Abstract: This work solves a practical decentralized track fusion problem where the global fusion center can not receive the real-time covariance information from local systems because of the limited communication bandwidth. The algorithms of track-to-track fusion were revisited, especially the recent new researching results, and the proper algorithms for the linear systems with incomplete covariance information were analyzed. The approximate optimal track fusion (AOTF) algorithm was proposed for steady or “slowly varying” system based on the optimal decentralized track fusion algorithm (OTF). This new designed algorithm has preferable performance than the maximum-likelihood-based fusion methods either in tracking error or in computational load, at the same time, the communication bandwidth needed is very small. Simulation results confirm its effectiveness.

Key-Words: Decentralized Track Fusion, Incomplete Covariance Information, Communication Bandwidth, “Slowly Varying” System, Linear System

1 Introduction

The use of multiple sensors for state estimation can lead to better quality estimates than when a single sensor is used, such as reported by [1], [2] and [3] etc. Taking into consideration the computational load sharing and the survivability and flexibility of the system, the distributed architecture becomes an attractive alternative. The problem of track-to-track association and track fusion has been considered in the literature where the fusion center has access to multiple track estimates and the associated estimation error covariance from local sensors, as well as their cross covariance. Due primarily to the communication constraints in real systems, some legacy trackers may only provide the local track estimates to the fusion center without any covariance information. In some cases, the local (sensor-level) trackers operate with fixed filter gain and do not have any self assessment of their estimation errors. In other cases, the network conveys a coarsely quantized root mean square (RMS) estimation error of each local tracker. Thus the fusion center needs to solve the track association and fusion problem with incomplete data from legacy local trackers.

From [4] we can see that if there is no covariance information than the optimal fusion result can not be gotten. Literature [5] first considered the approximation of the covariance of the estimation error from a legacy tracker with a fixed filter gain, and handled the cross covariance information by approximating this information through a modified Lyapunov equation. The fusion rule used is the Bar-Shalom/Campo combination (BC) fusion algorithm which is proved to be the maximum-likelihood (ML) estimator by [6] and [7].

In our work, we also meet the problem just like [5]. In order to save the bandwidth of wireless communication, no other or little information was transmitted to the center by local systems except for state estimates (tracks). We studied the track fusion algorithms up to now (assume the association has been completed), and proposed an approximate optimal track fusion (AOTF) algorithm based on the optimal track fusion (OTF) algorithm for steady or “slowly varying” system. Here “slowly varying” mainly means the asymptotical stability or very small variety of the tracking error covariance.

The rest of the paper is organized as follows. Section 2 revisits the track fusion problem. Section 3 presents general algorithms for fusing N tracks.
The comparison of Kalman filter (KF) and three distributed estimators with four sensors for a simulated target tracking is presented in Section 4. Concluding remarks are provided in Section 5.

2 Track-to-Track Fusion Algorithms with Local Tracks

Consider a linear dynamical system, such as a moving target,

\[
\begin{align*}
X_{k+1} &= F_k X_k + G_k w_k \\
Z_k &= H_k X_k + v_k
\end{align*}
\]

(1)

where \( X_k \) is the state system at time \( k \), \( F_k \) is the state transition matrix, \( H_k \) is the measurement matrix, \( Z_k \) is the measurement, \( G_k \) is the process noise matrix, \( w_k \) and \( v_k \) are process noise and measurement noise, the covariance are \( Q_k \) and \( R_k \), respectively.

Assume we have \( N \) local tracks and the state estimates at time \( k \) from the local centers \( i \) and \( j \) are the \( n \)-vectors \( \hat{X}_{i|k} \) and \( \hat{X}_{j|k} \), respectively. For steady state analysis, the time index may be dropped in the sequel. Without loss of generality, we denote the state estimates as \( \hat{X}^i \) and \( \hat{X}^j \), with covariance \( P^{ii} \), \( P^{jj} \), and cross-covariance \( P^{ij} \), respectively. Denote the true state of the target as \( X \) and assume the estimates from all tracks are purely target originated (no misassociations at any level). As summarized and discussed in [7], under Gaussian assumption, the simple convex combination (CC) fusion algorithm and the Bar-Shalom/Campo (BC) state vector combination are the two main state vector fusion algorithms. Track fusion algorithms are also compared using an analytical method in [8], and based on [8] we also enumerated the recent new researching results of the track fusion algorithms.

