic data to useful regulars series and inverse accumulated generic operation (IAGO); which is the other important tool of grey system theory, transforms this AGO generated regulars series to row data sequence. The basic idea of the grey model is to construct a regular differential equation with the help of AGO which denoted in general form as GM (n, m) of where, n denotes the order of ordinary differential equation and m denotes the number of grey variable defining the order of AGO and IAGO. As increases in n and m also increases the computation time exponentially causing likely correctness defects, most widely use model in grey system theory is GM (1, 1) which has important advantages those can be summarized as the usage for any kind of data distribution including small data sets and less requirement for computation [4, 33]. The system structure of the GrGM forecasting model which utilizes past data to establish a grey model to predict future is explained as follow [4, 33, 34].

GrGM model simply aims to obtain internal regularity for the available past data that will be used for forecasting and transfer the arranged sequence in to a differential equation to form grey model. Let $D^0$ show on hand data collected from the system as;

$$D^0 = (D_1^0, D_2^0, D_3^0, \ldots, D_n^0)$$

where n represents the number of data. The generated AGO series of $D^0$; $D^1$, then can be denoted as;

$$D^1 = (D_1^1, D_2^1, D_3^1, \ldots, D_n^1)$$

where

$$D_k^i = \sum_{i=1}^{k} D_i^0, \ \forall i = 1,2,\ldots,n$$

(i.e.; accumulated generating sequence data which increases monotonically) and $D_1^1 = D_0^1$. If the model is a GM (n, m) model and regularity can not be reached with one AGO, then operation has to be repeated m times till the data set became more regular.

Composing a differential equation for $D^1$ to establish internal regularity as in 3 the first-order differential equation (as $D_1^1$ increases monotonically enabling it to be approximated by an exponential function having dynamics of a first-order differential equation) can be given as in (4);

$$\frac{dD^1}{dk} + aD^1 = b$$  (3)

$$\frac{dD^1}{dk} = \lim_{h \to 0} \left( D_{k+1}^1 - D_k^1 \right) / h ; \forall k \geq 1$$  (4)

where $a$ and $b$ denotes the developed coefficient and the grey control variable respectively. By setting the sampling interval as one unit ($h = 1$), the first derivative of $D^1$ as a discrete time series can be written as follow;

$$\frac{dD^1}{dk} = \left( D_{k+1}^1 - D_k^1 \right) / h = D_{k+1}^1 - D_k^1 = D_0^1, \forall \geq 1$$  (5)

Setting the second part of the grey model to $D_{average}^1$ equation (3) can be redesign in a matrix form as follow.

$$\begin{bmatrix} D_2^0 \\ D_3^0 \\ \vdots \\ D_n^0 \end{bmatrix} = \begin{bmatrix} -D_{average(1)}^1 & 1 \\ -D_{average(2)}^1 & 1 \\ \vdots \ & \vdots \\ -D_{average(n)}^1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$  (6)

where,

$$D_{average(k+1)}^1 = 1/2 (D_k^1 + D_{k+1}^1)$$  (7)

After applying least square method values of the confidents $a$ and $b$ can be obtained with the following equation,

$$\hat{c} = \begin{bmatrix} a \\ b \end{bmatrix} = (B^T B)^{-1} B^T \bar{D}^0$$  (8)

with the corresponding matrices depicted below.
\[
B = \begin{bmatrix}
-0.5(D_1^0 + D_1^1) & 1 \\
-0.5(D_2^0 + D_2^1) & 1 \\
\ddots & \ddots \\
-0.5(D_n^0 + D_n^1) & 1 
\end{bmatrix}
\]  \quad (9)

\[
\bar{D}^0 = [D_2^0, D_3^0, \ldots, D_n^0]
\]  \quad (10)

where \(n\) denotes the number of data set used and \(\bar{D}^0\) is the raw sequenced data.

