Energy Distribution Modeling of Car Body Deformation Using LPV Representations and Fuzzy Reasoning

ISTVÁN Á. HARMATI
Széchenyi István University
Dept. of Mathematics and Computational Sciences
Egyetem tér 1, 9026 Győr
HUNGARY
harmati@sze.hu

ANDRÁS RÖVID
Budapest Tech
John von Neumann Faculty of Informatics
Bécsi út 96/B, 1034 Budapest
HUNGARY
rovid.andras@nik.bmf.hu

LÁSZLÓ SZEIDL
Budapest Tech
John von Neumann Faculty of Informatics
Bécsi út 96/B, 1034 Budapest
HUNGARY
Eötvös Loránd University
Faculty of Informatics
Budapest, Pázmány Péter sétány 1/C.
HUNGARY
szeidl@bmf.hu

PÉTER VÁRLAKI
Budapest University of Technolgy and Economics
Dept. of Chassis and Lightweight Structures
Bertalan L. u. 1, 1111 Budapest
HUNGARY
varlaki@kme.bme.hu

Abstract: The main aim of the paper is to introduce a novel method – based on fuzzy control and linear parameter varying (LPV) system representation transformable into HOSVD based Canonical form – for modeling deformation processes with respect to the distribution of the absorbed kinetic energy. Modeling such kind of processes requires many uncertain input parameters. Using the proposed concept we are able to handle them and keep the complexity of the models low by using higher order singular value decomposition (HOSVD) technique.

Key–Words: car crash, Takagi-Sugeno fuzzy model, LPV model, HOSVD, Canonical form

1 Introduction

On the roads of the European Union annually happen a vast number of traffic accidents. These claim more than 40000 deathly victims and 1.6 million injured. According to the directive of the EU these numbers have to decrease significantly. This includes the increasing both the active safety (accident prevention, driving morality) and the passive safety (as less as possible human injuries occur in an accident happened).

The manufacturers develop both the active and passive safety systems. Intelligent vehicle control systems (which analyze the moving of the other vehicles and the road conditions) could be ranked among the active safety systems, safety belt, various airbags and the so-called energy absorbing elements, energy absorbing zones.

These latter used for the absorption of deformation energy arising at the transformation of the before collision kinetic energy of the vehicle, and in this manner the passenger compartment is protected from the more serious consequences of the collision, at least up to certain velocity. [1] [2]

The development of each listed is an extremely difficult engineering task, in the resolution of which apply experimental data, mathematical models which at least approximately describe the examined system, and the results of simulation procedures, of course [3] [4] [5].

The experiential data may taken from real accidents, but at this time unfortunately only few parameters are known, and their values are in general uncertain. Because of this reasons it is more convenient to execute well-planned crash tests with known parameters, which are at least theoretically repeatable. In the course of this procedure at what more of the parameters of the examined vehicle and the collision process are registered, and then the theoretical simulation model's behavior is compared with the data measured here.

These experiments are exceptionally expensive, only some thousand are done annually of them, and it is almost impossible to measure all of the parameters of the system and the process simultaneously under the very short time of the crash. [1] [6]

Because of causes mentioned above, modeling processes and estimations of partial processes and parameters are of great importance. In what follows we deal with the energy absorption models of the front of vehicle, the so-called wrinkling zone.

From the point of view of the deformation energy absorbed by the crash process of the vehicle two main kinds of methods can be distinguished, i.e. there are methods, which aim is to estimate the absorbed energy using the final state of the deformed car-body and crash test data as input, while the other ones give alternatives for analyzing the behavior of the whole deformation process usually based on heuristic approaches.

The main aim of this investigation is to introduce models supporting the analysis of deformational processes, i.e. to model energy distribution of car body deformation using measurement data, fuzzy control and LPV representations.

The paper is organized as follows: In Section 2. the finite element method and the so called heuristic model based approach is confronted. Section 3. describes an LPV model based approach for modeling the energy distribution of car body deformation, while Section 4. discusses the same modeling problem using fuzzy reasoning. Finally Section 5 reports illustrative examples and conclusions.

2 FEM and Heuristic Models

For modeling the collisional deformation processes in the engineering practice usually a kind of finite element method (FEM) based softwares are applied. In these simulations the FE model of the vehicle rarely contain less than 500 000 elements. The model describes the dynamic process in time domain, and has to handle several contact conditions.

During the calculation it is necessary to apply non-linear laws for materials, since the components suffer plastic deformation in the course of the collision. Moreover, because of a process takes place with high velocity, the material features depend on the velocity of changing the deformational zone. [7] [8]

A huge amount of data arises during the calculation (esp. in all nodes 3 coordinates, the 9 elements of the tenseness tensor, etc.), and a model consists usually 100 000s of nodes. The distribution of the deformation energy, the quantity of the energy absorbed by certain parts is determinated from the values calculated in this way (force, shift), naturally together with many other features. The FE description, being based on differential equations is general, yields a model which can be used on every area, although this model is exceptional complicated and has high computational complexity. But also here the heuristic and the engineering experience receive an important role, for instance in generating the finite element net. [7] [8]

Contrary to this, the heuristic models handling only a certain part of the problem (for example energy distribution) have much smaller complexity, but of course not provide an overall description. Mathematical models based on those methods, which use less number of variables and include some heuristic knowledge of human experts are more efficient then the usual descriptions. From this reason our investigation is strongly related to these family of models.