2.1 Simple Convex Combination Algorithm (CC)

The simple convex combination algorithm is one of the simplest state vector fusion algorithms as it is very simple to implement. This method assumes that the cross covariance between two state estimates, \( \hat{X}^i \) and \( \hat{X}^j \), can be ignored and also that each individual tracks are independent. The combined state \( \hat{X} \) and its covariance \( P \) are given by:

\[
\hat{X} = \sum_{i=1}^{N} (P^{ii})^{-1} X^i
\]

\[
P^{-1} = \sum_{i=1}^{N} (P^{ii})^{-1}
\]

(2)

The \( \hat{X}_{i|k} \) (state estimation), \( P^{ii}_{i|k} \) (estimated covariance) should be transferred to the fusion center of this algorithm.

2.2 Bar-Shalom/Campo Algorithm (BC)

The individual tracks are not totally independent in the real world. The Bar-Shalom/Campo state vector combination algorithm was developed based on the track correlation on the same process noise [9]. This method is proved to be the maximum likelihood (ML) estimator by [6] and [7], not MMSE meaning because no prior information is used here. The combined state \( \hat{X} \) and its covariance \( P \) of the multiple tracks are given by:

\[
\hat{X} = (I^{T} P^{-1} I) \hat{X}
\]

\[
P = (I^{T} P^{-1} I)^{-1}
\]

(3)

where \( \hat{X} = [(\hat{X}^1)^{T}, (\hat{X}^2)^{T}, \ldots, (\hat{X}^N)^{T}]^{T} \), the superscript \( T \) denotes the transpose of the vector or matrix, \( I \) is the \( n \times n \) unit matrix, \( I = [1,1,\ldots,1]^{T} \),

\[
P = \begin{bmatrix}
p^{11} & p^{12} & \cdots & p^{1N} \\
p^{21} & p^{22} & \cdots & p^{2N} \\
\vdots & \vdots & \ddots & \vdots \\
p^{N1} & p^{N2} & \cdots & p^{NN}
\end{bmatrix}
\]

(4)

The \( \hat{X}_{i|k} \) (state estimation), \( P^{ii}_{i|k} \) (estimated covariance) and \( K^i \) (filter gain) should be transferred to the fusion center of this algorithm. And also the accuracy information of each local system, i.e. state transition matrix \( F^i_k \), process noise item \( G^i_k \) and \( Q^i_k \), measurement matrix \( H^i_k \), should also be known by the fusion center.

2.3 Covariance Intersection Algorithm (CI)

The paper [10] addressed the problem of estimation when the cross-correlation in the errors between different random variables are unknown and proposed the covariance intersection algorithm. The CI algorithm can be treated as a weighted form of the simple convex combination in a state estimation. This method is suitable for state vector fusion when there is unknown correlation among the tracks. The combined state \( \hat{X} \) and its covariance \( P \) are given by (\( N = 2 \) in [10]):
\[
\begin{align*}
\hat{X} &= \left[ \sum_{i=1}^{N} \omega^i (P^i)^{-1} \right]^{-1} \sum_{i=1}^{N} \omega^i (P^i)^{-1} \hat{X}^i \\
P^{-1} &= \sum_{i=1}^{N} \omega^i (P^i)^{-1} \\
\sum_{i=1}^{N} \omega^i &= 1
\end{align*}
\]  

(5)

where each \(\omega^i\) is a number that lies between 0 and 1. Usually, the \(\omega^i\) is determined by minimizing the determinant or trace of the combined state covariance \(P\). Note that this method cannot be applied to the case when individual tracks have the same error covariance \(P\) as the minimized functions are independent of the \(\omega^i\) shown in (5). The CI algorithm requires \(\omega^i\) to be optimized at every step by minimizing the trace or the determinant of \(P\).

[11] considered the case of five spacecraft with a decentralized estimation scheme where each spacecraft shares its own estimates with the rest of the fleet using a ring-type communication architecture. It is proved by [12] that the global optimal solution is actually given by the CI algorithm, which conducts the search only along a one-dimensional curve in the n-squared-dimensional space of combination gains.

In our opinion to say strictly, the CI algorithm is not the global optimal solution but only the estimator in approximate ML meaning. From [10], [11] and [12] we can see that: i) the consistent estimate of fusion covariance is the upper boundary of the true covariance, ii) even when the cross covariance is known, the CI estimates are equal to the BC estimates, iii) no prior information is used here also.

The CI algorithm requires \(\omega^i\) to be optimized at every step, for more than two local tracks, the complexity of this algorithm will be increased obviously. The \(\hat{X}^i_{k|k}\) (state estimation) and \(P^i_{k|k}\) (estimated covariance) should be transferred to the fusion center of this algorithm. A comparison between CC, BC and CI algorithms was described in [13].

