Research on the MPCG Algorithm applied in the three dimensional electric field calculation of SF_6 Circuit Breaker in three-phase-in-one-tank GIS

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Abstract: - With the structure miniaturization of SF_6 Circuit Breaker in three-phase-in-one-tank GIS, improvement of internal structure design and layout should be taken into account to balance the electric field intensity distribution among phases. By calculating the three-dimensional electric field intensity distribution in GIS with FEM to guide and optimize its internal structure will improve the electric field intensity distribution status among each phase or whole field domain and can avoid insulated medium breakdown due to excessiveness of local electric field intensity. Based on the character of the complex structure of the SF_6 circuit breaker in three-phase-in-one-tank GIS (Gas Insulated Switchgear), the modified preconditioned conjugate gradient (MPCG) algorithm applied to solve multi-nodes and multi-elements large sparse real positive defined symmetric matrix is analyzed. The program diagram of MPCG algorithm is also proposed. According to the sparse symmetric character of coefficient matrix, the method of storing coefficient matrix effectively and rapidly is studied and new data structure is adopted. The finite element calculation mathematical model and physical equations of SF_6 circuit breaker are founded in this paper. The large sparse matrix is solved by using the MPCG algorithm. The electric field intensity and potential distribution is calculated under power frequency voltage and their corresponding cutaway charts and planforms are given. They can provide a useful design parameter for designing the circuit breaker optimum configuration in GIS.

Key-Words: - MPCG(Modified Preconditioned Conjugate Gradient); Electric field calculation; Three dimensional; Three-phase-in-one-tank; GIS(gas insulated switchgear); Circuit breaker; Finite element method; Sparse matrix

1 Introduction

With the structure miniaturization of SF_6 Circuit Breaker in three-phase-in-one-tank GIS (in a following part of this paper is called GIS), improvement of internal structure design and layout should be taken into account to balance the electric field intensity distribution among phases. By calculating the three-dimensional electric field intensity distribution in GIS with FEM (Finite Element Method) to guide and optimize its internal structure will improve the electric field intensity distribution status among each phase or whole field domain and can avoid insulated medium breakdown due to excessiveness of local electric field intensity. It is especially important to design GIS structural dimension along the increasingly miniaturization direction.

Calculating static electromagnetic field by FEM can be equivalent to calculate a group of algebraic equations[2,3]. For the three dimensional project questions, the order of discrete equations is high and can get to hundreds of thousand. Direct methods, such as Gauss elimination method or direct triangle decomposed method, are hardly applicable; Iterative methods, such as Gauss-Seidel iterative method or over-relaxation iterative method, mainly depends on the selection of related optimum relaxation parameter, but can't ensure convergence and not be applicable for solving large finite elements equations[16]. Conjugate Gradient (CG) method can ensure convergence within limited iterative steps theoretically and need not estimate parameters in advance, but its convergence is determined by condition number of coefficient matrix. In 1970s, PCG (Preconditioned Conjugate Gradient) method brought by preconditioned technology has been used widely in electromagnetic numerical computation field. Reference[2] reduces the condition number by normalizing the diagonal elements of coefficient matrix; ICCG algorithm mentioned in reference[3-5] uses CG method to solve the linear equations after decomposing the coefficient matrix incompletely to form the preconditioned matrix; Algorithm mentioned in reference[6-8] also adopts CG method after processing coefficient matrix with Jacobi matrix as preconditioned matrix; Reference [9] is an adjustable-bandwidth ICCG algorithm in essential, because zero elements are injected into coefficient matrix artificially to reduce decomposed time. For the CG method exploiting different preconditioned technology, the purpose is to get better convergence in each iterative step by reducing the condition number of coefficient matrix and cuts down the memory occupied by the coefficient matrix and improves the calculation speed. But these algorithms are mostly applied for two dimensional or quarter three dimensional field domain, especially ICCG algorithm, it is widely used due to its low condition number, well convergence and stability. While as for a complicated three dimensional field domain, the order of coefficient matrix is so high that it requires more memory space, for example, field domain of GIS in this paper is not a symmetric field domain and can't be converted to a two-dimensional field or quarter three dimensional, it is required to calculation the whole three dimensional field based on GIS instances, the order of coefficient matrix will be great than 300,000 after meshing, although ICCG has well convergence, it is not applicable for calculating such super-large equations because of long calculation time caused by matrix operation during each iterative step.