3 LPV Model for Deformational Processes

During deformation processes of car-bodies in the force function characteristics fluctuations can be observed (see Fig. 1). This is caused by the inhomogeneous structure of car-body and all of its elements (e.g. engine, stiffeners, etc.). There are some basic models - used by many applications for deformation analysis - which do not take into consideration the above mentioned fluctuations of the force function. For example Campbell's method is based on approximating the above mentioned behavior of the force by function illustrated in (see Fig. 2).

Our main aim is to construct a model, which is able to handle the fluctuating behavior of the force, as well. To describe such a deformation process, measurements of certain parameters have to be performed at discrete time steps and locations on the specific parts of car body. These measured parameters may be the force and the displacement of a predefined number of measurement points located on the car-body (see Figs. 3 and 4).

Furthermore, we suppose that the mass of the vehicle and the impact speed are given.

Based on the above considerations our proposal is to fit a linear parameter varying (LPV) model to the measured data. In the followings the description of the proposed LPV model approach can be followed.

For determining the energy absorbed by the deformed car body we have to know the forces appearing against the deformation during the whole deformational process and the measure of deformation. In general, the absorbed energy is

$$E_d = \int_0^{x_0} F(x)dx, \qquad (1)$$

where x_0 is the maximal deformation, which is the same in the full width of the vehicle in case of totally

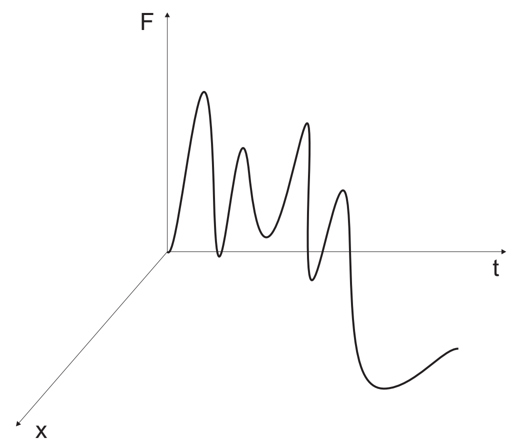

Figure 1: Illustration of a real force function. x axle represents the crush (displacement), the y axle stands for the time and the vertical axle represents the force.

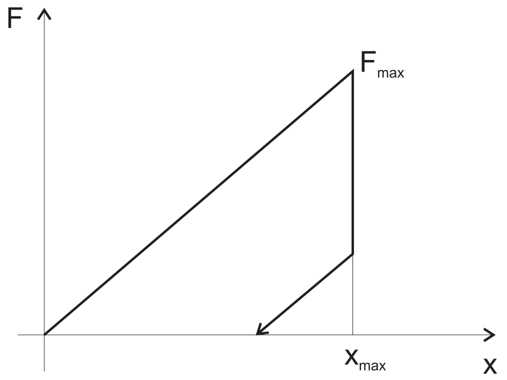

Figure 2: Illustration of a simple force model. Horizontal axle represents the crush (displacement) and the vertical axle stands for the force.

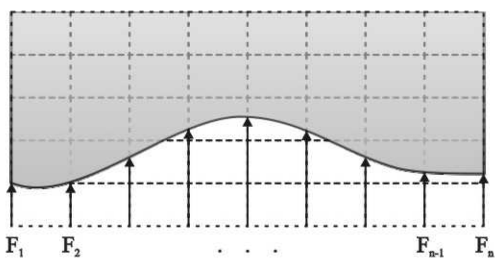

Figure 3: Illustration of forces. The grid represents the deformation zone of the front of car-body. The gray area stands for the deformed state of the car-body at a time t.

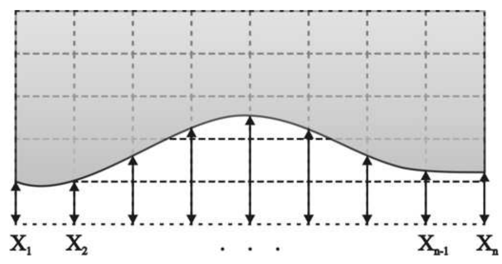

Figure 4: Illustration of crushes (displacements) corresponding to forces illustrated in the previous figure.

frontal collision. Force F is usually approximated by a simple model. Based on Emori's work ([9]) Campbell introduced first the linear force model ([10]), which is the basis of some crash simulational program (CRASH, SMAC). The main feature of the model is that constant stiffness parameter of the vehicle (or of a part the vehicle) is used for computation of deformational forces, the car body is taken into consideration during the collision as a simple linear spring with stiffness parameter k:

$$F = -kx. \tag{2}$$

This first approach was followed by number of models, which use more difficult force descriptions. For example bilinear force model, which deals with linear spring model until a certain level of permanent deformation and after this stage uses bilinear form ([11], [12], [13]), the saturation force model, which was applied successfully for investigation of frontal, lateral and rear impacts ([14], [15], [16], [17]).

