DEA Advanced Models for Geometric Evaluation of used Lathes

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Abstract: - This paper presents a preliminary study of the quality of lathes using Data Envelopment Analysis (DEA) as a multicriteria tool. Its purpose is to aggregate several measurements that show geometrical errors to obtain one single measurement that will be free of subjective opinions. Due to the small number of DMUs, additional techniques other than the DEA CCR model (super efficiency, cross-evaluation, inverted frontier and weights restrictions) are reviewed and used to differentiate between otherwise identical efficiency scores. The specificity of this problem required some changes in the above-mentioned methods. The models were applied to four lathes at the Fluminense Federal University Machine Shop.

Key-Words: - Data envelopment analysis; Machine tools; Decision aid

1 Introduction

The surface finish and geometrical deviations of a part being manufactured must satisfy design requirements, which demand rigorous standards in many cases. This depends on the quality of the machine tool being used. Thus, the increasing demand for better quality components has led to considerable research, to improve and maintain the performance of machine tools [1-4]. Thus machines lie at the heart of almost all manufacturing systems [5].

In order to improve quality, means are needed to measure it. The quality of a machine tool depends on several factors and, therefore, the quality measurement considers multiple criteria, which higher or lower importance is necessarily subjective and dependent upon the decision-maker's opinion.

To obtain a more reliable measurement, it is advisable to use a method that allows considering several factors with the least subjectivity. Data Envelopment Analysis (DEA) [6] allows this kind of evaluation, since the different factors under analysis are weighted by solving linear programming problems to compose an efficiency score without any decision-maker's subjective interference. Nevertheless, this method can not organise plainly the options owing to the existing identical efficiency scores. So, advanced models are required [7], [8] and [9].

In this paper, we use four of these models: Super efficiency [10], Weight Restrictions [11], Cross Evaluation [12, 13] and Inverted Frontier [14-16]. We have done some theoretical variations relative to the classic models, which are better suited to this problem. The DEA models are applied to evaluate four lathes belonging to the Fluminense Federal University Machine Shop. Those machines are quite old ones and used only for teaching purposes.

2 Machine Tool Metrology

2.1 Main Aspects

When manufacturing a component an engineer has to be aware of the machine tool operating performance, in order to reduce costs. Together with other precautions to avoid stoppages and failures, machine tool quality control is an important part of any organized production system. It measures the deviations that occur in the machine and their influences in the manufacturing tolerances. This determination must be objective and exact, and should involve, acceptance and comparison tests and periodic checking.

This evaluation is needed to guarantee that the machine follows the quality patterns required by the appropriate standards.

According to [1], experts estimate that 60-90% of the total quality costs are the result of internal failures. Some of these intrinsic internal failure costs can be estimated by a machine tool previous evaluation. A machine tool functional evaluation allows to:

– Detect errors while they are still small and do not affect the product quality;
– Know the manufacturing tolerances that are allowed for the part to proceed to a new production stage;
– Set the machine to quality standards, making it possible to standardise the production;
– Detect errors before they bring about serious damage to the machine itself;
– Plan the machine stoppages to correct the errors;
– Offer a safe base to decide either the recovery or substitution of the machine and its respective cost.

This evaluation is based upon a permanent control of the machine throughout its useful life. The fundamental goal of this analysis is to minimise the machine tool geometric errors, which generate non-conforming parts.

The increasing demand for better quality components has led to considerable research, to improve and maintain the performance of machine tools [17]. The machine tools manufacturers are requested more and more to reduce the time of component production and to improve their quality.

Part manufacture quality must depend solely on the following factors:

– Rigidity of machine, component and clamping devices;
– Alignment of the component;
– The quality and accuracy of the motor and control devices;

Each machine is submitted to acceptance tests relative to these items. The accuracy of machine tools is tested by means of geometric verifications and practical tests [18]. The geometric verifications evaluate the alignment of the several parts of a machine tool and are carried out whilst the machine is not in operation. The practical tests, that is the alignment tests, are carried out whilst the machine are in operation and shall be no concern of this paper.

Some measurements are chosen to evaluate the accuracy of each machine. Even one single measurement can be repeated at different points, thus supplying a series of evaluation criteria. Unitary output DEA models are able to synthesise these measurements in a single evaluation criterion, as explained in section 3. Special consideration will be given to the parallelism between the tailstock and the saddle guides in the following section.

2.2 Parallelism of tailstock and saddle guides

This test checks whether cylinders of different lengths have the same diameter deviation between both ends. In machines in use, it allows to measure the wear of the slide bars in relation to the tailstock guides. It can be expected that the wear in the slide bars is concentrated in the region next to the headstock, while the wear of the tailstock guide will be greater near the end of the bed.

The method of measurement is the following (Figure 1):

– The dial is fixed at the saddle;
– The sensor of the dial touches the tailstock guide near its end where the wear is the greatest;

– The saddle is displaced. At every 100 mm a reading of the dial is recorded.

