

# Implementation of the Stable States Transition Control Algorithm for a Four Free Joints Walking Robot

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*Abstract:* - The paper presents a systemic approach of a walking robot behavior and control in uncertain environments, with application to a hexapod robot. For simplicity, this paper considers only the vertical xz-plane evolution taking into account that the structure is symmetrical in the horizontal xy-plane and the results can be easily extended. Based on the mathematical model of the robot, determined considering all the points in the xz-plane as being complex numbers, a new concept of walking called SSTA, "Stable States Transition Approach", is proposed. Applications of this algorithm for a walking robot with four free joints are implemented on a user-friendly simulation and educational platform and also a control system development environment for control of walking robots systems - RoPa. All the examples demonstrate the efficacy of the proposed control algorithm.

*Key-Words:* - walking robot, mathematical model, control algorithm, variable causality dynamic system, stable states, free joints

## 1 Introduction

Today the mobile robotics field receives great attention. There is a wide range of industrial applications of autonomous mobile robots, including robots for automatic floor cleaning in buildings and factories, for mobile surveillance systems, for transporting parts in factories without the need for fixed installations, and for fruit collection and harvesting. These mobile robot applications are beyond the reach of current technology and show the inadequacy of traditional design methodologies.

Mobile robots control in uncertain environments represents still a challenge for real world applications. The robot should be able to gain its goal position facing the implicit uncertainty of the surrounding environment.

The walking robots, particularly the legged robots, allow many advantages with respect to the wheeled robots, especially regarding the autonomy in difficult environments. Unfortunately, a specific type of movement called legged locomotion, [1], [2], is characterized by strongly nonlinear mathematical models to allow describing both the fundamental aspects: leg movements and leg coordination. The problem of walking robots control

in uncertain environments has been deeply studied in literature and several techniques have been developed [3], [4], [5].

Several new control approaches have been attempted to improve robot interaction with the real world aimed at the autonomous achievement of tasks. An example is the architecture proposed by Brooks [6]. This approach is one of the first solutions systematically implemented on real robots with success.

Some approaches consider the robot having the necessary intelligence to operate in uncertain environments and use fuzzy logic or neural networks based techniques [7],[8], advanced control schemas, genetic algorithms [9],[10] etc., to develop the dynamic walking.

In [11],[12] a new concept of walking called SSTA "Stable States Transition Approach" based on the variable causality [13] mathematical model of the walking robot [14],[15] it was developed. According SSTA both the leg coordination and individual leg movements are entirely dependent on the robot goal and the environments only. The control structure of SSTA was proposed in order to

apply the best control with respect to safety issues and convergence to the goal.

The paper is structured as follows: in Section II, the geometrical structure of the walking robot is detail described; in Section III, the algorithm for walking robot control in SSTA strategy is presented. In Section IV, the implementation of the control algorithm for a four free joints robot is illustrated by many simulations. Last section closes the paper with conclusions and purposes for future activities.

## 2 Geometrical structure of a Walking Robot

It is considered the walking robot structure as depicted in Fig.1, having three normal legs  $L^i, L^j, L^p$  and a head equivalent to another leg,  $L_0$ , containing the robot centre of gravity,  $G$ , placed in its foot. The robot body  $RB$  is characterized by two position vectors  $O^0, O^1$  and the leg joining points denoted  $R^i, R^j, R^p$ . The joining point of the head,  $L^0$ , is the central point  $O^0, R^0 = O^0$ , so the robot body  $RB$  is univocally characterized by the set,

$$RB = \{O^0, O^1, \lambda^i, \lambda^j, \lambda^p, \lambda^0\}. \quad (1)$$

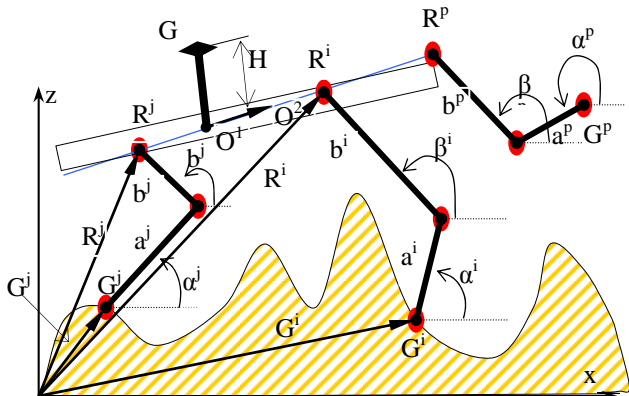


Fig. 1 The geometrical structure of the robot

A robot leg, let us consider of index  $i$ , has a body joining point  $R^i$  expressed by a complex number and the foot point denoted by the complex number  $G^i$ . It contains two joint segments defined by the lengths  $a^i, b^i$  with the angles  $\alpha^i, \beta^i$ . Here there are considered three legs:  $i, j, p$ .

The opposite side legs are denoted by  $i', j', p'$ . It is supposed that the ground shape of the robot evolution is expressed in the  $xz$ -plane by a function  $z = \psi(x)$ , which is the same for the right and the left legs. This function is analytically unknown but experimentally can be determined the

coordinates of the contact points between each support foot point and the ground.