However the results of crash tests show more difficult force and displacement behaviors than the simple models mentioned above. According to the observations, the stiffness parameter of a vehicle (or a part of the vehicle) is not a constant value, but depends on the measure of deformation (x), impact speed of the vehicle (v) and direction of the impact (φ) (this last can be out of consideration in case of purely frontal collisions). Based on this fact, we assume the force can be approximated well by a nonlinear form, which is a generalization of the linear spring model:

$$F = -k(x, v)x. \tag{3}$$

Or, in differential equation form:

$$m\ddot{x} = -k(x, v)x. \tag{4}$$

From this, with $k' = k(x, v)/m$, $x_1 = x$ and $x_2 = \dot{x}_1$ we obtain the following matrix form:

$$\begin{pmatrix} \dot{x_1} \\ \dot{x_2} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -k' & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}. \tag{5}$$

This is a parameter varying matrix and our main assumption is that the behavior of original system (force and displacement) can be described quite well using this kind of nonlinearity. In general state-space model form

$$\begin{aligned} \dot{\mathbf{x}}(t) &= f(\mathbf{x}(t)) \\ y(t) &= c(\mathbf{x}(t)) \end{aligned} \tag{6}$$

where

$$\begin{aligned} f(\mathbf{x}(t)) &= \begin{pmatrix} x_2(t) \\ -k\,(x_1(t), v) \end{pmatrix} \\ c(\mathbf{x}(t)) &= \begin{pmatrix} x_1(t) & 0 \end{pmatrix} \end{aligned} \tag{7}$$

If we write the above equation into the typical form of linear parameter-varying state-space model:

$$\begin{pmatrix} \dot{\mathbf{x}}(t) \\ y(t) \end{pmatrix} = \mathbf{S}(\mathbf{p}(t)) \begin{pmatrix} \mathbf{x}(t) \\ u(t) \end{pmatrix} \tag{8}$$

Here system matrix $\mathbf{S}(\mathbf{p}(t))$:

$$\mathbf{S}(\mathbf{p}(t)) = \begin{pmatrix} \mathbf{A}(\mathbf{p}(t)) & \mathbf{B}(\mathbf{p}(t)) \\ \mathbf{C}(\mathbf{p}(t)) & \mathbf{D}(\mathbf{p}(t)) \end{pmatrix} \tag{9}$$

where $\mathbf{p}(t) = (x_1(t), v)$ and

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -k' & 0 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{10}$$

$$\mathbf{C} = \begin{pmatrix} 1 & 0 \end{pmatrix} \quad \mathbf{D} = \begin{pmatrix} 0 \end{pmatrix}$$

The next task is to determine the function k'. The approach is similar to the methods introduced in [18] and [19]. Firstly the functional dependence of k' on the variables x (depth of deformation) and v (impact speed) must be specified, for example piecewise linear, polynomial, spline or other linear combinations of given functions of x and v.

The model identification includes two major steps: identification of the local models (LTI models) with the same structure of the LPV model and on the base of these models identification of the final LPV model.

For LTI model identification we need some data from well-measured crash tests. For example, a cascade of tests have to be taken at a speed of $v = 30km/h$ to register the deformational process: depth of deformation vs. time, force (at sensors) vs. time. From this data set for a certain deformation x (for example with 10 cms steps size) a linear spring model can be identified. Certainly, for other x an other model is valid. In this way, for a certain impact speed a set of simple linear models is determined. After that we have to repeat this measuring and identifying process at other impact speeds (for instance at $35km/h$, $40km/h$, $45km/h$, etc.), but with the same division of x. Finally we get a large amount of local LTI models in the space of the impact speed and the deformation, with the same structure of the searched LPV.

A set of linear models means a set of certain values of the parameter varying k' at different parameter values. From these points and using our assumption about the type of the functional dependence, the function k' identified. With the obtained LPV model, estimation of the energy distribution for crash deformational process is available at a point of the phase space of x and v even differents from those used for identification.

Because of the large amount of obtained LTI models our system may become very complex. In order to reduce the complexity of the system the number of LTI models have to be minimized by maintaining the accuracy as precise as possible. For this purpose the so called higher order singular value decomposition can advantageously be used, in order to get the minimal number of LTI models by keeping the error at a lower level [20] [21] [22]. The such obtained reduced number of LTI models form an orthonorm system. Our system can be expressed as a linear combination of such minimal set of orthonormal LTI models. In the next subsection we are going to introduce the mentioned HOSVD reduction technique for a general LPV system.

3.1 HOSVD Based Reduction of the LPV Model

Consider such LPV state-space model

$$\begin{pmatrix} \dot{\mathbf{x}}(t) \\ \mathbf{y}(t) \end{pmatrix} = \mathbf{S}(\mathbf{p}(t)) \begin{pmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{pmatrix} \tag{11}$$

where $\mathbf{p}(t) = (p_1(t), ..., p_N(t)) \in \Omega$ and which can be given in the form of

$$\begin{pmatrix} \dot{\mathbf{x}}(t) \\ \mathbf{y}(t) \end{pmatrix} = \left(\mathcal{S} \boxtimes_{n=1}^{N} \mathbf{w}_n^T(p_n))\right) \begin{pmatrix} \mathbf{x}(t) \\ \mathbf{u}(t), \end{pmatrix} \tag{12}$$

where column vector $\mathbf{w}_n(p_n) \in \mathbb{R}^{I_n}$ $n = 1, \ldots, N$ contains one variable bounded and continuous weighting functions $w_{n,i_n}(p_n)$, $(i_n = 1..I_n)$. The $(N+2)$-dimensional coefficient (system) tensor $\mathcal{S} \in \mathbb{R}^{I_1 \times \cdots \times I_{N+2}}$ is constructed from linear time invariant (LTI) vertex systems

$$\mathbf{S}_{i_1 \ldots i_N} = \{S_{i_1 \ldots i_N, \alpha, \beta}, 1 \le \alpha \le I_{N+1}, 1 \le \beta \le I_{N+2}\}$$

$$\mathbf{S}_{i_1 \ldots i_N} \in \mathbb{R}^{I_{N+1} \times I_{N+2}}.$$

Symbol $\boxtimes_n$ represents the n-mode tensor-matrix product. For further details we refer to [20].

