Properties of Fuzzy Systems

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Abstract: - The paper presents a short review of some main properties of the most general fuzzy systems used in a large class of practical applications. The fuzzy systems, implemented using different rule bases, fuzzy values, membership functions, fuzzyfication and defuzzification methods, may be classified based on these fuzzy methods used in their development. This paper proposes a unitary theory for describing Mamdani fuzzy controllers based on their characteristics. Fuzzy rules bases may be seen as fuzzy applications between fuzzy sets. These rule bases have algebraic properties as: commutative law, neutral element and symmetric elements. The fuzzy systems may be analytical described with MISO, SISO and gain transfer characteristics, which may be calculated using computer programs. The fuzzy systems have algebraic properties as: commutative law and symmetric elements. These algebraic properties may be noticed on their transfer characteristics. The fuzzy systems have a variable gain with their inputs. Some gain characteristics values are presented. The rule bases and the fuzzy systems have the sector property. The paper emphasizes, based on the transfer characteristics, planar and spatial sector properties useful in stability analysis with Lyapunov techniques. The transfer characteristics, system linear characteristic around the origin and the gain in origin are usefull in the design of fuzzy PID controllers. Based on transient characteristics of fuzzy control systems some quality criteria are presented. The fuzzy systems used in control assure better control quality criteria and a greater robustness at disturbances effects and at the error at the identification of controlled processes parameters.

Key-Words: - Fuzzy systems, fuzzy logic, nonlinear systems, algebraic properties, absolute global internal stability, fuzzy controllers, empirical control quality criteria, system robustness, circle criterion, fuzzy PID controllers.

1 Introduction

Fuzzy systems are developed using fuzzy logic techniques. They are used in many applications, control being one of the most important. In the world literature papers treating applications in this domain are reported for complex applications as optimal adaptive control [1] and adaptive tracking control [2] of nonlinear processes.

In the old literature books were treated fuzzy systems, presenting the basic theory. One of the most comprehensive text book in fuzzy control being [3], which gives the simplest and the most applicable theory to analyze and design fuzzy systems with application in control. Other textbooks on fuzzy control may be taken in consideration, as [4, 5] and for the basic of fuzz logic [7].

Manny papers are treating, in the main reference journals, the properties of fuzzy systems, emphasizing their advantages [8, 9, 10, 11, 12, 13].

The fuzzy systems, developed based on specific items of fuzzy logic as: fuzzy sets, membership functions, fuzzy operators, fuzzy values, fuzzification, rule bases, inference and defuzzificaton are nonlinear transfer elements, with multivariable inputs and single-variable output. In practice they are developed based on different fuzzy values: for example 3, 5 or 7, different membership functions, different inference methods, for example maxim-min or sum-prod methods and defuzzification may be done using different methods, for example the center of gravity or mean of maxima. A classification of these fuzzy systems is necessary.

The rule bases may be seen as algebraic applications between fuzzy sets. These applications have algebraic properties as: the commutative law, symmetric elements and neutral element. The algebraic properties of fuzzy rules are transposed also in the fuzzy systems, which are composed functions. Such multivariable systems, described with mathematical operations may be seen as algebraic applications between real sets. The real applications attached to a fuzzy system are composed functions, resulted with the composition laws from the rule bases, the inference engine and the defuzzyfication method. The algebraic applications have algebraic properties as the commutative law and symmetric elements.

An analytical description of fuzzy systems is difficult because the complexity of operations made in the interior of these systems. But, input-output transfer characteristics may be obtained using computer calculations. From these input-output characteristics we may obtain numerical and qualitative information about the variable gain of the fuzzy system, a graphic-analytical analysis may be easier done and the linear characterization of fuzzy systems for controller design.

The paper emphasizes some stability characteristics of fuzzy controller and how to use circle criterion in stability analysis.

From an example the paper shows that the fuzzy control system has better quality criteria and it is more robust then a control system based on a linear PI controller.

In the most references in the field of fuzzy systems [3 – 13] such characteristics and properties for fuzzy systems are not presented.

The paper develops this unitary theory as an extension of the conference paper [14].

The author used these properties in applications as: fuzzy control of electrical drives [15] and stabilization of fuzzy control systems [16].

2 Classification

The basic fuzzy systems FS, as it is presented in Fig. 1, has two crisp inputs x_1, x_2 and one crisp output y,

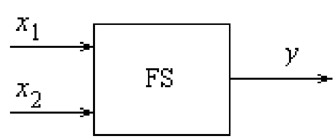


Fig. 1. The block diagram of the basic fuzzy system

described by the general relation:

$$y = FS(x_1, x_2) \quad (1)$$

This fuzzy system is used in fuzzy controllers with dynamics and other applications.

The physical system variables are defined on the following definition sets, called universes of discourse, which are real sets:

$$x_1 \in U_{x1},\ x_2 \in U_{x2},\ y \in U_y \quad (2)$$

For the above crisp variables respective fuzzy variables are defined and these fuzzy variables are taking fuzzy values.

In a theoretical case of study the definition sets are equal:

$$U_{x1} = U_{x2} = U_y = U \quad (3)$$

The fuzzy variables have a finite number n of fuzzy values. For symmetric universes of discourse the fuzzy values are symmetrical face to the origin. The input and output variables may take, for example, 3, 5 or 7 fuzzy values: $n \in \{3, 5, 7\}$. The fuzzy sets with the fuzzy values for these particular cases are:

$$\begin{aligned} U_3 &= \{NB,\ ZE,\ PB\} \\ U_5 &= \{NB,\ NS,\ ZE, PS, PB\} \\ U_7 &= \{NB, NM,\ NS,\ ZE,\ PS, PM,\ PB\} \end{aligned} \quad (4)$$

The fuzzy values are defined and described with membership functions. The membership functions are defined with relations as:

$$m_U(x) : U_{Rx} \rightarrow [0, 1] \quad (5)$$

were U_{Rx}=[-a, a] $\subset R$..

For the linguistic treatment a definition with membership functions of the input variable is needed. We may use equidistant membership functions, for the input variables and for the output variable, as they are shown in Fig. 2,

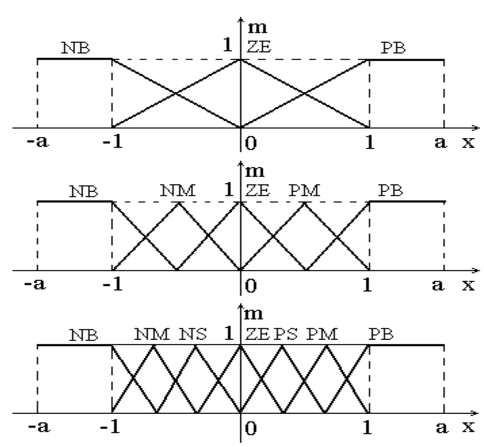


Fig. 2 Equidistant membership functions

or non-equidistant membership functions only for the output variable, as they are shown in Fig. 3.

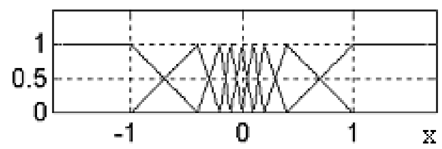


Fig. 3 Non-equidistant membership functions

For the input fuzzy variables we may use 3 to 7 fuzzy values and for the output fuzzy variables we may use 3 to 9 fuzzy values. The means of the fuzzy values are standard: *NB, NM, NS, ZE, PS, PM, PB*. The 8^{th} and the 9^{th} fuzzy values are *NVB* and *PVB*. The rising of the number of the fuzzy value does not ameliorate the dynamical behavior of the fuzzy controller and it complicates the formula of the inference rules. On the other hand the enlargement of the number of the fuzzy values tends to the linear character of the fuzzy system transfer characteristics. This aspect will be presented in this paper.

The membership functions have symmetrical forms face to zero, for a symmetrical command in four quadrants.

The membership functions have a unitary coverage of the universe of discourse. So, the presence of empty spaces between two next functions is avoided. These empty spaces will lead to the appearance of non-intervention zones (dead zones) of the controller. That often leads to the instability of the control. Also, this avoids the appearance of overlapped zones with membership grade greater then 1.

With these membership functions the fuzzy sets are defined, with relations like this:

$$A = \{x, m_U(x)\} \qquad (6)$$

The output variable is also scaled in the domain [-1, 1] that makes necessarily an adequate scaling factor at the output of the fuzzy controller. This scaling factor will be used in the control system as a part of the gain of the open loop. And it affects the internal stability of the closed loop.

For a fuzzy system several combinations may be developed, using the above membership functions. For example: *33, 35, 55, 57, 77, 79* membership functions, and other combinations like these. In this representation the first number represents the number of the input fuzzy value number and the second number represents the output fuzzy value number.

Also, the form of the output membership functions may be included in this representation: with an *e*, for equidistant forms and an *n* for non-equidistant forms. For example: *35e, 55e* or *79n* and other combinations like these. Different forms of the membership functions may be used for the output variables.

For the variables of the fuzzy system in this paper we are using trapezoidal and triangular forms. The variables of the fuzzy controller take values only in a scaled symmetrical universe of discourse [-1, 1] or [-*a*, *a*], were $a \in R_+^*$. The scaling factors may be chosen based on the operator knowledge about the controlled process. For example, in the case of a large transient regime the variables must not pass over the physical admissible domain. But in the time of a large transient regime the domain [-1, 1] will be exceeded. So, the membership functions must have values also out of the domain [-1, 1]. To counterattack the inconvenient overpasses of the imposed limits at the inputs of the controller we may use the limitations of the input variables x_1 and x_2.