Using this coefficients \(a\) and \(b\) equation (3) can also be solved and estimated cumulated value \(\hat{D}^i_{k+1}\) and output forecast; \(\hat{F}^0_{k+1}\), for the period \(k+1\) can be determined with the following equations.

\[
\hat{D}^i_{k+1} = (D_1^0 - (b/a)) - e^{-at} + (b/a) \quad \forall k \geq 1
\]  \quad (11)

\[
\hat{F}^0_{k+1} = \hat{D}^i_{k+1} - \hat{D}^i_k \quad \forall k \geq 1
\]  \quad (12)

### 2.2 ANFIS

Artificial neural networks (ANNs) are mathematical information processing systems which are constituted based on the functioning principles human brains in which neurons in biological neural systems correspond to nodes and synapses correspond to weighted links in ANN [35]. As ANNs are computational models constituted of many interconnected neurons, the basic processing element of the ANNs are neurons. Their way of interconnection also affects the ANN structure in addition to learning algorithm type, activation functions and number of layers. Using logical connections (weighted links) neurons in ANNs get the input from adjacent neurons with the input strength effected by the weight and; using the weighted input broadcasted from the adjacent neurons produce an output with the help of an activation function and broadcast the activation as an input; only one at a time, to other neurons [36]. In the input layer neurons receive input that is given to the system, contrarily the output layer neurons broadcast the ANN output to external environment while neurons in the hidden layers act as a black box providing links for the relation between the input and output [37, 38]. Due to their simple architecture, ability of learning, generalizing, adaptation, parallelism and also capability of to be implemented to software as well as hardware, ANNs are powerful information processing systems that yield successful results for many problems and systems in extensive fields, which made their usage widespread.

![Fig.4. General ANN architecture (feed-forward)](image)

Neuro-fuzzy systems (NFS); which also known as hybrid intelligent systems, can simply be defined as the combination of two complementary technologies: ANNs and FL. This combined system has the abilities of deducing knowledge from given rules (which come from the ability of fuzzy inference systems (FIS)), learning, generalization, adaptation and parallelism (which come from the abilities of ANN). So these hybrid systems cover the frailty of both FL (i.e., no ability of learning, difficulties in parameter selection and building appropriate membership function, etc.) and ANN (i.e., black box, difficulties in extracting knowledge, etc.) and became a robust technology using both systems powerful abilities.

![Fig.4. NFS](image)

The usage of hybrid NFS is rapidly increasing in many areas both civilian and military domain such as process controls, design, engineering applications, forecasting, modular integrated combat control systems, medical diagnosis, production planning and etc. This multilayer fuzzy inference integrated networks use neural networks to adjust membership functions of the fuzzy systems. This structure provides automation for designing and adjustment of membership functions improving desired output by extracting fuzzy rules from the input data with the trainable learning ability of ANNs and also overcomes the black box structure (i.e., difficulties of in understanding and explaining the way it deducts) of
learning process of ANNs. Many studies have been made using different architectures of these hybrid systems, such as architectures fuzzy logic based neurons [39], neuro-fuzzy adaptive models [40] and ANNs with fuzzy weights [41].

ANFIS is the implementation of FIS to adaptive networks for developing fuzzy rules with suitable membership functions to have required inputs and outputs. An adaptive network is a feed-forward multi-layer ANN with; partially or completely, adaptive nodes in which the outputs are predicated on the parameters of the adaptive nodes and the adjustment of parameters due to error term is specified by the learning rules (the other node type is named as fix node) [42, 43]. Generally learning type in ANFIS is hybrid learning. This learning model is appropriate for the systems having unsteady nature like ScNs. Jang [42] defined this learning type as the learning that involves parameter updating after each data is given to the system. In this paper ANFIS is used as a decision tool (together with GrGM) in every echelon to determine the order values to meet estimated demand with the selected input values. The model and the values are explained in the following section.

3 The Simulation Model

In the study, a near beer distribution game extended with ANFIS decision making process and GrGM forecasting model; which is improved from the base beer game of Sterman [7, 44] and its revised version of Paik’s [45] (that includes inventory/capacity restrictions and specific delay functions), is used to simulate a two stage ScN for evaluating the impacts of proposed system using MatLab as the simulation tool.

\[ BWE_{te+k} = \frac{\text{Max}[\sigma_i, \sigma_k]}{\text{Min}[\sigma_i, \sigma_k]}, \quad k = 2, 3 \]  

where, \( \sigma \) denotes the standard deviation of orders placed to upstream stage and subscripts 1, 2, 3 denote the customer, the retailer and the factory respectively.