For this model, we can assume that the functions $w_{n,i_n}(p_n)$, $i_n = 1,\ldots,I_n$, $n = 1,\ldots,.N$, are linearly independent over the intervals $[a_n, b_n]$, respectively.

The linearly independent functions $w_{n,i_n}(p_n)$ are determinable by the linear combinations of orthonormal functions (for instance by Gramm–Schmidt-type orthogonalization method): thus, one can determine such a system of orthonormal functions for all n as $\varphi_{n,i_n}(p_n)$, $1 \le i_n \le I_n$, respectively defined over the intervals $[a_n, b_n]$, where all $\varphi_{n,k_j}(p_n)$, $1 \le j \le I_n$ are the linear combination of w_{n,i_j}, where i_j is not larger than k_j for all j. The functions w_{n,i_j} can respectively be determined in the same way by functions φ_{n,k_j}. Thus, if the form (12) of (11) exists then we can determine it in equivalent form as follows:

$$\begin{pmatrix}\dot{\mathbf{x}}(t)\\ \mathbf{y}(t)\end{pmatrix} = \left(C \boxtimes_{n=1}^{N} \varphi_n^T(p_n(t))\right)\begin{pmatrix}\mathbf{x}(t)\\ \mathbf{u}(t)\end{pmatrix}, \qquad (13)$$

where tensor C has constant elements, and column vectors $\varphi_n(p_n(t))$ consists of elements $\varphi_{n,k_n}(p_n(t))$.

Corollary 1. *(see [20]) We can assume, without the loss of generality, that the functions w_{n,i_n} in the tensor-product representation of $\mathbf{S}(\mathbf{p})$ are given in orthonormal system:*

$$\forall n : \int_{a_n}^{b_n} w_{n,i}(p_n) w_{n,j}(p_n) dp_n = \delta_{i,j}, \quad 1 \le i, j \le I_n,$$

where $\delta_{i,j}$ is the Kronecker-function ($\delta_{ij} = 1$, if $i = j$ and $\delta_{ij} = 0$, if $i \ne j$).

Theorem 2. *(see [20]) [Higher Order SVD (HOSVD)] Every tensor $\mathcal{S} \in \mathbb{R}^{I_1\times\cdots\times I_L}$ can be written as the product*

$$\mathcal{S} = \mathcal{D} \boxtimes_{l=1}^{L} \mathbf{U}_l \qquad (14)$$

in which

1. $\mathbf{U}_l = \begin{bmatrix}\mathbf{u}_{1,l} & \mathbf{u}_{2,l} & \ldots & \mathbf{u}_{I_l,l}\end{bmatrix}$ is an orthogonal $(I_l \times I_l)$-matrix called l-mode singular matrix.

2. tensor $\mathcal{D} \in \mathbb{R}^{I_1\times\ldots\times I_L}$ whose subtensors $\mathcal{D}_{i_l=\alpha}$ have the properties of

(i) all-orthogonality: two subtensors $\mathcal{D}_{i_l=\alpha}$ and $\mathcal{D}_{i_l=\beta}$ are orthogonal for all possible values of l, α and β : $\left\langle \mathcal{D}_{i_l=\alpha}, \mathcal{D}_{i_l=\beta}\right\rangle = 0$ when $\alpha \ne \beta$,

(ii) ordering: $\left\|\mathcal{D}_{i_l=1}\right\| \ge \left\|\mathcal{D}_{i_l=2}\right\| \ge \cdots \ge \left\|\mathcal{D}_{i_l=I_l}\right\| \ge 0$ for all possible values of l.

The Frobenius-norm $\left\|\mathcal{D}_{i_l=i}\right\|$, symbolized by $\sigma_i^{(l)}$, are l-mode singular values of $\mathcal{D}$ and the vector $\mathbf{u}_{i,l}$ is an i-th singular vector. $\mathcal{D}$ is termed core tensor.

Theorem 3. *(see [20]) [Compact Higher Order SVD (CHOSVD)]For every tensor $\mathcal{S} \in \mathbb{R}^{I_1\times\cdots\times I_L}$ the HOSVD is computed via executing SVD on each dimension of $\mathcal{S}$. If we discard the zero singular values and the related singular vectors $\mathbf{u}_{r_l+1},\ldots,\mathbf{u}_{I_l}$, where $r_l = rank_l(\mathcal{S})$, during the SVD computation of each dimension then we obtain Compact HOSVD as:*

$$\mathcal{S} = \widetilde{\mathcal{D}} \boxtimes_{l=1}^{L} \tilde{\mathbf{U}}_l, \qquad (15)$$

which has all the properties as in the previous theorem except the size of $\mathbf{U}_l$ and $\mathcal{D}$. Here $\tilde{\mathbf{U}}_l$ has the size of $I_l \times r_l$ and $\widetilde{\mathcal{D}}$ has the size of $r_1 \times \ldots \times r_L$.

Consider (11) which has the form of (12). Then we can determine:

$$\begin{pmatrix}\dot{\mathbf{x}}(t)\\ \mathbf{y}(t)\end{pmatrix} = \left(\mathcal{D}_0 \boxtimes_{n=1}^{N} \mathbf{w}_n(p_n(t))\right)\begin{pmatrix}\mathbf{x}(t)\\ \mathbf{u}(t)\end{pmatrix}, \qquad (16)$$

via executing CHOSVD on the first N-dimension of $\mathcal{S}$. The resulting tensor $\mathcal{D}_0 = \tilde{\mathcal{D}} \boxtimes_{n=N+1}^{N+2} \tilde{\mathbf{U}}_n$ has the size of $r_1 \times \ldots \times r_N \times I_{N+1} \times I_{N+2}$, and the matrices $\tilde{\mathbf{U}}_k \in \mathbb{R}^{I_k\times r_k}$, $k = N+1, N+2$ are orthogonal [20].