The membership functions have quantification due to the analog to digital conversion of the measured variables from the controlled process and on the other hand due to the representation of the membership degree by a digital variable with a finite length. The inference is done using for the universes of discourse a finite number of values.

In the interior of the fuzzy system the linguistic variables are linked by rules, that are taking account of the static and dynamic behavior of the control system and also they are taken account of the limitations imposed to the controlled processed. Rule bases used in practice are developed taken account of the number of the fuzzy values for the input and output variables. For example: rule bases *33*, *55*, *57*, *77* or *79* mean: 3, 5, 5, 7 and 9 fuzzy values for the inputs and 3, 5, 7, 7 and 9 fuzzy values for the output, and other combinations like these. The rule bases may be developed for a symmetrical command in four quadrants.

The general diagram of a rule table is presented in Fig. 4.

Rule bases with 3x3, 5x5 or 7x7 rules are used in common applications. The rule base for 3 fuzzy values variables, called *the primary rule base*, is presented in Tab. 1.

The rule base for 5 fuzzy values variables is presented in Tab 2.

And the rule base for 7 fuzzy values variables is presented in Tab. 3.

In particular, the control system must be stable and it must assure a good amortization. After the inference we obtain fuzzy information for the output variable.

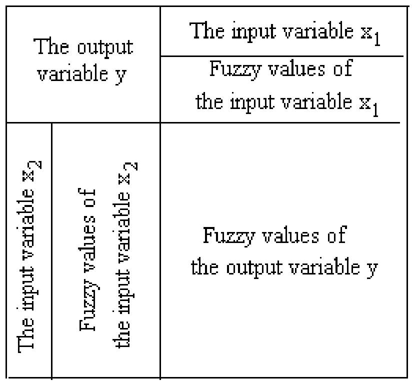


Fig. 4 The general diagram of the rule table

Tab. 1 The 3x3 (primary) rule base

y		*x_1*		
		NB	*ZE*	*PB*
x_2	*NB*	*NB*	*NB*	*ZE*
	ZE	*NB*	*ZE*	*PB*
	PB	*ZE*	*PB*	*PB*

Tab. 2 The 5x5 rule base

y		*x_1*				
		NB	*NM*	*ZE*	*PM*	*PB*
x_2	*NB*	*NB*	*NB*	*NB*	*NM*	*ZE*
	NM	*NB*	*NB*	*NM*	*ZE*	*PM*
	ZE	*NB*	*NM*	*ZE*	*PM*	*PB*
	PM	*NM*	*ZE*	*PM*	*PB*	*PB*
	PB	*ZE*	*PM*	*PB*	*PB*	*PB*

Tab. 3. The 7x7 rule base

y		x_1						
		NB	*NM*	*NS*	*ZE*	*PS*	*PM*	*PB*
x_2	*NB*	*NB*	*NB*	*NB*	*NB*	*NM*	*NS*	*ZE*
	NM	*NB*	*NB*	*NB*	*NM*	*NS*	*ZE*	*PS*
	NS	*NB*	*NB*	*NM*	*NS*	*ZE*	*PS*	*PM*
	ZE	*NB*	*NM*	*NS*	*ZE*	*PS*	*PM*	*PB*
	PS	*NM*	*NS*	*ZE*	*PS*	*PM*	*PB*	*PB*
	PM	*NS*	*ZE*	*PS*	*PM*	*PB*	*PB*	*PB*
	PB	*ZE*	*PS*	*PM*	*PB*	*PB*	*PB*	*PB*

The problem is to determine what algebraic properties are presented in the most common rule bases of the fuzzy controller, seen as a fuzzy application between fuzzy sets.

The crisp input variables are fuzzyfied. In the internal structure of the fuzzy system FS on the fuzzy variables an inference method is applied.

The rules from the rule bases are transposed into digital computation by the inference methods. The control strategy depends essentially on the adopted inference method. The inference links the measured variables, which are input variables (transformed in linguistic variables by fuzzyfication) by the output variable, which are also expressed as a linguistic variable.

In the general applications several methods to describe inference are used: matrix of inference, symbolic description or linguistic description.

Operators as AND, OR are used in the interior of the rule and to link rule one with another. In all cases more then one rule are activated.

Many inference methods may be used. For example: inference min-max (*mm*) or sum-prod (*sp*) and other combinations of these.

Because in general the actuator that must follow after the fuzzy controller must be commanded with a crisp value y_d, the defuzzification is used. The output variable furnished by the fuzzy system, is obtained by filtering the defuzzified variable y_d. The output variable of the fuzzy system used as a controller is the command input for the process.

Several defuzzification method as the center of gravity (*g*) or mean of maxima (*m*) are used in practice.

The fuzzy system does not treat a well-defined mathematical relation (an algorithm), as a linear system is treating, but it is using inferences with many rules, based on linguistic variables. The inferences are treated with the operators of the fuzzy logic. The fuzzy system has in its structure from Fig. 5 three distinctive parts: fuzzyfication, inference and defuzzification.

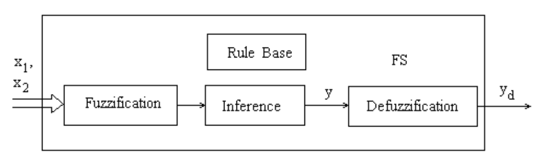


Fig. 5 The structure of a general fuzzy system

At the input there is the fuzzyfication interface. The inference is done using the rule base to obtain a fuzzy information *y*. At the output there is the defuzzyfication interface, which offers the output crisp value y_d.

The fuzzy system is a non-inertial system. The properties of the rule bases are transposed in the fuzzy system, defined as application on real sets. The fuzzy controllers are composed applications of the inference rules and defuzzification. The inference rules are made based on the rule bases. So, all the properties of the rule bases are transferred to the fuzzy controllers, by the composite laws.

The fuzzification, the inference and the defuzzification bring a nonlinear behavior of the fuzzy controller. For a better description of the fuzzy controllers some input-output transfer characteristics may be used. These characteristics are presented in a next paragraphs.

Using the input-output transfer characteristics we may linearized the fuzzy controller around the origin – for a permanent regime.

As we have seen, several types of controllers may be developed with the above characteristics. A classification of these controllers is necessary. In the next paragraph a simple classification is proposed for these controllers.

In this paper we propose the following way of classification of the fuzzy systems. The type of the fuzzy controllers is formed from a group of digits and letters, that is a symbol of the rule base (*rb*), the inference method (*im*) and the defuzzification method (*dm*), in this way:

$$\text{The type of the fuzzy system}: rb - im - dm \quad (7)$$

where:

-*rb*=*xxs*, for the rule base *xx*, were *xx* may be: 33, 35, 55, 57, 59, 77 and 79. In the case of the equidistant membership functions we use the suffix *s*=*e*. In the case of non-equidistant membership functions we use the suffix *s*=*n*.

-*im*=*yy*, were *yy* may be *mm* for inference with the method min-max or *sp* for the inference with the method sum-prod;

-*dm*=*z*, were *z* may be *g* for the center of gravity and *m* for the defuzzification with the method mean of maxima.

Based on the above classification some examples may be give: *33-mm-g* means a fuzzy controller with a 3 fuzzy values for the inputs and 3 fuzzy values for the output, max-min inference and defuzzification with center of gravity; *57e-sp-m* means a fuzzy controller with a 5 fuzzy values for the inputs and 7 fuzzy values for the output, the membership functions of the output are non-equidistant, sum-prod inference and defuzzification with mean of maxima, and *79n-mm-g* means a fuzzy controller with 7 fuzzy values for the input, 9 fuzzy values for the output, the membership functions of the output are non-equidistant, min-max inference and defuzzfication with center of gravity.

The problem is to determine what algebraic properties are presented in the most common fuzzy systems, with the structure from Fig. 6, from the above classification, seen as real applications between real sets.

3 Algebraic Properties of Rule Bases

The rule bases may be seen as algebraic applications between fuzzy sets. The algebraic applications have algebraic properties as the commutative law, symmetric elements and neutral element. The algebraic properties of the fuzzy rules are transposed also in the systems that are composed functions, resulted with the composition laws from the rule bases. In the most important references in the field of the fuzzy sets such properties for the rule bases are not presented.

3.1 Rule Bases as Fuzzy Composition Laws

With the definition sets for the fuzzy values, as (3, 4) a fuzzy application may be defined:

$$f : UxU \to U,\ (x_1, x_2) \to f(x_1, x_2) \quad (8)$$

that we call a fuzzy composition law defined over the set U. The unique defined element $y=f(x_1,x_2)\in U$ through the application f we call the composed of x_1 and x_2 through the law f. A rule table describes the law f, for example the rule bases from Tab. 1, 2 and 3. We will demonstrate some algebraic properties of the basic fuzzy controller. We use the symbol * for the composition law f:

$$UxU \to U, (x_1, x_2) \to x_1 * x_2 \quad (9)$$

3.2 Algebraic properties

3.2.1 Commutative law

The commutative law is verified on the rule tables. For example on the rule table 1:

$$x_1 * x_2 = x_2 * x_1,\ \forall x_1, x_2 \in U \quad (10)$$

The element x_1*x_2 from the crossing of row x_2 with the column of x_1 is equal to the element x_2*x_1, from the crossing of column x_1 with the line x_2, for any x_1, $x_2\in U$. This is equivalent to the property that the rule table is symmetric face to the main diagonal. All the above rule bases have the commutatively law, also in the case that the input and output variables have not the same definition sets.