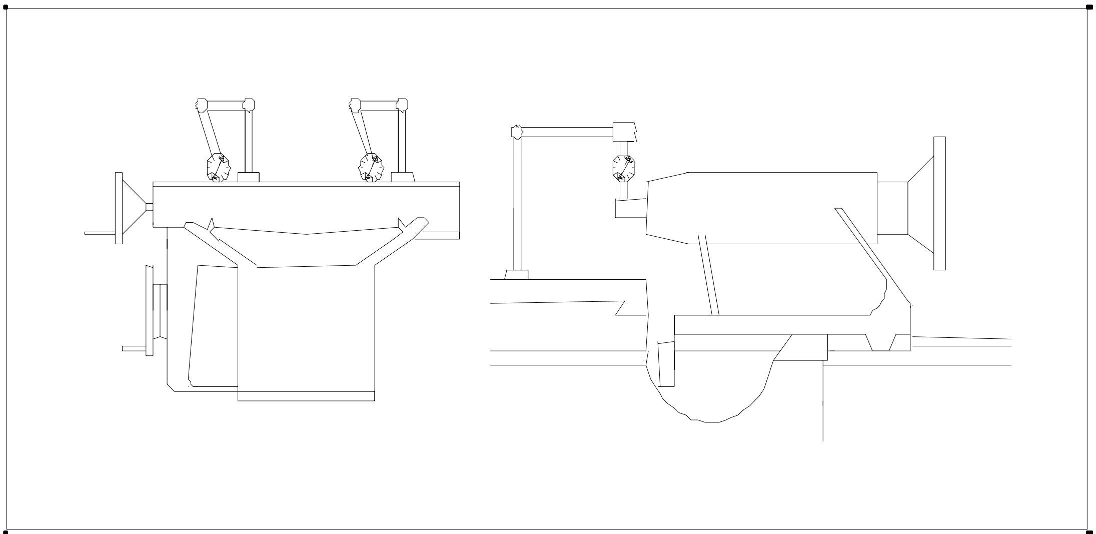

Figure 1

3 Data Envelopment Analysis

DEA measures the efficiency of productive units, named Decision-Making Units (DMUs), in the presence of multiple inputs (resources or production factors) and multiple outputs (goods or products). In this paper, the DEA model used is the CCR, also known as CRS, or constant returns to scale [6]. Its mathematical formulation considers that each DMU k, $k = 1,..., n$, is a production unit that uses m inputs i, x_{ik}, $i = 1,..., m$, to produce s outputs j, y_{jk}, $j = 1,..., s$. This model maximises the ratio between the linear combination of outputs and the linear combination of inputs, the constraint being that, for any DMU, this quotient cannot be bigger than 1.

This model can be turned into a Linear Programming Problem (LPP) presented in (1), where h_o is the efficiency of the DMU o under analysis; x_{io} and y_{jo} are inputs and outputs of the DMU o; v_i and u_j are the weights calculated by the model for inputs and outputs.

$$\max \ h_o = \sum_{j=1}^{s} u_j y_{jo}$$

subject to

$$\sum_{i=1}^{m} v_i x_{io} = 1 \qquad (1)$$

$$\sum_{j=1}^{s} u_j y_{jk} - \sum_{i=1}^{m} v_i x_{ik} \le 0, \forall k$$

$$v_i, u_j \ge 0, \forall i, j$$

Although DEA models have the advantage of ranking DMUs with no dependence on the decision-makers' opinions, they are favourable to the evaluated units. These can be efficient even not considering several evaluation criteria, which means that those weights are nil. Thus, it is usual to have a great number of units, which are 100% efficient. To obtain a tie free ranking as possible the total number of units must be, at least, the double or the triple of the number of variables. In the present paper, the number of variables is greater than the number of units. Such a situation does not hamper the use of DEA, but it is advisable the use of additional techniques to increase the discrimination among them. A review of these techniques can be found in [7] or [8].

4 Modelling

The DEA CCR model was used to evaluate the four lathes considering the parallelism measured between guides. The use of classic DEA model allows the efficiency for each lathe to be found only by taking into account its best operation band.

The deviations in parallelism are now assumed to be the price to pay for the lathe to function, and thus become the inputs. In a non-operating evaluation, there are no outputs, as nothing is produced. This is a situation that leads to paradoxes [19], which can be avoided by assuming that for all the lathes there is but one single output whose value is one. The output represents, thus, the very existence of the lathe, an approach supported by [20]. These models are equivalent to a particular weight sum multicriteria model in which every alternative acts as if it ascribes weights to each criterion, ignoring any opinion of an eventual decision-maker. Other models with unitary inputs or outputs are studied by [21] and used by [22]. Thus, DEA is used here only as a multicriteria tool, and not as a measure of classic efficiency neither as a benchmarking tool. Studies on this type of use for DEA can be found in [23], [24], [25], [26], [27] [28] and [29].

Table 1 condenses the data used. The measures between 0 and 100 mm are ignored, as they present similar and small errors that do not contribute to differentiate one lathe from another. We have made the assumption that all the data were known exactly, i.e., the errors are so small that can be ignored. If this was not possible, we would need to use a different DEA model in order to deal with imprecise data, for instance the model used in [30] and [31].

Regarding the numerical aspects, this problem is very simple due to the presence of only 4 DMUs. If a large number of DMUs is present on the problem. We shall use high speed algorithms for DEA such us [32, 33].