The weighting functions have the property of:

1. The r_n number of weighting functions $w_{n,i_n}(p_n)$ contained in vector $\mathbf{w}_n(p_n)$ form an orthonormal system. The weighting function $\mathbf{w}_{i,n}(p_n)$ is an i-th singular function on dimension $n = 1..N$.

Tensor $\mathcal{D}$ has the properties as:

2. Tensor $\mathcal{D} \in \mathbb{R}^{r_1\times\ldots\times r_{N+2}}$ whose subtensors $\mathcal{D}_{i_n=i}$ have the properties of

(i) all-orthogonality: two subtensors $\mathcal{D}_{i_n=i}$ and $\mathcal{D}_{i_n=j}$ are orthogonal for all possible values of n, i and j : $\left\langle \mathcal{D}_{i_n=i}, \mathcal{D}_{i_n=j}\right\rangle = 0$ when $i \ne j$,

(ii) ordering: $\left\|\mathcal{D}_{i_n=1}\right\| \ge \left\|\mathcal{D}_{i_n=2}\right\| \ge \ldots \ge \left\|\mathcal{D}_{i_n=r_n}\right\| > 0$ for all possible values of $n = 1,\ldots,N+2$.

3. The Frobenius-norm $\left\|\mathcal{D}_{i_n=i}\right\|$, symbolized by $\sigma_i^{(n)}$, are n-mode singular values of $\mathcal{D}$.

4. $\mathcal{D}$ is termed core tensor consisting the LTI systems.

As result we obtain a much more simple system, which includes minimal number of orthonormal LTI systems. The number of these vertex systems depends on the number of kept singular values. As higher the number of kept singular values as precise the approximation will be. Even if the number of discarded singular values is small, significant decrease in complexity can be observed [23].

4 Takagi–Sugeno Fuzzy Model

4.1 TS Fuzzy Model Description

In this section we give a brief review on the fundamental form of Takagi-Sugeno (TS) fuzzy models

[24] [25] [26]. A TS model consists a number of local linear models assigned to fuzzy regions, which are designed to approximate the dynamic features at the corresponding operating fuzzy points in vector space P. Fig. 5 shows the structure of a TS fuzzy model. The model varies according to vector $\boldsymbol{p} \in R^N$, which may contain some values of the state vector $\boldsymbol{x}$ as well. The TS fuzzy inference engine is responsible for combining the local linear models according to vector $\boldsymbol{p}$ in order to find a proper model, which is assumed to be the momentary linear descriptor of the system capable of generating output vector $\boldsymbol{y}$ from state vector $\boldsymbol{x}$ and input vector $\boldsymbol{u}$.

Consequently, the original system is approximated by a convex combination of a number of local linear models assigned to regions defined by basis functions $\mu_j(p)$. In case of TS model approximations coefficients $\mu_j(p)$ are computed as the firing probability of the fuzzy rules, based on the product operator t-norm. Usually the antecedent sets are given in Ruspini-partition, so for every j: $\sum_i \mu_{A_{j,i}}(p_j) = 1$.

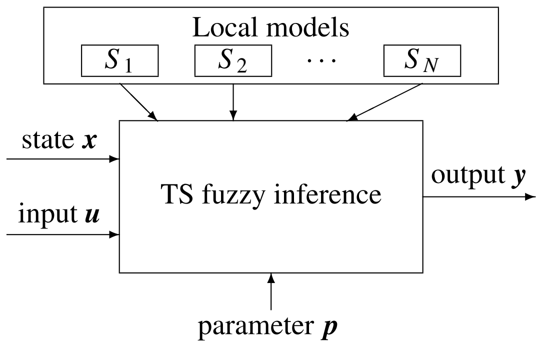

Figure 5: Scheme of the Takagi-Sugeno fuzzy inference model

The fuzzy rules are formed by all combination of the antecedent sets. So a typical rule is:

IF p_1 is A_{1,i_1} AND ... AND p_N is A_{N,i_N}

THEN model $S_{i_1 i_2 \ldots i_N}$

The range of the indeces $i_n = 1 \ldots V_n$, where V_n denotes the number of antecedent sets in the n-th universe. The output of a rule is:

$$\hat{S}_{i_1,i_2,\ldots,i_N} = S_{i_1 i_2 \ldots i_N} \prod_{j=1}^{N} \mu_{A_{j,i_j}}(p_j) \tag{17}$$

The final conclusion is the weighted sum of the outputs:

$$\hat{S}(p) = \frac{\sum_{i_1=1}^{V_1} \cdots \sum_{i_N=1}^{V_N} \hat{S}_{i_1,i_2,\ldots,i_N}}{\sum_{i_1=1}^{V_1} \cdots \sum_{i_N=1}^{V_N} \prod_{j=1}^{N} \mu_{A_{j,i_j}}(p_j)} = \tag{18}$$

$$= \frac{\sum_{i_1=1}^{V_1} \cdots \sum_{i_N=1}^{V_N} \prod_{j=1}^{N} \mu_{A_{j,i_j}}(p_j) S_{i_1 i_2 \ldots i_N}}{\sum_{i_1=1}^{V_1} \cdots \sum_{i_N=1}^{V_N} \prod_{j=1}^{N} \mu_{A_{j,i_j}}(p_j)} \tag{19}$$

If the antecedents sets are in Ruspini partition then

$$\sum_{i_1=1}^{V_1} \cdots \sum_{i_N=1}^{V_N} \prod_{j=1}^{N} \mu_{A_{j,i_j}}(p_j) = 1 \tag{20}$$

so the approximated model is

$$\hat{S}(p) = \sum_{i_1=1}^{V_1} \cdots \sum_{i_N=1}^{V_N} \prod_{j=1}^{N} \mu_{A_{j,i_j}}(p_j) S_{i_1 i_2 \ldots i_N} \tag{21}$$

4.2 TS Fuzzy Model for Crash Process

The main task is generating a fuzzy rule base which includes partitioning the parameter spaces and identifying the local linear models, too. Because of the reason that exact matemathical of the system, or rather of the process is unknown (if we were know that, it would be too difficult to deal with), determination of the rules is based on available measurment data and heuristic assumptions.