Based on ICCG algorithm, MPCG introduced in this paper uses a control parameter δ to reduce nonzero elements of the lower triangle decomposition matrix by normalizing the diagonal elements of sparse matrix[1-2]. Without adding iterative time, MPCG has almost same convergence speed as ICCG when reducing the super-large equations calculation time by decreasing the coefficient matrix condition number and matrix operation effort during each iterative step.

Based on the three dimensional calculation mathematical model of GIS and its physical equations, MPCG is employed to calculate the finite equations to get the inner electric field intensity distribution of GIS. The calculate result indicates that MPCG algorithm has short calculation time and high precision. According to the calculation result, inner structure of GIS can be optimized to meet the miniaturization and stable performance requirement.

2 MPCG Algorithm

Consider the system of linear equations

$$AX = b \tag{1}$$

which arises from the finite element analysis of electromagnetic fields, where A is a sparse real positive definite symmetric matrix.

Using conventional ICCG algorithm to solve the equations, we need to decompose the matrix A to form the preconditioned matrix. By implement incomplete cholesky decomposition on matrix A, its condition number is not only reduced, but also its sparse character is keep. Next, we suggest to solve the equations using conjugate gradient (CG) method, in this case:

$$A = LDL^T + E$$

where L is a lower triangle matrix which has same sparse as A, D is a diagonal matrix, E is a error matrix which can normally be ignored.

For a matrix A with rather high order, solving (1) takes long CPU time even if the CG iteration could converge in a reasonable number of iterative steps. In order to improve it we should reduce the order difference as soon as possible. Firstly, normalize the diagonal elements, a_{jj}, of matrix A, to reduce the difference between the magnitude order of them. That is, transform (1) into

$$A'X' = b' \tag{2}$$

Secondly, implement the incomplete decomposition of A' with the controlling parameter δ, i.e., $A' = L_1 D_1 L_1^T + E_1$, where L_1 is lower triangle matrix, D_1 is diagonal matrix. During the decomposition process the nonzero elements of the matrix A' are reduced by use of parameter δ. Finally, we use CG method to solve equations. In this case, (2) has to be transformed into

$$CY = d \tag{3}$$

Elements of (2) and (3) are determined by:

1) A' is preconditioned matrix， $A' = BAB$ ， and $A' = \{a_{ij}'\}$， its main diagonal elements are 1， other elements are:

$$a_{ij}' = \begin{cases} b_{ii} a_{ij} b_{jj} & i \neq j = 1, 2, \cdots n \\ 1 & i = j = 1, 2, \cdots n \end{cases} \tag{4}$$

$$b_{ij} = \begin{cases} 0 & i \neq j = 1, 2, \cdots n \\ \sqrt{\dfrac{1}{a_{ii}}} & i = j = 1, 2, \cdots n \end{cases} \tag{5}$$

2) $$\begin{cases} C = (L_1 D_1)^{-1} A' (L_1 D_1)^{-T} \\ Y = (L_1 D_1)^T X' \\ d = (L_1 D_1)^{-1} b' \end{cases} \tag{6}$$

L_1 、 D_1 is determined by $G = L_1 + D_1 - I$, where I is unit matrix. The filled-in entries g_{ij} of G are determined in the following way: if the element of A' , a_{ij}' , is equal to zero, then g_{ij} , the element of G, is ignored; if $a_{ij}' \neq 0$, calculate

$$g_{ij} = \frac{[a_{ij}' - \sum_{m=1}^{j-1} g_{im} \times g_{mm} \times g_{jm}]}{g_{jj}} \qquad j = 1, 2, \cdots, i \tag{7}$$

When $i > j$ and $|g_{ij}| < \delta$ g_{ij} is ignored, or else filled in.

