Research on the Model of Ship Parametrical-Highly Excitation Nonlinear Dynamics System

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Abstract: - Chaotic and periodic motion of ship parametrical- highly excitation rolling dynamics system is researched by qualitative analysis method. Firstly, approximately harmonic solution is gained by researching the system model's harmonic solution bifurcation with the theorem of Poincaré. Secondly, approximately sub-harmonic solution is gained by researching system's sub-harmonic solution bifurcation. Lastly, chaotic motion performance of dynamics system is talked by function.

Key-Words: - Nonlinear differential equation; harmonic solution; chaos

1 Introduction

Applying nonlinear dynamics theory, pitching influences on rolling is expressed as parametric excitation term. Blocki , Nayfeh, Dongyanqiu, Tangyougang have researched the ship stability and dynamic behaviour in longitudinal waves according to parametric resonance and main parametric resonance[1-5].Literature [6,7] has respectively researched the ship capsizing in rolling waves and the wide range rolling motion. But up to now, the ship's dynamic characteristic suffered from parametric excitation and forced rolling excitation, is researched scarcely. According to parametrical-highly excitation nonlinear rolling dynamics system, Literature [8] sets up the model of differential equation. And literature [9] gets second order approximately solution of the system model, and also discusses the phenomenon of 1/2 meta harmonic resonance of system and receives condition of rolling losing stability. This article on the basis of the system model of ship parametrical-highly excitation nonlinear dynamic which is founded in literature [8], applying qualitative analysis method, researches harmonic solution bifurcation and sub- harmonic solution bifurcation of ship parametrical-highly excitation nonlinear dynamic system. That is periodic motion performance of system. Finally, applying function, chaotic motion of dynamics system is talked about.

The balance of the paper is organized as follows. Harmonic solution bifurcation of system is stated in Section 2. Sub-harmonic sulution bifurcation of system is described in Section 3. Heteroxenous robit and chaos of system is given in Section 4. Ideogenous robit and chaos of system is presented in Section 5. Finally, concluding remarks are drawn in Section 6.

2 Harmonic Solution Bifurcation of System

Differential equation model of ship parametrical-highly excitation nonlinear dynamic system [8] is

$$(I+\Delta I)\ddot{\varphi}+D_1\dot{\varphi}+D_3\dot{\varphi}^3+\begin{bmatrix} D\cdot\overline{GM}_0+k_3\varphi^2+k_5\varphi^4 \\ +h_1\cos(\Omega t)\end{bmatrix}\varphi = E_0\sin(\Omega t+\delta_0) \quad (1)$$

where, φ is rolling angel, I is rolling rotor inertia, $D_i(i=1,3)$ is nonlinear damping coefficient, D is displacement, $\overline{GM_0}$ is initial stability height, $E_0\sin(\Omega t+\delta_0)$ is regular wave forced rolling moment,

h is parametric excitation amplitude, Ω is interference frequency.

In equation (1), set

$$\omega^2 = \frac{D \cdot \overline{GM_0}}{I + \Delta I}$$

Other coefficients calling small parameter ε, then we have

$$\ddot{\varphi} + \omega^2\varphi + \varepsilon u_1\dot{\varphi} + \varepsilon u_3\dot{\varphi}^3 + \varepsilon[a_3\varphi^2 + a_5\varphi^4 + h\cos(\Omega t)]\varphi = \varepsilon K_e \sin(\Omega t + \delta_0) \tag{2}$$

To simplify calculation, set

$\Omega = \omega$

Then equation (2) will be

$$\ddot{\varphi} + \omega^2\varphi + \varepsilon u_1\dot{\varphi} + \varepsilon u_3\dot{\varphi}^3 + \varepsilon[a_3\varphi^2 + a_5\varphi^4 + h\cos(\omega t)]\varphi = \varepsilon K_e \sin(\omega t + \delta_0) \tag{3}$$

Set $u = \varphi, \quad \dot{u} = v$

We obtain

$$\begin{cases} \dot{u} = \nu \\ \dot{\nu} = -\omega^2 u + \varepsilon[K_e \sin(\omega t + \delta_0) - \\ u_1\nu - u_3\nu^3 - a_3u^3 - \\ a_5u^5 - h\cos(wt) \end{cases} \tag{4}$$

To make equation (4) to *Van der Pol* transformation, we reach

$$\begin{cases} u = x\sin(\omega t) + y\cos(\omega t) \\ \nu = \omega(x\cos(\omega t) - y\sin(\omega t)) \end{cases} \tag{5}$$

Then derivation to both sides of equation (5), we have

$$\begin{cases} \dot{u} = \dot{x}\sin(\omega t) + \dot{y}\cos(\omega t) + \nu \\ \dot{\nu} = \omega(\dot{x}\cos(\omega t) - \dot{y}\sin(\omega t) - \omega u) \end{cases} \tag{6}$$

To put equation (5) and equation (6) into equation (3) and arrange to

$$\begin{cases} \dot{x} = \frac{\varepsilon}{\omega}\left[K_e\sin(\omega t + \delta_0) - u_1\nu - u_3\nu^3 - a_3u^3 - a_5u^5 - h_1\cos(\omega t)u\right]\cdot \\ \cos(\omega t) \\ \dot{y} = -\frac{\varepsilon}{\omega}\left[K_e\sin(\omega t + \delta_0) - u_1\nu - u_3\nu^3 - a_3u^3 - a_5u^5 - h_1\cos(\omega t)u\right]\cdot \\ \sin(\omega t) \end{cases} \tag{7}$$

To simplify calculation, set

$x = r\cos\theta, \quad y = r\sin\theta$

Then we have

$$\begin{cases} u = x\cos(\omega t) + y\sin(\omega t) = r\cos(\omega t - \theta) \\ \nu = \omega(x\cos(\omega t) - y\sin(\omega t)) = -r\sin(\omega t - \theta) \end{cases} \tag{8}$$