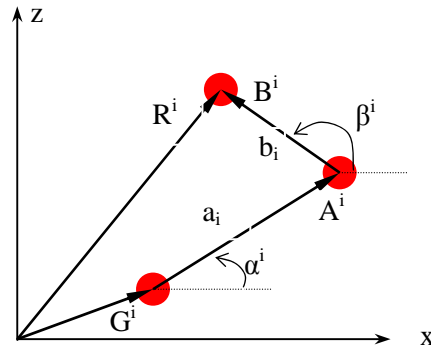


Fig. 2 The "i" leg structure

One leg structure is represented as in Fig. 2, where,

$$A^i = a^i \cdot e^{j\alpha^i} = x^{A^i} + j \cdot z^{A^i}, \quad (2)$$

$$B^i = b^i \cdot e^{j\beta^i} = x^{B^i} + j \cdot z^{B^i}, \quad (3)$$

$$G^i = g_i \cdot e^{j\gamma^i} = x^{G^i} + j \cdot z^{G^i}, \quad (4)$$

$$R^i = r_i \cdot e^{j\rho^i} = x^{R^i} + j \cdot z^{R^i} \quad (5)$$

express the "i" leg parameters as complex numbers.

Its position is controlled by the angles  $\alpha^i, \beta^i$ , from mathematical point of view. These two input variables are related to some physical angles not represented here. The same aspects are encountered for the other legs "j" and "p".

Each leg can be in two states: fixed leg and free leg.

If "i" leg is a fixed leg then the point  $G^i$  is fix, and the input variables can affect the point  $R^i$  by the relation

$$R^i = G^i + A^i + B^i \quad (6)$$

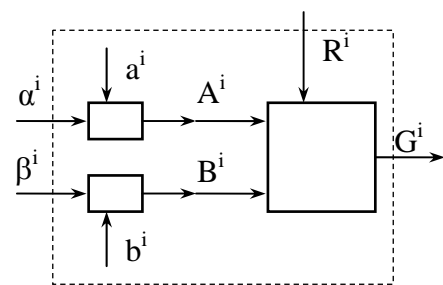


Fig. 3 Block schema of the "i" robot leg

Physically, the legs of the robot have actuators. They develop joint torques which ensure  $\alpha_i$  and  $\beta_i$  angles. Each leg can be found in two states: free leg and rigid support leg. Thus a state is experimentally detected by the resisting torque value from the position systems.

When the point  $G^i$  touch a rigid point detected on the abrupt increasing of the resisting torque, this means that the coordinates  $G^i = (x^{G^i}, z^{G^i})$  of the point  $G^i$  are on the contact plane of the uncertain environments where the robot evolves.

If the leg "i" is free, this means that the coordinates  $(x^{G^i}, z^{G^i})$  can be chosen at any values from the possible domain of values considering that the length  $a_i, b_i$  and

$\alpha^i \in [\alpha_{\min}^i, \alpha_{\max}^i], \beta^i \in [\beta_{\min}^i, \beta_{\max}^i]$  are known.

The coordinates  $(x^{G^i}, z^{G^i})$  are fixed if  $G^i$  is rigid supported. It is supposed that there is no sliding in the support point and the angles  $\alpha^i$  and  $\beta^i$  as command variables determines the coordinates of joint points  $R^i = (x^{R^i}, z^{R^i})$ .

The robot body position is defined by two points,  $O^1, O^2$  of the coordinates  $O^1 = (x^{O^1}, z^{O^1})$ ,  $O^2 = (x^{O^2}, z^{O^2})$  and the center of gravity is in the point  $G = (x^G, z^G)$ .

If the legs  $i, j$  are fixed legs, the robot is in the state  $S^{ij}$ . It is proved that in this state the stability condition is measured by a so called stability index  $\varepsilon^{ij}$  of the state  $S^{ij}$ , where

$$\varepsilon^{ij} = \frac{\text{Re}(O^1 - G^i) - H \cdot \sin(\theta)}{\text{Re}(G^j - G^i)}, \quad (7)$$

$$\theta = \arg(O^2 - O^1) \quad (8)$$

and this state is stable if and only if  $\varepsilon^{ij} \in (0, 1)$ .

The stability index  $\varepsilon^{ij}$  depends on the input variables  $\alpha^i, \beta^i, \alpha^j, \beta^j$  which affect the robot position in the state  $S^{ij}$ . In the same time, during the state  $S^{ij}$  the other future possible index of stability  $\varepsilon^{pi}, \varepsilon^{pj}$  are evaluated. They depend on the active manipulated variables  $\alpha^i, \beta^i, \alpha^j, \beta^j$  and also on the variables  $\alpha^p, \beta^p$  of the free leg of the index  $p$ .

In the state  $S^{ij}$ , the free leg  $p$  is testing the ground for finding the future fixing point in such a way to ensure the final goal of the robot, that means a desired time evolution of the points  $O^1, O^2$ .

Throughout of the new control algorithm SSTA, the succession of the three possible states  $S^{ij}, S^{pj}$  and  $S^{pi}$ , that means the succession of the legs on the ground, depends on the stability indexes which depends on the shape of the ground.