Table 1

DMU	Output	Inputs			
	"Existence of the lathe"	**d200**	**d300**	**d400**	**d500**
T1	1	35	58	52	35
T2	1	2	5	40	50
T3	1	10	30	48	49
T4	1	25	40	35	22

Table 2 presents the results of the classic DEA CCR model. The mathematical linear programme for Lathe T1 is in (2). This programme is called the multipliers formulation.

$$
\begin{aligned}
& max\ \ h_o = u \\
& subject\ to \\
& 35v_1 + 58v_2 + 52v_3 + 35v_4 = 1 \\
& u - 35v_1 - 58v_2 - 52v_3 - 35v_4 \le 0 \\
& u - 2v_1 - 5v_2 - 40v_3 - 50v_4 \le 0 \\
& u - 10v_1 - 30v_2 - 48v_3 - 49v_4 \le 0 \\
& u - 25v_1 - 40v_2 - 35v_3 - 22v_4 \le 0 \\
& u, v_1, v_2, v_3, v_4 \ge 0
\end{aligned} \qquad (2)
$$

There is a tie between DMUs T2 and T4. To differentiate between these two efficient DMUs, additional models must be used.

Table 2

DMU	Efficiency (%)
T1	67,8
T2	100,0
T3	85,7
T4	100,0

5 Improving Discrimination between Efficient Units

5.1 Theoretical formulation

In [7] is presented a review of models to increase discrimination in DEA. The authors separate these models in two big groups: models that incorporate *a priori* information, and models that do not use any *a priori* information for its calculations. In the first group, there are weight restrictions models and a DEA model originated by a fusion with multicriteria decision making, called Value Efficiency Analysis. Three models compose the second group: Super Efficiency, Cross Evaluation and Multiobjective DEA models.

In this paper, were chosen four models to increase discrimination in addition to DEA CCR classic model and the results are compared. The models employed, super efficiency, weight restrictions, cross evaluation and inverted frontier, are described succinctly in the following subsections. It is important to notice that the inverted frontier, as used here, is a new approach not described in [7].

5.2 Super Efficiency

The basic idea of this model is to compare the DMU being evaluated with a linear combination of all the other DMUs excluding the above mentioned DMU [34]. Thus, due to the fact that the DMU being evaluated is removed from the set of the comparison units, the resulting efficiency score can be bigger than 100%. As this model allows DMUs to get efficiency scores bigger than 100%, it manages to untie the efficient units. This method's major advantage is supplying discrimination among efficient units, without modifying the ranking of the inefficient ones. The drawback is that the order obtained depends only on local conditions of the efficient frontier, not on the general properties of the DMUs or the frontier. Furthermore, the authors in [35] noticed that the super efficiency model is god for outlier identification but rather unsatisfactory to rank efficient DMUs. Besides, the model can lead to unfeasibility [36, 37]. Nevertheless, we include this model in this study for comparison purposes only. A study where such an approach was used to rank technologies can be found in [38]

Figure 2 shows an example in which the evaluation of DMU C is presented using the super efficiency model.

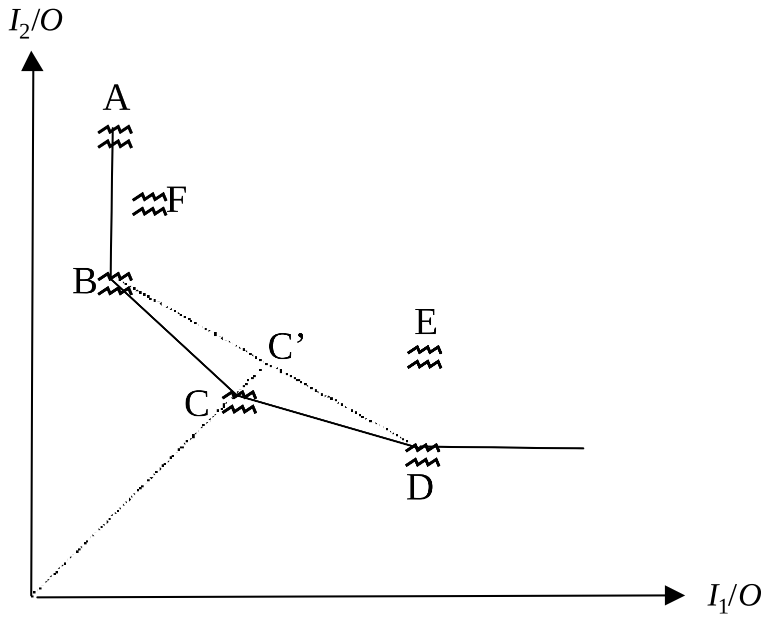

Figure 2

In (3), the linear programming model (called envelope formulation) for the super efficiency calculation is presented.

$$\min \quad \theta$$

subject to

$$\theta X_j \geq \sum_{\substack{k=1 \\ k\neq j}}^{n} \lambda_k X_k \qquad (3)$$

$$Y_j \leq \sum_{\substack{k=1 \\ k\neq j}}^{n} \lambda_k Y_k$$

$$\lambda_k \geq 0$$

In this model, θ is the efficiency; e is a unitary vector; x_k and y_k represent, respectively, the set of inputs and outputs; λ_k represents the contribution of DMU k in composing the target for DMU under evaluation.

5.3 Weights Restrictions

When there are value judgements about the relative importance of inputs and/or outputs, these judgements can be incorporated into the DEA models through weight restrictions associated to the DMUs inputs and/or outputs. In [11] is presented a complete survey of the evolution of value judgements incorporation using weights restrictions. These restrictions can be divided in three groups: direct restrictions on the multipliers; adjustment of the observed levels of input-output to capture of the value judgements; and restriction to virtual inputs and outputs. This last one often requires a lot of information from the decision maker. Some multicriteria techniques may be needed to help the decision agent to properly express its preferences [39].

