Cross Evaluation using Weight Restrictions in Unitary Input DEA Models: Theoretical Aspects and Application to Olympic Games Ranking

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Abstract: There is no official method to establish a final ranking for the Olympic Games. The usual ranking is based on the Lexicographic Multicriteria Method, the main drawback of which is to overvalue gold medals. Furthermore, it does not take into account the results of the Winter Games, which are also part of the Olympic Games. This paper proposes a method based on DEA, in which the outputs are the number of all three medals (gold, silver and bronze) that each country won at both the Salt Lake City and Sydney games; a constant input is considered for all countries. Theoretical aspects of this model are explained hereinbelow. Restrictions based on the importance of each medal are defined for this model. A weight average for each output, based on a modified Cross Evaluation model, is developed and is used as the coefficient in the weighted sum that establishes the final ranking.

Keywords: Data Envelopment Analysis – Cross evaluation – Weights restrictions – Unitary input – Ranking.

1 Introduction

The Olympic Games were born in ancient Greece and were designed for individual contests. In [1] is presented a complete history of the modern games in what concerns their summer edition. The Winter Games were incorporated to the Olympic Games in 1908 with the inclusion of figure skating. In 1924, the first Winter Olympic Games took place in Chamonix, France. Since 1994, the Winter Olympic Games and Summer Olympic Games have taken place at two year scattered intervals.

Despite their national character, the Olympic Committee has never issued an official ranking to pick an overall Olympic winner country. The mass media, however, did so. Their ranking has become a quasi-official ranking. It is based on the Lexicografic Multicriteria method, as explained in [2]. This ranking does not deal properly with the possible existence of countries that have won a large number of silver and bronze medals but no gold medal.

If we take Summer and Winter Games together, we have neither an official ranking nor a quasi-official one. In this paper we use a Data Envelopment Analysis approach to study the results of the Summer and Winter Olympic Games. We wish to establish a ranking that takes into account the results of both games together. With this ranking a country is able to know is complete (Summer and Winter) Olympic performance. As all the medals are take into account, we believe that, this is a fairer ranking than the one based on the Lexicografic Method.

In the next section we review in brief the Olympic rankings. In section 3, we establish the fundamentals of our ranking. In section 4, we consider the theoretical aspects of the DEA model used in our ranking. In section 5, we present the results and, finally in section 6, we draw some conclusions and point to future developments.

2 A Review on the Analyses of Olympic Games Results

The Lexicografic Method is not the sole method used to rank countries in the Olympic Games. Some newspapers produce a ranking determining the total number of medals earned by each country. They simply add up bronze, silver and gold medals. The obvious drawback of this method is to under-evaluate gold medals.

An alternative approach is to make an arbitrary evaluation of each medal, for instance, 1 point for bronze, 2 for silver and 3 for gold. This is a much unsophisticated approach, as it assumes all medals to be equally desired, albeit in proportion to their value.

The previous approaches follow contradictory assumptions. It is important to study alternative ways to rank competitors in the Olympic Games. In [3] is used a statistical methods to determine “who wins the Olympics”. Other statistical approach considering socio economical variables and the number of medal earned has been performed by [4].

There are already some approaches using DEA to establish Olympic rankings. The very first one was proposed by [5]. They used population and GNP as inputs and the medals as outputs. In a similar approach, [2] built a new model taking in account one more constraint: the total amount of medals is a constant. This resulted in the development of a new model, the so-called Zero Sum Gains DEA model (ZSG-DEA). In [6] DEA was used to establish a ranking, the inputs of which were some social economics variables. Instead of using as outputs the number of gold, silver and bronze medal, they used four linear combinations of these figures. This approach eliminates the problem of nil valued weights. For each country, they determined which output has the greatest weight, in order to divide the countries into clusters. They also made a classical cluster analysis using socio economical variables, and compared the two classifications. The authors emphasized the importance of Olympic rankings and asked for new studies on this subject.

All the works mentioned hereabove take into account the results in the Olympics and the socio economical conditions of each country.