$k (k = 0, 1, 2, \cdots)$ at figure.1 is iterative time. From the analysis above, MPCG algorithm can not only

decrease the condition number of coefficient matrix, but also reduce the number of non-zero element. Although the process of MPCG algorithm is similar to ICCG algorithm, it can decrease the computation effort at each CG iterative step on the basis of keeping the same convergence speed as ICCG algorithm. It is a good method to compute super large scale equations.

The iterative procedure of MPCG algorithm is shown as figure 1:

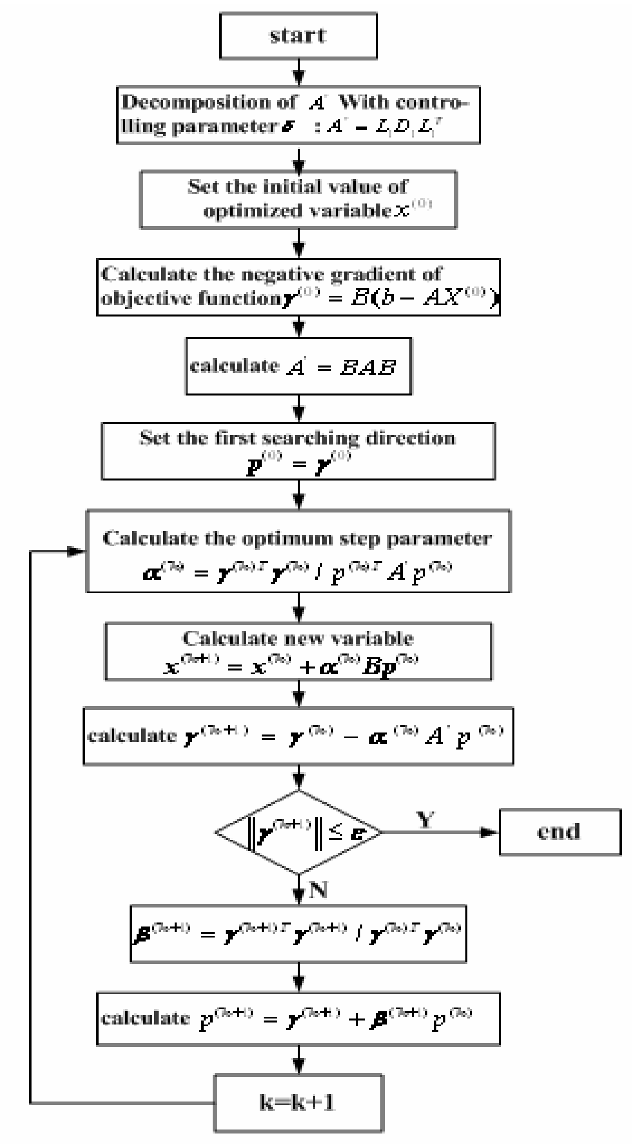


Fig.1 the program diagram of MPCG algorithm

3. Storing mode

By analyzing the calculation process of incompletely Cholesky decomposition and CG iteration, most time is actually used to address the non-zero elements of coefficient matrix, and arithmetical operation of matrix only takes little time. Designing a reasonable and effective storing mode for coefficient matrix can reduce matrix calculation time greatly.

Zero elements of matrix $\boldsymbol{A}$ can be ignored in matrix calculation process. Aiming at the sparseness character of matrix $\boldsymbol{A}$, the occupied memory can be cut down greatly if storing only non-zero elements. Based on irregular distribution of non-zero elements and sparseness character of matrix $\boldsymbol{A}$, two arrays are designed to store the coefficient matrix $\boldsymbol{A}$ elements. The occupied memory of matrix $\boldsymbol{A}$ with new storing mode will only be two times $\boldsymbol{m}$, where $\boldsymbol{m}$ is the number of non-zero elements in matrix $\boldsymbol{A}$. Suppose the order of matrix $\boldsymbol{A}$ is $\boldsymbol{N}$, the number of non-zero elements m. One-dimension real array called **tab_val**[] is used to store all non-zero elements, another one-dimension integral array called **tab_position**[] is used to store address in **tab_val**[] and column number of all non-diagonal elements in matrix $\boldsymbol{A}$. For **tab_val**[], the storing rules is as follows:

(1) The value of diagonal elements in matrix $\boldsymbol{A}$ is stored orderly from **tab_val**[1] to **tab_val**[N]. The purpose is to use the diagonal characteristic, namely for a matrix which order is N, the row number and column number of diagonal elements must be equal. When storing diagonal elements by row, the array subscript is just the row number of diagonal elements, i.e. the column number.