In this paper, direct weights restrictions are employed. In this approach, initially developed by [40], and generalised by [41], we impose numerical bounds to the weights, also called multipliers, with the objective of neither overestimating nor ignoring inputs and outputs in the analysis.

Let $I_o = \sum_i v_i x_{io}$ be the numerator of the objective function in the original formulation, in which I_o is the virtual input consumed by the DMU *o*. The limits imposed to the inputs and outputs multipliers, v_i and u_j, are given by the relations presented in (4), where *II*, *SI*, *IO*, *SO* are the inferior and superior limits for inputs and outputs.

$$\begin{aligned} II_i \le v_i \le SI_i \\ IO_i \le u_i \le SO_i \end{aligned} \quad (4)$$

This kind of restriction can lead to an unfeasible model, because the need to establish an upper bound to an input weight implies a threshold for the virtual input of the remaining variables. In this paper the maximum values that do not lead unfeasible models are searched by trial and error.

5.4 Cross Evaluation

The idea behind Cross Evaluation, introduced by [42] and extended by [13], is a peer evaluation. This means that in Cross Evaluation DMUs are self-evaluated (classic DEA) and are also evaluated by the other DMUs. This is achieved using the optimum weights given by the classic model. One can say that, while in classic DEA each DMU is evaluated only from its own point of view, in Cross Evaluation it is also evaluated from the other DMUs points of view. Finally, Cross Efficiency is the average of all DMUs points of view.

There are a lot of papers on the use of cross evaluation. For instance, in [43] were used cross efficiencies as an alternate methodology for technology selection and in [44] it was used for educational evaluation.

On the other hand, the linear programming model that determines the efficiency score of each DMU can have multiple optimal solutions, that is, the weights (or multipliers) could not be unique. To choose among the several possible weights for each DMU, a new model is formulated where the weights must minimise the efficiency score (aggressive formulation) or, in contrast, maximise it (benevolent formulation), when these weights are applied to the other DMUs. In [13] was established the model (5) for weights calculation in the aggressive formulation, where the efficiency of DMU *s* using the DMU *k* weights is given in (6).

$$\begin{aligned} &\min \sum_i \left(u_{ki} \sum_{s \neq k} y_{si} \right) - \sum_j \left(v_{kj} \sum_{s \neq k} x_{sj} \right) \\ &\text{subject to} \\ &\sum_j v_{kj} x_{kj} = 1 \\ &\sum_i u_{ki} y_{ki} - E_{kk} \sum_j v_{kj} x_{kj} = 0 \\ &\sum_i u_{ki} y_{si} - \sum_j v_{kj} x_{sj} \le 0, \forall s \neq k \\ &u_{ki}, v_{kj} \ge 0 \end{aligned} \quad (5)$$

$$E_{ks} = \frac{\sum_i u_{ki} y_{si}}{\sum_j v_{kj} x_{sj}} \quad (6)$$

This approach forces the decision-maker to choose one of the formulations (aggressive or benevolent one). This fact contradicts the main characteristic of the cross evaluation method that is to minimise the interference of the decision-maker in the efficiency calculation process. However this is not an important drawback, since, almost always, both formulations lead to the same ranking.

A DEA model that assigns single weights to the extreme efficient DMUs was proposed by [45] and [46] through the use of a smoothed DEA frontier. An alternative approach is the one proposed by [47], which evaluates the average weights for each variable and, in sequence, the efficiency of each DMU with these weights. The theoretical foundations for the use of average weights instead of average efficiencies are given by [34], who proved that the cross evaluation method is equivalent to a fixed weight sum, when using one input and multiple outputs.

5.5 Inverted Frontier

The fourth model used for improving discrimination among the units is the inverted frontier. The concept of inverted frontier was initially proposed by [48]; it was adapted by [14] and [49] to evaluate each DMU pessimistically to obtain an interval efficiency. In [15] was used the inverted frontier as a opposite point of view, i.e., the classic DEA frontier represents the DMUs efficient for sellers and the inverted frontier represent the buyers points of view. The SIAD software [16] uses the inverted frontier to build an index for improving discrimination in DEA. The inverted frontier concept consists of considering outputs as inputs and inputs as outputs and, thus, determines DMUs with the worst managerial practices (and so we call it an inefficient frontier). In this paper, we use the inverted frontier as used in [16], [50] and [51].

Therefore, to establish a ranking of DMUs a composed efficiency score is calculated, that is the arithmetic average between the efficiency relating the original frontier and the inefficiency (1 minus efficiency) relating to the inverted frontier. This score can be presented in a normalised form, just by dividing all efficiency scores by the biggest calculated score. Figure 3 shows the graphic representation of the frontier and the inverted frontier to the BCC bi-dimensional case.