Analysing crash test results one can establish that the stiffness parameter of a certain vehicle is not a constant value, but depends on impact velocity and measure of deformation. Namely, if we take into consideration a simple linear spring model, our system is a spring with changing stiffness parameter, where this non-constant stiffness is unknown. Based on measurment data (force, displacment), assign different simple linear spring models to different points of the (X, V) parameter space, which describe approximately well the behavior of the system at these certain points.

Construct the partition of parameter spaces (fuzzy universes) X and V in such a way, that every measured point let be a core of a certain fuzzy set. For simplicity, determine the fuzzy sets to form a Ruspini partition. Since in case of given x and v the local linear spring model depends on parameter k only, therefore a general fuzzy rule is the following:

IF x is x^* AND v is v^* THEN $k = k^*$

Applying this rule base, the value of parameter k is interpolated at arbitrary value of x and v, and using

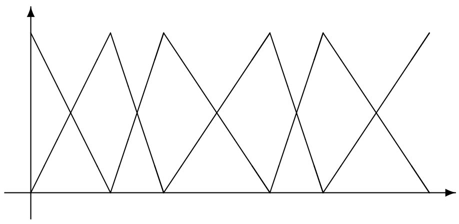

Figure 6: Fuzzy partition of parameter space V

this, the appearing force is approximated. By this way, based on measured data of force and displacement, one can estimate the amount of energy absorbed by the deformation for arbitrary value of x and v.

5 Examples

The input data used in this investigation were generated on the bases of real crash test charts corresponding to 35, 40, 50, 60, 70 km/h impact velocities.

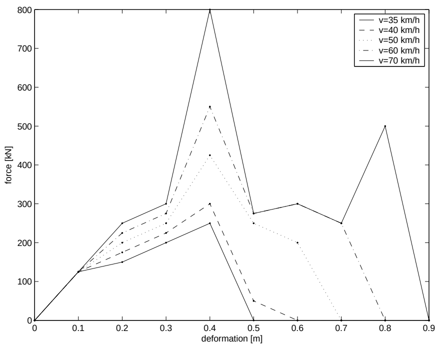

Figure 7: Force data for different velocities and deformations.

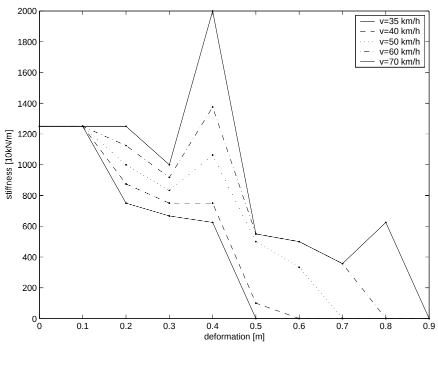

Figure 8: Stiffness data for different velocities and deformations.

Figs. 7 and 8 illustrate the force as function of displacement and velocity. The below examples are based on these generated data.

5.1 Example I

This example belongs to our proposed fuzzy based modeling technique. The measured data as mentioned in the previous section are the force and the corresponding displacement. Based on these data our fuzzy rule base can be created with the corresponding membership functions illustrated in Fig.9 and 10. In Fig. 11 the obtained surface is illustrated describing the force distribution as function of the displacement and velocity, while Fig. 12 shows the surface corresponding to the stiffness.

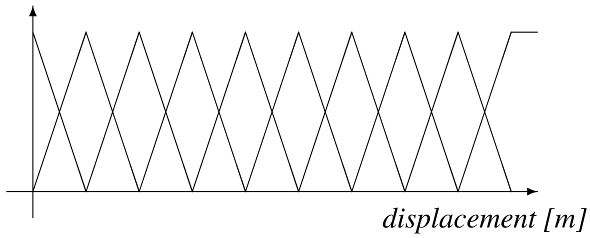

Figure 9: Fuzzy partition of the displacement (the cores are at points 0.1, 0.2, etc.)

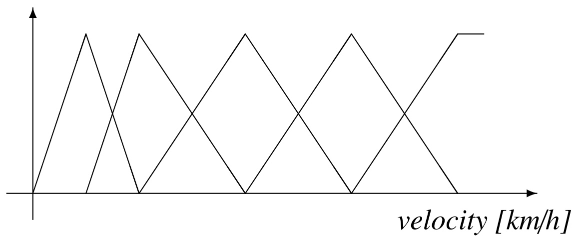

Figure 10: Fuzzy partition of the velocity (the cores are at 35, 40, 50, 60 and 70)