(2) No data is stored in **tab_val**[N +1].

(3) All non-zero elements of each row except diagonal elements in matrix $\boldsymbol{A}$ will be stored from **tab_val**[N +2], perspicuously the last element stored in **tab_val**[] is the last non-diagonal element of the last row, address of this element is called $\boldsymbol{k}$. If matrix $\boldsymbol{A}$ has $\boldsymbol{N}+1$ row, the start address of non-diagonal elements of $N+1$ row in **tab_val**[] should be $\boldsymbol{k}+1$.

For **tab_position**[], the storing rule is as follows:

(1) The start address of non-diagonal elements of each row in **tab_val**[] is stored orderly from **tab_position**[**1**] to **tab_position**[$\boldsymbol{N}$].

(2) If matrix $\boldsymbol{A}$ has $\boldsymbol{N}$ **+1** row, the start address of non-diagonal elements of $\boldsymbol{N}$ **+1** row in **tab_val**[] should be $\boldsymbol{k}$**+1**, this address is stored at **tab_position**[$\boldsymbol{N}$ **+1**]. The purpose of the storing rules is to obtain the number of non-diagonal elements of each row according to the start address stored in **tab_position**[]. The number of i row non-diagonal elements is **tab_position**[$\boldsymbol{i}$+1]−**tab_position**[$\boldsymbol{i}$], $\boldsymbol{i}$ ∈ **1,2,…,**$\boldsymbol{N}$.

(3) The column number of all non-zero elements of each row in matrix $\boldsymbol{A}$ is stored from **tab_position**[N+2] by row. The value of non-diagonal elements in matrix $\boldsymbol{A}$ is stored from **tab_val**[N+2] in order to keep addresses of these two arrays alignment, which is very important to address conveniently, and it is also why **tab_val**[N+1] is reserved.

The position of the non-diagonal elements in **tab_val**[] is mapped to **tab_position**[]. Owe to **tab_position**[], not addressing but basic arithmetical operation is carried on in each loop. Convenient addressing will speed up matrix calculation greatly.

In conclusion, the size of **tab_val**[] is m+1, and the size of **tab_position**[] is N+1+m−N, i.e. m+1. Total occupied memory is 2m+2, normally m>>2, so the total occupied memory could be regarded as 2m approximatly. For the GIS circuit breaker presented in this paper, the order of matrix $\boldsymbol{A}$ after meshing reaches three hundred thousand and total number of non-zero elements is close to four million. If the coefficient matrix elements are single-precision, the occupied memory is about 32M bytes, if the coefficient matrix elements are double-precision, the occupied memory is about 48M bytes. So it is essential to design the novel storing mode.

4 Three-dimension static electric field analysis of GIS circuit breaker

4.1 Calculation model

Computation model of GIS is shown as Fig.2 (a), (b) and Fig.3, in order to get the whole structure of GIS to be more clear, two lids at the right end are not covered, as shown fig.2 (a). While implementing calculation actually, the three-phase inputs at the right end will be all closed, as shown fig2. (b). In the case of actual running and testing conditions, outer electric field intensity of GIS tank can be regarded as zero since GIS tank is a close field domain, which will not affect the precision of calculation result. So the close problem of calculation field domain can be solved effectively by making use of the characteristic of GIS sealed tank.