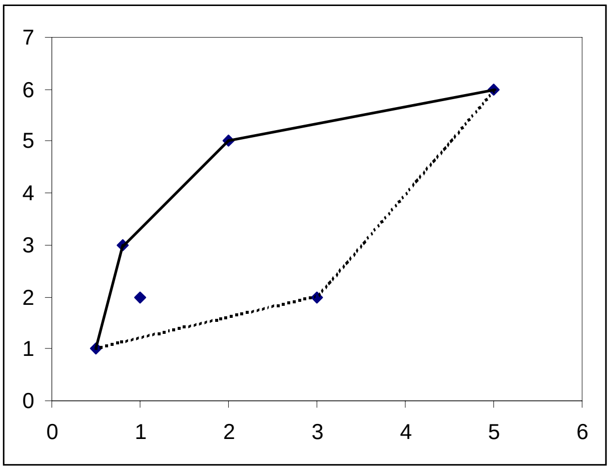

Figure 3. Frontier and inverted frontier for the BCC bi-dimensional case

6 Adaptations and results

The first additional model used to distinguish between DMUs T2 and T4 was the super efficiency model. Table 3 presents the results for this model and it can be seen that lathe 2 is better than lathe 4. Model (7) shows the multipliers formulation for lathes T2.

$$
\begin{aligned}
& max\ \ h_{T2} = u \\
& subject\ to \\
& 2v_1 + 5v_2 + 40v_3 + 50v_4 = 1 \\
& u - 35v_1 - 58v_2 - 52v_3 - 35v_4 \leq 0 \qquad (7) \\
& u - 10v_1 - 30v_2 - 48v_3 - 49v_4 \leq 0 \\
& u - 25v_1 - 40v_2 - 35v_3 - 22v_4 \leq 0 \\
& u, v_1, v_2, v_3, v_4 \geq 0
\end{aligned}
$$

To confirm the resulting superiority of DMU T2 compared to DMU T4, the other three models were applied: Weights Restrictions, Cross Evaluations and Inverted Frontier. In the first two cases, the evaluation was carried out in a different way from the one used in literature, that is, with methodological changes proposed by the authors for this particular case.

Table 3

DMU	Efficiency (%)
T1	67,8
T2	600,0
T3	85,7
T4	159,0

In the weight restrictions model, direct restrictions were used for inputs weights. We determined for each lathe the maximum value that we can impose as the minimum limit for all the multipliers value that still ascribes 100% efficiency to the lathe under evaluation. In such a methodology, the significant parameter is the admissible weight restriction, rather than the efficiency score, which is the value used in classical models. The best DMU is the one that admits the greater restriction to the multipliers. So, there was no need for the decision-maker to choose arbitrary weights, which allows the evaluation to be independent from any subjective opinion. Another weights restriction model based on objective formulation and using optimal values is found in [52].

In our model, T2 lathe stopped being efficient when we imposed a minimum input weight value of at least 0,01. For T4 lathe this parameter was 0,005 and, so, T2 lathe is the best, as it allowed more rigid weight restrictions maintaining its efficiency score. In mathematical terms this means that program (8) is unfeasible and program (9) is feasible and its efficiency equals one.

$$
\begin{aligned}
& max\ \ h_{T2} = u \\
& subject\ to \\
& 2v_1 + 5v_2 + 40v_3 + 50v_4 = 1 \\
& u - 35v_1 - 58v_2 - 52v_3 - 35v_4 \leq 0 \\
& u - 2v_1 - 5v_2 - 40v_3 - 50v_4 \leq 0 \qquad (8) \\
& u - 10v_1 - 30v_2 - 48v_3 - 49v_4 \leq 0 \\
& u - 25v_1 - 40v_2 - 35v_3 - 22v_4 \leq 0 \\
& v_1, v_2, v_3, v_4 \geq 0{,}01 \\
& u \geq 0
\end{aligned}
$$

$$
\begin{aligned}
&max\ \ h_{T4} = u \\
&subject\ to \\
&25v_1 + 40v_2 + 35v_3 + 22v_4 = 1 \\
&u - 35v_1 - 58v_2 - 52v_3 - 35v_4 \le 0 \\
&u - 2v_1 - 5v_2 - 40v_3 - 50v_4 \le 0 \\
&u - 10v_1 - 30v_2 - 48v_3 - 49v_4 \le 0 \\
&u - 25v_1 - 40v_2 - 35v_3 - 22v_4 \le 0 \\
&v_1, v_2, v_3, v_4 \ge 0{,}01 \\
&u \ge 0
\end{aligned}
\qquad (9)
$$

As described previously, in the cross evaluation approach, all DMUs are evaluated by its peers, generating an average score of evaluations. This procedure makes the ranking and scores extremely sensitive to the inclusion or exclusion of any DMU, even inefficient ones. To lessen this drawback, only efficient DMUs were used for mutual evaluation [45]. If DMU T2 evaluates DMU T4 (model 10), T4 efficiency score is 8%. In the opposite case, if T4 evaluates T2 (model 11), the T2 efficiency score is 44%. As there are only two evaluating DMUs, there is no need to calculate average efficiencies to confirm the previous conclusion about lathe T2 being better.