### 3 SSTA Control Algorithm Implementation

By SSTA algorithm is assured the walking robots evolution in uncertain environments subordinated to two goals:

- achievement of the desired trajectory expressed by the functions  $O_z^0 = f(x)$  and  $\theta = \theta(x)$ , where  $x$  is the ground abscissa and  $O_x^0 = x$ ; it is considered the evolution from left to right;
- assurance of the system stability that is, in any moment of the evolution the centre of gravity has to be in the stability area.

Considering the walking robot as a variable causality dynamic system it is possible to realize this desideratum in different variants of assurance the steps succession. The steps succession supposes a series of elementary actions that are accomplished only if the stability condition exists.

Continuously, by sensorial means or using the passive leg, the robot has informations about its capacity of evolving on the ground. Every time it is considered that the legs  $i, j$  are on the ground and the system is stable ( $\varepsilon_{ij} \in [0, 1]$ ). The passive leg

$G^p$  is which realises the walking.

There are two situations in which is impossible to touch a point on the ground: external blockage (when the distance from robot to  $G^p$ , the foot of the passive leg, is bigger than  $|a - b|$ ) and internal blockage (when the distance from robot to  $G^p$ , the foot of the passive leg, is smaller than  $|a - b|$ ).

By testing the ground is realized its division in lots representing the fields on  $x$  axis which constitute the abscissas of some points that can be touched by the  $G^p$  leg. The leg will always touch the ground only on an admitted lot.

A next support point given by the free  $G^p$  leg, is chosen so that to existe a next stable state  $\varepsilon_{ip}$  or  $\varepsilon_{jp}$ , taking into account the actual state of legs activity. For example, if  $q=132$ , passive leg (which tests) is  $G^p = G^2$  and assures  $\varepsilon_{12} \in [0, 1]$  or  $\varepsilon_{23} \in [0, 1]$ . When the change of legs activity is realised ( $q=123$  or  $q=321$  or  $q=231$  etc.), the present passive leg  $G^p$  will become the leg  $i$  or the leg  $j$ .

In this paper, a variant of movements succession, composed by 12 steps, is proposed.

Fig. 4 presents the graphical representation of SSTA algorithm.