To put equation (8) into equation (7), we obtain

$$\begin{cases} \dot{x} = \frac{\varepsilon}{\omega}\cos(\omega t)[K_e\sin(\omega t + \delta_0) + u_1\omega r\sin(\omega t - \theta) + \\ u_3\omega^3r^3\sin^3(\omega t - \theta) - a_3r^3\sin^3(\omega t - \theta) - \\ a_5r^5\cos^5(\omega t - \theta) - h_1 r\cos(\omega t - \theta)] \\ \dot{y} = -\frac{\varepsilon}{\omega}\sin(\omega t)[K_e\sin(\omega t + \delta_0) + u_1\omega r\sin(\omega t - \theta) + \\ u_3\omega^3r^3\sin^3(\omega t - \theta) - a_3r^3\sin^3(\omega t - \theta) - \\ a_5r^5\cos^5(\omega t - \theta) - h_1 r\cos(\omega t - \theta)] \end{cases} \tag{9}$$

By calculating, we obtain average equation of equation (9)

$$\begin{cases} \dot{x} = \frac{\varepsilon}{2}\left[K_e\sin\delta_0 - u_1\omega x + \frac{3}{4}u_3\omega^3r^2x - \frac{3}{4}a_3r^2y - \frac{5}{8}a_5r^4y\right] \\ \dot{y} = -\frac{\varepsilon}{2}\left[K_e\cos\delta_0 + u_1\omega y + \frac{3}{4}u_3\omega^3r^2y - \frac{3}{4}a_3r^2x - \frac{5}{8}a_5r^4x\right] \end{cases} \tag{10}$$

Among them $r^2 = x^2 + y^2$, we have

$$\begin{cases} F(x,y,0) = \frac{1}{2}\left[K_e\sin\delta_0 - u_1\omega x + \frac{3}{4}u_3\omega^3r^2x - \frac{3}{4}a_3r^2y - \frac{5}{8}a_5r^4y\right] \\ = 0 \\ G(x,y,0) = -\frac{1}{2}\left[K_e\cos\delta_0 + u_1\omega y + \frac{3}{4}u_3\omega^3r^2y - \frac{3}{4}a_3r^2x - \frac{5}{8}a_5r^4x\right] \\ = 0 \end{cases}$$

That is

$$\begin{cases} \left(u_1\omega - \frac{3}{4}u_3\omega^3r^2\right)x + \left(\frac{3}{4}a_3r^2 + \frac{5}{8}a_5r^4\right)y = K_e\sin\delta_0 \\ \left(\frac{3}{4}a_3r^2 + \frac{5}{8}a_5r^4\right)x - \left(u_1\omega + \frac{3}{4}u_3\omega^3r^2\right)y = K_e\cos\delta_0 \end{cases} \tag{11}$$

To arrange equation (11) to

$$\begin{cases} x=\dfrac{K_e\left[\sin\delta_0\left(u_1\omega+\dfrac{3}{4}u_3\omega^3r^2\right)\right]-\cos\delta_0\left(\dfrac{3}{4}a_3r^2+\dfrac{5}{8}a_5r^4\right)}{\omega^2u_1{}^2-\dfrac{9}{16}\omega^6u_3{}^2r^4-\left(\dfrac{3}{4}a_3r^2+\dfrac{5}{8}a_5r^4\right)^2} \\ y=\dfrac{K_e\left[\cos\delta_0\left(u_1\omega-\dfrac{3}{4}u_3\omega^3r^2\right)\right]-\sin\delta_0\left(\dfrac{3}{4}a_3r^2+\dfrac{5}{8}a_5r^4\right)}{\omega^2u_1{}^2-\dfrac{9}{16}\omega^6u_3{}^2r^4-\left(\dfrac{3}{4}a_3r^2+\dfrac{5}{8}a_5r^4\right)^2} \end{cases} \tag{12}$$

So that r satisfies

$$\begin{aligned} & r^2\left[\omega^2u_1{}^2-\frac{9}{16}\omega^6u_3{}^2r^4-\left(\frac{3}{4}a_3r^2+\frac{5}{8}a_5r^4\right)^2\right]^2 \\ &= K_e{}^2\omega^2u_1{}^2-\frac{3}{2}\omega^4u_3u_1r^2.\cos 2\delta_0+\frac{9}{16}\omega^6u_3{}^2r^4 \\ &-2\sin 2\delta_0.\omega u_1\left(\frac{3}{4}a_3r^2+\frac{5}{8}a_5r^4\right)+\left(\frac{3}{4}a_3r^4+\frac{5}{8}a_5r^4\right) \end{aligned} \tag{13}$$

And

$$\begin{cases} \dfrac{\partial F}{\partial x}=\dfrac{1}{2}\left[-u_1\omega+\dfrac{3}{4}u_3\omega^3(r^2+2x^2)-\dfrac{3}{2}a_3xy-\dfrac{5}{2}a_5xyr^2\right] \\ \dfrac{\partial F}{\partial y}=\dfrac{1}{2}\left[-\dfrac{3}{4}a_3(r^2+2y^2)+\dfrac{3}{2}u_3\omega^3xy-\dfrac{5}{8}a_5\left(r^4+4y^2r^2\right)\right] \\ \dfrac{\partial G}{\partial x}=-\dfrac{1}{2}\left[-\dfrac{3}{4}a_3(r^2+2x^2)+\dfrac{3}{2}u_3\omega^3xy-\dfrac{5}{8}a_5\left(r^4+4x^2r^2\right)\right] \\ \dfrac{\partial G}{\partial y}=-\dfrac{1}{2}\left[u_1\omega+\dfrac{3}{4}u_3\omega^3(r^2+2y^2)-\dfrac{3}{2}a_3xy-\dfrac{5}{2}a_5xyr^2\right] \end{cases} \tag{14}$$

Therefore, according to literature [10], if only *Jacobi* determinant

$$J_0=\left.\frac{\partial(F,G)}{\partial(x,y)}\right|_{\varepsilon=0}=\left.\left(\frac{\partial F}{\partial x}\frac{\partial G}{\partial y}-\frac{\partial F}{\partial y}\frac{\partial G}{\partial x}\right)\right|_{\varepsilon=0}$$

is not equal to zero, equation （9）must exist harmonic solution. And constant x and y ,which defined by equation（12）and equation（13）, can be considered approximately harmonic solution of equation（12）(if only $|\varepsilon|$ sufficient small).So that , $u=r\cos(\omega t-\theta)$ is approximately harmonic solution of equation（17）.Next, we will prove equation (4) definitely existing harmonic solution by example.