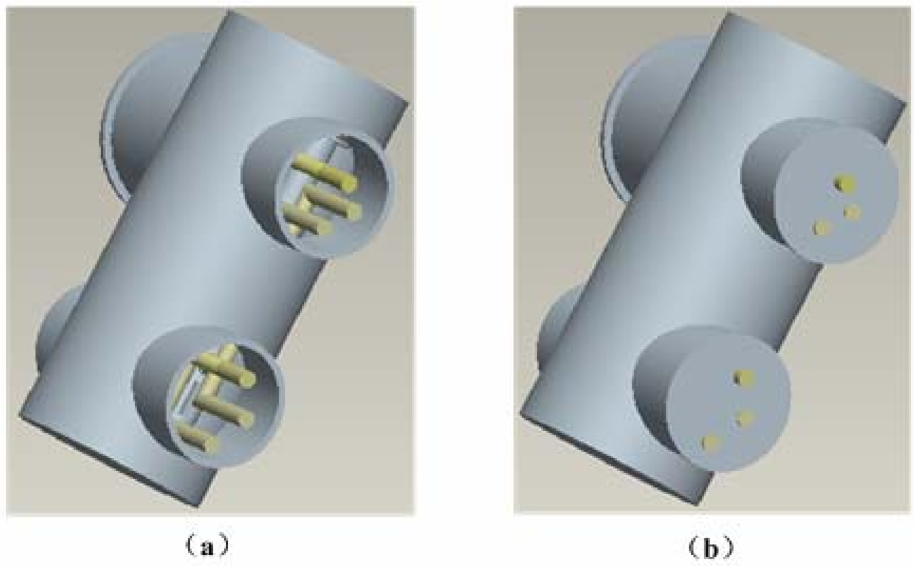


Fig.2 The structure of high voltage circuit breaker of GIS

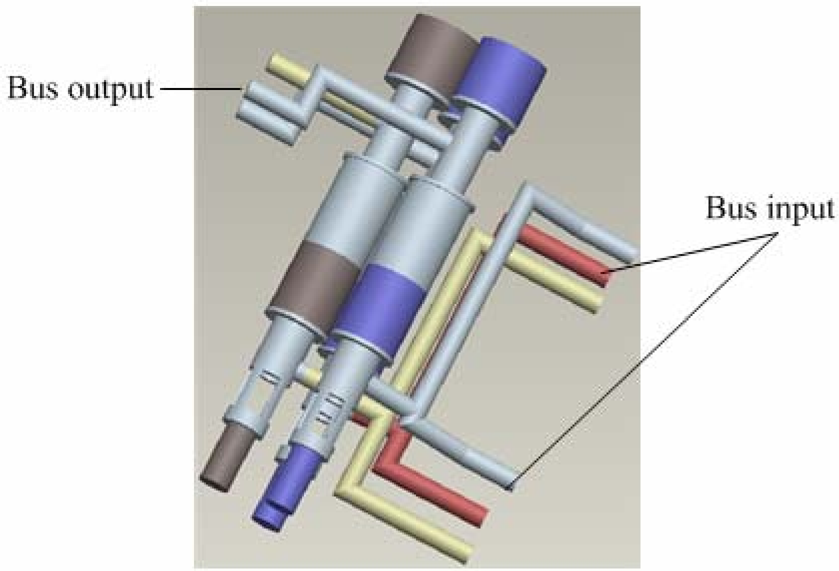


Fig.3 inner structure of CB in GIS

Electric potential equation of GIS inner electrical field intensity is:

$$\nabla^2\varphi = 0 \tag{8}$$

When GIS circuit breaker disconnecting or closing at power frequency voltage, the inner electrical field distribution of GIS can be regarded as a three dimensional static electric field approximately, and the space charge effect of inner electric field can be ignored. Boundary condition of GIS should be:

$$\begin{cases} \varphi_1\big|_{\Gamma_1} = U \\ \dfrac{\partial \varphi_2}{\partial n}\Big|_{\Gamma_2} = 0 \end{cases} \tag{9}$$

U is a known zero electric potential or a known power-frequency test voltage.

The three dimensional model in this paper is a sealed tank and is used to calculate the inner electric field intensity distribution of whole GIS tank in test power-frequency voltage working conditions.