3.2.2 Neutral element

Definition: An element $o\in U$ is called neutral for a composition law Ux$U\to U$, $(x_1, x_2)\to x_1*x_2$ if

$$o * x = x * o = x,\ \forall x \in U \quad (11)$$

The neutral element is unique.

We notice from the rule table that the neutral element is the fuzzy value *ZE*.

3.2.3 Symmetric elements

Definition: An element $x \in U$ is called symmetric through the composition rule Ux$U \rightarrow U$, $(x_1, x_2) \rightarrow x_1 * x_2$, (with a neutral element) if there is $x' \in U$ with the property:

$$x' * x = x * x' = o \qquad (12)$$

Then $x' \in U$ is called the symmetric of x.

We notice that the symmetric element is the opposed element. For example, in the 3x3 rule base *NB*PB=PB*NB=ZE*, in the 5x5 rule base *PM*NM=ZE*, or in the 7x7 rule base *NS*PS=ZE*.

The symmetrical elements verify the algebraic definition:

$$(-x) * x = x * (-x) = ZE \qquad (13)$$

If *ZE*ZE=ZE*, it results that the neutral element is symmetrical face to *ZE*, and his symmetrical is *ZE*.

3.2.4 Associatively law

From the study of the rule bases results that the fuzzy compositional laws that generate that rule bases have not the associatively law. This thing may be demonstrated taken, for example from the 3x3 rule base (*NB*PB*)**PB=PB* and *NB**(*PB*PB*)*=ZE*. Also there are cases when we cannot define $(x_1 * x_2) * x_3$

So, because the fuzzy composition laws f have not the associatively law, they can define neither the simplest algebraic structure on the definition sets U.

4 Transfer Characteristics

4.1 Generalities

An analytical description of the fuzzy systems is difficult because of the complexity of operations made in the interior of these systems: fuzzyfication, inference and defuzzyfication. In this paper some input-output transfer characteristics fuzzy systems are illustrated. These transfer characteristics may be calculated using computer programs. And, based on the graphical characteristics presented in this paper a grapho-analytical analysis may be easier done. Some properties of the fuzzy systems may be noticed on these characteristics. Some important values and properties of the transfer characteristics may be used in the design of fuzzy PID controllers and in the stability analysis of fuzzy control systems. In the most important references in the field of the fuzzy sets such characteristics and properties for the fuzzy systems are not presented. This paper proposes an unitary theory for describing fuzzy systems based on their input-output transfer characteristics.

The fuzzy systems have in the most common cases two input variables x_1 and x_2 and one output variable y, as Fig. 1, 5 and 6 show. The input variables are taken from the control system. The inference is doing a digital treatment, based on the linguistic rules from the rule base, of the input variables x_1 and x_2, obtained by the filtration of the system input variables and offers a linguistic variable y of the output. The linguistic variable y is sent to the output interface of defuzzification, to obtain a crisp value y_d.

The membership functions taken for this study for the input and output variables of the fuzzy controllers have general shapes. They are defined in concrete on specific universes of discourse.

The characteristics of the fuzzy systems were obtained for different membership functions in the cases of two inference methods: min-max and sum-prod, and the two defuzzification methods: center of gravity and mean of maxima.

So, for a fuzzy system the following transfer characteristics may be obtained.

In the case that the fuzzy system is used for control, its output will be the control variable: $y_d = u_d$, the notation used in the following presentation.

4.2 SISO Transfer Characteristics

Families of SISO transfer characteristics, defined with the function $f_1(x_1; x_2)$ considering for the fuzzy controller as an output variable u_d, a input variable x_1 and x_2 as a parameter:

$$u_d = f_1(x_1; x_2),\ x_1 \in [-2;2], \qquad (14)$$
$$x_2 \in \{-1;-0,75;-0,5;-0,25;0;0,25;0,5;0,75;1\}$$

4.3 MISO Transfer Characteristics

The MISO transfer characteristic of the fuzzy controller as a three-dimensional surface, considering as an output variable u_d and x_1 and x_2 as input variables:

$$u_d = f_{FC}(x_1, x_2),\ x_1 \in [-2;2],\ x_2 \in [-2;2] \qquad (15)$$

4.4 Translated SISO Characteristics

Families of transfer characteristics $u_d = f(x_t;\ x_2)$, with

x_2 as parameter:

$$u_d = f(x_t; x_2), x_t \in [-4;4], \quad x_2 \in \{-1;-0,75;-0,5;-0,25;0;0,25;0,5;0,75;1\} \tag{16}$$

were x_t is a new variable, composed, defined by summation of x_1 with x_2:

$$x_t = x_1 + x_2,\ x_1 \in [-2;2],\ x_2 \in [-2;2] \tag{17}$$

If we analyze these characteristics we may notice that they are obtained from the SISO transfer characteristics by translation. So, the characteristics situated at the left of the origin are translated to the right, in the origin. And the characteristics situated at the right of the origin are translated to the left, in the same way to the origin.

4.5 Gain Characteristics

Families of characteristics of the gain function:

$$K_{FC}(x_t; x_2) = \frac{u_d}{x_t}, x_t \in (0;4], \quad x_2 \in \{-1;-0,75;-0,5;-0,25;0;0,25;0,5;0,75;1\} \tag{18}$$

Utility of these transfer characteristics is in the design of the fuzzy control systems based on fuzzy controllers:

-For the design of the parameters of the PID controllers which are using in their structure such fuzzy controllers.

-For the stability analysis of the fuzzy control systems that are using this type of fuzzy controllers. Some of characteristics are presented on.

The transfer characteristics are obtained for specific scaled universes of discourse.

4.6 Exemplifications

We offer some examples of characteristics for the rule bases 33 and 79, min-max and sum-prod inferences, defuzzification with center of gravity and mean of maxima, and equidistant and non-equidistant membership functions. For an easier identification of the characteristics we make the following notation from Fig. 6.

According to the above notation the characteristic types may be: *cs* – SISO characteristics; *ct* – translated SISO characteristics; *k* – gain characteristics; *s* – MISO characteristics. The rule base may be: 33, 35, 55, 57, … , 77, 79. The membership functions may be: *e* – equidistant; *n* – non-equidistant. The inference method may be: *mm* – min-max; *sp* – sum-prod. The defuzzification may be: *g* – center of gravity; *m* – mean of maxima. For example: *cs79emmg* means the SISO characteristics of the fuzzy controller with the rule base 79, with equidistant membership functions, min-max inference and defuzzification with center of gravity.

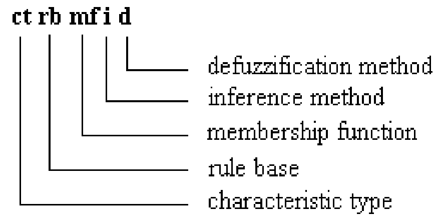


Fig. 6 Notation of transfer characteristics

In fig. 7 - 42 we present some examples of transfer characteristics.

4.7 Analysis of Transfer Characteristics

In this paper a classification of the fuzzy systems was made. The classification is taken account of membership functions, rule bases, inferences and defuzzification types. For the general no inertial fuzzy systems the MISO, SISO, translated and gain characteristics were presented. From these characteristics some general properties for all these fuzzy systems were illustrated.

The examples were obtained combining 7 rule bases, membership function equidistant or non-equidistant, two inference methods and two defuzzification methods.

Many fuzzy systems may be obtained, using other rule bases, other inference methods, and other defuzzification methods. But, the fuzzy systems analyzed in this paper are the most used in practice. We may take Other forms for the membership functions may be chosen, for example Gauss shape, or Λ, instead of triangle and Γ and *L* instead of trapeze. The non-equidistant membership functions may have different forms.a great number of fuzzy values, greater then 7. As inference method other combination of max and min with sum and prod, for example max-prod or sum-min. But, for all these kind of fuzzy systems the transfer characteristics from this paper are available, and the properties illustrated are presented in all.