Example 1 in equation (4), set

$$K_e=1, \delta_0=\frac{\pi}{2}, \omega u_1=1, \frac{3}{4}\omega^3u_3=1, \frac{3}{4}a_3=1, \frac{5}{8}a_5=1$$

Then according to equation (12), we have

$$\begin{cases} x=\dfrac{1+r^2}{1-r^4-\left(r^2+r^4\right)^2} \\ y=-\dfrac{r^2+r^4}{1-r^4-\left(r^2+r^4\right)^2} \end{cases} \tag{15}$$

And $r^2=x^2+y^2$ must also satisfy

$$r^2\left[1-r^4-\left(r^2+r^4\right)^2\right]^2=\left(1+r^2\right)^2+\left(r^2+r^4\right)^2$$

That is

$$r^2\left[1-r^4-\left(r^2+r^4\right)^2\right]^2-\left(1+r^2\right)^2-\left(r^2+r^4\right)^2=0 \tag{16}$$

Set

$$f(r)=r^2\left[1-r^4-\left(r^2+r^4\right)^2\right]^2-\left(1+r^2\right)^2-\left(r^2+r^4\right)^2$$

Clearly, $f(r)$ is continued, and

$f(0)=-1<0, f(1)=8>0$

According to Intermediate Value Theorem, exist $r_0\in(0,1)$ to satisfy

$$f(r_0)=r_0{}^2\left[1-r_0{}^4-\left(r_0{}^2+r_0{}^4\right)^2\right]^2-\left(1+r_0{}^2\right)^2-\left(r_0{}^2+r_0{}^4\right)^2=0$$

That is, exist $r_0\in(0,1)$ to make equation (12) and equation (13) be founded. To calculate *Jacobi* Determinant, can make

$$x_0=-y_0=\frac{r_0}{\sqrt{2}}$$

And according to equation (14)

We obtain

$$\begin{cases} \dfrac{\partial F}{\partial x}=\dfrac{1}{2}\left(-1+3r_0{}^2+2r_0{}^4\right) \\ \dfrac{\partial F}{\partial y}=-\dfrac{3}{2}r_0{}^2\left(1+r_0{}^2\right) \\ \dfrac{\partial G}{\partial x}=\dfrac{3}{2}r_0{}^2\left(1+r_0{}^2\right) \\ \dfrac{\partial G}{\partial y}=-\dfrac{1}{2}\left(1+3r_0{}^2+2r_0{}^4\right) \end{cases}$$

Jacobi Determinant is

$$J_0=\left.\begin{vmatrix} \dfrac{\partial F}{\partial x} & \dfrac{\partial F}{\partial y} \\ \dfrac{\partial G}{\partial x} & \dfrac{\partial G}{\partial y} \end{vmatrix}\right|_{x_0=-y_0=\frac{r_0}{\sqrt{2}}}=\frac{1}{4}\left(1+6r_0{}^6+5r_0{}^8\right)\neq 0 \tag{17}$$

Because $J_0\neq 0$, equation (14) exists harmonic solution, that is harmonic solution bifurcation. To make a comprehensive survey, we obtain the following theorem

Theorem 1 Equation (4) must exist harmonic solution, and corresponding approximately harmonic solution is

$$u = r\cos(\omega t - \theta)$$

Where,

$$\theta = \arctan\frac{y}{x},$$

r is defined by equation (12) and equation (13)

3 Sub-harmonic Solution Bifurcation of System

Leading $\tau = \omega_0 t$ to make equation (1) to be dimensionless, where, Time is still expressed to be h_1, rolling angle is still expressed to be φ, frequency $\hat{\Omega}$ is still expressed to be Ω. Leading small parameter ε, rewrite equation (1) to be

$$\ddot{\varphi} + \varphi + \varepsilon u_1\dot{\varphi} + \varepsilon u_3\dot{\varphi}^3 + \varepsilon[a_3\varphi^2 + a_5\varphi^4 + h\cos(\Omega t)]\varphi = E_0\sin(\Omega t + \delta_0) \tag{18}$$

Where

$$U_1 = \frac{D_1}{2(I+\Delta I)}, U_3 = \frac{D_1}{I+\Delta I}, \omega_0^{\ 2} = \frac{D\overline{GM}_0}{I+\Delta I}, a_3 = \frac{k_3}{I+\Delta I},$$

$$a_5 = \frac{k_5}{I+\Delta I}, H = \frac{h_1}{I+\Delta I}, u_1 = \frac{U_1}{\omega_0}, u_3 = U_3\times\omega_0$$

Now discussing equation (18) exists sub-harmonic solution. Not losing generality, To simplify calculation, can set

$$\Omega = 2, E_0 = 1, \delta_0 = 0$$

Then we have

$$\ddot{\varphi} + \varphi = \sin 2t - \varepsilon\left(u_1\dot{\varphi} + u_3\dot{\varphi}^3 + a_3\varphi^3 + a_5\varphi^5 + h_0\cos 2t.\varphi\right) \tag{19}$$

First of all, when $\varepsilon = 0$, Equation (19) has harmonic solution $\varphi_0(t) = -\frac{1}{3}\cos 2t$, set

$x_1 = \varphi - \varphi_0(t)$, $\varphi_2 = \dot{\varphi} - \dot{\varphi}_0(t)$

According to Equation (19), we have

$$\begin{cases}\dot{x}_1 = x_2\\ \dot{x}_2 = -x_1 - \varepsilon\left[a_3\left(x_1 - \frac{1}{3}\cos 2t\right)^3 + a_5\left(x_1 - \frac{1}{3}\cos 2t\right)^5 + h_1\cos 2t\cdot\right.\\ \left.\left(x_1 - \frac{1}{3}\cos 2t\right) + u_1\left(x_2 + \frac{2}{3}\sin 2t\right) + u_3\left(x_2 + \frac{2}{3}\sin 2t\right)^3\right]\end{cases} \tag{20}$$

When $\varepsilon = 0$, Equation (20) has first integr

$$H(x_1, x_2) = \frac{1}{2}\left(x_1^{\ 2} + x_2^{\ 2}\right)$$

Corresponding closed locus is

$$L_h:\ x = q(t,h) = \left(\sqrt{2h}\sin t, \sqrt{2h}\cos t\right)^T, \quad h > 0$$