### 2.4 General Fusion Algorithm for Asynchronous Tracks (GF)

[14] solves a practical sensor to sensor track fusion problem when the sensors used are asynchronous, and tracks may arrive out-of-sequence. A new general fusion algorithm was proposed which is also suitable for sequence tracks. Using the new solution, sensor to sensor track fusion can be performed without the additional computational cost of track synchronization. In addition, the user can include latent and out-of-sequence data in the fusion process without filter reinitialization or backtracking. But the optimality of minimize mean square error (MMSE) meaning is not global because no prior information is used. It is shown from the special case of Bar-Shalom/Campo fusion rule which is proved to be the maximum likelihood (ML) estimator by [6] and [7]. We omit the complex fusion formula in order to saving space. The \(\hat{X}^i_{k|k}\), \(P^i_{k|k}\) and \(K^i_{k}\) should be transferred to the fusion center of the algorithm of [14]. And also the accuracy information of each local system, i.e. state transition matrix \(F^i_k\), process noise item \(G^i_k\) and \(Q^i_k\), measurement matrix \(H^i_k\) and noise item \(R^i_k\), should also be known by the fusion center.

### 2.5 Optimal Track Fusion Algorithm (OTF)

From the optimal centralized measurements fusion algorithm, the optimal decentralized track fusion algorithm can be derived with sensor noises uncorrelated. The combined state \(\hat{X}\) and its covariance \(P\) are given by:

\[
\begin{align*}
\hat{X}_{k|k} &= P_{k|k}^{-1} \hat{X}_{k|k-1} + \sum_{i=1}^{N} \left( (P^i_{k|k})^{-1} \hat{X}^i_{k|k-1} - (P^i_{k|k})^{-1} \hat{X}^i_{k|k-1} \right) \\
P_{k|k}^{-1} &= P_{k|k-1}^{-1} + \sum_{i=1}^{N} ((P^i_{k|k})^{-1} - (P^i_{k|k})^{-1})^{-1}
\end{align*}
\]  

(6)

The cross covariance and the prior information are all considered in this algorithm. It is actually global optimal. The proof can be found in many books such as [7]. The \(\hat{X}^i_{k|k}\), \(\hat{X}^i_{k|k-1}\), \(P^i_{k|k}\) and \(P^i_{k|k-1}\) should be transferred to the fusion center of the algorithm.

The new paper [15] presented a distributed Kalman filtering fusion formula for linear dynamic systems with sensor noises cross-correlated, and proved that under a mild condition the fused state estimate is equivalent to the centralized Kalman filtering using all sensor measurements, therefore, it achieves the best performance.

### 3 Track Fusion with Incomplete Covariance Information
There are always many kinds of data sources in tracking of air or space targets, such as ground radar datum, space satellite measurements and inertial measurements of the target itself, etc. The information is transferred to the center processor or between each other by wireless communication generally. In order to save the bandwidth of wireless communication the error information especially covariance and cross covariance are always ignored. We studied this problem distinguishing four situations.

### 3.1 Without Any Information

In this situation no information can be used except for local tracks, no any prior information also. The fusion is very simple.

\[
\hat{X}_{k|k} = \frac{1}{N} \sum_{i=1}^{N} \hat{X}_{i|k}
\]

(7)

### 3.2 With Prior Error Information

When the local tracking variance \( \sigma^i \), e.g. of the position error, is known by prior statistical result, then the weighted form of the simple convex combination can be

\[
\hat{X}_{k|k} = \sum_{i=1}^{N} \omega^i \hat{X}_{i|k}
\]

(8)

where \( \omega^i = (\sigma^i)^{-1} \sum_{i=1}^{N} (\sigma^i)^{-1} \).

### 3.3 For Steady System without Covariance Received

We know that many systems can reach steady state or at least approximate steady state. This characteristic can be used in the fusion center. As

\[
P_{k|k-1}^u = F^T \hat{X}_{k|k-1}^u (F^T)^T + G^T Q (G^T)^T
\]

(9)

the covariance updated equation

\[
(P_{k|k}^u)^{-1} = (P_{k|k-1}^u)^{-1} + (H^T R^{-1} H)^T
\]

(10)

can be written as

\[
(P_{k|k}^u)^{-1} = [F^T \hat{X}_{k|k-1}^u (F^T)^T + G^T Q (G^T)^T]^{-1} + (H^T R^{-1} H)^T
\]

(11)

We can solve the equation by recursive method off line. So the covariance of each sensor can be computed. Of course, the local models should be known or simulated by the center and it is preferable for the local systems to choose the same linear time-invariant models.