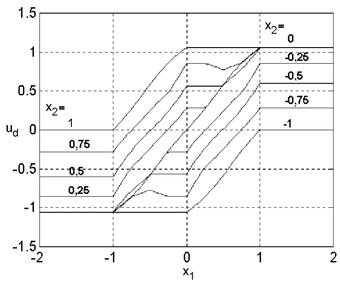


Fig. 7. Cs33mmg

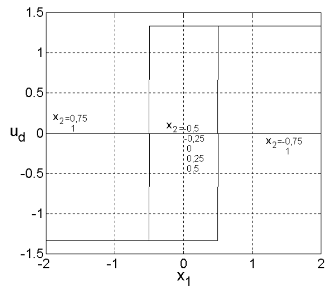


Fig. 8. Cs33mmm

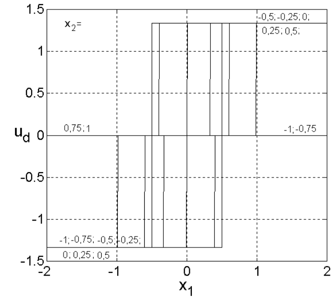


Fig. 9. Cs33spm

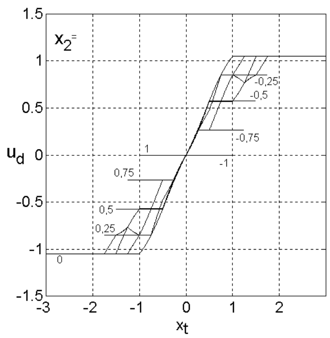


Fig. 10. Ct33mmg

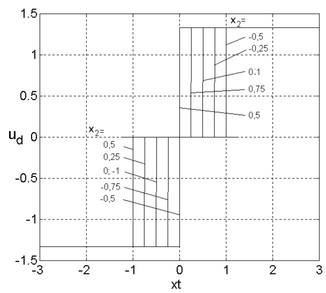


Fig. 11. Ct33mmm

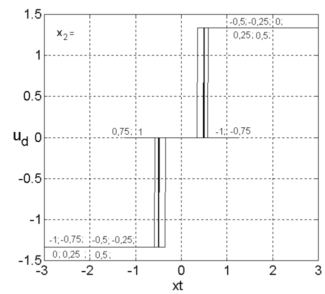


Fig. 12. Ct33spm

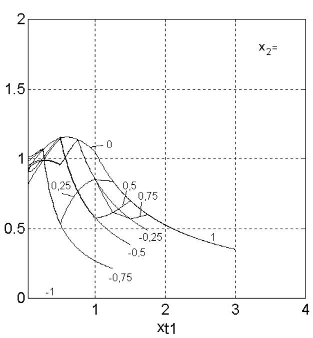


Fig. 13. K33mmg

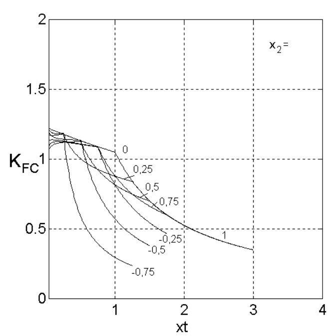


Fig. 14. K33spg

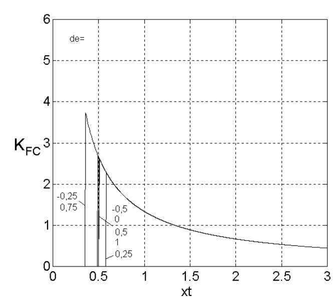


Fig. 15. K33spm

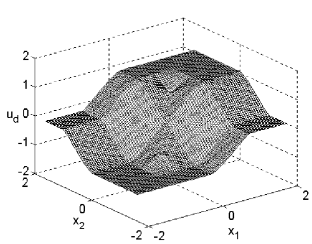


Fig. 16. S33mmg

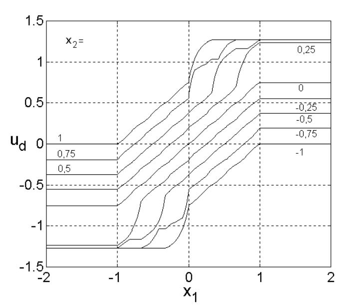


Fig. 17. Cs79emmg

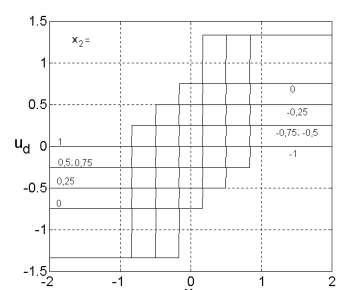


Fig. 18. Cs79emmm

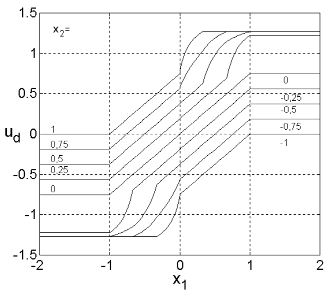


Fig. 19. Cs79espg

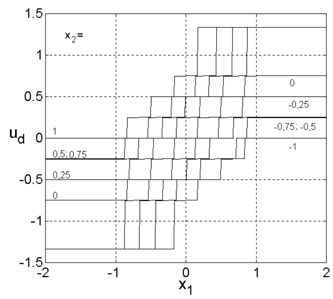


Fig. 20. Cs79espm

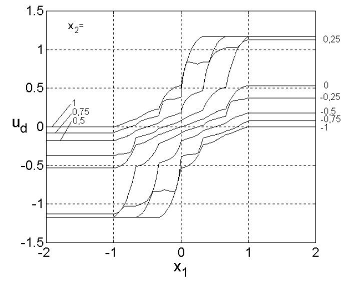


Fig. 21. Cs79nmmg

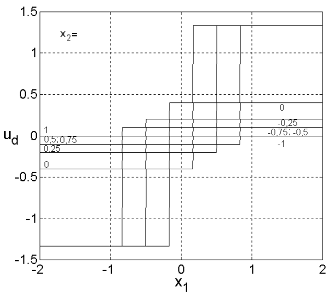


Fig. 22. Cs79nmmm

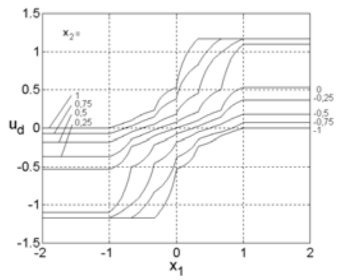
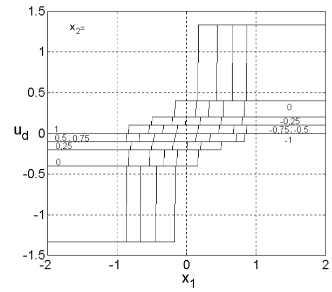


Fig. 23. Cs79nspg



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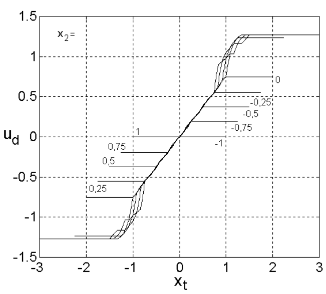