In [7], [8] and [9] were used only the results themselves. In [7} authors study the existence of “home advantage” and in [8] they intend to establish a ranking for both Summer and Winter Games. In [9] it is also postulated that the difference in importance between a gold medal and a silver medal must be greater or equal to the difference in importance between a silver medal and a bronze medal. This statement was also used by [10].

Another work comparing summer and winter Games is the one of [11]. The uncertainty in Olympic Games was studied in [12].

We shall also mention that there are DEA based rankings for some other sports. Among them we can mention [13], [14], [15], [16] and [17].

3 Building the New Ranking

In order to establish a general ranking for the Olympic Games, we have to solve some problems. The different competitions within the Games have to be valued, and within each competition, the positions obtained by each country in the Summer and Winter Games have to be valued as well. The first of these problems deals with the popularity of each sport, or its Olympic tradition or even the number of participating athletes.

In this paper we shall assume that all competitions have the same importance. The reason behind this assumption is the impossibility of an agreement among all the countries: for some countries, soccer, for instance, will never be important, for others, it is the very essence of the games. We assume that the International Olympic Committee considers the sport important enough when it is included in the Olympic Games. We intend to establish a ranking suitable for the international community and not only for any particular country. Nevertheless, we will consider every medal won in the Summer Games as more important than a similar medal won in the Winter Games. This is because Summer Games are more popular than Winter Games all over the world.

How to use the results that were obtained is the very essence of the remaining problem. As mentioned earlier, we wish to rank countries taking into consideration Olympic results only and not the conditions that make individual countries achieve those results. We also want to take into account both Winter and Summer Games. Finally, we would like to rank countries based on a single weighted sum (easily understood by everyone) with no weights arbitrarily imposed by anyone. So, the weights shall be obtained from a mathematical model involving as little subjectivity as possible.

To do so, we start off by proposing an Olympic ranking based on Data Envelopment Analysis (DEA) that uses the results of the Sydney 2000 Olympic Games and the Salt Lake City 2002 Winter Olympic Games. The choice of the Sydney Games rather than the more recent Athens Games was made as the analysis of the 2002 Games results was of greater interest. The Sydney Games results included, indeed, a number of countries with a considerable amount of silver and bronze medals and no gold medals.

A total of 10,651 athletes, from 199 nations, competing in 300 events participated in the Sydney Games. In the Salt Lake City Games, there were 78 events and 2,399 athletes from 77 nations. Athletes from 18 nations earned gold medals. These figures show that the Winter Olympic Games are not as popular as the traditional Summer Games are. Our mathematical model ought to take this fact into account.

In our DEA model, we have three outputs: the number of gold, silver and bronze medals earned by each country in both games. A constant unitary input is considered for all countries, as we wish to consider only the results, not the means used to obtain them.

A well-known property of the DEA models is that they allow for an excessive degree of freedom for the weights assigned to each variable. To avoid that excessive freedom, restrictions based on the importance of each medal ought to be incorporated in the model.

On the other hand, a ranking based on different weights for each country is not easily accepted by the general public. To prevent this situation, we present a final ranking based on a weighted sum in which we use the average of the weights assigned by the DEA model to each variable. As we have just one single input, that weighted sum is a Cross Evaluation model, as demonstrated by [18].

This paper will also discuss the theoretical aspects of our proposed model.

4 Modeling with DEA

4.1 Fundamentals

The aim of DEA is to compare a certain number of production units usually named Decision Making Units (DMUs) that perform similar tasks using different levels of inputs to achieve different levels of outputs. Besides identifying efficient DMUs, DEA models allow inefficiencies to be measured and diagnosed. Efficient DMUs define a piecewise linear efficient frontier. In this paper a DMU will be a country that won a medal in either Sydney or Salt Lake City.

Let us recall that DEA models allow each DMU to choose in complete freedom the weight assigned to each variable. This means, in our case, that some DMUs may overvalue the silver or bronze medals, and in some cases they can even ignore the gold medals, to obtain their efficiency scores. This is clearly unfair.