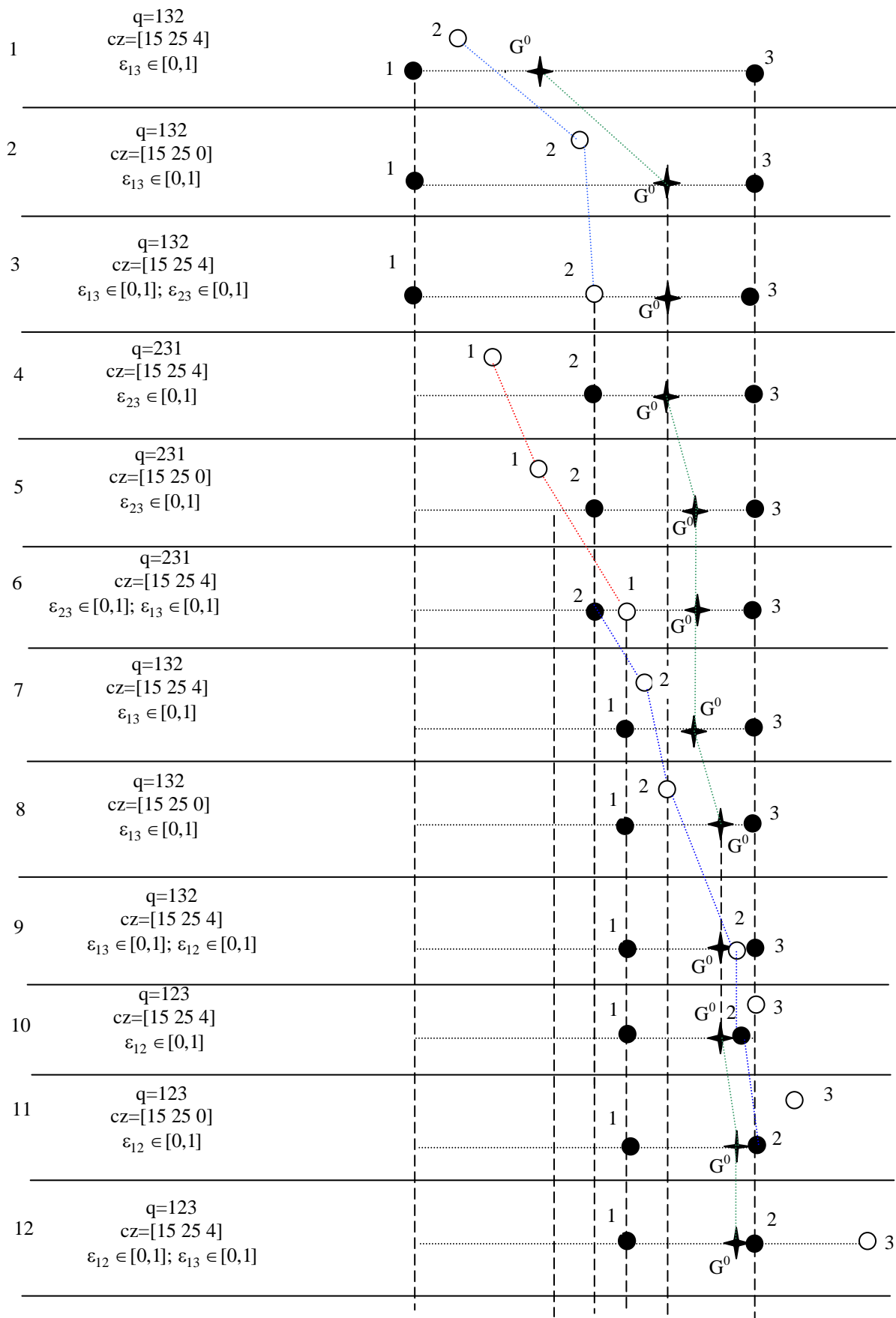


Fig. 4 Graphical representation of SSTA walking algorithm

**Step 1:** It is considered the causality  $cz=[15\ 25\ 4]$  and  $q=132$ . In this state  $L_1$  and  $L_3$  are active legs and the passive legs  $L_2$  is up. The stability condition is assured by  $\varepsilon_{13} \in [0,1]$ .

**Step 2:** The causality is changed in  $cz=[15\ 25\ 0]$  and  $q=132$ . The stability condition is also assured by  $\varepsilon_{13} \in [0,1]$ . The robot centre of gravity  $G^0$  is behind the leg  $L_3$ .

**Step 3:**  $q=132$  is maintained but the causality is change in  $cz=[15\ 25\ 4]$ . The leg  $L_2$  is situated behind the robot centre of gravity  $G^0$ , so that  $\varepsilon_{13} \in [0,1]$  and  $\varepsilon_{23} \in [0,1]$ .

**Step 4:** The causality  $cz=[15\ 25\ 4]$  is maintained but the index of activity is changed from  $q=132$  in  $q=231$ .  $L_1$  becomes passive legs and is up. The stability condition is assured by  $\varepsilon_{23} \in [0,1]$ .

**Step 5:**  $q=231$  is maintained but the causality is changed in  $cz=[15\ 25\ 0]$ . The stability condition is previous assured ( $\varepsilon_{23} \in [0,1]$ ).

**Step 6:**  $q=231$  is maintained but the causality is changed in  $cz=[15\ 25\ 4]$ . The passive legs  $L_2$  evolves behind the robot centre of gravity  $G^0$  so that  $\varepsilon_{23} \in [0,1]$  and  $\varepsilon_{13} \in [0,1]$ .

**Step 7:** The causality  $cz=[15\ 25\ 4]$  is maintained but the index of activity is changed from  $q=231$  in  $q=132$ . The passive legs  $L_2$  is up, the stability condition being assured ( $\varepsilon_{13} \in [0,1]$ ).

**Step 8:**  $q=132$  is maintained but the causality is changed in  $cz=[15\ 25\ 0]$ . The active legs are fixed but the robot body evolves on the trajectory until the centre of gravity arrives behind the leg  $L_3$ , so that  $\varepsilon_{13} \in [0,1]$ .

**Step 9:**  $q=132$  is maintained but the causality is changed in  $cz=[15\ 25\ 4]$ . The passive leg  $L_2$  is positioned in front of the centre of gravity. The stability condition is assured by  $\varepsilon_{13} \in [0,1]$  and  $\varepsilon_{12} \in [0,1]$ .

**Step 10:** The causality  $cz=[15\ 25\ 4]$  is maintained but the index of activity is changed from  $q=132$  in  $q=123$ . The passive legs  $L_3$  is up, the stability condition being previous assured ( $\varepsilon_{12} \in [0,1]$ ).

**Step 11:**  $q=123$  is maintained but the causality is changed in  $cz=[15\ 25\ 0]$ . The robot body evolves on the trajectory until the centre of gravity arrives behind the leg  $L_2$ , so that  $\varepsilon_{12} \in [0,1]$  is keeping.

**Step 12:**  $q=123$  is maintained but the causality is changed in  $cz=[15\ 25\ 4]$ . The passive legs  $L_3$  evolves maxim possible and test the ground, the stability being assured by  $\varepsilon_{12} \in [0,1]$  and  $\varepsilon_{13} \in [0,1]$ .

Then, it comes back to step 1.

## 4 Experimental Results

An experimental platform, called RoPa, has been conceived. The RoPa platform is a complex of MATLAB programs for simulation and control of walking robots evolving in uncertain environments according to SSTA control algorithm.

A number of eight causality orderings of the robot structure have been implemented on RoPa.

Fig. 5 presents the interface of this application for the causality structure with four free joints. The four degrees of freedom are thus consumed: one to fulfill the kinematics restriction; one to ensure the desired value of the  $\theta$  angle of the robot body and two for the desired values  $\hat{O}^0(O_x^0, O_z^0)$  of the robot body.

The causal ordering is activated by selecting the causal variable  $cz=[15\ 25\ 0]$ .