Fig. 25. Ct79emmg

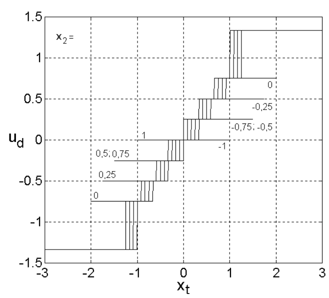

Fig. 26. Ct79emmm

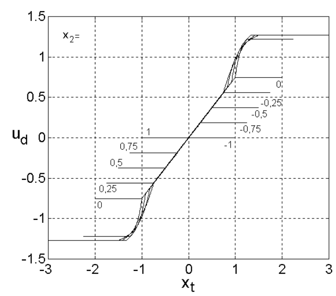

Fig. 27. Ct79espg

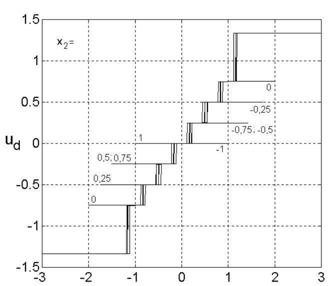

Fig. 28. Ct79espm

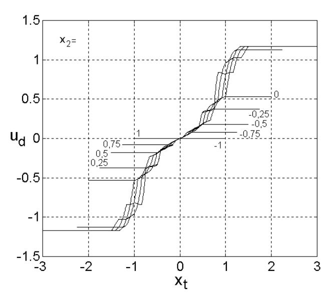

Fig. 29. Ct79nmmg

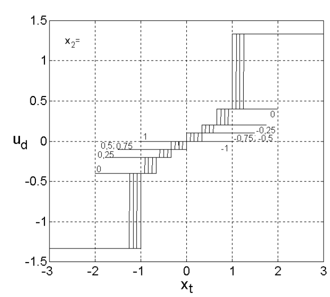

Fig. 30. Ct79nmmm

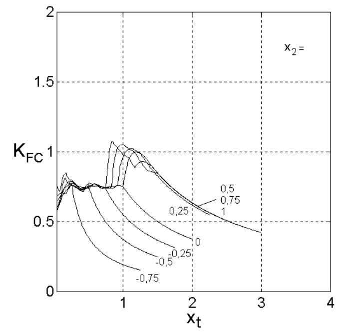

Fig. 31. K79emmg

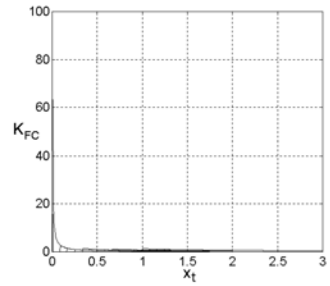

Fig. 32. K79emmm

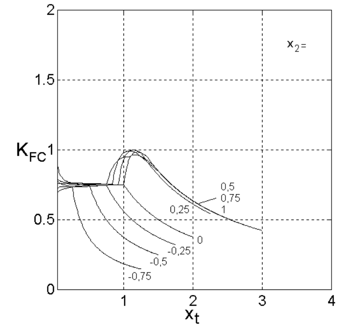

Fig. 33. K79espg

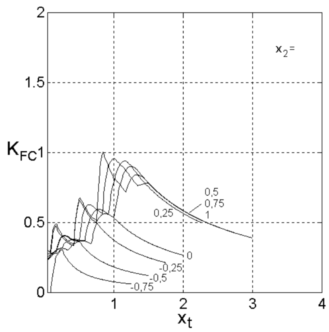

Fig. 34. K79nmmg

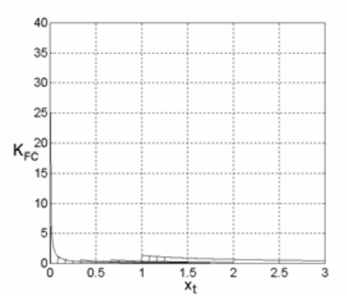

Fig. 35. K79nmmm

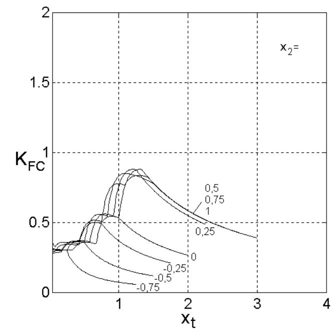

Fig. 36. K79nspg

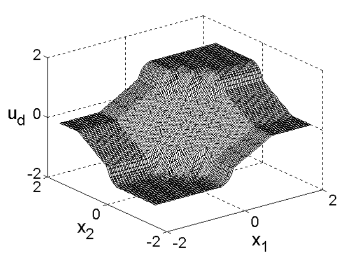

Fig. 37. S79emmg

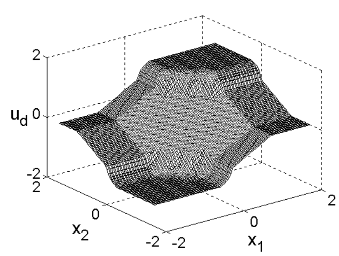

Fig. 38. S79espg

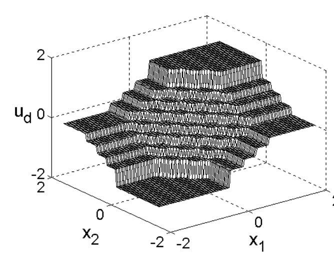

Fig. 39. S79espm

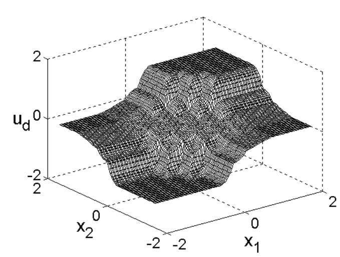

Fig. 40. S79nmmg

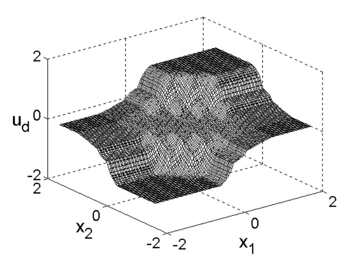

Fig. 41. S79nspg

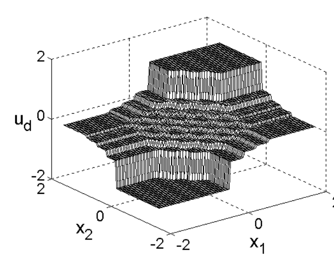

Fig. 42. S79nspm

Comparing the characteristics shown in this paper we may present the following conclusion.

-The transfer characteristics are according to the base rule of the fuzzy systems;

-For all methods used for developing fuzzy systems, these characteristics are non-linear. There are non-linear parts, more or less marked. Non-linear characteristics are obtained even the membership functions for the input variables are triangular and equidistantly. The non-linearity is more marked if the method max-min is used. The use of the AND operator provokes an important non-linear characteristics.

-The inference sum-prod leads to more settled non-linear characteristics. The fuzzy systems that use sum-prod inference have characteristics with large linear parts.

-If the number of the fuzzy values increases the characteristics of the fuzzy systems become more linear. They are close to the characteristics of a state space controller.

-The fuzzy systems with the same rule base and which are using the same inference method min-max or sum-prod have very closed transfer characteristics. Their characteristics are different only by simple displacements from a straight line.

All the characteristics of the family $y_d=f(x_t;\ x_2)$ are placed only in the first and the third quadrant. The function $y_d=f(x_t;\ x_2)$ has the following properties:

$$\begin{aligned} u_d &= 0 \quad for \quad x_t = 0 \\ u_d &> 0 \quad for \quad x_t > 0 \\ u_d &< 0 \quad for \quad x_t < 0 \end{aligned} \qquad (19)$$

The function $K_{FC}(x_t;\ x_2)$ is always a positive function. It is an even function, its graphic is placed symmetrical face to the ordinate axis. It fulfills *the sector condition* in a plane, described by the double inequality:

$$0 \le K_{FC}(x_t; x_2) \le K_M \qquad (20)$$

The function of two variables $y_d=f_{FC}(x_1,\ x_2)$ attached to the fuzzy system fulfills a *sector condition in space*, by the form:

$$\begin{aligned} & f_{FC}(x_1, x_2)\{ f_{FC}(x_1, x_2) - \\ & - K_M \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T \} \le 0 \end{aligned} \qquad (21)$$

-The fuzzy systems that are using defuzzification with the method mean of maxima presents on there characteristics $y_d=f(x_t;\ x_2)$ leaps on many levels. The number of the leaps is equal to the number of the fuzzy values of the output variable. The values of the output leaps are correlated with the values from the universe of discourse of the output for those the membership functions take their maximum values.

-The fuzzy systems which are using the max-min inference and defuzzification with mean of maxima have characteristics $y_d=f(x_t;\ x_2)$ which are placed in the sector [0, +∞]. All the other systems have these characteristics in a sector [0, K_m].

-In general, the fuzzy systems which are using defuzzification with center of gravity do not present a constant values around the origin of the variable x_t of the function $K_{FC}(x_t;\ x_2)$, that is next to the bisectorial plane of the MISO transfer characteristic. Otherwise said, the characteristics of the family $f(x_t;\ x_2)$ has tangent line in the origin with different slopes. This thing is more pronounced to the fuzzy systems that are using non-equidistant membership functions.

-For the values greater then 1 of the variable x_t the values of the function $K_{FC}(x_t;\ x_2)$ are decreasing due to the phenomenon of saturation presented in the transfer characteristics of the fuzzy controllers.

Comparing the values determinate from the characteristics K_{FC}, we may take the following conclusions:

-The fuzzy system that are using the max-min inference and defuzzification with mean of maxima presents very great values of the gain around the origin of variable x_t. This means that the function K_{BF} of these fuzzy systems has the maximum value when the input variables x_1 and x_2 are both zero or they have opposed values, that is they are situated on the bisector plane of MISO characteristics. This fact results also from the rule bases of these fuzzy systems.

-The fuzzy systems that are using the sum-prod inference and defuzzification with mean of maxima have zero values of the function K_{FC} in origin and in its proximity.

-The fuzzy systems that are using the center of gravity presents maxim values of the function K_{FC} in general in the zones situated at the middle of the positive domains and respective of the negative domains of the universe of discourse.

The forms of the transfer characteristics may influence the behavior of a fuzzy controller and the fuzzy control system, seen in the quality criteria of the control.

These characteristics are a powerful tool for designing and analyzing fuzzy control systems.

5 Variable Gain

The fuzzy systems have variable transfer gain, that may be notice on the family of gain transfer characteristics.

In Tab. 4 we present the maximum values K_M of the function $K_{FC}(x_t;\ x_2)$. Also, for this function the values at the limit in origin:

$$K_0 = \lim_{x_t \to 0} K_{FC}(x_t;0) \quad (22)$$

are enumerated.

The value of K_0 is appreciated from the characteristic $K_{FC}(x_t;\ x_2)$ obtained for x_2=0, for small values of x_t. Also, for the fuzzy systems that are using defuzzification with mean of maxima in this table the number of levels of the SISO characteristics.

Tab. 4 Specific gain values of fuzzy systems

Fuzzy system type	K_M	K_0	Levels
33-mm-g	1,17	0,9	-
33-mm-m	350	Maxim	3
33-sp-g	1,22	1,22 - Maxim	-
33-sp-m	3,8	Minim (0)	3
35e-mm-g	0,9	0,34	-
35n-mm-g	1,2	0,75	-
35e-mm-m	125	Maxim	5
35n-mm-m	70	Maxim	5
35e-sp-g	0,7	0,50	-
35n-sp-g	0,83	0,83 – Maxim	-
35e-sp-m	1,4	Minim (0)	5
35n-sp-m	1	Minim (0)	5
55e-mm-g	1,6	0,68	-
55n-mm-g	1,7	1,49	-
55e-mm-m	180	Maxim	5
55n-mm-m	110	Maxim	5
55e-sp-g	1,35	1,00	-
55n-sp-g	1,49	1,49 – Maxim	-
55e-sp-m	2,0	Minim (0)	5
55n-sp-m	1,8	Minim (0)	5
57e-mm-g	0,9	0,45	-
57n-mm-g	0,9	0,34	-
57e-mm-m	130	Maxim	7
57n-mm-m	36	Maxim	7
57e-sp-g	0,85	0,67	-
57n-sp-g	0,8	0,40	-
57e-sp-m	1,3	Minim (0)	7
57n-sp-m	1,1	Minim (0)	7
59e-mm-g	0,7	0,34	-
59n-mm-g	0,6	0,15	-
59e-mm-m	105	Maxim	7
59n-mm-m	40	Maxim	7
59e-sp-g	0,65	0,50	-
59n-sp-g	0,6	0,20	-
59e-sp-m	1,0	Minim (0)	9
59n-sp-m	0,75	Minim (0)	9
77e-mm-g	1,45	0,71	-
77n-mm-g	1,7	0,91	-
77e-mm-m	85	Maxim	7
77n-mm-m	25	Maxim	7
77e-sp-g	1,3	1,02	-
77n-sp-g	1,2	0,92	-
77e-sp-m	2,6	Minim (0)	9
77n-sp-m	1,7	Minim (0)	9
79e-mm-g	1,1	0,52	-
79n-mm-g	1	0,21	-
79e-mm-m	65	Maxim	
79n-mm-m	25	Maxim	
79e-sp-g	1,0	0,76	-
79n-sp-g	0,8	0,30	-
79e-sp-m	2,1	Minim (0)	9
79n-sp-m	1,2	Minim (0)	9

As we may see on the input-output transfer characteristics the gain from the input to the output of the fuzzy system is variable function of the inputs. This is an important property in the stability analysis of the fuzzy control systems [16].