From the second equation of (6) the covariance recursive equation of the fusion center is

\[
(P_{k|k})^{-1} = (FP_{k|k-1} F^T + GQG^T)^{-1} + \sum_{i=1}^{N} (P_{k|i}^{-1} - (P_{k|i}^{-1})^{-1})
\]

(12)

So the steady global covariance can be calculated also.

When the global covariance is gotten, we can estimate the global states vector by the first equation of (6), which is

\[
\hat{X}_{i|k} = P_{k|k}^{-1}(P_{k|i-1}^{-1} \hat{X}_{i|k-1} + \sum_{i=1}^{N} (P_{k|i}^{-1} - (P_{k|i})^{-1}) \hat{X}_{i|k-1})
\]

(13)

where \( X_{i|k-1}^i = F^i X_{i-1|k}^i \).

### 3.4 With Intermittent Covariance Information Update

If the local system is “slowly varying” and the covariance varies obviously from the last one, the local center can transfer the new covariance to the fusion center intermittently. When received a new covariance, the fusion center should update its global covariance with (12) correspondingly, and then continue to estimate the states using (13).

A threshold such as the variable quantity of \( \delta = e P_{k|k}^{-1} e^T \) can be defined to determine whether to retransfer the local covariance by the local systems, where \( e = [1,1,\cdots,1]_n \).

We call the last two algorithms for steady system and “slowly varying” system as the approximate optimal track fusion (AOTF) algorithms. They are all based on the OTF algorithm with sensor noises uncorrelated.

In this section we studied the problem of track fusion with incomplete real-time input information distinguishing four situations. Using the AOTF algorithms we can get approximate global optimal performance and the communication bandwidth is saved obviously. Furthermore survivability and flexibility of the fusion center is stronger. When one local track was lacked the fusion center only need to update the global covariance and continue to estimate omitting the corresponding track.

The complexity of the proposed AOTF algorithm is reduced than the BC, CI and GF algorithms. The later three algorithms need to calculate the cross covariance or weights in every fusing step, but the former only need to update the covariance occasionally.

If the system is unsteady or “quickly varying”, then the AOTF algorithm may not work well. We
can consider the methods of 3.1 and 3.2 if the communication bandwidth is still limited.

4 Simulation

As the performance of CC, BC and CI has been compared in [13], and GF algorithm is near to BC, we only compare KF, BC, OTF with AOTF in this section. The method for steady system without covariance received is used here (method 3.3 in section 3), which means the fusing covariance is fixed.

The sensor measurements use the Cartesian coordinate-system (CS). There are four sensors in this simulation. In order to make the comparison easily, coordinate conversion is omitted. The stationary Kalman Filter (KF) was used to track the position of a particular stationary space target by the sensors. Hence, the experimental results are based on 3D simulation. The simulation results show the mean position errors of fused tracks over a number of Monte Carlos runs. The tracking model of approximate “current” model which we proposed in [16] was used here (a time-invariant system, no use of the adaptive rule). The single coordinate dynamic system is

\[
X_{k+1} = \begin{bmatrix}
1 & T & T^2 / 2 & -aT^3 / 6 \\
0 & 1 & T - aT^2 / 2 & aT^3 / 6 \\
0 & 0 & 1 - aT & aT \\
\end{bmatrix}X_k + \begin{bmatrix}
aT^3 / 6 \\
aT^2 / 2 \\
aT \\
\end{bmatrix}\alpha_k + w_k
\] (14)

In this simulation, the three coordinates are tracking simultaneously. The state vector is

\[
X = [x, \dot{x}, x, y, \dot{y}, y, z, \dot{z}, z]^T.
\]

Denotes

\[
F_C = \begin{bmatrix}
1 & T & T^2 / 2 & -aT^3 / 6 \\
0 & 1 & T - aT^2 / 2 & aT^3 / 6 \\
0 & 0 & 1 - aT & aT \\
\end{bmatrix}F
\] (15)

The state transition matrix is

\[
F = F^T = \text{diag}(F_C, F_C, F_C)
\] (16)

The process noise matrix of single coordinate dynamic system is

\[
Q_k = 2\alpha^2 \begin{bmatrix}
T^5 / 20 & T^4 / 8 & T^3 / 6 \\
T^4 / 8 & T^3 / 3 & T^2 / 2 \\
T^3 / 6 & T^2 / 2 & T
\end{bmatrix}
\] (17)

where \(\alpha = 0.1\), interval \(T = 1\) s, \(\sigma = 3\) m/s².

The measurement matrix is

\[
H^I = \begin{bmatrix}
1,0,0,0,0,0,0,0,0 \\
0,0,1,0,0,0,0,0,0 \\
0,0,0,0,0,1,0,0,0
\end{bmatrix}
\] (18)

4.1 Scenario Generation

The scenario of one target moving in the 3D spaces tracked by four sensors was simulated. The target moves in x-y plane with strong maneuvering mode and in z direction with uniform motion (nonmaneuvering mode). The trajectory in x-y plane is shown in Fig.1 which simulates from 0-400s. 50-70s, 180-198s and 330-398s constantly turns right with 9°/s angle velocity, 100-125s and 250-272s constantly turns left with 7°/s angle velocity, other time moves in straight line with constant velocity (uniform motion). The initial state is \([10000, 200, 0, 10000, 250, 0, 20000, -30, 0]^T\).

The standard deviations (square root of variance) of measurement noise of the four sensors are shown in Table 1.

![Fig.1: Target trajectory in x-y plane](image.png)

### Table 1. The standard deviation of measurement noise of the four sensors (m)

<table>
<thead>
<tr>
<th>Sensor 1</th>
<th>Sensor 2</th>
<th>Sensor 3</th>
<th>Sensor 4</th>
</tr>
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<tbody>
<tr>
<td>x</td>
<td>100</td>
<td>100</td>
<td>200</td>
</tr>
<tr>
<td>y</td>
<td>100</td>
<td>100</td>
<td>300</td>
</tr>
<tr>
<td>z</td>
<td>100</td>
<td>100</td>
<td>300</td>
</tr>
</tbody>
</table>

4.2 Simulation Results
The root mean square (RMS) of x, y and z position error of 100 Monte Carlo runs is shown in Fig. 2, Fig. 3 and Fig. 4 respectively. The RMS of BC is plotted with dot line, of OTF with dash-dot line, and of AOTF with solid line. Table 2 gives the means of the RMS over all simulation time. Table 3 gives the computational load of each algorithm with 100 Monte Carlo runs. In the two tables we also give the performance of Kalman filter tracking by the sensor 1, but not plot in the figure because of the obvious difference between KF and the other three. We can compare the performance of BC, OTF and AOTF more clearly from the figures.

From the three figures and Table 2 we can see that: i) the performance of OTF is true optimal and the fusion algorithms are better than the single Kalman filter, ii) in x and y direction AOTF is better than BC, in z direction the two algorithms are near to each other, iii) when the target maneuvering or the model being not proper the tracking error and fusing error will increase. The difference between different directions is due to the maneuverability and the matching performance of the model used with the system motion mode.

From Table 3 we can see that the computational load of the new AOTF algorithm is the smallest, even lower than Kalman filter. It is obvious that the main inverse matrix calculating needs only once with the method of 3.3 in section 3. If we take the method of 3.4 in section 3, the computational load will increase, which lies on the covariance updating rate.

| Table 2. The means of RMS of the position over all time (m) |
|-----------------|-----------|-----------|-----------|
|                 | KF1       | BC        | OTF       | AOTF      |
| x                | 81.0256   | 69.7688   | 62.2479   | 65.9214   |
| y                | 80.2867   | 67.6485   | 63.5436   | 66.5729   |
| z                | 67.9831   | 57.1964   | 56.1520   | 57.4630   |

<table>
<thead>
<tr>
<th>Table 3. Computational load (s)</th>
</tr>
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<tbody>
<tr>
<td>KF1</td>
</tr>
<tr>
<td>time</td>
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</tbody>
</table>

In this section we use the method of 3.3 in section 3 which means the fusing covariance is steady here. It is not difficult to believe that the
method of 3.4 in section 3 can fuse better than that of 3.3, but the computational load will also increase. Considering the ultimate situation of 3.4, when the local covariance can transmit to the center in real time the AOTF is equivalent to the OTF.

5 Conclusion

The decentralized track-to-track fusion algorithms were reviewed in this paper, and the performance is analyzed. The new proposed AOTF algorithm is effective for “slowly varying” system and superior to the ML methods, and can be used in aerospace targets tracking. Different from the CI algorithm this new algorithm needs to use the system state transition matrix, so the performance is relative to the dynamic model. There is no need for consistency of global model and local model, but it is preferable if possible. If the system is unsteady or “quickly varying”, then the AOTF algorithm may not work well. We can consider the methods of 3.1 and 3.2 in section 3 if the communication bandwidth is still limited.

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