On the basis of

$$T(h) = 2\pi, \quad \Omega(h) = \frac{2\pi}{T(h)} = 1$$

We have

$$G(\theta, h) = q\left(\frac{\theta}{\Omega(h)}, h\right) = q(\theta, h) \tag{21}$$

And

$$g(t,x) = \left[0, -a_3\left(x_1 - \frac{1}{3}\cos 2t\right)^3 - a_5\left(x_1 - \frac{1}{3}\cos 2t\right)^5 - h_1\cos 2t\cdot\right.$$
$$\left.\left(x_1 - \frac{1}{3}\cos 2t\right) - u_1\left(x_2 + \frac{2}{3}\sin 2t\right) + u_3\left(x_2 + \frac{2}{3}\sin 2t\right)^3\right]^T \tag{22}$$

According to Equation (4) 、 (20) and (22), we have

$$\alpha(t+t_0,h)g(t,q(t+t_0,h)) =$$
$$[\Omega(h)g(t,q(t+t_0,h)) \wedge q_n(t+t_0,h)/f(q(t+t_0,h)) \wedge$$
$$q_h(t+t_0,h) = \frac{1}{\sqrt{2h}}\sin(t+t_0)a_3\left(\sqrt{2h}\sin(t+t_0) - \frac{1}{3}\cos(2t)\right)^3 + a_5\cdot$$
$$\left(\sqrt{2h}\sin(t+t_0) - \frac{1}{3}\cos(2t)\right)^5 + h_1\left(\sqrt{2h}\sin(t+t_0) - \frac{1}{3}\cos(2t)\right)\cos 2t$$
$$\left.+u_1\left(\sqrt{2h}\cos(t+t_0) + \frac{2}{3}\sin(2t)\right) + u_3\left(\sqrt{2h}\cos(t+t_0) + \frac{2}{3}\sin(2t)\right)^3\right]$$

Meanwhile, we easily obtain

$$DH(q(t+t_0,h))g(t,q(t+t_0,h)) = \left[-a_3\left(\sqrt{2h}\sin(t+t_0) - \frac{1}{3}\cos(2t)\right)^3 +\right.$$
$$a_5\cdot\left(\sqrt{2h}\sin(t+t_0) - \frac{1}{3}\cos(2t)\right)^5 + h_1\left(\sqrt{2h}\sin(t+t_0) - \frac{1}{3}\cos(2t)\right)\cos 2t$$
$$\left.+u_1\left(\sqrt{2h}\cos(t+t_0) + \frac{2}{3}\sin(2t)\right) + u_3\left(\sqrt{2h}\cos(t+t_0) + \frac{2}{3}\sin(2t)\right)^3\right]$$

According to period of g $T = \pi, \quad \Omega(h_0) = 1$, we have

$$\frac{m}{k} = \frac{2\pi}{\Omega(h_0)T} = 2 = \frac{2}{1}$$

That is

$$m = 2, k = 1$$

Therefore, we obtain second order harmonic $Me\ln ikov$ function of Equation (20) is

$$M(t_0,h)=\int_0^{2\pi} DH(q(t+t_0,h))g(t,q(t+t_0,h))dt \tag{23}$$

And

$$N(t_0,h)=\int_0^{2\pi} \alpha(t+t_0,h)g(t,q(t+t_0,h))dt \tag{24}$$

After calculating, we gain

$$\begin{cases} M(t_0,h)=2\left(\frac{3}{4}-\pi\right)a_3 h\cos t_0+\frac{5}{12}\pi a_5 h\cdot \\ (5h\cos t_0-\cos 2t_0)+2u_3\pi h\sin 2t_0+\frac{1}{3}\pi h_1 \\ N(t_0,h)=\frac{\pi}{2}a_3\left(\frac{3}{2}+\frac{\sqrt{2}}{3}h^{\frac{1}{2}}\right)+a_5\pi h^{\frac{1}{2}}\cdot \\ \left(\frac{5}{216}\sqrt{2}+\frac{20}{9}\sqrt{2}h+\frac{1}{3}\sqrt{2}h^2\right)-\frac{1}{2}\pi h_1\cos 2t_0 \end{cases} \tag{25}$$

According to literature [11], if existing

$h_0\in J,\ t_0^*\in(0.2\pi)$ to satisfy

$$M(t_0^*,h_0)=N(t_0^*,h_0)=0,\quad \det\frac{\partial(M,N)}{\partial(t_0,h)}\bigg|_{(t_0^*,h_0)}\neq 0$$

Equation（19）has second order harmonic solution when sufficient small $\varepsilon>0$ is considered. Next, we will prove Equation（19）definitely exist second order harmonic solution by example 2.

Example 2 In Equation（25）, set $h_0=1,\quad t_0=0$, and set $M(t_0^*,h_0)=N(t_0^*,h_0)=0$

Then we have

$$\begin{cases} 2\left(\frac{3}{4}-\pi\right)a_3+\frac{5}{3}a_5\pi=-\frac{1}{3}\pi h_1 \\ \frac{\pi}{2}a_3\left(\frac{3}{2}+\frac{\sqrt{2}}{3}\right)+\sqrt{2}a_5\pi\left(\frac{5}{216}+\frac{20}{9}+\frac{1}{3}\right)=\frac{1}{2}\pi h_1 \end{cases} \tag{26}$$

For certain h_1, Equation（26）is linear equation group about variable a_3,a_5 .And whose coefficient determinant is clearly unequal to zero. To set it's solution to be

$a_3=a_3(h_1),\ a_5=a_5(h_1)$,

that is, for certain h_1, if only

$a_3=a_3(h_1),\ a_5=a_5(h_1)$,

Then we must have $M(t_0^*,h_0)=N(t_0^*,h_0)=0$

When

$a_3=a_3(h_1),\ a_5=a_5(h_1),\ h_0=1,\quad t_0=0$

We have

$$\det\frac{\partial(M,N)}{\partial(t_0,h)}\bigg|_{(t_0^*,h_0)}=u_3\left(\sqrt{2}a_3\pi+\frac{365}{108}\sqrt{2}\pi+\frac{5}{6}\right)\neq 0$$

So that, Equation（22） exists second order sub-harmonic solution bifurcation.

To make a comprehensive survey, we obtain the following theorem.

Theorem 2 Equation（23） must exist second order sub-harmonic solution and corresponding approximately harmonic solution is

$$x(t)=q(t+t_0^*,h_0)=[\sqrt{2h_0}\sin(t+t_0),\sqrt{2h_0}\cos(t+t_0)]^T$$

Where $t^*,h_0\ t^*,h_0$ satisfy

$$M(t_0^*,h_0)=N(t_0^*,h_0)=0,\quad \det\frac{\partial(M,N)}{\partial(t_0,h)}\bigg|_{(t_0^*,h_0)}\neq 0$$

4 Heteroxenous Robit and Chaos of System

To make Equation（1）to be dimensionless, we have

$$\begin{aligned}&\ddot{\varphi}+\omega_0^2\varphi+K_3\varphi^3+u_1\dot{\varphi}+u_3\dot{\varphi}^3+ \\ &K_5\varphi^4+H\cos(\Omega t)\varphi=K_e\sin(\Omega t+\delta_0)\end{aligned} \tag{27}$$

Where

$$U_1=\frac{D_1}{2(I+\Delta I)},U_3=\frac{D_1}{I+\Delta I},\omega_0{}^2=\frac{D\overline{GM_0}}{I+\Delta I},K_3=\frac{k_3}{I+\Delta I},$$

$$K_5=\frac{k_5}{I+\Delta I},H=\frac{h_1}{I+\Delta I},K_e=\frac{E_0}{I+\Delta I}$$

In Equation（27）, set

$\omega_0^2=1,K_3=-1$

Other coefficients calling small parameter ε, then Equation（27）rewrite to be

$$\begin{aligned}&\ddot{\varphi}+\varphi-\varphi^3+\varepsilon(u_1\dot{\varphi}+u_3\dot{\varphi}^3+K_5\varphi^4+H\cos(\Omega t)\varphi)= \\ &\varepsilon K_e\sin(\Omega t+\delta_0)\end{aligned} \tag{28}$$

Set

$x=\varphi, y=\dot{x}$

Then according to Equation（28）, we have

$$\begin{cases} \dot{x} = y \\ \dot{y} = -x + x^3 + \varepsilon(K_e \sin(\Omega t + \delta_0) - u_1 y - \\ \quad u_3 y^3 - hx\cos(\Omega t) - K_5 x^5) \end{cases} \tag{29}$$

To rewrite Equation (29) as matrix

$$\dot{X} = f(X) + \varepsilon g(X,t) \tag{30}$$

Where

$$X = (x, y)^T,\ f(X) = (y, -x + x^3)^T,$$

$$g(X,t) = [0, K_e \sin(\Omega t + \delta_0) - u_1 y - u_3 y^3 - hx\cos(\Omega t) - K_5 x^5]^T$$

When $\varepsilon = 0$, Equation (30) is *Hamilton* system, whose *Hamilton* magnitude is

$$H(x,y) = \frac{1}{2}y^2 + \frac{1}{2}x^2 - \frac{1}{4}x^4 \tag{31}$$

Characteristic equation of linear approximately system is $\lambda^2 + 1 = 0$, Characteristic root is $\lambda_{1,2} = \pm i$, so that ,$(0,0)$ is the center, and also has two singular point $(\pm 1, 0)$. Characteristic equation is

$$\begin{vmatrix} 0-\lambda & 1 \\ -1+3x^2 & 0-\lambda \end{vmatrix} = 0$$

$$\lambda^2 = 2,\ \lambda_{1,2} = \pm\sqrt{2}$$

Therefore $(\pm 1, 0)$ are two saddle points and two heteroxenous robits are

$$q_i^0(t) = [x_\pm^0(t), y_\pm^0(t)]^T = [\pm th(\frac{\sqrt{2}}{2}t), \pm\frac{\sqrt{2}}{2}\sec h^2(\frac{\sqrt{2}}{2}t)]^T \tag{32}$$

according to Equation (32), we have

$$f(q_i^0(t)) = [y_\pm^0(t), -x_\pm^0(t) + (x_\pm^0(t))^3]^T$$

$$g(q_\pm^0(t), t+t_0) = [0, K_e \sin(\Omega(t+t_0)+\delta_0) - u_1 y_\pm^0(t) - u_3(y_\pm^0(t))^3 - hx_\pm^0(t)\cos(\Omega(t+t_0)) - K_5(x_\pm^0(t))^5]^T$$

$$f(q_i^0(t)) \wedge g(q_i^0(t), t+t_0) = [K_e \sin(\Omega(t+t_0)+\delta_0) - u_1 y_\pm^0(t) - u_3(y_\pm^0(t))^3 - hx_\pm^0(t)\cos(\Omega(t+t_0)) - K_5(x_\pm^0(t))^5] y_\pm^0(t)$$

So that, *Melnikov* function of Equation (30) is

$$M_\pm^0(t_0) = \int_{-\infty}^{+\infty} f(q_i^0(t)) \wedge g(q_i^0(t), t+t_0)dt = \int_{-\infty}^{+\infty}[K_e \sin(\Omega(t+t_0)+\delta_0) - u_1 y_\pm^0(t) - u_3(y_\pm^0(t))^3 - hx_\pm^0(t)\cos(\Omega(t+t_0)) - K_5(x_\pm^0(t))^5 y_\pm^0(t)] = K_e I_1 - u_1 I_2 - u_3 I_3 - hI_4 - K_5 I_5 \tag{33}$$

On the basis of Residue Theorem

$$\int_{-\infty}^{+\infty} y_\pm^0(t)\cos(\Omega t)dt = \pm\sqrt{2}\pi \csc h(\frac{\sqrt{2}}{2}\Omega\pi)$$

So that

$$I_1 = \int_{-\infty}^{+\infty} y_\pm^0(t)\sin(\Omega(t+t_0)+\delta_0)dt$$
$$= \int_{-\infty}^{+\infty}(\sin(\Omega t)\cos(\Omega t_0+\delta_0) + \cos(\Omega t)\sin(\Omega t_0+\delta_0))y_\pm^0(t)dt$$
$$= \sin(\Omega t_0+\delta_0)\int_{-\infty}^{+\infty}\cos(\Omega t)y_\pm^0(t)dt$$
$$= \pm\sqrt{2}\pi\csc h(\frac{\sqrt{2}}{2}\Omega\pi)\sin(\Omega t_0+\delta_0)$$

I_2, I_3, I_5 in Equation (30) can be gained by applying odevity of function and variable substitution method of integration.

$$I_2 = \int_{-\infty}^{+\infty}(y_\pm^0(t))^2 dt = \frac{1}{2}\int_{-\infty}^{+\infty}\sec h^4\frac{\sqrt{2}}{2}tdt$$
$$= \frac{\sqrt{2}}{2}\int_{-\infty}^{+\infty}\sec h^4 u du = \frac{\sqrt{2}}{2}\int_{-\infty}^{+\infty}(1 - th^2 u)dthu$$
$$= \frac{2}{3}\sqrt{2}$$

$$I_3 = \int_{-\infty}^{+\infty}(y_\pm^0(t))^4 dt = \frac{1}{4}\int_{-\infty}^{+\infty}\sec h^8\frac{\sqrt{2}}{2}tdt$$
$$= \frac{\sqrt{2}}{4}\int_{-\infty}^{+\infty}\sec h^6 u du$$
$$= \frac{\sqrt{2}}{4}\int_{-\infty}^{+\infty}(1 - 3thu + 3th^2u - th^3u)dthu$$
$$= \frac{\sqrt{2}}{6}$$

$$I_5 = \int_{-\infty}^{+\infty}(x_\pm^0(t))^5 y_\pm^0(t)dt$$
$$= \frac{\sqrt{2}}{2}\int_{-\infty}^{+\infty}th^5(\frac{\sqrt{2}}{2}t)\sec h^2(\frac{\sqrt{2}}{2}t)dt$$
$$= 0$$

Calculation of I_4 in Equation (33) is quite complicated. Firstly, considering odevity of function, we have

$$I_4 = \int_{-\infty}^{+\infty} x_\pm^0(t)y_\pm^0(t)\cos(\Omega(t+t_0))dt =$$
$$\int_{-\infty}^{+\infty} x_\pm^0(t)y_\pm^0(t)(\cos\Omega(t)\cos(\Omega t_0) - \sin\Omega(t)\sin(\Omega t_0))dt$$
$$= -\sin(\Omega t_0)\int_{-\infty}^{+\infty} x_\pm^0(t)y_\pm^0(t)\sin\Omega(t)dt =$$
$$-\frac{\sqrt{2}}{2}\sin(\Omega t_0)\int_{-\infty}^{+\infty} x_\pm^0(t)y_\pm^0(t)\sin\Omega(t)\sec h^2\left(\frac{\sqrt{2}}{2}t\right)th\left(\frac{\sqrt{2}}{2}t\right)dt$$
$$= -\sin(\Omega t_0)\int_{-\infty}^{+\infty}\sin(\sqrt{2}\Omega u)\sec h^2(u)th(u)du$$

Calculation of integration $\int_{-\infty}^{+\infty}\sin(\sqrt{2}\Omega u)\sec h^2(u)th(u)du$ also need apply

Residue Theorem which is belongs to theory of functions of a complex variable.

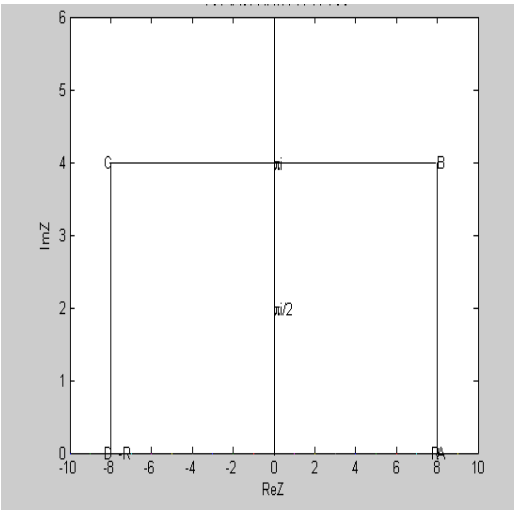

Fig.1 The closed curvel computed by Integral

According to the closed curve L which is made up of by line AB, BC, CD and DA in Fig.1, function of the complex variable $\sin(\sqrt{2}\Omega Z)\sec h^2(Z)th(Z)$ has third order pole $(0, \pi i/2)$ in L, residue is $i(\Omega^2-1)sh(\frac{\sqrt{2}}{2}\pi\Omega)$ .On the basis of Residue Theorem

$$\oint_L \sin(\sqrt{2}\Omega Z)\sec h^2 ZthZdZ = 2\pi i(i(\Omega^2-1)sh(\frac{\sqrt{2}}{2}\Omega\pi))$$

Set

$R \to \infty$

We get

$$(1+ch(\sqrt{2}\pi\Omega))\int_{-\infty}^{+\infty}\sin(\sqrt{2}\Omega u)\sec h^2 uthudu =$$

$$2\pi(1-\Omega^2)sh(\frac{\sqrt{2}}{2}\pi\Omega)$$

That is

$$\int_{-\infty}^{+\infty}\sin(\sqrt{2}\Omega u)\sec h^2 uthudu = \pi(1-\Omega^2)\sec h(\frac{\sqrt{2}}{2}\pi\Omega)th(\frac{\sqrt{2}}{2}\pi\Omega)$$

So

$$I_4 = \pi(\Omega^2-1)\sec h(\frac{\sqrt{2}}{2}\pi\Omega)th(\frac{\sqrt{2}}{2}\pi\Omega)\sin(\Omega t_0) \quad (34)$$

To synthesize the derived results, we obtain

$$M_\pm^0 = \pm\sqrt{2}\pi K_e\sin(\Omega t_0+\delta_0) - \frac{2}{3}\sqrt{2}u_1 - \frac{\sqrt{2}}{6}u_3 +$$
$$\pi(1-\Omega^2)h\sec h(\frac{\sqrt{2}}{2}\pi\Omega)th(\frac{\sqrt{2}}{2}\pi\Omega)\sin(\Omega t_0) =$$
$$[\pi(1-\Omega^2)h\sec h(\frac{\sqrt{2}}{2}\pi\Omega)th(\frac{\sqrt{2}}{2}\pi\Omega) \pm \sqrt{2}\pi K_e\cos(\delta_0)]\sin(\Omega t_0) \pm$$
$$\sqrt{2}\pi K_e\sin(\delta_0)\cos(\Omega t_0) - \frac{2}{3}\sqrt{2}u_1 - \frac{\sqrt{2}}{6}u \quad (35)$$

In Equation（35）, set

$$\begin{cases}\lambda_1 = \pi(1-\Omega^2)h\sec h(\frac{\sqrt{2}}{2}\pi\Omega)th(\frac{\sqrt{2}}{2}\pi\Omega) \pm \sqrt{2}\pi K_e\cos(\delta_0) \\ \lambda_2 = \pm\sqrt{2}\pi K_e\sin(\delta_0) \\ a = \frac{2}{3}\sqrt{2}u_1 + \frac{\sqrt{2}}{6}u\end{cases} \quad (36)$$

When λ_1, λ_2 is not zero at the same time, set again

$$\cos(\theta_0) = \frac{\lambda_1}{\sqrt{\lambda_1^2+\lambda_2^2}}, \quad \sin(\theta_0) = \frac{\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}} \quad (37)$$

Get

$$M_\pm(t_0) = \sqrt{\lambda_1^2+\lambda_2^2}[\sin(\Omega t_0+\theta_0) - \frac{a}{\sqrt{\lambda_1^2+\lambda_2^2}}] \quad (38)$$

When $|a| < \sqrt{\lambda_1^2+\lambda_2^2}$ exist $t_0 = 0$ to satisfy $M_\pm(t_0^\pm) = 0$, and $M_\pm^{'}(t_0) = \Omega\sqrt{\lambda_1^2+\lambda_2^2}\cos(\Omega t_0+\theta_0) \neq 0$,So $M_\pm(t_0)$ has simple repeated root zero point, therefore *Smale* horsehoe and chaos happen.[61~63].For which we have the following conclusion:

Theorem 3 When λ_1 and λ_2 are not zero at the same time, and $|a| < \sqrt{\lambda_1^2+\lambda_2^2}$, then equation (28) exists *Smale* horsehoe and appears chaotic phenomenon; when λ_1 and λ_2 are zero at the same time or $|a| > \sqrt{\lambda_1^2+\lambda_2^2}$, then according to all t_0, $M(t_0) \neq 0$, so there is no chaos.

5 Ideogenous robit and chaos of system

To make Equation (1) anamorphosis

$$\ddot{\varphi}+\frac{D_1}{I+\Delta I}\dot{\varphi}+\frac{D_1}{I+\Delta I}\dot{\varphi}^3+\left[\frac{D.\overline{GM}_0}{I+\Delta I}+\frac{k_3}{I+\Delta I}\varphi^2+\frac{k_5}{I+\Delta I}\varphi^4+\frac{h}{I+\Delta I}\cos(\Omega t)\right]\cdot\varphi=\frac{E_0}{I+\Delta I}\sin(\Omega t+\delta_0) \quad (39)$$

Considering initial stability height is negative, also set

$$\frac{D.\overline{GM}_0}{I+\Delta I}=-1,\ \frac{k_3}{I+\Delta I}=1$$

Other coefficients using small parameter ε,

We obtain

$$\ddot{\varphi}-\varphi+\varphi^3+\varepsilon[u_1\dot{\varphi}+u_3\dot{\varphi}^3+h\varphi\cos(\Omega t+a_5\varphi^5)]=\varepsilon K_e\sin(\Omega t+\delta_0) \quad (40)$$

Then set

$$x=\varphi,\ y=\dot{x}$$

$$\begin{cases}\dot{x}=y\\ \dot{y}=x-x^3+\varepsilon(K_e\sin(\Omega t+\delta_0)-u_1y-u_3y^3-hx\cos(\Omega t)-a_5x^5)\end{cases} \quad (41)$$

Set

$$X=(x,y)^T$$

$$f(X)=[y,x-x^3]^T$$

$$g(X,t)=[0,K_e\sin(\Omega t+\delta_0)-u_1y-u_3y^3-hx\cos(\Omega t)-K_5x^5]^T$$

Get

$$\dot{X}=f(X)+\varepsilon g(X,t) \quad (42)$$

According to $\varepsilon=0$, Equation (41) is *Hamilton* system, whose *Hamilton* magnitude is

$$H(x,y)=\frac{1}{2}y^2-\frac{1}{2}x^2+\frac{1}{4}x^4$$

Characteristic equation of linear approximately system is $\lambda^2-1=0$, characteristic root is $\lambda_{1,2}=\pm1$, so (0,0) is saddle point and also has symmetric ineogenous robit

$$q_i^0(t)=[x_\pm^0(t),y_\pm^0(t)]^T=\pm[\sqrt{2}\sec h(t),-\sqrt{2}\sec h(t)th(t)]^T \quad (43)$$

On the basis of equation (43),

$$f(q_\pm^0(t))=[y_\pm^0(t),-x_\pm^0(t)+(x_\pm^0(t))^3]^T$$

$$g(q_\pm^0(t),t+t_0)=[0,K_e\sin(\Omega(t+t_0)+\delta_0)-u_1y_\pm^0(t)-u_3(y_\pm^0(t))^3-hx_\pm^0(t)\cos(\Omega(t+t_0))-a_5(x_\pm^0(t))^5]^T$$

$$f(q_i^0(t))\wedge g(q_i^0(t),t+t_0)=[K_e\sin(\Omega(t+t_0)+\delta_0)-u_1y_\pm^0(t)-u_3(y_\pm^0(t))^3-hx_\pm^0(t)\cos(\Omega(t+t_0))-a_5(x_\pm^0(t))^5]y_\pm^0(t)$$

Therefore, *Melnikov* function in Equation (41) is

$$M_\pm^0(t_0)=\int_{-\infty}^{+\infty}f(q_i^0(t))\wedge g(q_i^0(t),t+t_0)dt=\int_{-\infty}^{+\infty}[K_e\sin(\Omega(t+t_0)+\delta_0)-u_1y_\pm^0(t)-u_3(y_\pm^0(t))^3-hx_\pm^0(t)\cos(\Omega(t+t_0))-a_5(x_\pm^0(t))^5y_\pm^0(t)]=K_eI_1-u_1I_2-u_3I_3-hI_4-a_5I_5 \quad (44)$$