6 Linear Transfer Characteristic

With the translated characteristics $K_{FC}(x_t;\ x_2)$

$$K_{FC}(x_t;x_2) = f_t(x_t;x_2)/x_t,\ x_t \neq 0 \quad (23)$$

the MISO transfer characteristic of the fuzzy block may be written as follows:

$$u_d = f_{FC}(x_1,x_2) = K_{FC}(x_1,x_2).(x_1+x_2) = \\ = K_{FC}(x_t;x_2).x_t \quad (24)$$

If the fuzzy system is characterized with the linear transfer characteristic around the point of the permanent regime x_1=0, x_2=0 and y_d=0, the following relation is obtained:

$$u_d = K_0(x_1+x_2) \quad (25)$$

The value K_0 is the value at the limit, in origin of the characteristic $K_{FC}(x_t;\ X_2)$:

$$K_0 = \lim_{xt \to 0} K_{FC}(x_t;x_2), x_2 = 0 \quad (26)$$

This value may be determined with a good approximation, at the limit, from the gain characteristics.

7 System Properties

7.1 Fuzzy System Seen as Application Between Real Sets

A fuzzy systems FS most common in practice has two input real variables x_1 and x_2 and one output real variable y. A fuzzy system has the structure presented in Fig. 6, in which we may see the fuzzyfication, the rule base, the inference and the defuzzification procedures. We may attach to this fuzzy system a real application f.

The physical variables of the fuzzy system has membership functions:

$$m_U(x) : U_{Rx} \to [0, 1] \quad (27)$$

were U_{Rx} =[-a, a]$\subset R$. The most common membership functions used in applications are presented in fig. 3. The variables x_1, x_2 and y take values in the real universes of discourse U_{x1}, U_{x2} and U_y. In general, in theory, the universes of discourse are taken with a norm and then we may consider

$$U_{x1} = U_{x2} = U_y = U = [-1, 1] \quad (28)$$

The application f is defined on the Cartesian product UxU, with values in U:

$$f : UxU \to U, (x_1, x_2) \to f(x_1, x_2) \quad (29)$$

and it is a composition law on U.

The application f, based on the way that we defined it above is an application on real sets with values on a real set. This application may be called as a real fuzzy compositional law, taken in consideration the fact that its definition domains and the value domain are real sets. The application f describes the input output transfer characteristic of the fuzzy system, seen as an application between real sets.

The compositional laws may be called and annotated as the fuzzy systems, related to the inference and defuzzyfication methods used in the fuzzy system. For example a compositional law of a fuzzy system with 3x3 rule base, with min-max inference (*mm*) and defuzzyification with the center of gravity (*g*) may be called the compositional law *33-mm-g*. In general we may attached the following symbol: *rb-im-dm* to a fuzzy system, were *rb* means the rule base type, *im* means the inference method and *dm* means the defuzzyication method.

A fuzzy system is characterized by the multi input single output (MISO) transfer characteristic.

The unique element determined $y=f(x_1,x_2)\in U$ through the application f is called the composed of x_1 with x_2 through the fuzzy compositional law f.

An enlarged universe of discourse of the basic universe of discourse U=[-1, 1] may be

$$\underline{U} = [-a, a],\ a > 1 \quad (30)$$

In the shown figures a=2.

If the compositional law f is defined on $\underline{U}$x$\underline{U}$ with values in $\underline{U}$, then the universe of discourse U is a stable part of $\underline{U}$ in the enlarged universe $\underline{U}$, related to the fuzzy compositional law f, if there is the following property:

$$\forall x_1, x_2 \in U \Rightarrow f(x_1, x_2) \in U \quad (31)$$

If the compositional law f is defined on $\underline{U}$x$\underline{U}$ with values on $\underline{U}$, then we obtain the multi input, single output transfer characteristic of the fuzzy system $f_{FS}(x_1, x_2)$.

If U is a stable part of $\underline{U}$ related to the fuzzy compositional law f:$\underline{U}$x$\underline{U}\to\underline{U}$, then on U we may define the compositional law f:Ux$U\to U$ taken

$$y = f(x_1, x_2) \in U,\ \forall x_1, x_2 \in U \quad (32)$$

We may say f is a real fuzzy compositional law inducted on U by f.

The universes of discourse U_{x1}, U_{x2}, U_y and U may be continuous or discrete. Because the fuzzy systems are digital implemented, these universes of discourse are considered discrete. A discrete universe of discourse ha a finite number of discrete values

$$u_i\in U,\ i=1,...,n:\ U=\{u_1, u_2,...,u_n\} \quad (33)$$

A compositional law that describes a fuzzy system may be represented by a surface with 3 dimensions. Such surfaces are presented in the above figures.

In general, in the definition of a compositional law we may ignore the nature of the elements of the set U. Also, we may ignore the effective action way of f on UxU. The only restriction imposed is f to associates to any ordered couple of elements (x_1, x_2) from U an element $f(x_1, x_2)$ from U and only one.

We shall analyze as follow if the algebraic properties of the rule bases may be found in the fuzzy systems.

For the real fuzzy compositional law we use the symbol ∘. We are saying that the definition set U, different from the vide set Φ is made with the compositional law ∘:

$$UxU \to U, (x_1, x_2) \to x_1 \circ x_2 \quad (34)$$

7.2 Commutative Law

By definition:

$$x_1 \circ x_2 = x_2 \circ x_1, \; \forall x_1, x_2 \in U \quad (35)$$

This property may be verified on the surface associated to the to the compositional law of the fuzzy system. The element $x_1 \circ x_2$ from the intersection of the coordinate line of x_1 with the coordinate line of x_2 must be equal to the element $x_2 \circ x_1$ from the intersection of the line coordinate of x_2 with the line coordinate of x_1, for any x_1, $x_2 \in U$. This is related to the property that the surface of the fuzzy system, defined on then universe UxU is symmetrically face to the main diagonal of the three dimensional Cartesian coordinate system (x_1, x_2, y). The most commonly surfaces defined in practice have commutative compositional laws, on the some parts on that the input variables are defined on the same real definition sets.

In the following paragraph the commutative law will be analytical proofed. Also, the necessarily conditions in the definitions of the universes of discourse and the membership functions will be defined, to assure this property. The application f is composed from an operation of inference and an operation of deffuzification. We annotate the inference function with f_I and the defuzzification function with f_d.

$$y = f(x_1, x_2) = f_d(f_I(x_1, x_2) \quad (36)$$

These functions are defined as follows. The inference function is defined with the following relations:

$$y = f_I(x_1, x_2), \; x_1, x_2 \in [-1, 1] \quad (37)$$

and y results as a fuzzy sets of elements y_i, i=1,...,n: y={y_1, y_2,...,y_n}. The values y_i are membership values resulted after an inference operation, for all n values of the universe of discours of the fuzzy variable y.

The defuzification function is uniquely defined as follow:

$$y_d = f_d(y), \; y = \{y_i\}, \; i = 1,...,n, \; y_d \in U \quad (38)$$

By the composition of two function f_I and f_d the fuzzy controller results as a composed function:

$$y_d = f_d(f_I(x_1, x_2)), \; x_1, x_2, y_d \in U \quad (39)$$

A first necessary condition is that the membership functions of x_1 and x_2 to be equal, that is to have the same definition relations and of course the same universes of discourse. If m_{NB}, m_{ZE}, m_{PB} are membership functions defined as m:U→[-1, 1]:

$$\begin{aligned} & m_{NB}(x_1) = m_{NB}(x_2), \\ & m_{ZE}(x_1) = m_{ZE}(x_2), \\ & m_{PB}(x_1) = m_{PB}(x_2), \; for \; \forall x_1 = x_2 \in U \end{aligned} \quad (40)$$

The output fuzzy variable y has the following membership functions:

$$\begin{aligned} & y_{NB} = \{y_{NBi} = m_{NB}(y_{di}) | y_{di} \in U, \; i = 1,...,n\}, \\ & y_{ZE} = \{y_{ZEi} = m_{ZE}(y_{di}) | y_{di} \in U, \; i = 1,...,n\}, \\ & y_{PB} = \{y_{PBi} = m_{PB}(y_{di}) | y_{di} \in U, \; i = 1,...,n\} \end{aligned} \quad (41)$$

In the definition relation of the inference the operation minimum, for min-max inference, and the operation product, for sum-prod inference, are symbolized by ∧. The operations of summation and maximum are symbolized by ∨.