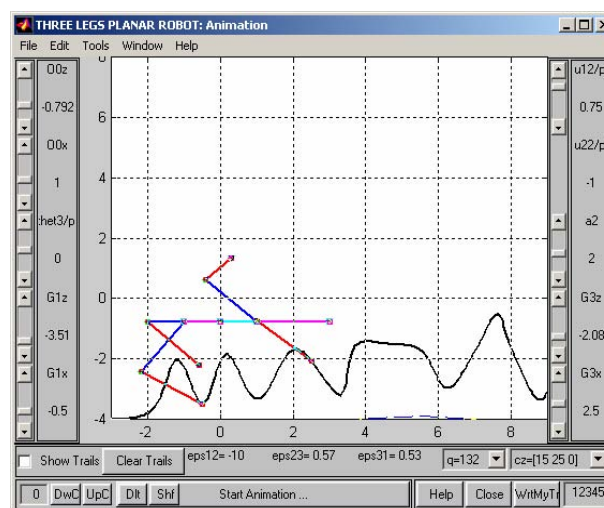


Fig. 5 RoPa Graphic User Interface

The robot evolution in this causality is presented in Fig. 6.

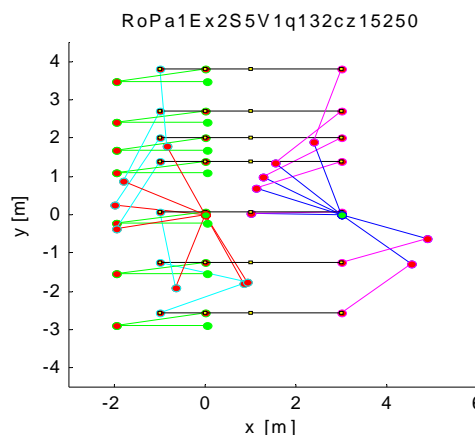


Fig. 6 The robot kinematics evolution

The stability of this evolution is graphical represented by a stability certificate of the evolution (Fig.7, 8). This certificate attests the stability index of the active pair of legs in any moment.

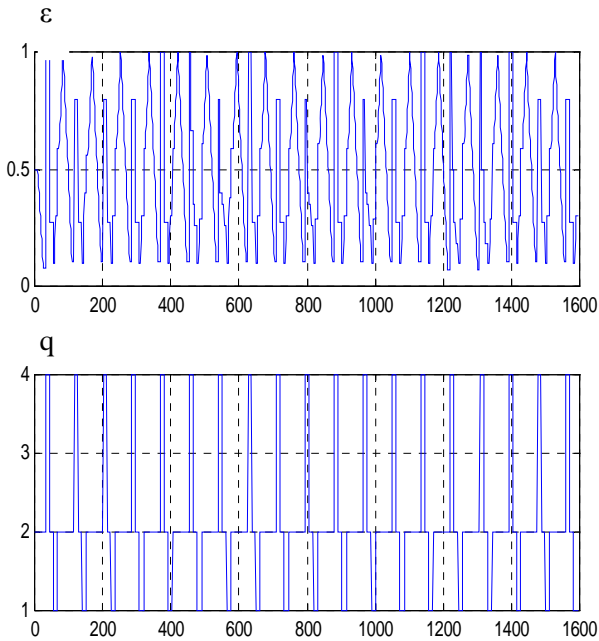


Fig. 7 The stability certificate of the evolution in different graphics

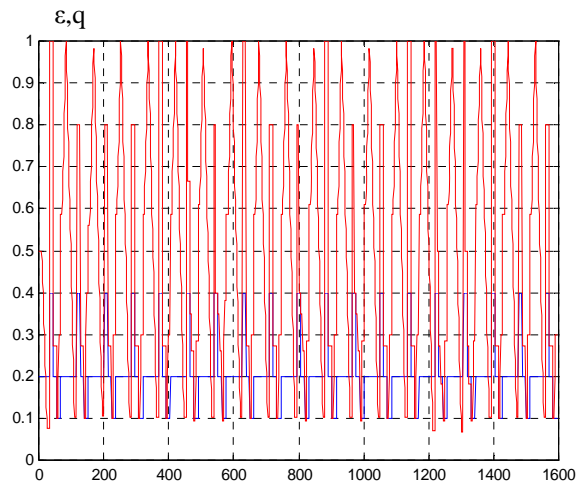


Fig. 8 The stability certificate of the evolution in the same graphic

In Fig.7,8 the following notation agreement has been adopted for the active pair of legs:

- Pair {123} → 1;
- Pair {132} → 2;
- Pair {213} → 3;
- Pair {231} → 4.

In the following there are presented some experimental results of walking robot behaviour, considering the causal orderings for four free joints.