The game begins with the demand orders placed from the customer to retailer. Retailer tries to meet the demand from its own inventory upon the availability of the stocks. If demand exceeds the inventory level, retailer place order to wholesaler. Also for maintaining appropriate inventory level for the future customer demand, the ordering decision of the retailer must also comprehend customer demand rate for the upcoming periods. And in the same manner the demand and distribution processes go on through the ScN system of the game till the factory stage where beers produced to meet the demand of distributor. So, in each stage except factory, the participants of the game receives demand orders from downstream stage, tries to meet the demand from its own inventory (actual inventory), ships orders to downstream stage, receives shipments from upstream stage and places orders to upstream stage by taking, future demand from downstream stage, desired inventory level together with shipment and orders that have been placed but not received yet into consideration. The only difference in factory; which is the final stage of the game stage, is that the orders placed from the wholesaler are attempt to be met from either factory inventory or by production made in factory.

The ordering/production decision process rule in each phase of the base model is simple but effective as it takes almost all factors reflecting behaviors of ScN [7, 45].

\[ \text{Upstream Order Quantity} = [\text{Forecast Value} + \text{Correction of Inventory} + \text{Correction for Supply Line}] \]  

Simple exponential smoothing (ES) model is used as a crisp forecasting technique for comparison. The formulation of ES is as follow;

\[ F_{t+1} = \alpha D_{t-1} + F_{t-1}(1 - \alpha) \]  

where, \( F_t \) is the forecast value for period \( t \), \( D_{t-1} \) is observation of demand in period \( t-1 \), \( F_{t-1} \) is the calculated forecast value of the previous period \( t-1 \) and \( \alpha \) is the smoothing constant; \( 0 < \alpha \leq 1 \).

The total formulation of system structure in each stage is as follow [4].
\[ OD_{i,t} = FW_t + \alpha(DINV_{i,t} - INV_{i,t}) + \theta \beta(SD_{i,t} - SA_{i,t}) \]  
(16)

\[ OB_{i,t} = OB_{i,t-1} + IO_{i,t-1} - OS_{i,t-1} \]  
(17)

\[ OS_{i,t} = \begin{cases} OB_{i,t} & \text{if } INV_{i,t} \geq OB_{i,t} \\ INV_{i,t} & \text{else} \end{cases} \]  
(18)

\[ INV_{i,t} = INV_{i,t-1} + (IS_{i,t-1} - OS_{i,t-1}) \]  
(19)

\[ DINV_{i,t} = SC_i x FW_{i,t} \]  
(20)

\[ SD_{i,t} = FW_{i,t} x DL_i \]  
(21)

\[ SA_{i,t} = BD_{i,t} + OM_{i,t} + MA_{i,t} + OB_{i,t} \]  
(22)

\[ BD_{i,t} = BD_{i,t-1} + (OD_{i,t} + IO_{i,t+1}) \]  
(23)

\[ OM_{i,t} = OM_{i,t-1} + (DS_{i,t} - IO_{i,t+1}) \]  
(24)

\[ MA_{i,t} = MA_{i,t-1} + (OS_{i,t+1} - IS_{i,t}) \]  
(25)

where, \( OD \) is the order decision, \( FW \) is the forecast value determined from the selected forecasting model, \( INV \) and \( DINV \) are inventory and desired inventory, \( SA \) and \( SD \) are the supply line and desired supply line, \( OB \) is the orders backlogged, \( IO \) is the incoming orders, \( OS \) and \( IS \) are outgoing and incoming shipments, \( SC \) is the safety constant, \( DL \) is the total delay in stage \( i \) at period \( t \). \( \theta \) and \( \beta \) represents the adjustment parameters for inventory and supply line respectively.

The maximum, mean and minimum values of the parameters used in the decision rule which have been estimated by Sterman [7, 45] are illustrated in the following table.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>( \alpha )</th>
<th>( \theta )</th>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Mean</td>
<td>0.36</td>
<td>0.26</td>
<td>0.34</td>
</tr>
<tr>
<td>Maximum</td>
<td>1.00</td>
<td>0.80</td>
<td>1.05</td>
</tr>
</tbody>
</table>