The inference function may be defined by the operations made in the frame of the inference on the input values. For example, in the case of the system with a 3x3 rule base we may write the following definition relation of the inference:

$$\begin{aligned} y = & \{[m_{NB}(x_1) \wedge m_{NB}(x_2)] \wedge x_{NBi} | i = 1,...,n\} \vee \\ & \vee \{[m_{ZE}(x_1) \wedge m_{NB}(x_2)] \wedge x_{NBi} | i = 1,...,n\} \vee \\ & ... \\ & \vee \{[m_{PB}(x_1) \wedge m_{PB}(x_2)] \wedge x_{PBi} | i = 1,...,n\} \end{aligned} \quad (42)$$

Because the inference function f_d is unique defined, it results that for the same output fuzzy set y results the same defuzzificated value y_d and so y_d=f_d(f_I(x_1, x_2))= f_d(f_I(x_2, x_1)).
Same reasoning may be done also for the other types of fuzzy systems.

7.3 Neutral Element

The application f has no neutral element. This fact is observed also on the families of the SISO transfer characteristics y=f_e(x_1; x_2), with x_2 as a parameter, presented above.

7.4 Symmetrical Elements

Though the application f has no neutral element however, it has a property that is close to the symmetry. For values of the input variable taken on the universe of discourse [-1, 1] there is the relation:

$$f(x,-x) = f(-x,x) = 0,\ x \in [-1, 1] \qquad (43)$$

We may notice that in the composition law *f* the symmetrical of *x* is –*x*, the opposed of him, as in the case of an additive composition law.

This property may be proofed graphically based on Fig. 43, in the case of the fuzzy system *33-mm-g*.

For x_1=-*x* and x_2=*x*, with *x*>0, from the primary rule base from the Tab. 1, there are activated only the rules 1, 5, 7 and 8.

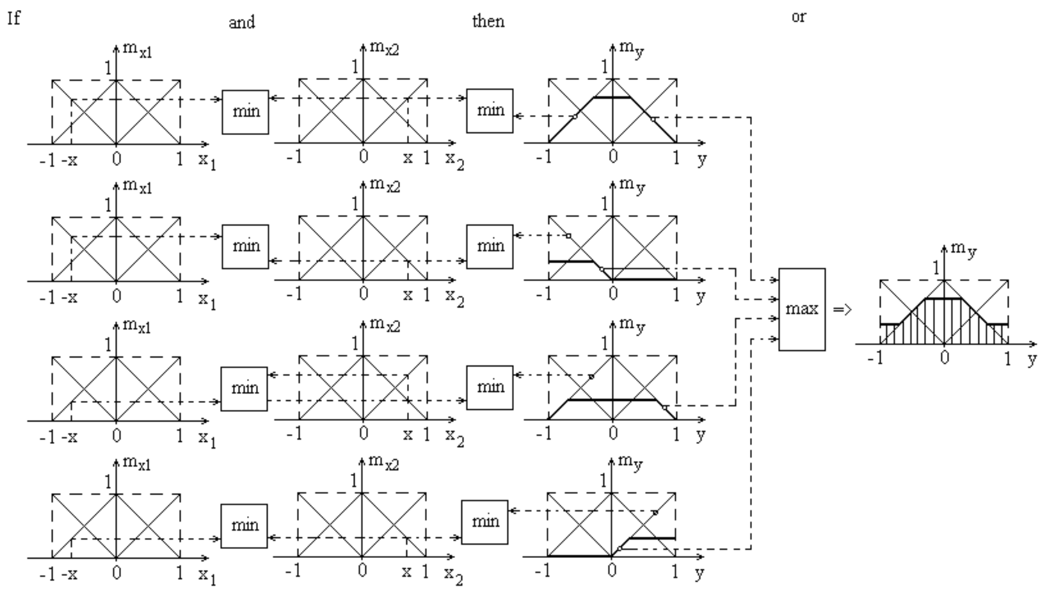

Fig. 43. Inference with symmetric elements in the case of min-max inference for the fuzzy system *33-mm-g*

In fig. 43 we present graphically the operations of inference of type min-max for the above four activated rules, from the top to the bottom, in the order 7, 8, 1 and 5. We may notice the abscise of the center of gravity of the resulted output fuzzy set is 0.

7.5 Associatively Law

From the study of the composition tables it resulted that the composition law *, that generates the tables, is not an associatively law.

Because the composition law ∘ is not an associatively law, they do not define either the simplest algebraic structure on the definition set *U*.

7.6 Continuity

Another important property of the fuzzy systems is the property of continuity. The notion of continuity is usually infirmed by the existence of some singular discontinuity points, from the definition sets, in which the functions are discontinuous. In this way, we know that a function that results as a composition of some continuous functions is continuous. So, the inference function f_I, that results after the application of the min-max operation or sum-prod operation that determines continuous functions on the universe of discourse *U* is a continuous function.

The defuzzification function is a continuous function for the defuzzication method with the center of gravity, but it is not continuous for the defuzzification with mean of maxima. These aspects may be seen on the MISO transfer characteristics, for a fuzzy system *33-mm-m*, as an example.

We may notice that in the case of the fuzzy systems that use defuzzification with the center of gravity the MISO characteristic has no discontinuities. But the MISO characteristics of the fuzzy systems with defuzzification with mean of maxima have sudden leaps between the levels of the output variable.

8 Sector Property

8.1 Sector Property for Rule Bases

A fuzzy relation may be developed for some fuzzy systems, corresponding to a composed characteristic:

$$y = f_1(x_1 \oplus x_2) \tag{44}$$

For the primary rule base the variables of the above relation take the following values on the following sets:

$$\begin{aligned} & x_1 \in \{N, ZE, P\}, x_2 \in \{N, ZE, P\}, \\ & x_1 \oplus x_2 \in \{MN, N, ZE, P, MP\}, \\ & y \in \{N, ZE, P\} \end{aligned} \tag{45}$$

In the sets (45) the fuzzy values *MN* and *MP* have the means of "more negative then *N*" and "more positive then *P*". For the relation (44), defined for the primary rule base on the sets (45), it is possible to build the Tab 5.

Tab. 5. Table of the fuzzy relation $y=f_1(x_1 \oplus x_2)$

Rule no.	x_1	x_2	$x_1 \oplus x_2$	y
1	*ZE*	*ZE*	*ZE*	*Z*
2	*N*	*N*	*MN*	*N*
3	*P*	*P*	*MP*	*P*
4	*ZE*	*N*	*N*	*N*
5	*ZE*	*P*	*P*	*P*
6	*P*	*N*	*ZE*	*ZE*
7	*N*	*P*	*ZE*	*ZE*
8	*N*	*ZE*	*N*	*N*
9	*P*	*ZE*	*P*	*P*

The graphical representation of the relation (44), on the definition set (45), based on the Tab. 5, is presented in Fig. 44.

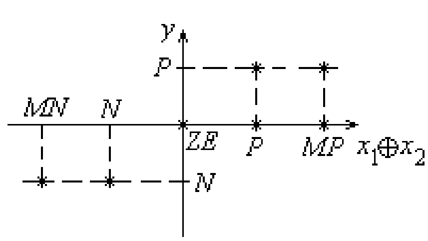

Fig. 44. The sector characteristic

On this figure we may notice that the characteristic is situated only in the first and the third quadrants. We may say that the rule base is characterized by a sector property.

The same considerations may be done for the other rule bases.

8.2. Sector Property for Fuzzy Systems

The fuzzy systems defined above have the sector property such the rule bases.

All the characteristics $y=f_t(x_t;\ x_2)$ are placed only in the first and the third quadrants. The function $y=f_t(x_t;\ x_2)$ has the following properties:

$$\begin{aligned} & y = 0 \quad for \quad x_t = 0 \\ & y > 0 \quad for \quad x_t > 0 \\ & y < 0 \quad for \quad x_t < 0 \end{aligned} \tag{46}$$

The function of two variables $y=f(x_1,\ x_2)$ attached to the fuzzy system has a property of a sector in a three dimensions space:

$$f(x_1, x_2)[f(x_1, x_2) - K_M[1 \quad 1][x_1 \quad x_2]^T)] \le 0 \tag{47}$$

9 Stability

In the case that fuzzy systems are used in control stability analysis there is the problem of internal stability. We present an example of a general fuzzy control system based on a fuzzy PI controller in Fig. 45 [16].

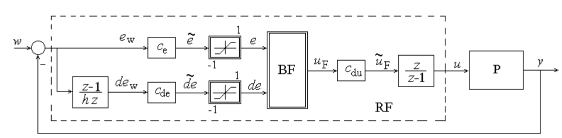

Fig. 45 A basic fuzzy control system

The fuzzy controller RF has a fuzzy block BF with two scaling coefficients c_e and c_{de} and two saturations blocks. It is a nonlinear part in this control system. The stability analysis may be framed in the theory of the nonlinear systems. The transfer characteristic of this fuzzy block is presented in Fig. 46.

The characteristic has a part placed only on the *x*-axis. This does not assure stability and does not allow the usage of circle criterion in stability analysis.