$$
\begin{aligned}
&min\ \ h_{T2} = u \\
&subject\ to \\
&2v_1 + 5v_2 + 40v_3 + 50v_4 = 1 \\
&u - 35v_1 - 58v_2 - 52v_3 - 35v_4 \le 0 \\
&u - 2v_1 - 5v_2 - 40v_3 - 50v_4 \le 0 \\
&u - 10v_1 - 30v_2 - 48v_3 - 49v_4 \le 0 \\
&u - 25v_1 - 40v_2 - 35v_3 - 22v_4 = 0 \\
&v_1, v_2, v_3, v_4, u \ge 0
\end{aligned}
\qquad (10)
$$

$$
\begin{aligned}
&min\ \ h_{T4} = u \\
&subject\ to \\
&25v_1 + 40v_2 + 35v_3 + 22v_4 = 1 \\
&u - 35v_1 - 58v_2 - 52v_3 - 35v_4 \le 0 \\
&u - 2v_1 - 5v_2 - 40v_3 - 50v_4 = 0 \\
&u - 10v_1 - 30v_2 - 48v_3 - 49v_4 \le 0 \\
&u - 25v_1 - 40v_2 - 35v_3 - 22v_4 \le 0 \\
&v_1, v_2, v_3, v_4, u \ge 0
\end{aligned}
\qquad (11)
$$

Finally, it was used the inverted frontier method. For this method, the SIAD software [16] was used. The mathematical linear program for lathe T2 inverted frontier is shown in model (12).

$$
\begin{aligned}
&max\ \ h_{invT2} = 2u_1 + 5u_2 + 40u_3 + 50u_4 \\
&subject\ to \\
&v = 1 \\
&35u_1 + 58u_2 + 52u_3 + 35u_4 - v \le 0 \\
&2u_1 + 5u_2 + 40u_3 + 50u_4 - v \le 0 \\
&10u_1 + 30u_2 + 48u_3 + 49u_4 - v \le 0 \\
&25u_1 + 40u_2 + 35u_3 + 22u_4 - v \le 0 \\
&u, v_1, v_2, v_3, v_4 \ge 0
\end{aligned}
\qquad (12)
$$

This software supplies the composed efficiency score, already normalised, besides the classic efficiency score. Table 4 condenses the results obtained.

Table 4

DMU	Inverted efficiency (%)	Normalised aggregated score (%)
T1	100,0	52,8
T2	100,0	77,8
T3	100,0	66,7
T4	71,4	100,0

It should be noticed that this method inverted the relative position of T2 and T4, when compared to the previous methods. This result can be explained by the fact that the other methods used information related to all inputs whereas the composed score uses only the most optimistic (classic efficiency DEA) and pessimist (inverted frontier) information. In this case, lathes T1, T2 and T3 have a 100% efficiency score in relation to the inverted frontier, which means that every one of them has at least one worst parallelism deviation. As the T4 lathe is never the worst in any measure, and is the best in relation to some of the measures, it had the best result according to this method.

7 Conclusions

The quality of a machine tool depends on several factors. These factors can be evaluated jointly using the DEA approach that carries through an evaluation using Linear Programming Problems, with no additional information from the decision-maker.

In the case study, due to the great number of variables compared to the number of evaluated units, two DMUs were considered efficient, the basic model being unable to distinguish between the two of them. To untie them, additional approaches were used that led mostly to the superiority of DMU T2 in relation to DMU T4.

Other models for increase discrimination were not used, mainly due to the unitary output nature of our model. Models based on benchmarks and canonical correlations [8] as well as those based on the calculus of partial productivities geometric averages [53] are meaningless in this situation. On the other hand, MCDEA model [54] only provides analysis, not a complete rank, even with the improvements of [55]. Additional improvements for the use of MCDEA model in machine tools evaluation are suggested for future works.

The question of choosing between T2 and T4 is conditional to the engineer's objective. If the goal is to guarantee that, in worst cases, the machine will not have a very deficient operation (pessimistic attitude), he may prefer T4, as the inverted frontier model showed. Any other goal leads the engineer to choose T2.

The results presented here are a preliminary study. Further developments must take into account a complete machine tool evaluation. Specifically, the test evaluation must be the subject of further study, where DEA will be used as a true measure of efficiency, and, instead of using a unitary output, the metal working speed, for instance, could be taken into account.

References

[1] S. Dolinsek, J. Kopac, Linkage between quality assurance tools and machinability criteria, *Journal of Materials Processing Technology,* Vol. 118, 2001,

[2] J. M. Zhou, M. Andersson, J. E. Ståhl, Identification of cutting errors in precision hard turning process, *Journal of Materials Processing Technology*, Vol. 153-154, 2004, pp. 746-750.

[3] M. Zhou, C. Zhao, An optimization model and multiple matching heuristics for quality planning in manufacturing systems, *Computer & Industrial Engineering*, Vol. 42, 2002, pp. 91-101.

[4] A. R. Motorcu, A. Güllü, Statistical process control in machining, a case study for machine tool capability and process capability, *Materials and Design*, Vol. 27, 2006, pp. 364-372.

[5] S. W. Lee, J. Y. Song, Construction and Operation of a Knowledge Base on Intelligent Machine Tools, *WSEAS Transactions on Systems,* Vol. 7, 2008, pp. 148-155.

[6] A. Charnes, W. W. Cooper, E. Rhodes, Measuring the efficiency of decision-making units, *European Journal of Operational Research,* Vol. 2, 1978, pp. 429-444.

[7] L. Angulo-Meza, M. P. E. Lins, Review of methods for increasing discrimination in data envelopment analysis, *Annals of Operations Research,* Vol. 116, 2002, pp. 225-242.

[8] N. Adler, L. Friedman, Z. Sinuany-Stern, Review of ranking methods in the data envelopment analysis context, *European Journal of Operational Research,* Vol. 140, 2002, pp. 249-265.

[9] V. V. Podinovski, E. Thanassoulis, Improving discrimination in data envelopment analysis: Some practical suggestions, *Journal of Productivity Analysis,* Vol. 28, 2007, pp. 117-126.

[10] P. Andersen, N. C. Petersen, A procedure for ranking efficient units in data envelopment analysis, *Management Science*, Vol. 39, 1993, pp. 1261-1264.

[11] R. Allen, A. Athanassopoulos, R. G. Dyson, E. Thanassoulis, Weights restrictions and value judgements in data envelopment analysis: evolution, development and future directions, *Annals of Operations Research,* Vol. 73, 1997, pp. 13-34.

[12] T. R. Sexton, R. H. Silkman, A. J. Logan, "Data Envelopment Analysis: Critique and extensions," in *Measuring efficiency: An assessment of data envelopment analysis*, H. Silkman, Ed. San Francisco: Jossey-Bass Editor, 1986.

[13] J. Doyle, R. H. Green, Efficiency and cross-efficiency in DEA derivations, meanings and uses, *Journal of the Operational*

Research Society, Vol. 45, 1994, pp. 567-578.

[14] T. Entani, Y. Maeda, H. Tanaka, Dual models of interval DEA and its extensions to interval data, *European Journal of Operational Research,* Vol. 136, 2002, pp. 32-45.

[15] M. P. E. Lins, L. F. L. Novaes, L. F. L. Legey, Real estate appraisal: A double perspective data envelopment analysis approach, *Annals of Operations Research,* Vol. 138, 2005, pp. 79-96.

[16] L. Angulo-Meza, L. Biondi Neto, J. C. C. B. Soares de Mello, E. G. Gomes, ISYDS - Integrated System for Decision Support (SIAD Sistema Integrado de Apoio a Decisão): A Software Package for Data Envelopment Analysis Model, *Pesquisa Operacional,* Vol. 25, 2005, pp. 493-503.

[17] A. D. Jenning, P. R. Drake, Machine Tool Condition Monitoring Using Statistical Quality Control Charts, *International Journal of Machining Tools Manufacturing,* Vol. 37, 1997, pp. 1243-1249.

[18] R. K. Jain, *Engineering Metrology*. New Delhi: Khanna Publisher, 1981.

[19] C. A. K. Lovell, J. T. Pastor, Radial DEA models without inputs or without outputs, *European Journal of Operational Research,* Vol. 118, 1999, pp. 46-51.

[20] R. G. Thompson, F. D. Singleton Junior, R. M. Thrall, B. A. Smith, Comparative evaluation for locating a high-energy physics lab in Texas, *Interfaces,* Vol. 16, 1986, pp. 35-49.

[21] L. E. Caporaletti, J. H. Dulá, N. K. Womer, Performance evaluation based on multiple attributes with nonparametric frontiers, *Omega,* Vol. 27, 1999, pp. 637-645.

[22] J. C. C. B. Soares de Mello, E. G. Gomes, L. Angulo-Meza, L. Biondi Neto, Cross Evaluation using Weight Restrictions in Unitary Input DEA Models: Theoretical Aspects and Application to Olympic Games Ranking, *WSEAS Transactions on Systems,* Vol. 7, 2008, pp. 31-39.

[23] T. J. Stewart, Relationships between Data Envelopment Analysis and Multicriteria Decision Analysis, *Journal of the Operational Research Society,* Vol. 47, 1996, pp. 654-665.

[24] F. Farinaccio, A. Ostanello, "Evaluation of DEA Validity as a MCDA/M Tool: Some Problems and Issues," Dipartimento de Informatica, Università di Pisa, Pisa, Italy TR-99-06, 1999.

[25] D. Bouyssou, Using DEA as a tool for MCDM: some remarks, *Journal of the Operational Research Society,* Vol. 50, 1999 1999, pp. 974-978.

[26] J. Sarkis, Comparative analysis of DEA as a discrete alternative multiple criteria decision tool, *European Journal of Operational Research,* Vol. 123, 2000, pp. 543-557.

[27] W. D. Cook, J. Zhu, *Modeling Performance Measurement: Applications and Implementation Issues in DEA*: Springer, 2005.

[28] L. F. A. d. C. Senra, L. C. Nanci, J. C. C. B. Soares de Mello, L. Angulo-Meza, Estudo sobre métodos de seleção de variáveis em DEA, *Pesquisa Operacional,* Vol. 27, 2007, pp. 191-207.

[29] P. H. Lin, Multiple criteria ranking by fuzzy data envelopment analysis, *WSEAS Transactions on Computers,* Vol. 5, 2006, pp. 810-816.

[30] D. V. Derpanis, E. Foundas, Decision's problem with the help of imprecise data envelopment analysis (IDEA), *WSEAS Transactions on Systems,* Vol. 4, 2005, pp. 745-750.

[31] D. V. Derpanis, E. Foundas, The problem of intervals in imprecise data envelopment analysis (IDEA), *WSEAS Transactions on Information Science and Applications,* Vol. 2, 2005, pp. 1673-1678.

[32] J. H. Dulá, Computations in DEA, *Pesquisa Operacional,* Vol. 22, 2002, pp. 165-182.

[33] L. Biondi Neto, M. P. E. Lins, E. G. Gomes, J. C. C. B. Soares de Mello, F. S. Oliveira, Neural data envelopment analysis: A simulation, *International Journal of Industrial Engineering: Theory Applications and Practice,* Vol. 11, 2004, pp. 14-24.

[34] T. R. Anderson, K. Hollingworth, L. Inman, The Fixed Weighting Nature of A Cross-Evaluation Model, *Journal of Productivity Analysis,* Vol. 17, 2002, pp. 249-255.

[35] R. D. Banker, ., H. Chang, The super-efficiency procedure for outlier identification, not for ranking efficient units, *European Journal of Operational Research,* Vol. 175, 2006, pp. 1311-1320.

[36] M. Thrall, Duality, classification and slacks in DEA, *Annals of Operations Research,* Vol. 66, 1996,

[37] J. Zhu, Super-efficiency and DEA sensitivity analysis, *European Journal of Operational Research*, Vol. 129, 2001,

[38] R. Farzipoor Saen, Using super-efficiency analysis for ranking technologies, *WSEAS Transactions on Mathematics,* Vol. 5, 2006, pp. 599-604.

[39] J. C. C. B. Soares de Mello, M. P. E. Lins, M. H. C. Soares de Mello, E. G. Gomes, Evaluating the performance of Calculus classes using operational research tools, *European Journal Of Engineering Education* Vol. 27, 2002, pp. 209-218

[40] R. G. Dyson, E. Thanassoulis, Reducing Weight Flexibility in Data Envelopment Analysis, *Journal of the Operational Research Society,* Vol. 39, 1988, pp. 563-576.

[41] Y. Roll, W. D. Cook, B. Golany, Controlling factor weights in DEA, *IIE Transactions,* Vol. 23, 1991, pp. 2-9.

[42] T. R. Sexton, *Measuring Efficiency: An assessment of Data Envelopment Analysis. New Directions For Program Evaluation.* San Francisco: Jossey-Bass, 1986.

[43] R. C. Baker, S. Talluri, A closer look at the use of data envelopment analysis for technology selection, *Computers & Industrial Engineering*, Vol. 32, 1997, pp. 101-108.

[44] L. Angulo-Meza, E. G. Gomes, L. Biondi Neto, P. H. G. Coelho, Avaliação do ensino nos cursos de Pós-graduação em engenharia: Um enfoque quantitativo de avaliação em conjunto, *Engevista,* Vol. 5, 2003, pp. 41-49.

[45] J. C. C. B. Soares de Mello, M. P. E. Lins, E. G. Gomes, Construction of a smoothed DEA frontier, *Pesquisa Operacional*, Vol. 28, 2002, pp. 183-201.

[46] J. C. C. B. Soares de Mello, E. G. Gomes, L. Biondi Neto, M. P. E. Lins, Suavização da Fronteira DEA: O caso BCC tridimensional, *Investigação Operacional*, Vol. 24, 2004, pp. 89-107.

[47] M. P. E. Lins, E. G. Gomes, J. C. C. B. Soares de Mello, A. J. R. Soares de Mello, Olympic ranking based on a zero sum gains DEA model, *European Journal of Operational Research*, Vol. 148, 2003, pp. 312-322.

[48] Y. Yamada, T. Matui, M. Sugiyama, New analysis of efficiency based on DEA, *Journal of the Operations Research Society of Japan*, Vol. 37, 1994, pp. 158-167.

[49] T. Entani, H. Tanaka, Improvement of efficiency intervals based on DEA by adjusting inputs and outputs, *European Journal of Operational Research*, Vol. 172, 2006, pp. 1004-1017.

[50] J. C. C. B. Soares de Mello, L. Angulo-Meza, E. G. Gomes, L. Biondi Neto, Evaluación de la concentración en una ruta aérea brasilera con modelo DEA y frontera invertida, *Revista de la Facultad de Ingeniería,* Vol. 14, 2006, pp. 64-71.

[51] L. Angulo-Meza, J. C. C. B. Soares de Mello, E. G. Gomes, A. J. S. Fernandes, Selecção de variáveis em DEA aplicada a uma análise do mercado de energia elétrica, *Investigação Operacional,* Vol. 27, 2007, pp. 21-36.

[52] C. C. Liu, Applying optimal weights restriction model in analyzing the efficiency of organization, *WSEAS Transactions on Mathematics*, Vol. 5, 2006, pp. 236-241.

[53] J. C. C. B. Soares de Mello, E. G. Gomes, L. Angulo-Meza, M. H. C. Soares de Mello, A. J. R. Soares de Mello, Engineering Post-Graduate Programmes: A Quality and Productivity Analysis, *Studies in Educational Evaluation,* Vol. 32, 2006, pp. 136-152.

[54] X. B. Li, G. R. Reeves, Multiple criteria approach to data envelopment analysis, *European Journal of Operational Research,* Vol. 115, 1999, pp. 507-517.

[55] J. C. N. Clímaco, J. C. C. B. Soares de Mello, L. Angulo-Meza, "Performance Measurement – From DEA to MOLP," in *Encyclopedia of Decision Making and Decision Support Technologies*. vol. 2, F. Adam and P. Humphreys, Eds. Hershey: Information Science Reference, 2008, pp. 709-715.