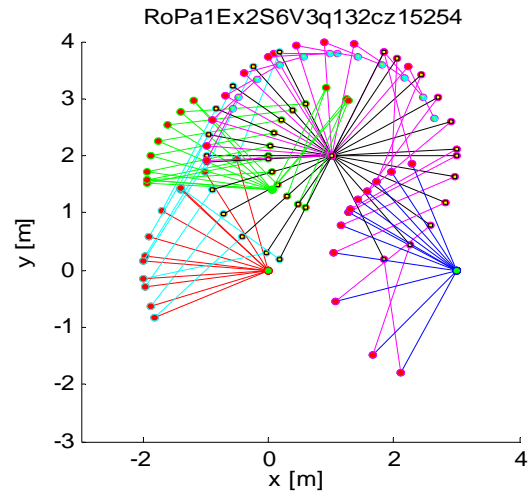


Fig. 9 The robot kinematics evolution

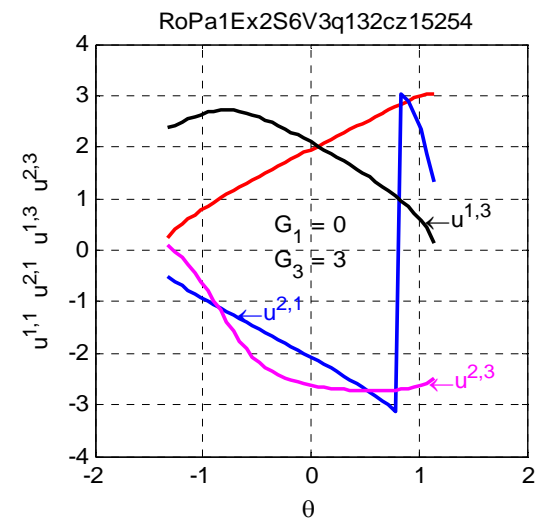


Fig. 10 Controlled angles with respect to the input variable

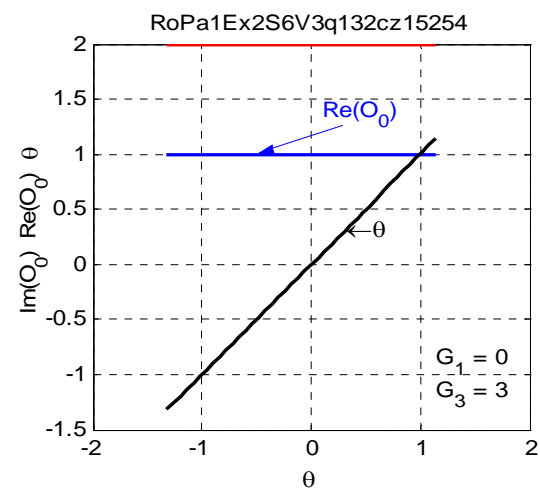


Fig. 11 Robot body position with respect to the input variable

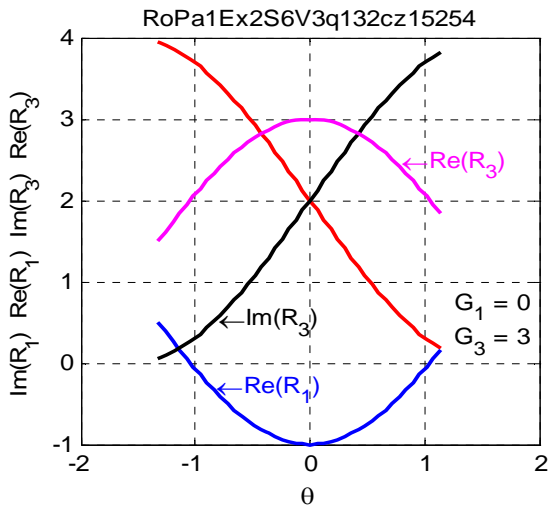


Fig. 12 Joints positions with respect to the input variable

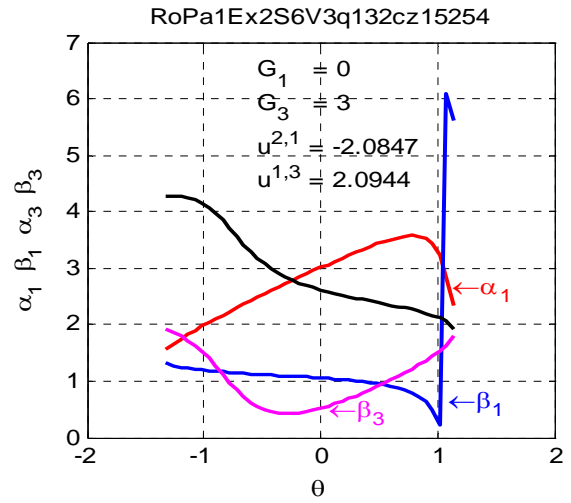


Fig. 15 Legs angular coordinates with respect to the input variable

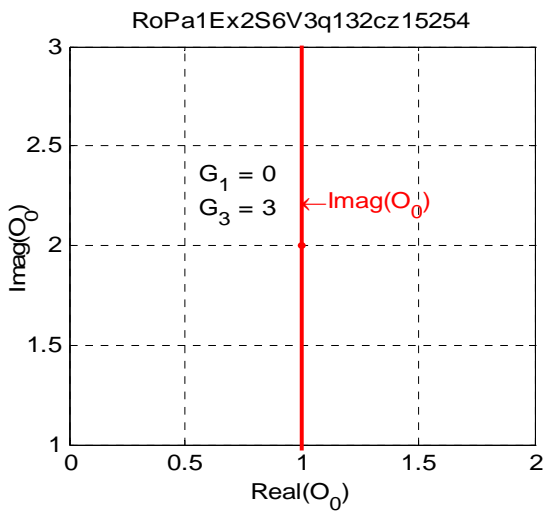


Fig. 13 Reference point  $O^0$  evolution scene

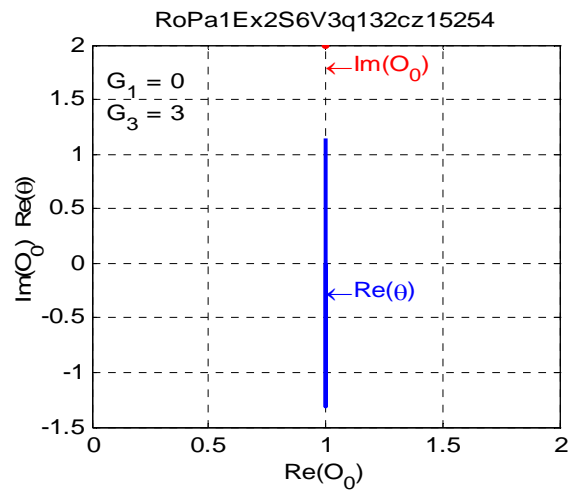


Fig. 16 Robot body angular position with respect to horizontal position

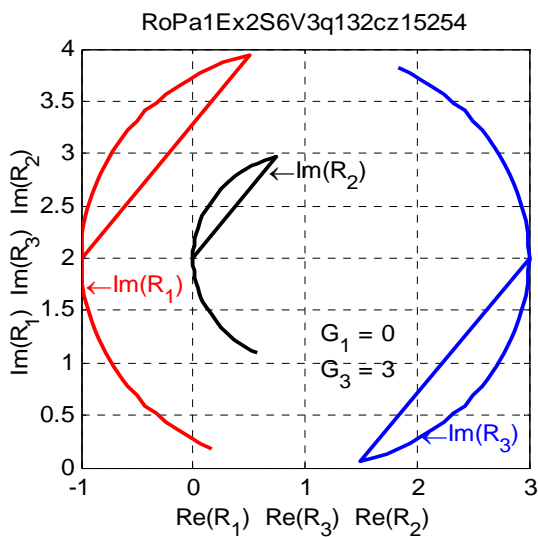


Fig. 14 Joints points' locus in evolution scene