The possibility of nil weights for some outputs usually leads to a larger number of ties among DMUs. To avoid this lack of discrimination among efficient DMUs, several approaches were developed. Some of these developments were reviewed by [19] and [20]. Two of the methods for the improvement of discrimination will be combined to build our Olympic ranking.

One of those methods is the so-called “weight restrictions” method. Two main approaches are available in this method: the Cone Ratio and the restriction on the importance of each variable (virtual inputs and virtual outputs). This one requires more information from the decision-maker, and may need a multicriteria approach for a reliable assignment of the weigh, as done in [21]. The Cone Ratio method needs only ordinal considerations on the importance of each variable and, for that reason, it will be used in this paper.

The second technique we use to increase discrimination among DMUs is based on the Cross Evaluation method. The idea behind Cross Evaluation, introduced by [22] and extended by [23], is a peer evaluation. This means that in Cross Evaluation DMUs are self evaluated (the classic DEA) and are also evaluated by the complete set of DMUs using the optimum weights given by the classic model. It can be said that, while in the classic DEA each DMU is evaluated only from its own point of view, in Cross Evaluation it is also evaluated from the other DMUs points of view. Finally, Cross Efficiency is the average of all DMUs points of view. We use Cross Evaluation with some modifications as explained in Section 3.

4.2 Modeling the Case Study

Our model aims at ranking together all the countries that have won medals in both the Winter and Summer Olympic Games. As mentioned earlier, the DMUs are those countries (80 nations). The objective of each country is to obtain the largest possible number of medals. The number of gold, silver and bronze medals won in Sydney and the number of gold, silver and bronze in Salt Lake City are the outputs. So we have a total of 6 outputs for each DMU.

As our goal is to rank the countries based only on their results, no input should be taken into consideration. However, this model leads to mathematical inconsistencies [24]. To avoid such inconsistencies and to be sure that the model will rank the DMUs based only on their results, we assume that the mere existence of each individual DMU is its own input. In other words, we have considered a unitary constant input for all DMUs in a framework similar to the one used by [25].

4.3 Theoretical Aspects of Unitary Input DEA Model

Owing to the existence of a single constant input, we use the Constant Returns to Scale DEA model (DEA CCR) [26]. In (1) the mathematical formulation is shown for the DEA CCR model, where h_0 is the DMU 0 efficiency under evaluation; y_{jk} is the j-th output (j=1,...,s) of the k-th DMU (k=1,...,n); x_{ik} is the i-th input (i=1,...,r) of the k-th DMU; μ_j and v_i are the output and the input weights, respectively.

$$
\begin{aligned}
&\text{Maximise } h_0 = \sum_{j=1}^{s} \mu_j y_{j0} \\
&\text{subject to} \\
&\sum_{i=1}^{r} v_i x_{i0} = 1 \\
&\sum_{j=1}^{s} \mu_j y_{jk} - \sum_{i=1}^{r} v_i x_{ik} \le 0, \quad k = 1,\dots,n \\
&\mu_j, v_i \ge 0, \quad j = 1,\dots, s, \quad i = 1,\dots,r
\end{aligned} \tag{1}
$$

In the particular case of a constant input, model (1) becomes (2).

$$
\begin{aligned}
&\text{Maximise } h_0 = \sum_{j=1}^{s} \mu_j y_{j0} \\
&\text{subject to} \\
&\sum_{j=1}^{s} \mu_j y_{jk} \le 1, \quad k = 1,\dots,n \\
&\mu_j \ge 0, \quad j = 1,\dots, s,
\end{aligned} \tag{2}
$$

We must emphasize that model (1) is an output maximization model. As a matter of fact, the dual of model (2) is (3).

$$
\begin{aligned}
&\text{Minimise } \sum_{k=1}^{n} \lambda_k \\
&\text{subject to} \\
&\sum_{k=1}^{n} y_{jk} \lambda_k \ge y_{jo}, \ j=1,\dots,s \\
&\lambda_k \ge 0, \ \forall k
\end{aligned} \tag{3}
$$

As there is no input reduction in model (3), model (2) may have an alternative interpretation that does not into account the input reduction. The minimization of the share sum interpretation makes the model a meaningful one even in the presence of a constant input. This model has already been derived by [27]. The authors interpreted this model as a multi-attribute one, in the spirit of DEA, only with outputs. This is the same as considering a unitary and constant input. In [28] was used an analogous model but they missed the theoretical considerations. A model with the same objective function and different constraints is used by [29].