On the basis of Residue Theorem, we get

$$\int_{-\infty}^{+\infty}\sin(\Omega t)\sec h(t)th(t)dt=\Omega\pi\csc h(\frac{\pi}{2}\Omega)$$

So that

$$I_1=\int_{-\infty}^{+\infty}y_\pm^0(t)\sin(\Omega(t+t_0)+\delta_0)dt$$
$$=\int_{-\infty}^{+\infty}(\sin(\Omega t)\cos(\Omega t_0+\delta_0)+\cos(\Omega t)\sin(\Omega t_0+\delta_0))y_\pm^0(t)dt$$
$$=\mp\sqrt{2}\cos(\Omega t_0+\delta_0)\int_{-\infty}^{+\infty}\sin(\Omega t)\sec h(t)th(t)dt$$
$$=\mp\sqrt{2}\pi\Omega\csc h(\frac{\pi}{2}\Omega)\cos(\Omega t_0+\delta_0)$$

I_2,I_3,I_5 of equation (44) can be gained by applying odevity of function and substitution method of integration

$$I_2=\int_{-\infty}^{+\infty}(y_\pm^0(t))^2dt=2\int_{-\infty}^{+\infty}\sec h^2tthh2tdt=2\int_{-\infty}^{+\infty}th^2tdtht=\frac{4}{3}$$

$$I_3=\int_{-\infty}^{+\infty}(y_\pm^0(t))^4dt=4\int_{-\infty}^{+\infty}\sec h^4tth^4tdt=4\int_{-\infty}^{+\infty}th^4t(1-th^2t)dtht=\frac{16}{35}$$

$$I_5=\int_{-\infty}^{+\infty}(x_\pm^0(t))^5y_\pm^0(t)dt=0\int_{-\infty}^{+\infty}th(t)\sec h^6(t)dt=0$$

According to equation (38), we have

$$I_4 = \int_{-\infty}^{+\infty} x_{\pm}^0(t) y_{\pm}^0(t) \cos(\Omega(t+t_0))dt$$
$$= \int_{-\infty}^{+\infty} x_{\pm}^0(t) y_{\pm}^0(t)(\cos(\Omega t)\cos(\Omega t_0) - \sin(\Omega t)\sin(\Omega t_0))dt$$
$$= \sin(\Omega t_0)\int_{-\infty}^{+\infty} \sec h^2 t th(t)\sin(\Omega t)dt$$
$$= \pi(1-\frac{\Omega^2}{2})\sec h(\Omega\pi) th(\Omega\pi)\sin(\Omega t_0)$$

To synthesize the derived results, we obtain

$$M_{\pm}^0 = \pm\sqrt{2}\pi K_e \csc(\frac{\pi}{2}\Omega)\cos(\Omega t_0 + \delta_0) - \frac{4}{3}u_1 - \frac{16}{35}u_3 + \pi(\frac{\Omega^2}{2}-1)h\sec h(\pi\Omega)th(\pi\Omega)\sin(\Omega t_0)$$
$$= \pm\sqrt{2}\pi K_e \csc(\frac{\pi}{2}\Omega)\cos(\delta_0)\cos(\Omega t_0) + [\pi(\frac{\Omega^2}{2}-1)h\sec h(\pi\Omega)th(\pi\Omega) \mp \sqrt{2}\pi K_e \csc(\frac{\pi}{2}\Omega)\sin(\delta_0)] \sin(\Omega t_0) - \frac{4}{3}u_1 - \frac{16}{35}u_3 \quad (45)$$

In Equation (45), set

$$u_1 = \pm\sqrt{2}\pi K_e \csc(\frac{\pi}{2}\Omega)\cos(\delta_0)$$
$$u_2 = \pi(\frac{\Omega^2}{2}-1)h\sec h(\pi\Omega)th(\pi\Omega) \mp \sqrt{2}\pi K_e \csc(\frac{\pi}{2}\Omega)\sin(\delta_0)$$
$$b = \frac{4}{3}u_1 + \frac{16}{35}u_3$$

When u_1, u_2 are not zero at the same time, set again

$$\sin\alpha_0 = \frac{u_1}{\sqrt{u_1^2+u_2^2}}, \quad \cos\alpha_0 = \frac{u_2}{\sqrt{u_1^2+u_2^2}} \quad (46)$$

We get

$$M_{\pm}(t_0) = \sqrt{u_1^2+u_2^2}[\sin(\Omega t_0 + \alpha_0) - \frac{b}{\sqrt{u_1^2+u_2^2}}] \quad (47)$$

When $|b| < \sqrt{u_1^2+u_2^2}$,

exist $t_0 = 0$ to satisfy $M_{\pm}(t_0^{\pm}) = 0$,

And

$M_{\pm}^{'}(t_0) = \Omega\sqrt{u_1^2+u_2^2}\cos(\Omega t_0 + \theta_0) \neq 0$, so $M_{\pm}(t_0)$ has simple repeated root zero point, therefore *Smale* horsehoe and chaos happen. For which we get the following conclusion:

Theorem 4 When u_1, u_2 are not zero at the same time, and $|b| < \sqrt{u_1^2+u_2^2}$,then Equation (40) exists *Smale* horsehoe and appears chaotic phenomenon; when u_1, u_2 are zero at the same time or $|b| > \sqrt{u_1^2+u_2^2}$, then according to all t_0 , $M(t_0) \neq 0$, so there is no chaos.

6 Conclusions

This article applying Poincaré Theorem of plan periodic system and *Melnikov* function, obtains the following achievements for ship parametric-highly excitation rolling dynamics system:

(1) Proved the system has harmonic solution bifurcation and gained approximately harmonic solution
(2) Proved the system exists sub- harmonic solution bifurcation and gained corresponding approximately sub- harmonic solution.
(3) Founded ideogenous robit and heteroxnous robit of the system, Proved the system would bring out chaotic motion under *Smale* horsehoe.

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