4.2 Electric field calculation

Fig.2 is the structure diagram of GIS, the three-dimension model which includes all components of GIS except for some unimportant screws is mainly used to analyze the electric field distribution of GIS at disconnection or close status. GIS internal structure is complex which include two groups of three-phase connection poles, one disk-type insulator and one support pedestal and one three-phase circuit breaker pole at center position, the left port is outputs of three-phase electrode and disk-type insulators, there is many crook corners. All connection poles within GIS tank need to be more compact in order to make the size of GIS tank more miniaturization in the case of ensuring reasonable insulation, this require more precision as to electric field distribution calculation. Obviously, this GIS structure model should be calculated as whole field instead of two-dimension field, thus its coefficient matrix will be a super-large, linear, positive definition, sparse matrix. In this paper, FEM (Finite Element Method) is used to calculate the three-dimension electric field distribution of a 126kv GIS, ICCG and MPCG are employed to calculate finite element algebraic equations. The calculation platform configuration is 2.40GHz CPU and 1G memory, iterative error ε is 1×10^{-7}, the result is shown at table 1. The final calculation time is decrease determined by convergence speed and operation effort at each iterative step. From table1, when the value of δ is lesser, calculation time will be long due to more matrix operation effort since there are still more nonzero elements within matrix A after implementing decomposition incompletely, while the value of δ is bigger, although the number of nonzero elements within matrix A after incomplete cholesky decomposition will decrease, the calculation time will still be longer because convergence speed will slower, thus, the controlling parameter δ is exploited to make a compromise between the convergence speed and matrix operation effort. In this example, when setting δ to 1×10^{-4} we will get well convergence speed and short calculation time.

Table 1 comparison of MPCG and ICCG

Algorithm	δ	Non-zero elements	Iteration times	CPU time (s)
ICCG	0.0	3770891	63	75.532
MPCG	1×10^{-3}	1204021	66	41.264
MPCG	1×10^{-4}	1553202	62	37.886
MPCG	1×10^{-5}	1782170	60	41.156

Fig.4(a) and Fig.4(b) are three-dimensional meshing figures of electric field calculation model, in Fig.4(a), meshing element of SF_6 is a three-dimension 10 nodes tetrahedron, the total nodes of whole calculation field domain after meshing are 364,984, the total elements are 298,601.