Sterman [8] concluded that “anchoring and adjustment” (i.e.; decision process of the base model) heuristic inconsequent as this heuristic is lack of sensibility to delays and repercussions of ScN system or; as to generalize, lack of “System Thinking”. For all systems used in management determination of the appropriate decision making process is crucial. But especially in ScN, the importance of ordering or production decision is vital as whole ScN system mainly depends on these decisions. In this study, the proposed model contains an ANFIS based decision process in each phase of ScN to determine order quantities (or, the quantity of production in factory stage) using forecast values gathered from the selected forecasting model (GrGM) together with inventory and pipeline information which also are the same input used in the base model. For the proposed decision process 5 inputs are taken into consideration (including received demand data) and Upstream Order Quantity is the output which
determines the order quantity of the stage. The output of the generated FIS in the proposed model is the quantity of orders that will be placed to the upstream phase of the ScN. But, for the factory stage, the output represents the quantity of production for the current time period $t$.

After the performed trials of the simulation the hybrid method (which is a combination of back propagation and least square estimation (the sum of the squared errors between the input and output)) is selected and used for membership function parameter estimation of FIS [46].

The hybrid method optimizes consequent parameter with the premise parameters fixed exploiting the least square estimation in forward pass and, exploiting the gradient in backward pass, adjusts the premise parameters corresponding to the fuzzy sets in the input domain [47]. The appropriate membership functions for the parameters are defined as Gaussian after trials. The selected inference system is Sugeno-type which also is a must arises from the restrictions of the ANFIS editor [46].

The output membership functions of the FIS are evaluated with the performed trial and constant type is chosen.

The following figure illustrates the GrGM process performed in each stage.

**4 Application**

For this specific application, number of randomly generated training data set ($D_{train}$) is 200 (periods), randomly generated demand data ($D_{set}$) for 100 periods, time horizon for the simulation runs are the same as the time horizon of demand data. All delay functions (delivery, clerical and mailing) are 2 periods. Smoothing constant for ES, and the adjustment parameters inventory levels and supply lines are taken as 0.5. Safety stock time horizon is 5 periods. On hand inventory for all stages at the beginning are 100 units and factory capacity is 500 units per period. Number of fuzzy rules for ANFIS applied model in each stage are both 243. Number of epochs are 3500 and 655 for the stages respectively.

The membership function selected for all inputs is Gaussian membership function and for output is constant. The partition method used is Grid partition. Generated and calculated demand values derived from the simulations are illustrated in the following figures.
The response of system are given with standard deviation values for customer orders ($\sigma_c$), retailer demand ($\sigma_r$) and factory production order ($\sigma_f$) are illustrated with Table-2.

**Table-2. Standard deviation values**

<table>
<thead>
<tr>
<th>Model ($\mu = 50$)</th>
<th>Base &amp; ES</th>
<th>GrGM &amp; ANFIS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_c$</td>
<td>8.685</td>
<td>8.685</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>50.6</td>
<td>17.91</td>
</tr>
<tr>
<td>$\sigma_f$</td>
<td>72.2</td>
<td>20.25</td>
</tr>
</tbody>
</table>

BE values for the base and proposed models are illustrated with the following table.

**Table-3. BE values**

<table>
<thead>
<tr>
<th>Model ($\mu = 50$)</th>
<th>Base &amp; ES</th>
<th>GrGM &amp; ANFIS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retailer</td>
<td>5.816</td>
<td>2.02</td>
</tr>
<tr>
<td>Factory</td>
<td>8.126</td>
<td>2.33</td>
</tr>
</tbody>
</table>

Two surface view examples from the hybrid approach; one for retailer and one for factory are illustrated with figure 11 and 12 respectively for the same inputs.

5 **Research Findings and Conclusion**

Ordering decision and forecasting based demand variability is a major factor negatively influencing stability of ScNs. In this study, a hybrid approach consists of ANFIS and GrGM are used together for decision and forecasting processes in a simple two echelon ScN simulation and the response of BE is examined in terms of standard deviations. The simple application in Section 4 showed that; by comparing results gathered from ES model and proposed application, usage of ANFIS together with GrGM forecasting model easily monitored the demand pattern and provided remarkable decreases in demand variability through the ScN which also result in cost and inventory level degreases. Another study; in which other fuzzy grey regression model is used together with ANFIS, is also made by us and similar results obtained, but those results are not illustrated as they are beyond the scope of this of paper.
Further researches can be made using fuzzy cost, time and also inventory policies for improving proposed models applicability complex real ScNs.

References:


