Pricing Strategy of Ecological Industry Chain Based on Game Theory

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Abstract: -The pricing decision of three stages ecological industry chain of which is consisted of manufacturer producing mainproduct and byproduct, mainproduct seller and byproduct buyer is studied. Four kinds of pricing decision were discussed: manufacturer stackeberg equilibrium, mainproduct seller-manufacturer-byproduct buyer stackeberg equilibrium, buyer stackeberg equilibrium, alliance of mainproduct seller and byproduct buyer stackeberg equilibrium. Computational instances show that the system profit of manufacturer stackeberg equilibrium equals that of alliance of mainproduct seller and byproduct buyer stackeberg equilibrium, and is larger than that of other two equilibriums. Leader can obtain more profit than follower. The profit of buyer stackeberg equilibrium is less than other three equilibriums because both of mainproduct seller and byproduct buyer makes the price decision with manufacturer independently in the situation. Moreover, it is possible that the deal in the chain can not be made in buyer stackeberg equilibrium. Alliance of mainproduct seller and byproduct buyer should be the good way by which the system profit can be improved.

Key-Words: - Ecological industry chain; Game theory; Pricing decision; Stackelberg model

1 Introduction

Ecological industry chain is industry chain or network which is composed of many industries with ecological dependency relationship. It is different from traditional supply chain because it has attribute of the natural ecosystem. Enterprises alliance is constructed by the link of resource, and there are no wastes in the ecological link. The byproducts of a plant are the raw materials of another plant. A closed-loop system is formed by this kind of material flow[1]. We called it ecological industry chain. Ecological industry chain provides a new approach for environment protection, efficient usage of resource, and sustainable development. More and more researchers have studied the ecological industry chain recently. However, most of researches focus on the design of ecological industry chain, performance indicator, and management or plan of the ecological industry park. Many problems and risks in the running stage of ecological industry chain, for instance, conflict of private interests and objectives among members, have been neglected [2-3].

Wang Xiu-li analyzed the different private interests of upstream and downstream enterprise in ecological industrial chain constructing process， and proposed constructing methods of ecological industry chain according to different players' interests[4]. Chen Jie analyzed the environmental purchasing model under the market condition in which the green raw material and non-green raw material appear at the same time based on game theory. Some results are obtained[5]. The methods for improving environmental purchasing are given. But few researcher focused their attention on pricing problem in the phase of running of the ecological industry chain, this kind of problem does exist is very important to enterprises cooperation in ecological industry chain. Thus, three stages ecological industry chain was took as a background. Pricing decisions between manufacturers and main product buyers and byproduct buyers were studied in this paper.

In three stages ecological industry chain, the manufacturer produces one kind mainproduct and byproduct. Production proportion between mainproduct and byproduct is a certain number. The manufacturer sells main products to one buyer (retailer), and sells byproducts to the downstream enterprise. Because there is a certain proportion between main products and byproducts, profit of the manufacturer comes from mainproduct sale revenue and byproduct sale revenue. The pricing decisions of mainproduct need be considered together with the pricing decision of byproduct. In this paper, pricing decisions are researched based on game theory.

Though few researchers focused their attentions on the pricing problem of the ecological industry chain, there are a lot researches about pricing strategy of supply chain. Ertek (2002) studied the pricing strategy of a two stages supply chain with a single product and a supplier and a buyer, and obtained the optimal price value of supplier-driven problem and buyer-driven problem[6]. Bian Xu (2003) studied the optimal price-discount contract between a single seller and a single buyer[7]. Choi (1991) studied the price competition of two manufacturers and a retailer, discussed Manufacturer-Stackelberg, Retailer- Stackelberg and Vertical-Nash game models under the linear demand and nonlinear demand[8]. Chen (2001) discussed the pricing and coordination mechanisms for a distribution system with one supplier and multiple retailers[9]. Lu (2006) discussed the pricing problem of the supply chain with one manufacturer and two retailers. Furthermore, the pricing problem of supply chain with asymmetric information was studied in research papers [10-13]. Zhao (2002) studied the coordinate decision of production and pricing[14]. Liu (2007) studied the pricing incentive model with sharing information[15]. Gu (2005) studied the pricing problem in reverse logistics[16].

In all above articles, a common assumption is that manufacturer/supplier provides downstream enterprise one kind of product. However, manufacturer produces not only mainproduct but also byproduct in many circumstances. If the pricing problem of mainproduct is studied independent of the pricing problem of byproduct, production quantity of the mainproduct and byproduct may not be equal to output proportion, and may not be the optimal. Thus, the pricing decision of mainproduct and byproduct should be made together.

2 Problem description and assumption

In fig.1, a three stages ecological industry chain is took as background with a manufacturer M, a mainproduct buyer R1 and a byproduct buyer R2. Manufacturer M produces not only mainproduct (product 1) but also byproduct (product 2). Manufacturer M sells main products to buyer R1, and buyer R1 sells these to market. Moreover, manufacturer M sells byproducts to buyer R2, and buyer R2 sells these to market after some processing. The following notations are parameters for the problem.

p_i = Sale price of the buyer i (i = 1, 2) sell its products to market

w_i = Transfer price of manufacturer sell product i (i = 1, 2) to buyer i (i = 1, 2)

q_i = Quantity of product i (i = 1, 2) that manufacturer sell to buyer i (i = 1, 2)

d_i = Market demand of product i (i = 1, 2)

k = Output proportion between mainproduct and byproduct, manufacturer produce one unit mainproduct and k unit byproduct.

c_i = Cost of manufacturer produce one unit product i (i = 1, 2)

π_m = Profit function of manufacturer

π_i = Profit function of buyer i (i = 1, 2)

We suppose that the market demand of any mainproduct or byproduct is inversely proportional to its sell price. That is to say that $d_i = a_i - b_i p_i$ (i = 1, 2), in which ai and bi are constants respectively, and $a_i > 0$, $b_i > 0$. Unit profit of buyer i (i = 1, 2) is denoted by $m_i = p_i - w_i$. Profit function of manufacturer is shown as follows.

$$\pi_m = q_1 w_1 + q_2 w_2 - c_1 q_1 - c_2 q_2$$

Profit function of buyer i (i = 1, 2) is shown as follows:

$$\pi_i = q_i \left(p_i - w_i \right)$$

q_i is equal to d_i, in the profit function of buyer. Processing costs of byproducts of buyer are neglected. In the follow pricing model, costs have no essential effect on the result. Assumptions of this paper are shown as follows.

(1) Manufacturer must sell all byproducts to downstream enterprise because byproducts or wastes in ecological industry chain do harm to environment. This assumption is reasonable in many circumstances.

(2) Information is common knowledge to manufacturer and two buyers. It means that all of manufacturer and two buyers know about the value of all parameters in the problem.

To above ecological industry chain structure, manufacturer and two buyers maybe have different relative influence power. The first situation is that manufacturer has stronger influence power than buyer in pricing decision process. The second situation is that buyer has stronger influence power than manufacturer in pricing decision process. The third situation is that manufacturer has moderate influence power (stronger than one buyer, weaker than another buyer). Different relative influence power between manufacturer and two buyers has different pricing model and optimal price solutions. There are four kinds of pricing decision models below.

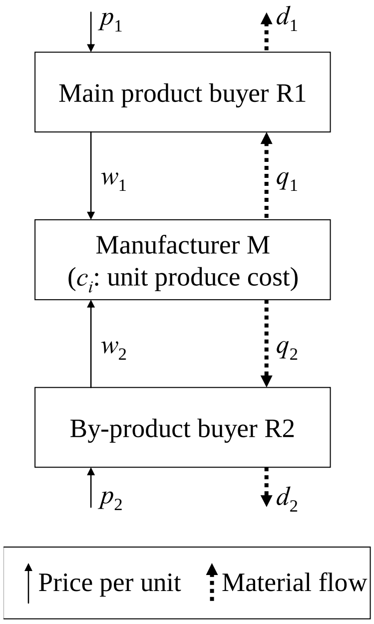

Fig.1 Three stages ecological industry chain structure

3 Pricing decision of ecological industry chain

3.1 Manufacturer Stackelberg model

In this circumstance, manufacturer is the leader of pricing decision. Buyer is the follower of pricing decision. Manufacturer makes his own transfer price w_i, and considers the buyer's reaction function to w_i. The buyers' reaction function can be obtained by the first order condition of buyers' profit function π_i (i = 1, 2).

$$\pi_i = (p_i - w_i) q_i = (p_i - w_i)(a_i - b_i p_i) \quad i = 1, 2 \tag{1}$$

Let

$$\frac{\partial \pi_i}{\partial p_i} = -b_i (p_i - w_i) + (a_i - b_i p_i) = 0 \quad i = 1, 2$$

We can obtain the equation (2).

$$p_i = \frac{b_i w_i + a_i}{2b_i} = \frac{w_i}{2} + \frac{a_i}{2b_i} \quad i = 1, 2 \tag{2}$$

According to pi, manufacturer's profit function π_m is denoted as follows.

$$\pi_m = (w_1 - c_1)(a_1 - b_1 p_1) + (w_2 - c_2)(a_2 - b_2 p_2) \tag{3}$$

Equation (4) can be obtained by substituting p_i denoted in equation (2) into equation (3).

$$\pi_m = (w_1 - c_1)\left(\frac{a_1}{2} - \frac{b_1 w_1}{2}\right) + (w_2 - c_2)\left(\frac{a_2}{2} - \frac{b_2 w_2}{2}\right) \tag{4}$$

Profit function π_m above is constrained with equation (5).

$$a_1 - b_1 p_1 = k(a_2 - b_2 p_2) \tag{5}$$

Equation (6) can be obtained by substituting p_i (i = 1, 2) into equation (5).

$$w_1 = \frac{a_1 - ka_2 + kb_2 w_2}{b_1} \tag{6}$$

We get the following equation after equation (6) is substituted into equation (4).

$$\pi_m(w_2) = \left(\frac{a_1 - ka_2 + kb_2 w_2}{b_1} - c_1\right)\left(\frac{ka_2}{2} - \frac{kb_2 w_2}{2}\right) + (w_2 - c_2)\left(\frac{a_2}{2} - \frac{b_2 w_2}{2}\right)$$

The value of w_2 can be obtained by solving the first order condition of $\pi_m(w_2)$.

$$w_2 = \frac{2k^2 a_2 b_2 - ka_1 b_2 + b_1 b_2 c_1 + kb_1 b_2 c_1 + a_2 b_1}{2b_2 (b_1 + k^2 b_2)}$$

According to w_2 and equation (6), (2), (1), (3), we can obtain the values of w_1、p_1、p_2、π_1、π_2 and π_m.

$$w_1 = \frac{2a_1 b_1 + k^2 a_1 b_2 - ka_2 b_1 + kb_1 b_2 c_2 + k^2 b_1 b_2 c_1}{2b_1 (b_1 + k^2 b_2)}$$

$$p_1 = \frac{4a_1 b_1 + 3k^2 a_1 b_2 - ka_2 b_1 + kb_1 b_2 c_2 + k^2 c_1 b_1 b_2}{4b_1 (b_1 + k^2 b_2)}$$

$$p_2 = \frac{4k^2 a_2 b_2 - ka_1 b_2 + b_1 b_2 c_2 + kb_1 b_2 c_1 + 3b_1 a_2}{4b_2 (b_1 + k^2 b_2)}$$

$$\pi_1 = \frac{k^2 (ka_1 b_2 + a_2 b_1 - c_2 b_1 b_2 - kb_1 b_2 c_1)^2}{16b_1 (b_1 + k^2 b_2)^2}$$

$$\pi_2 = \frac{(a_2 b_1 + ka_1 b_2 - b_1 b_2 c_2 - kb_1 b_2 c_1)^2}{16b_2 (b_1 + k^2 b_2)^2}$$

$$\pi_m = \frac{1}{8b_1 b_2 (b_1 + k^2 b_2)^2} * \begin{pmatrix} c_2^2 b_1^2 b_2^2 + k^2 c_1^2 b_1^2 b_2^2 - 2a_2 b_1^2 b_2 c_2 - 2ka_2 b_1^2 b_2 c_1 \\ +2kb_1^2 b_2^2 c_1 c_2 + a_2^2 b_1^2 - 2ka_1 b_1 b_2^2 c_2 \\ +2ka_1 b_1 a_2 b_2 - 2k^2 a_1 b_1 b_2^2 c_1 + k^2 a_1^2 b_2^2 \end{pmatrix}$$

3.2 Buyer R1- manufacturer-buyer R2 Stackelberg model

In this situation, manufacturer is follower of buyer R1 and leader of buyer R2. Firstly, buyer R1 gives the price of mainproduct. Then, the value of q_1 and q_2 can be easily obtained. In order to sell all byproducts to buyer R2, manufacturer needs make the appropriate value of w_2 according to the reaction function of buyer R2. After w_2 is determined, manufacturer computes the optimal of w_1 according to profit function π_m. Finally, buyer R1 determines the optimal value of p_1 according to profit function π_1.

The value of byproduct q_2 is a certain number as long as buyer R1 gives the value of p_1.

$$p_2 = \left(\frac{a_2}{b_2} - \frac{\left(a_1 - b_1 p_1\right)}{k b_2} \right) \tag{7}$$

According to equation (2), we can obtain equation (8).

$$w_2 = \frac{2 b_2 p_2 - a_2}{b_2} \tag{8}$$

Equation (9) can be obtained after equation (7) is substituted into equation (8).

$$w_2 = \frac{k a_2 - 2\left(a_1 - b_1 p_1\right)}{k b_2} \tag{9}$$

Equation (9) is substituted into the profit function of manufacturer π_m.

$$\pi_m = \left(w_1 - c_1 + \frac{k a_2 - 2\left(a_1 - b_1 m_1 - b_1 w_1\right)}{k^2 b_2} - \frac{c_2}{k} \right) * \left(a_1 - b_1 m_1 - b_1 w_1\right) \tag{10}$$

Let $\partial \pi_m / \partial w_1 = 0$, we can obtain the value of w_1.

$$w_1 = \frac{k^2 b_2 a_1 - k^2 b_1 b_2 m_1 + 4 a_1 b_1 - 4 b_1^2 m_1 + k b_1 b_2 c_2 + k^2 b_1 b_2 c_1 - k a_2 b_1}{2 b_1 \left(k^2 b_2 + 2 b_1\right)}$$

or

$$w_1 = \frac{k^2 b_2 a_1 - k^2 b_1 b_2 p_1 + 4 a_1 b_1 - 4 b_1^2 p_1 + k b_1 b_2 c_2 + k^2 b_1 b_2 c_1 - k a_2 b_1}{k^2 b_1 b_2} \tag{11}$$

Substituted the value of w_1 into $\pi_1 = \left(p_1 - w_1\right)\left(a_1 - b_1 p_1\right)$, and make $\partial \pi_1 / \partial p_1 = 0$.

$$p_1 = \frac{3 k^2 b_2 a_1 + 8 a_1 b_1 - k a_2 b_1 + k^2 b_1 b_2 c_1 + k b_1 b_2 c_2}{4 b_1 \left(k^2 b_2 + 2 b_1\right)}$$

We can obtain values of p_2、w_1、w_2、π_1、π_2 and π_m according to equation (7)、(8)、(11)、(1)、(10) and (9).

$$p_2 = -\frac{k a_1 b_2 - 7 a_2 b_1 - k b_1 b_2 c_1 - b_1 b_2 c_2 - 4 k^2 b_2 a_2}{4 b_2 \left(k^2 b_2 + 2 b_1\right)}$$

$$w_2 = -\frac{k a_1 b_2 - 3 a_2 b_1 - k b_1 b_2 c_1 - b_1 b_2 c_2 - 2 k^2 b_2 a_2}{2 b_2 \left(k^2 b_2 + 2 b_1\right)}$$

$$w_1 = \frac{\begin{gathered}(3 k^3 b_2^2 a_1 + 4 k a_1 b_1 b_2 - 3 k^2 a_2 b_1 b_2 + 3 k^3 b_1 b_2^2 c_1 + \\ 3 k^2 b_1 b_2^2 c_2 - 4 b_1^2 a_2 + 4 k b_1^2 b_2 c_1 + 4 b_1^2 b_2 c_2)\end{gathered}}{4 b_1 \left(k^2 b_2 + b_1\right)}$$

$$\pi_1 = \frac{\left(k a_1 b_2 + a_2 b_1 - k b_1 b_2 c_1 - b_1 b_2 c_2\right)^2}{8 b_1 b_2 \left(k^2 b_2 + 2 b_1\right)}$$

$$\pi_2 = \frac{\left(k a_1 b_2 + a_2 b_1 - k b_1 b_2 c_1 - b_1 b_2 c_2\right)^2}{16 b_2 \left(k^2 b_2 + 2 b_1\right)}$$

$$\pi_m = \frac{\left(k b_2 a_1 + a_2 b_1 - k b_1 b_2 c_1 - b_1 b_2 c_2\right)^2}{16 b_1 b_2 \left(k^2 b_2 + 2 b_1\right)}$$

3.3 Buyer-Stackelberg model

In this situation, buyer R1 and R2 are leaders of pricing decision. Manufacturer is the follower of pricing decision. Buyer R1 and buyer R2 set their own sale price p_i (i = 1, 2) and sale quantity because buyer R1 and buyer R2 have the same status. Manufacturer can't control another buyer's sale quantity when he reacts to one buyer's price. It is difficult to ensure that two buyers' sale quantities are equal to the proportion between mainproduct and byproduct. Thus, manufacturer's profit function is separated into two parts. One part is the profit function of buyer R1 $\pi_{m,1} = (w_1 - c_1)\, q_1$. Another part is the profit function of buyer R2 $\pi_{m,2} = (w_2 - c_2)\, q_2$. The values of w1 and w2 can be obtained respectively. Then, buyer R1 and buyer R2 determine the values of q_1 and q_2. If the proportion between q_1 and q_2 is not equal to the proportion between mainproduct and byproduct, the buyer whose sale quantity is fewer than another one increases sale quantity or change production quantity.

Two buyers don't have direct relationship because manufacturer is the follower of pricing decision. Manufacturer has different reaction function to mainproduct buyer R1 and byproduct buyer R2. Similar to two stages supply chain pricing problem. Profit function of manufacturer is denoted as two parts.

$$\pi_{m,i} = (w_i - c_i)(a_i - b_i(m_i + w_i))$$

(12)

$$\frac{\partial(\pi_{m,i})}{\partial w_i} = a_i - b_i(m_i + w_i) - b_i w_i - b_i c_i = 0$$

$$w_i = \frac{a_i}{b_i} - p_i + c_i$$

(13)

Substituted w_i into the profit function of buyer R_i ($i = 1, 2$), then

$$\pi_i = (p_i - w_i)(a_i - b_i p_i) = \left(2p_i - \frac{a_i}{b_i} - c_i\right)(a_i - b_i p_i)$$

(14)

Let $\frac{\partial(\pi_i)}{\partial p_i} = -4b_i p_i + 3a_i + b_i c_i = 0$, we can obtain the value of p_i.

$$p_i = \frac{3a_i}{4b_i} + \frac{c_i}{4}$$

(15)

Substituted p_i denoted in equation (15) into equation (13) and (14), then

$$w_i = \frac{a_i}{4b_i} + \frac{3c_i}{4}$$

(16)

$$\pi_i = \frac{1}{8}\frac{(a_i - b_i c_i)^2}{b_i}$$

(17)

$$\pi_{m,i} = \frac{1}{16}\frac{(a_i - b_i c_i)^2}{b_i}$$

(18)

According to p_i denoted in equation (15), we know $q_i = (a_i - b_i c_i)/4$. As mentioned above, manufacturer produces unit main product and k unit byproducts simultaneously. There are three situations. Firstly, all main products and byproducts are sold out where $a_1 - b_1 c_1 = k(a_2 - b_2 c_2)$. Secondly, byproducts are surplus and can not be sold totally where $a_1 - b_1 c_1 < k(a_2 - b_2 c_2)$. Finally, byproducts are shortage where $a_1 - b_1 c_1 > k(a_2 - b_2 c_2)$.

In the situation with redundant byproduct, one can take some price discount strategies to stimulate buyer R2 purchase more byproducts or decrease production quantity. We may choose a strategy or both two strategies. The reduced amount of byproduct transfer price and the reduced product quantity need to be determined if both of two strategies are considered.

If buyer gives sale price p_i ($i = 1, 2$), manufacturer will compute reaction function w_i. The reaction function influences buyer's optimal price p_i (w_i is inversely proportional to p_i), and influences market demand of product i (w_i is in directly proportional to d_i). In order to stimulate byproduct buyer purchase more products, we add reduction gene α to manufacturer's reaction function. Manufacturer's new reaction transfer price w_2' is denoted as below:

$$w_2' = \frac{a_2}{b_2} - p_2 + c_2 - \alpha$$

(19)

R2's new profit function can be got after w_2' is substituted into π_2.

$$\pi_2' = \left(2p_2 - \frac{a_2}{b_2} - c_2 + \alpha\right)(a_2 - b_2 p_2)$$

(20)

Let $\frac{\partial(\pi_2')}{\partial p_2} = -4b_2 p_2 + 3a_2 + b_2 c_2 - b_2\alpha = 0$.

p_2' 、 π_2' 、 w_2' and $\pi_{m,2}'$ denote sale price of buyer R2 after manufacturer use price discount strategy, profit of R2, byproduct's transfer price and profit of manufacturer sell byproducts respectively.

$$p_2' = \frac{3a_2}{4b_2} + \frac{c_2 - \alpha}{4}$$

$$w_2' = \frac{a_2}{4b_2} + \frac{3}{4}(c_2 - \alpha)$$

$$\pi_{m,2}' = \frac{1}{16}\left(\frac{a_2 - c_2 b_2 - 3\alpha b_2}{b_2}\right)(a_2 - c_2 b_2 + \alpha b_2)$$

$$\pi_2' = \frac{(a_2 - c_2 b_2 + \alpha b_2)}{8b_2}$$

Let q_2' 、 $\Delta_{m,2}$ and Δ_2 denote byproduct sale quantity after manufacturer use price discount strategy, difference of manufacturer and R2 between profit without price discount strategy and after use price discount strategy respectively.

$$q_2' = \frac{1}{4}(a_2 - b_2 c_2 + b_2\alpha)$$

$$\Delta_{m,2} = \pi_{m,2} - \pi'_{m,2} = \frac{1}{16}\alpha\left(3b_2\alpha - 2c_2b_2 + 2a_2\right)$$

$$\Delta_2 = \pi_2 - \pi'_2 = -\frac{1}{8}\alpha\left(b_2\alpha - 2b_2c_2 + 2a_2\right)$$

Manufacturer need change transfer price of product if one use production quantity decrease strategy. Let p'_1 、 π'_1 、 w'_1 and $\pi'_{m,1}$ denote sale price of after manufacturer use price decrease strategy, profit of R1, transfer price of product and profit of manufacturer obtained by selling products respectively.

$$w'_1 = w_1 + \beta = \frac{a_1}{b_1} - p_1 + c_1 + \beta$$

$$\pi'_1 = \left(2p_1 - \frac{a_1}{b_1} - c_1 - \beta\right)\left(a_1 - b_1p_1\right)$$

Make $\frac{\partial\left(\pi'_1\right)}{\partial p_1} = 0$,

$$p_1 = \frac{3a_1 + b_1c_1 + b_1\beta}{4b_1}$$

$$w'_1 = \frac{a_1 + 3b_1c_1 + 3b_1\beta}{4b_1}$$

$$\pi'_{m,1} = \frac{1}{16}\left(\frac{a_1 - c_1b_1 + 3b_1\beta}{b_1}\right)\left(a_1 - c_1b_1 - b_1\beta\right)$$

$$\pi'_1 = \frac{\left(a_1 - c_1b_1 - b_1\beta\right)}{8b_1}$$

Let q'_1 、 $\Delta_{m,1}$ and Δ_1 denote sale quantity after changing product transfer price, profit difference of manufacturer and R1 between before changing transfer price and after improving transfer price respectively.

$$q'_1 = \frac{1}{4}\left(a_1 - b_1c_1 - b_1\beta\right)$$

$$\Delta_{m,1} = \pi_{m,1} - \pi'_{m,1} = \frac{1}{16}\beta\left(3b_1\beta + 2c_1b_1 - 2a_1\right)$$

$$\Delta_1 = \pi_1 - \pi'_1 = -\frac{1}{8}\beta\left(b_1\beta + 2b_1c_1 - 2a_1\right)$$

Strategy that manufacturer will use can be got from the following optimization problem.

$$Min \quad \Delta_{m,1} + \Delta_{m,2}$$

$$s.t. \quad \begin{cases} k\left(a_2 - b_2c_2 + b_2\alpha\right) = a_1 - b_1c_1 - b_1\beta \\ \alpha \geq 0, \beta \geq 0 \end{cases}$$

The optimization problem above is a line programming problem. It is easy to obtain optimal solution of α and β. Lingo software is used to resolve the problem in this paper.

The third situation may be thought as the reverse question of the second situation. Byproducts demand quantity can be thought as production quantity in the situation. There are some products don't be sell out. Using production quantity decrease and product price decrease strategy can make proportion between main product demand quantity and byproduct demand quantity fit with the production proportion of main product and byproduct.

As a conclusion, some strategies need be used in order to make proportion between main product demand quantity and byproduct demand quantity fit with production proportion of main product and byproduct when the two buyers are leaders and make price decision with manufacturer independently. Profit of manufacturer and buyers and whole industry chain are influenced by these strategies. Profit of manufacturer and buyers and whole industry chain may be decreased if two buyers are alliance. When R1 and R2 are alliance and leaders, Stackeberg game model is constructed as follow.

3.4 Alliance of R1 and R2 stackelberg model

We assume that product sale price p_1 is a certain number. According to the production proportion between product and byproduct, $k(a_2 - b_2p_2) = (a_1 - b_1p_1)$.

$$p_2 = \frac{b_1p_1 + ka_2 - a_1}{kb_2} \quad (21)$$

According to equation (13) and (21), w_1 and w_2 can be denoted by p_1 and p_2.

$$w_1 = \frac{a_1}{b_1} + c_1 - p_1 \quad (22)$$

$$w_2 = \frac{kc_2b_2 + a_1 - b_1p_1}{kb_2}$$

(23)

Let π_{12} denotes the whole profit of R1 and R2.

$$\pi_{12} = -\frac{\begin{matrix}(k^2b_2a_1 - 2k^2b_2b_1p_1 + k^2b_1b_2c_1 \\ +2a_1b_1 - 2b_1^2p_1 - ka_2b_1 + kb_1b_2c_2)\end{matrix}}{k^2b_1b_2} * (a_1 - b_1p_1)$$

Let the first order condition of π_{12} to p_1 is equal zero. We can obtain the values of p_1 and p_2.

$$p_1 = \frac{3k^2b_2a_1 + k^2b_1b_2c_1 + 4a_1b_1 - ka_2b_1 + kb_1b_2c_2}{4(k^2b_2 + b_1)}$$

$$p_2 = \frac{k^2b_2a_1 + 3k^2b_1b_2c_1 + 4b_1^2c_1 + ka_2b_1 - kb_1b_2c_2}{4(k^2b_2 + b_1)b_2}$$

p_1 and p_2 are substituted into equation (22) and (23). We can obtain the values of w_1, w_2, w_m, w_{12}.

$$w_1 = \frac{k^2b_2a_1 + 3k^2b_1b_2c_1 + 4b_1^2c_1 + kb_1a_2 - kb_1b_2c_2}{4b_1\left(k^2b_2 + b_1\right)}$$

$$w_2 = \frac{kb_2c_2 - b_1p_1}{kb_2}$$

$$\pi_m = \frac{(ka_1b_2 - kb_1b_2c_1 + a_2b_1 - b_1b_2c_2)^2}{16b_1b_2(b_1 + k^2b_2)}$$

$$\pi_{12} = \frac{(ka_1b_2 - kb_1b_2c_1 + a_2b_1 - b_1b_2c_2)^2}{8b_1b_2(b_1 + k^2b_2)}$$

4 Computational instances

Let a_1=500 , a_2=300 , b_1=4 , b_2=5 , c_1=20 , c_2=30 and k=2 in the computational instances. M-Stackelberg denotes Manufacturer Stackelberg equilibrium. R1-M-R2 denotes buyer R1-manufacturer M- buyer R2 Stackelberg equilibrium. R-Stackelberg denotes Stackelberg equilibrium when buyers are leaders. R1+R2 denotes alliance of R1 and R2 Stackelberg equilibrium. Table 1 shows that the values of w_1、p_1、w_2 and p_2 of every equilibrium. Table 2~ table7 show profits of every equilibrium when value of a_1、a_2、b_1、b_2、c_1 and k are fine-tuned.

As shown in table 2 ~ table7, whole profit of manufacturer in M-Stackelberg is equal to the total profit of R1 and R2 in R1+R2. The total profit of M-Stackelberg is more than the whole profit of R1-M-R2. The total profit of R-Stackelberg is least because R1 and R2 are not combined. Furthermore, profit $\pi_{1,2}$ of R1+R2 is equal to the manufacturer's profit of M-Stackelberg. It shows leader in pricing has initiative right and can obtain more profit than follower.

Comparing π_1 、 π_2 and π_m of Stackelberg equilibrium R1-M-R2, we know that the influence power is stronger, profit is more. That is to say that $\pi_1 > \pi_m > \pi_2$ in R1-M-R2. The phenomenon is consistent with the conclusions of Stackelberg equilibrium when manufacturer is leader and alliance of R1 and R2 Stackelberg equilibrium.

We can know that manufacturer needs use production quantity decrease strategy or byproduct price decrease strategy in most calculation examples from R column of table 2 ~ table 7. There are some instances that can not make a deal ("×"in rows of table2 and table3). In these situations, both of them can't make deal with manufacturer if R1 and R2 don't combine into one unit. Profits of three sides are influenced. a and β are equal to zero in last row of table 5. It shows that the proportion between sale quantity of main product and that of byproduct is fit with production proportion. π_1+π_2 is equal to $\pi_{1,2}$ of Stackelberg equilibrium of R1-M-R2. It shows also that the profit of R1 and R2 under independent pricing between buyers and manufacturer is equal to the profit of R1and R2 under cooperation pricing between buyers and manufacturer.

We can know that the profit will change while the value of k changes. Byproduct quantity is few than or equal to mainproduct quantity and $\pi_1 > \pi_2$ when $k \geq 1$. Byproduct quantity is more than mainproduct quantity and $\pi_1 < \pi_2$ when $k = 0.5$. It shows that the profits of buyers are influenced by production quantity of mainproduct and byproduct in R equilibrium.

Table 1. Values of w_1、p_1、w_2 and p_2

M				R1-M-R2				R				R1+R2			
w_1	p_1	w_2	p_2	w_1	p_1	w_2	p_2	w_1	p_1	w_2	p_2	w_1	p_1	w_2	p_2
75	100	40	50	44	104	43	51	76	106	38	53	45	100	40	50

Table 2. Effects on profit of a1 change

	M	R1-M-R2	R	R1+R2

a_1	π_1	π_2	π_m	π_1	π_2	π_m	π_1	π_2	π_m			$\pi_{1,2}$	π_m
500	2500	500	6000	5143	367	2571	2813	563	3938	0	30.00	6000	3000
450	2006	401	4815	4127	295	2063	2812	563	3000	0	17.50	4815	2408
400	1566	313	3760	3223	230	1611	2812	563	2062	0	5.00	3760	1880
350	1182	236	2836	2431	174	1215	×	×	×	×	×	2836	1418
300	851	170	2042	1750	125	875	×	×	×	×	×	2042	1021

Table 3. Effects on profit of a_2 change

	M			R1-M-R2			R					R1+R2	
a_2	π_1	π_2	π_m	π_1	π_2	π_m	π_1	π_2	π_m	α	β	$\pi_{1,2}$	π_m
400	2934	587	7042	6036	431	3018	×	×	×	×	×	7042	3521
350	2713	543	6510	5580	399	2790	5513	1000	3253	0	5.00	6510	3255
300	2500	500	6000	5143	367	2571	2813	563	3938	0	30.00	6000	3000
250	2296	459	5510	4723	337	2362	3599	250	3674	5.56	41.11	5510	2755
200	2101	420	5042	4321	309	2161	2179	62.5	3362	14.44	43.89	5042	2521

Table 4. Effects on profit of b_1 change

	M			R1-M-R2			R					R1+R2	
b_1	π_1	π_2	π_m	π_1	π_2	π_m	π_1	π_2	π_m	α	β	$\pi_{1,2}$	π_m
4	2500	500	6000	5143	367	2571	2813	563	3938	0	30.00	6000	3000
3.5	3011	527	7076	6159	399	3079	3215	563	4676	0	37.14	7076	3538
3	3705	556	8523	7539	435	3770	3750	563	5656	0	46.67	8523	4261
2.5	4694	587	10563	9506	475	4753	4500	563	7032	0	60.00	10563	5281
2	6201	620	13642	12505	521	6252	5625	563	9094	0	80.00	13642	6821

Table 5. Effects on profit of b_2 change

	M			R1-M-R2			R					R1+R2	
b_2	π_1	π_2	π_m	π_1	π_2	π_m	π_1	π_2	π_m	α	β	$\pi_{1,2}$	π_m
5	2500	500	6000	5143	367	2571	2813	563	3938	0	30.00	6000	3000
4.5	2546	566	6223	5265	405	2632	3404	756	3935	0	22.50	6223	3111
4	2601	650	6503	5419	452	2709	4050	1013	3882	0	25.00	6503	3251
3.5	2669	763	6864	5616	511	2808	4753	1358	3751	0	7.50	6864	3432
3	2756	919	7350	5880	588	2940	5513	1838	3675	0	0	7350	3675

Table 6. Effects on profit of c_1 change

	M			R1-M-R2			R					R1+R2	
c_1	π_1	π_2	π_m	π_1	π_2	π_m	π_1	π_2	π_m	α	β	$\pi_{1,2}$	π_m
40	1736	347	4167	3571	255	1786	2813	563	2437	0	10.00	4167	2088
30	2101	420	5042	4321	309	2161	2813	563	3188	0	20.00	5042	2521
20	2500	500	6000	5143	367	2571	2813	563	3938	0	30.00	6000	3000
10	2934	587	7042	6036	431	3018	2813	563	4688	0	40.00	7042	3521

Table 7. Effects on profit of k change

	M			R1-M-R2			R					R1+R2	
k	π_1	π_2	π_m	π_1	π_2	π_m	π_1	π_2	π_m	α	β	$\pi_{1,2}$	π_m
2	2500	500	6000	5143	367	2571	2813	563	3938	0	30.00	6000	3000
1.5	2126	756	5763	4566	474	2283	1890	672	3561	2.79	43.52	5763	2882
1	1406	1125	5063	3505	539	1752	1250	1001	3094	10.00	55.00	5063	2531
0.5	386	1235	3241	1840	398	920	343	1098	1879	11.90	78.81	3241	1621

5 Conclusion

Pricing strategy of three stages ecological industry chain was researched in this paper. Manufacturer produces simultaneously mainproduct and byproduct in the ecological industry chain. Compare with pricing problem of two stages ecological industry chain consisting of manufacturer and retailer, more complex pricing decision problem among manufacturer, mainproduct buyer and byproduct buyer was considered. Four kinds of price decision, manufacturer stackeberg equilibrium, mainproduct seller-manufacturer-byproduct buyer stackeberg equilibrium, buyer stackeberg equilibrium, alliance of mainproduct seller and byproduct buyer stackeberg equilibrium, were discussed. Results show that optimal price decision of any player in three stages ecological industry chain is influenced by the other two sides. System profit of manufacturer stackeberg equilibrium equals that of alliance of mainproduct seller and byproduct buyer stackeberg equilibrium, and is larger than that of other two equilibriums. Leader can obtain more profit than follower.

The profit of buyer stackeberg equilibrium is less than that of the other three equilibriums because both of mainproduct seller and byproduct buyer make the price decision with manufacturer independently in the situation. Moreover, it is possible that the deal in the chain can not be made in buyer stackeberg equilibrium. Alliance of mainproduct seller and byproduct buyer should be the good way by which the system profit can be improved.

The future research should be the pricing decision of ecological industry chain under incomplete information. The pricing decision of the ecological industry chain with one leader and multi-follower is also an interesting problem.

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