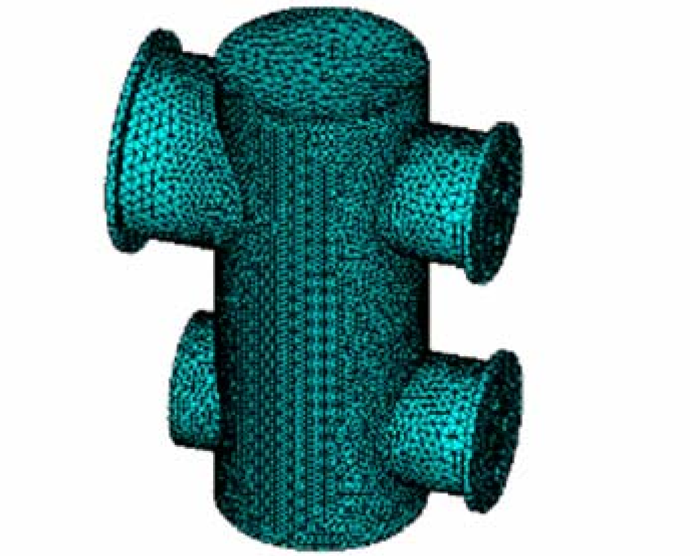


Fig4(a) the three-dimensional mesh of SF6 in GIS

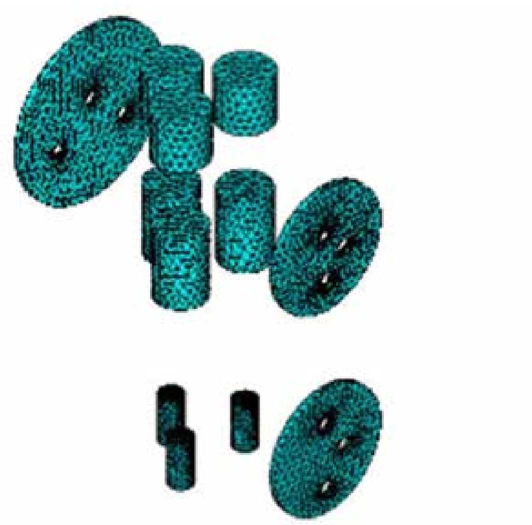
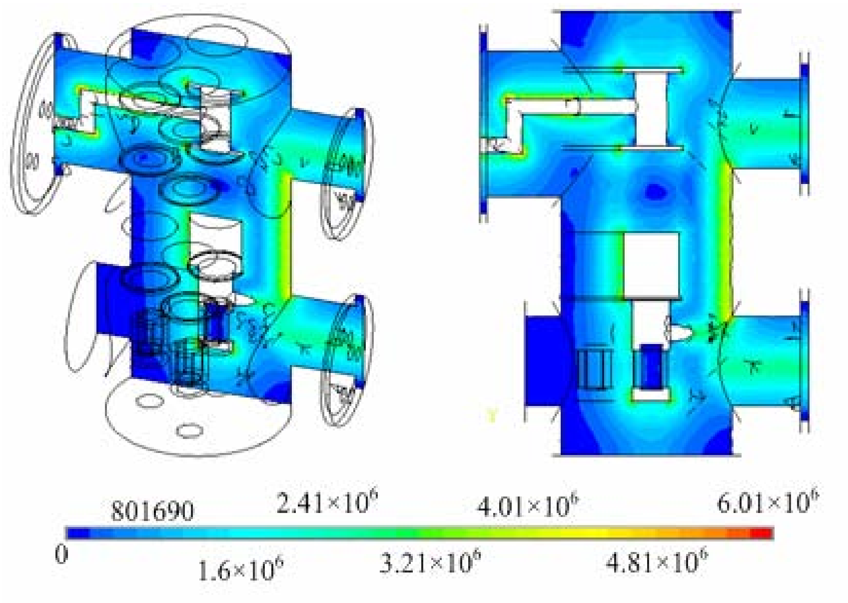
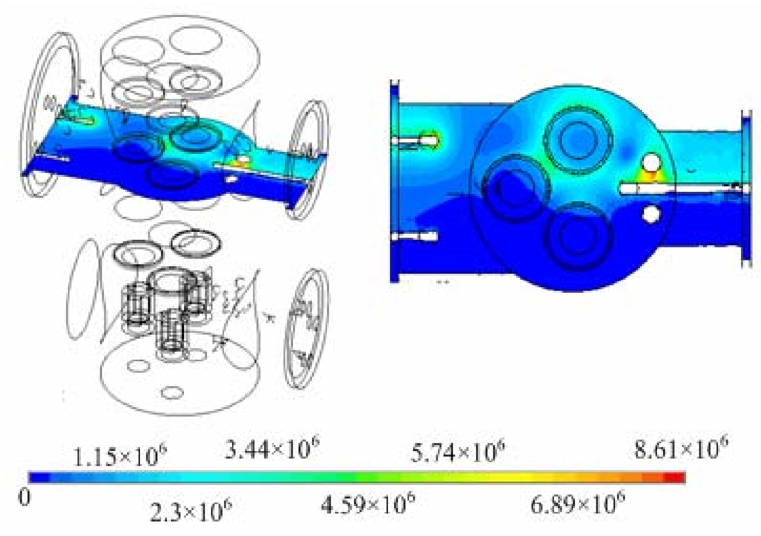


Fig.4(b) the three dimensional mesh of epoxy resin

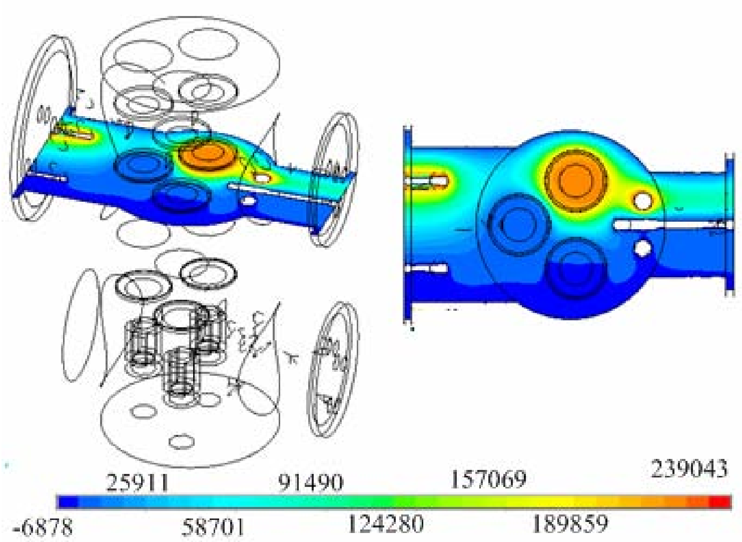
In this paper, electric potential distribution and electric field intensity distribution of GIS at sealed status is introduced, cutaway diagrams and planforms are shown as Fig5.(a), (b), (c) and (d). The unit is V/m. According to the electric field intensity distribution and potential distribution of GIS, electrodes layout within GIS and its insulation performance can be analyzed or optimized. Based on the maximum broken down electric field intensity, the insulated dielectric will not be broken down. The results show that the structure of GIS is available.



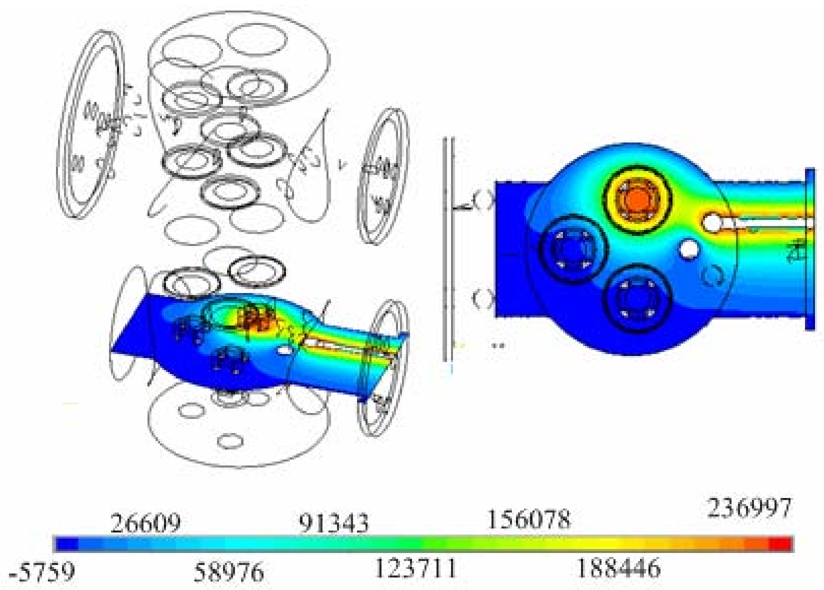
(a)


1.15×10^6
3.44×10^6
5.74×10^6
8.61×10^6
0
2.3×10^6
4.59×10^6
6.89×10^6

(b)


25911
91490
157069
239043
-6878
58701
124280
189859

(c)


26609
91343
156078
236997
-5759
58976
123711
188446

(d)

Fig.5 (a) and (b) are electric field distribution figures of circuit breaker
Fig.5(c) and (d) are potential distribution figures of circuit breaker

5 Conclusion

In this paper, a modified PCG algorithm that can calculate super-large sparse real positive definite symmetric matrix is introduced. The algorithm is simple, and needs much less memory due to reducing number of non-zero elements. From tab.1, the calculation time can be reduced 50% when controlling parameter δ is 1×10^{4}. Thus, this method is an effective method for calculating super-large finite element equations.

A novel storing mode of coefficient matrix is provided. It can improve effectively speed of solving equations by way of saving memory occupied and quick addressing.

The calculation mathematical model and physical equations of GIS circuit breaker is built. The three-dimension electric field intensity distribution of GIS is calculated by finite element method. The electric field intensity distribution can be effectively as a reference of GIS circuit breaker structure miniaturization design.

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