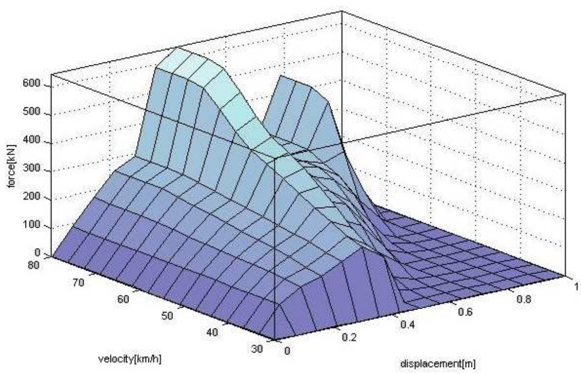

Figure 11: Surface of the fuzzy inference system. The output is the force

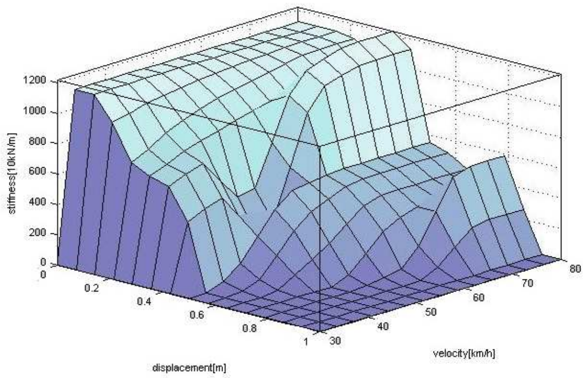

Figure 12: Surface of the fuzzy inference system. The output is the stiffness

5.2 Example II

This example is connected with our proposed LPV model. As described in section 6, first the function k has to be estimated. Using the proposed LPV based approach the force and the stiffness as functions of the displacement and velocity have been determined (see Figs. 13 and 14. For the impact velocity 55 km/h the absorbed energy distribution has been modeled, as well (see Figs. 15, 16, 17).

The LPV system can reduced by discarding some of singular values, for example if only two singular values are kept for each dimension then we get the following vertex systems and waiting functions (see Figs. 18 and 19. The computations were carried out by TPToolbox [27].

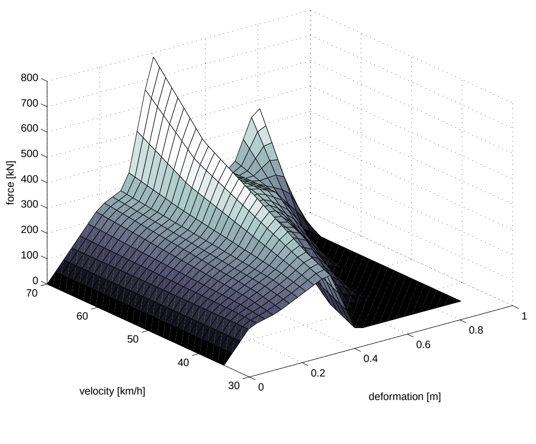

Figure 13: The force in function of the velocity and displacement (deformation).

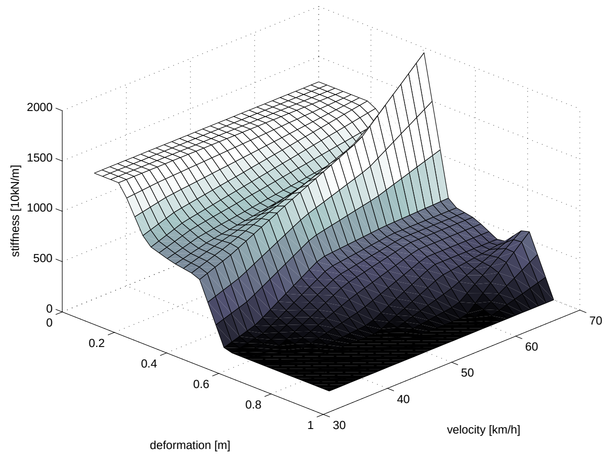

Figure 14: The stiffness in function of the velocity and displacement (deformation).

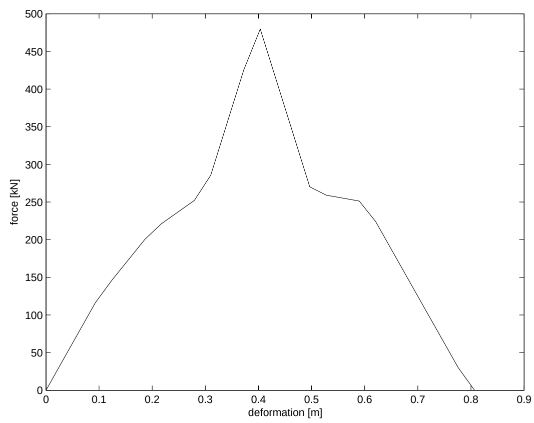

Figure 15: The force function at corresponding to the impact velocity 55 km/h.

$$S_{11} = \begin{pmatrix} 0.0000 & 11.3459 \\ -1.1597 & 0.0000 \end{pmatrix}$$

$$S_{12} = \begin{pmatrix} 0.0000 & 1.5225 \\ 0.0000 & 0.0000 \end{pmatrix}$$

$$S_{21} = \begin{pmatrix} 0.0000 & -4.2236 \\ 0.0000 & 0.0000 \end{pmatrix}$$

$$S_{22} = \begin{pmatrix} 0.0000 & -0.5667 \\ -0.1896 & 0.0000 \end{pmatrix}$$

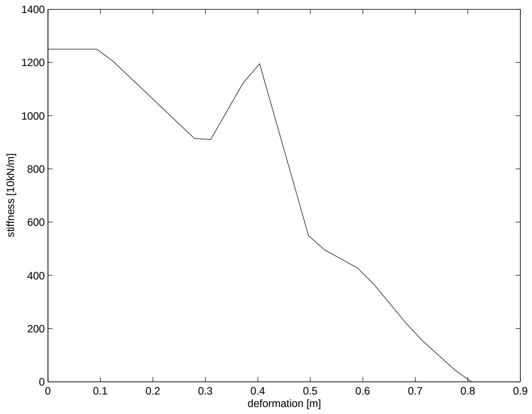

Figure 16: The stiffness function corresponding to the impact velocity 55 km/h.