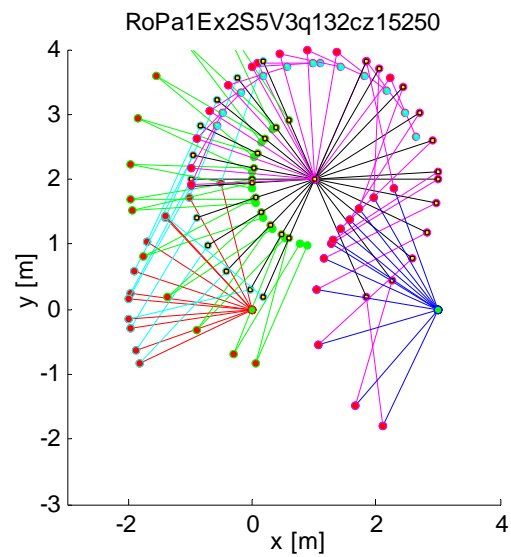


Fig. 17 The robot kinematics evolution

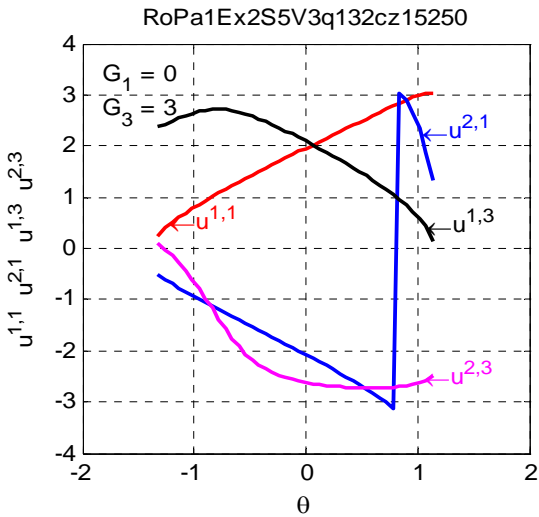


Fig. 18 Controlled angles with respect to the input variable

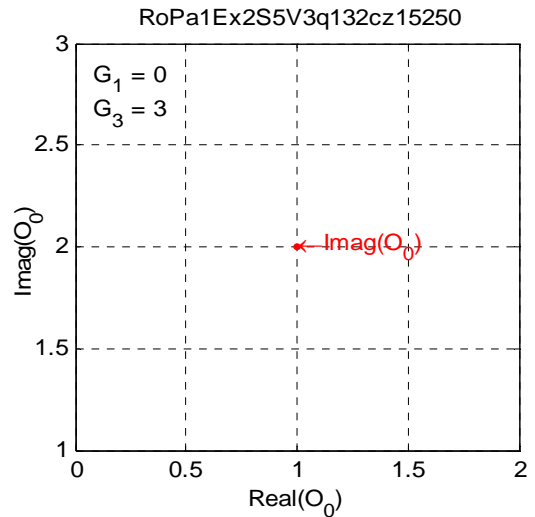


Fig. 21 Reference point  $O^0$  evolution scene

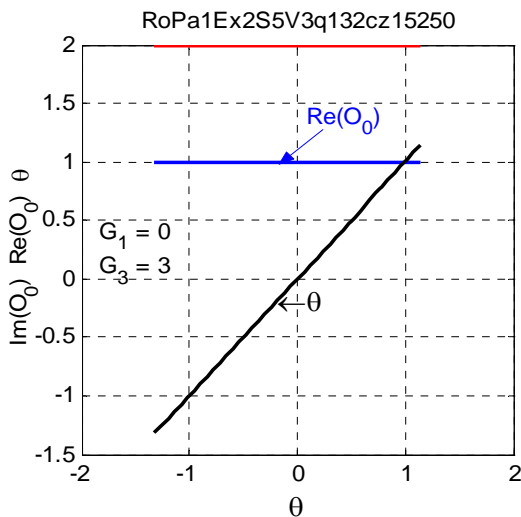


Fig. 19 Robot body position with respect to the input variable

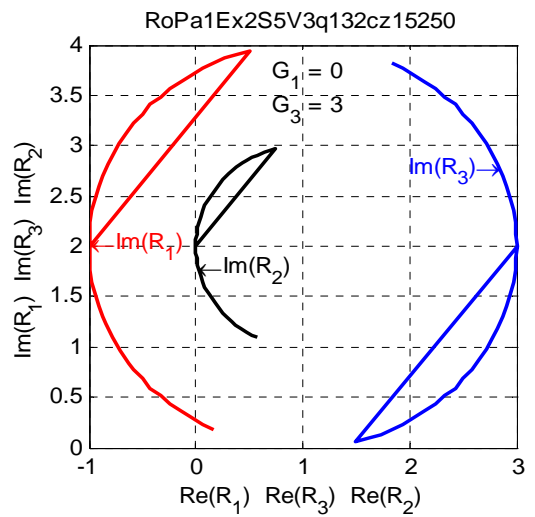


Fig. 22 Joints points' locus in evolution scene

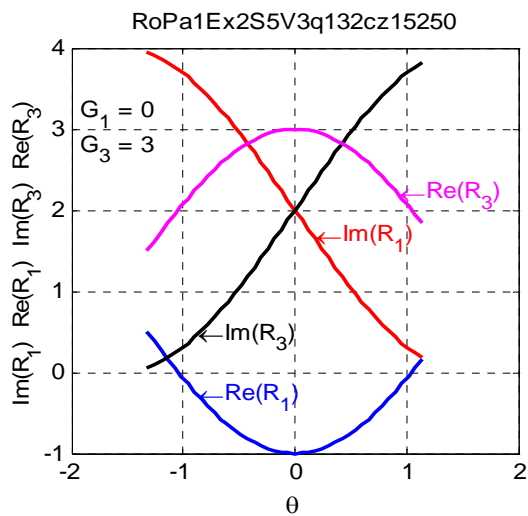


Fig. 20 Joints positions with respect to the input variable

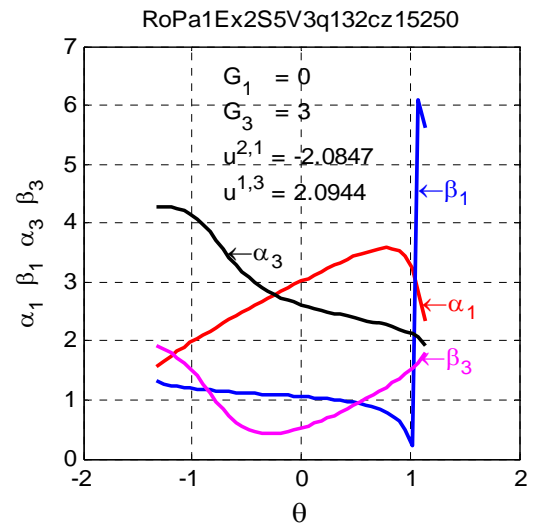


Fig. 23 Legs angular coordinates with respect to the input variable



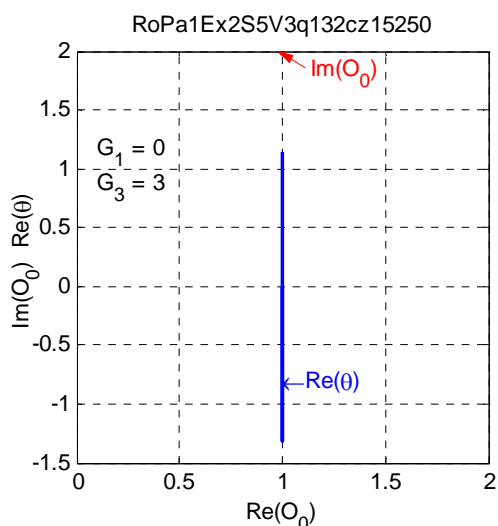


Fig. 24 Robot body angular position with respect to horizontal position

## 5 Conclusion

The experiments performed on RoPa, optimal platform to test SSTA control algorithm, demonstrate the efficacy and adaptability of the proposed method when the walking robots evolve in uncertain environments.

All the causal orderings are perfectly integrated in RoPa structure proving the correctness of the theoretical results.

Simulation results show how the developed walking control algorithm allows the robot to navigate safely, in uncertain environments, toward the goal and to modify its behavior according to the SSTA control algorithm.

Further investigations will be directed towards a hexapod robot performing a task in uncertain environment. This will be the main research activity in the near future.

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