So, the author proposed a correction of the nonlinear part [16] to assure stability and to allow the usage of circle criterion in stability analysis.

This correction is presented in Fig. 47.

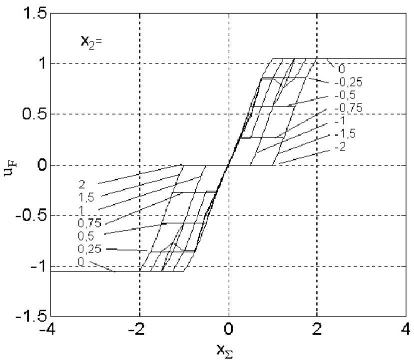

Fig. 46 Transfer characteristic of non-linear part

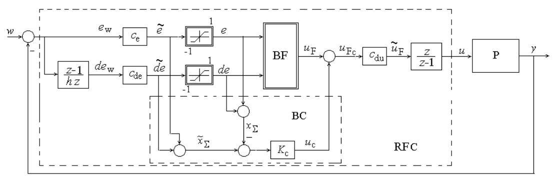

Fig. 47 Fuzzy control system with controller correction

It is made with a correction coefficient K_c and a modified fuzzy controller RFC results.

$$u_c = K_c[(\tilde{e} + \tilde{de}) - (e + de)] \tag{48}$$

The characteristic of the modified nonlinear part is presented in Fig. 48.

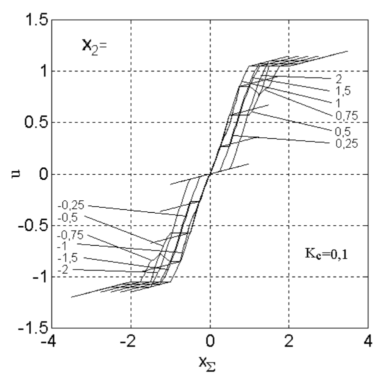

Fig. 48 Transfer characteristic of the modified nonlinear part

This new controller may be used in general control application. For all these control systems the global absolute internal stability may be proven using the circle criterion. The block diagram for stability analysis is presented in Fig. 49.

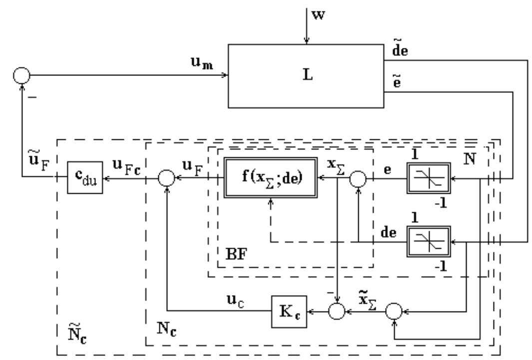

Fig. 49 The structure for stability analysis

With this approach absolute global internal stability may be assured and demonstrated.

10 Control Quality Criteria

In this paragraph we show by an example that a fuzzy PI controller assures better quality criteria then the linear PI controllers. An example of a fuzzy control system based on a fuzzy PI controller for speed control to a d.c. motor [15] is presented in Fig. 50.

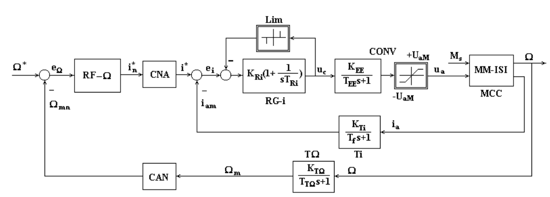

Fig. 50 Fuzzy control system of a dc motor

The fuzzy controller from Fig. 47 is used. The design of this fuzzy controller may be done using the above transfer characteristics. We may use the linear relation of the controller (25, 26) to obtain a transfer function of the PI fuzzy controller:

$$u_d(z) = c_{du}\frac{z}{z-1}K_0(x_e + x_{de})e(z) = = c_{du}K_0\left(c_e\frac{z}{z-1} + \frac{c_{de}}{h}\right)e(z) \tag{49}$$

Also, the scaling coefficient c_{du} may be chosen to assure stability according to the above presentation. With this fuzzy control system we obtained the transient characteristics from Fig. 51, 52.

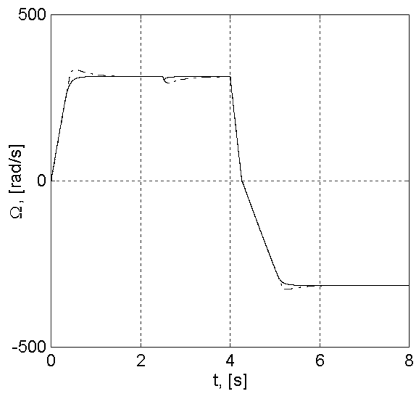

Fig. 51 Speed transient characteristic for fuzzy and linear control, tuned parameters

A step reference at starting, a step disturbance load torque at the moment 2,5 s and a reversing of the motor at the moment 4 s give the transient regime.

The simulations were made for a conventional (c) linear PI controller and for a fuzzy (f) PI controller, for tuned (t) and detuned (d) parameter of the motor.

The characteristics from Fig 51 are for tuned parameter.

The same characteristics may be obtained for detuned parameter (Fig. 52), in the case of errors in parameter estimation.

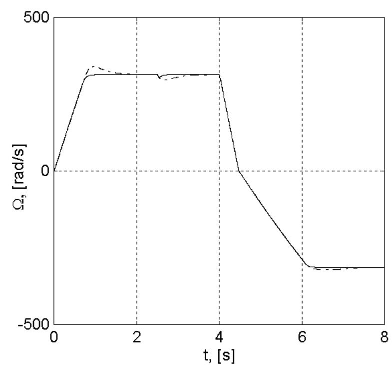

Fig. 52 Speed transient characteristic for fuzzy and linear control, detuned parameters

The values of the main empirical quality criteria are presented in Tab. 6. From this table the following appreciations may be done: -Zero overshoot at start up under the reference action and at reversing; -Reduced settling time; -Reduced deviation at the disturbance action; -Reduced settling time at the elimination of disturbance effect; -High robustness at disturbances; -High robustness at errors at the identification of process parameters. The comparison made above shows better quality criteria for fuzzy control.

Tab. 6 Values of quality criteria for conventional (c) and fuzzy (f) control, for tuned (a) and detuned (d) parameters

Analysis case	$\sigma_{1\Omega}$ [%]	$t_{r\Omega}$ [s]	σ_{1M} [%]	t_{rM} [s]	σ_{1r} [%]	t_{rr} [s]	$\Im$ 10^{-5}	$\Delta\sigma_{1\Omega}$ [%]	$\Delta\sigma_{1M}$ [%]	$\Delta t_{r\Omega}$ [s]	Δt_{rM} [s]
c-t	6,7	1	6,1	0,6	4,1	1,5	1,1	6,7	2,3	0,5	0,46
f-t	0	0,5	3,8	0,14	0	1,2	1,03				
c-d	8,3	1,5	6,1	0,65	4,1	3	2,0	8,3	2,3	0,7	0,51
f-d	0	0,8	3,8	0,14	0	2,2	1,89				

11 Conclusion

In this paper some main properties of fuzzy systems were presented. All these properties may be used in fuzzy system applications, the most important applications being process control.

A classification of the most common fuzzy systems used in practice is proposed.

Some input-output transfer characteristics for these fuzzy controllers are discussed and some exemplifications are done. These characteristics were analyzed and some important properties of them were emphasized. In this paper a classification of the most common fuzzy controller used in practice is done.

The input-output transfer characteristics allowed an easier graphic-analytical analysis of fuzzy control systems.

Based on these transfer characteristics and unitary theory of the fuzzy control systems is proposed.

Two important applications of these characteristics are PID fuzzy controller designing and global internal stability analysis of fuzzy control systems.

In this paper some algebraic characteristics –

commutative law, neutral element and symmetrical elements, were presented for the most common symmetrical rule bases seen as applications between fuzzy sets.

Some properties – commutative law, symmetrical elements and continuity were presented for the most general fuzzy systems, with Mamdani structure, seen as application between real sets. The properties of the rule bases are transposed in the fuzzy controllers, defined as application on real sets. The fuzzy controllers are composed applications of the inference rules and defuzzification. The inference rules are made based on the rule bases. The defuzzification has also the above algebraic laws. So, all the algebraic properties are transferred to the fuzzy controllers, by the composite laws.

The algebraic properties of the fuzzy systems are important for analytical characterization of the fuzzy systems with the input-output transfer characteristics.

An important property – the sector property was emphasized for the rule bases and for the fuzzy systems. This sector property was illustrated in 2 dimensions plane and also in3 dimension space.

The sector property is important for the global absolute internal stability analysis of the fuzzy control systems, based on the circle criterion.

The properties of the rule bases are transposed in the fuzzy controllers, defined as application on real sets.

The fuzzy systems are nonlinear systems and their gain is variable with the inputs. From the gain transfer characteristics we may chose useful values in fuzzy controller design.

The gain in origin is used in the linear transfer characteristic to design fuzzy PID fuzzy controllers.

The fuzzy control systems assure better empirical quality criteria and they are more robust then the linear control systems.

The forms of the transfer characteristics do not influence the behavior of a fuzzy controller and the fuzzy control system, if the scaling coefficients are well chosen.

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