For the Olympic ranking, model (2) is transformed into model (4), where *gS*, *sS* and *bS* refer to the gold, silver and bronze medals in the Sydney 2000 Olympic Games and *gSL*, *sSL* and *bSL* refer to the gold, silver and bronze medals in the Salt Lake City 2002 Olympic Games.

$$
\begin{aligned}
&\text{Maximise } h_0 = \mu_{gS} y_{gS0} + \mu_{sS} y_{sS0} + \mu_{bS} y_{bS0} + \\
&\qquad \mu_{gSL} y_{gSL0} + \mu_{sSL} y_{sSL0} + \mu_{bSL} y_{bSL0} \\
&\text{subject to} \\
&\mu_{gS} y_{gSk} + \mu_{sS} y_{sSk} + \mu_{bS} y_{bSk} + \mu_{gSL} y_{gSLk} + \\
&\qquad \mu_{sSL} y_{sSLk} + \mu_{bSL} y_{bSLk} \le 1, \ k = 1,\dots,80 \\
&\mu_j \ge 0, \ j = gS, sS, bS, gSL, sSL, bSL
\end{aligned} \tag{4}
$$

4.4 The DEA Model with Weight Restrictions

Obviously the medals are not equally important. To take that fact into account we will use weight restrictions in our DEA model. For sure, a gold medal is more important than a silver one and this one is more important than a bronze one. However, the difference in their relative importance is not the same. In opposition to the ideals of Baron de Coubertin, victory is the main goal of the competitors. So the difference in importance between gold and silver medals should not be smaller than the difference between silver and bronze medals.

We can also assume that a medal earned in the Winter Games has less impact than an equivalent medal earned in the Summer Olympic Games, owing to the significant difference between the numbers of participants in those games.

Having these assumptions in mind, the unitary input DEA model is shown in (5).

$$
\begin{aligned}
&\text{Maximise } h_0 = \sum_{j=1}^{6} \mu_j y_{j0} \\
&\text{subject to} \\
&\sum_{j=1}^{6} \mu_j y_{jk} \le 1, \quad k = 1,...,80 \\
&\mu_{gS} \ge \mu_{sS} \\
&\mu_{sS} \ge \mu_{bS} \\
&\mu_{gS} - \mu_{sS} \ge \mu_{sS} - \mu_{bS} \\
&\mu_{gSl} \ge \mu_{sSL} \\
&\mu_{sSL} \ge \mu_{bSL} \\
&\mu_{gSL} - \mu_{sSL} \ge \mu_{sSL} - \mu_{bSL} \\
&\mu_{gS} \ge \mu_{gSL} \\
&\mu_{sS} \ge \mu_{sSL} \\
&\mu_{bS} \ge \mu_{bSL} \\
&\mu_j \ge 0, \quad j = gS, sS, bS, gSL, sSL, bSL
\end{aligned} \tag{5}
$$

4.5 Theoretical Aspects of the Modified Cross Evaluation Method with Weight Restrictions

In [22] was proposed a cross evaluation method that avoids the poor discrimination among the DMUs. In that model, each DMU, besides being self evaluated as in the classical DEA models, is evaluated by all other DMUs. In other words, an average efficiency is computed based on the weights assigned to each variable by the complete set of DMUs. This approach has a drawback: the existence of multiple results, as the obtained weights are not unique for the extreme efficient DMUs (see instance [30], [31], [32]).

In [22] that problem was solved with the introduction of a lexicographic multiobjetive model. First, they maximize the efficiency of a DMU, as in a CCR model. After that, they make efficiency a constraint and minimize the average efficiency of the complete set of DMUs using the weights of the DMU under analysis. They called this the aggressive model. They pointed out that instead of minimizing the average efficiency it is possible to maximize it. This new model is called the benevolent one. Neither model is linear. The model presented in [22] were linearized by [23].

An alternative Cross Evaluation model was proposed by [31] making use of a smoothed DEA frontier. However, this technique has two requirements: the existence of at least three efficient DMUs and the use of the BCC model [33]. As we are using the CCR model with only one efficient DMU, this approach does not applies to the present study.

Another approach was proposed in [2]. This approach starts to determine the average weights for each variable. With these weights a new efficiency index for each DMU is calculated. The theoretical aspects of this method were studied by [18]. They proved that, under certain conditions, the cross evaluation method is equivalent to a fixed weighted sum.

In [2], the authors did not deal with the multiple solutions problem. They explained this assuming that there was only one extreme efficient DMU. So the existence of a multiple set of weights for this DMU is not relevant when the average weights are calculated.

We will propose a modification to that approach: we will not consider the weights of the extreme efficient DMU, since they do not affect meaningfully the calculation of the average weight. This approach can only be used when the number of inefficient DMUs is much larger than that of the efficient ones.

Traditionally, weight restrictions and cross evaluation are two independent models. An important characteristic introduced by [2] and also used by our model is the simultaneous use of those approaches. We have thus increased the discrimination in the DEA model.

5 Results

Using the restriction weight model and the SIAD software [34], we reached the results.In this model, the United States is still at the top of the ranking. In an opposite situation we have countries such as Cameroon and Mozambique. These countries won a single gold medal at the Sydney Games. The gold medal overvaluation of the lexicographic method results in the relatively good position of these countries in the popular ranking. As the DEA model does not over evaluate the gold medal, those countries are now in a worse position.

The top of the ranking does not show any important difference between the results obtained from the weight restrictions DEA model and the lexicographic method.

Even with weight restrictions, there is a significant number of DMUs that assign zero weights to some medals. To avoid that, we used the modified version of Cross Evaluation as explained in Section 4.5. Accordingly, we have removed the United States to calculate the average weight for the Olympic medals.

The average weights obtained for each medal are shown in Table 1.

Table 1. Average Weights.

	Weight
g-S	0,013503
s-S	0,008281
b-S	0,007081
g-SL	0,001911
s-SL	0,000748
b-SL	0,000598

It should be emphasized that the medal weights for the Salt Lake City Games were much smaller than the ones obtained for the Sydney Games. This is caused by the constraints we chose and because too few countries earned medals in the Salt Lake City Games.

The final ranking using average weights is shown in Table 2.

Table 2. Final Olympic Ranking Using Average Weights.

DMU	***Weighted sum***	***DMU***	***Weighted sum***	***DMU***	***Weighted sum***
United States	1,0000	Kazakhstan	0,0736	Slovenia	0,0276
Russia	0,8788	Kenya	0,0660	Croatia	0,0271
China	0,6214	Denmark	0,0589	Nigeria	0,0248
Germany	0,5511	Jamaica	0,0544	Bahamas	0,0218
Australia	0,5473	Indonesia	0,0525	Saudi Arabia	0,0154
France	0,3819	Finland	0,0521	Moldavia	0,0154
Italy	0,3435	Mexico	0,0513	Trinidad and Tobago	0,0154
Cuba	0,2892	Lithuania	0,0482	Costa Rica	0,0142
United Kingdom	0,2840	Austria	0,0481	Portugal	0,0142
Netherlands	0,2744	Iran	0,0476	Cameroon	0,0135
South Korea	0,2650	Turkey	0,0476	Colombia	0,0135
Romania	0,2619	Slovakia	0,0454	Mozambique	0,0135
Ukraine	0,1941	Algeria	0,0430	Ireland	0,0083
Hungary	0,1790	Georgia	0,0425	Uruguay	0,0083
Japan	0,1705	South Africa	0,0378	Vietnam	0,0083
Poland	0,1450	Belgium	0,0378	India	0,0071
Byelorussia	0,1438	Morocco	0,0366	Armenia	0,0071
Canada	0,1340	Taiwan	0,0366	Barbados	0,0071
Bulgaria	0,1333	Uzbekistan	0,0359	Chile	0,0071
Norway	0,1287	New Zealand	0,0347	Iceland	0,0071
Greece	0,1249	Azerbaijan	0,0341	Israel	0,0071
Sweden	0,1205	Estonia	0,0308	Kuwait	0,0071
Spain	0,1065	Argentina	0,0307	Qatar	0,0071
Brazil	0,0922	North Korea	0,0295	Kirgizstan	0,0071
Switzerland	0,0882	Yugoslavia	0,0289	Macedonian Republic	0,0071
Ethiopia	0,0835	Letonia	0,0289	Sri Lanka	0,0071
Czech Republic	0,0756	Thailand	0,0277		

Even without significant in the Winter Games, Brazil and Georgia are the countries that benefit the most from the use of our model. This is so because neither country earned any gold medal in Sydney and both of them earned a significant number of silver and bronze medals in the Sidney Games.

6 Conclusions

We believe that the results obtained with our model are fairer than those of the traditional Lexicographic Multicriteria Method. The main reason is the Lexicographic Method gold medal overvaluation and our method takes into account all the medals earned. Another advantage of our method is to have a general Olympics Ranking, i.e., a ranking taking into account the results of both Summer and Winter Games. Due to the mathematics used we do not believe that our method could provide an official ranking. It is a toll for results analysis only, which provides full information about the Olympic results (Summer and Winter) of each country.

The proposed model ranked better those countries that had won fewer gold medals but, on the other hand, had won a considerable number of silver and bronze medals. Of course the countries that are in such a situation are not the same in different editions of the Games. For instance, Brazil would benefit four our method in Sydney and Salt lake City Games but will loose several positions if the method was applied to Athens and Torino Games. In the fixed weights model, the Salt Lake City 2002 Winter Olympic Games results have little influence because of the low average weight assigned to the medals earned in those games. Thus, some countries, particularly Norway, ended up in a better position with the use of the single weight restriction model.

An interesting question is whether or not the importance of the Winter Games is as small as our model implies. Tropical countries, with no tradition in winter sports, benefited from this approach. Naturally, this can be a topic for wide debate, particularly in countries with a strong tradition in winter sports, e.g. Switzerland, Austria and others.

Our approach is a two-group multidecision makers' problem. The authors, who defined the weight restrictions based on their own opinions, make up the first set of decision makers. All the DMUs referred to in this paper (with the exception of United States) make up the second group of decision makers.

As far as theoretical aspects are concerned, it should be emphasized that not having taken into account the sole efficient DMU in the average weight calculation led to a model with a single solution which is free from the complexity of the classical cross evaluation approach. Furthermore**,** using weight restrictions together with average weights avoids the problem of unreal weights mentioned by [18].

As DEA is a linear well-studied technique, future research in this field is not likely to be in the optimization aspects of the problem, unless we take into account that the number of medals is somewhat imprecise due to, for instance, doping problems. In such a case, we may use Imprecise DEA [35]. An improved rank with a DEA approach should take into account the differences among the various sports in the Olympics. Such an approach will probably need to use experts' opinions. One method to do that is incorporating fuzzy theory into DEA, as done, for instance, by [36] and [37]. Rankings like these may be useful in studies concerning the "home advantage" phenomena.

It is also possible to take into account all the results obtained, and not only the number of medals earned for each country. To perform such a complete model we will need faster algorithms for DEA, due to the use of a massive data set. A possible method to do that is the Neuro-DEA algorithm [38].

Acknowledgements

We acknowledge the financial support of CNPq (Brazilian Ministry of Science and Technology).

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