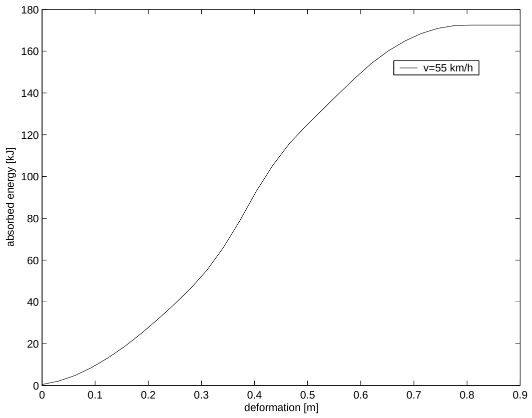

Figure 17: The absorbed energy corresponding to the impact velocity 55 km/h.

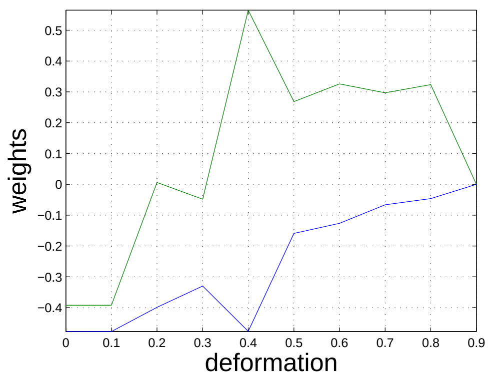

Figure 18: The obtained weighting functions of the HOSVD reduced LPV model.

6 Conclusion

LPV and fuzzy models were introduced to support the modeling of vehicle crash deformational processes

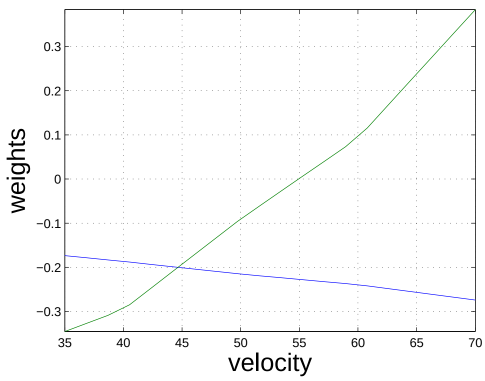

Figure 19: The obtained weighting functions of the HOSVD reduced LPV model.

with respect to the distribution of the absorbed kinetic energy. These kind of models provide an estimation of the real (or simulated) case, but they require less computational capacity than the usually applied FEM based procedures.

The methods described have to be validated by comparing with real crash test results or accurate simulations. The data we used are based on real measurements, but in order to achieve better approximation more detailed measurement data are required.

Acknowledgements: This paper was supported by the János Bolyai Research Scholarship of the Hungarian Academy of Sciences and by OTKA T048756.

References:

[1] A. Rövid, A.R. Várkonyi-Kóczy, P. Várlaki: Intelligent methods for car deformation modeling and crash speed estimation,*Proceedings of the 1st Romanian-Hungarian Joint Symposium on Applied Computational Intelligence*, 2004., pp. 75-84.

[2] H. Steffan, B.C. Geigl, A. Moser, H. Hoschopf: Comparison of 10 to 100 km/h rigid barrier impacts, Paper No. 98-S3-P-12

[3] M. Huang: *Vehicle Crash Mechanics*, CRC Press, 2002.

[4] R.M. Brach (ed.): *Accident Reconstruction: Crash Analysis*, SAE, 2001.

[5] Martin, O., Numerical Solutions of Differential Equations Using Fast Fourier Transform, *WSEAS Transactions on Mathematics*, Vol. 3, Nr. 2, 2004, pp. 383–389.

[6] T. Péter: Reduction of parameters of spatial nonlinear vehicle swinging systems, for identifica-

tion and purposes, Periodica Polytechnica, Vol. 36 No. 1., 1992., pp. 131-141.

[7] P. Griškevičius, A. Žiliukas: The crash energy absorption of vehicles front structures, *Transport* Vol. 18., No. 2., 2003., pp. 97-101.

[8] I. Zobory, H.G. Reimerdes, E. Békefi, J. Marsolek, I. Németh, Longitudinal dynamics of train collison – crash analysis, *7th Mini Conference on Vehicle System Dynamics, Identification and Anomalies*, Budapest, 2000, pp. 89-110.

[9] R. Emori: Mechanics of automobil collision, *1st International Conference on Vehicle Mechanics*, Wayne State University, Detroit, Michigan, July, 1974.

[10] K. Campbell: Energy basis for collision severity, *SAE*, 740565, 1974.

[11] C.E. Strother, R.L. Woolley, and M.B. James: Crush energy in accident reconstuction, *SAE*, 860371, 1986.

[12] J.A. Neptune: A comparison of crush stifness characteristics from partial-overlap and full-overlap frontal crash tests, *SAE*, 1999-01-0105, 1999.

[13] M.S. Varat, S.E. Husher, and J.F. Kerkhoff: An analysis of trends of vehicle frontal impact stiffness, *SAE*, 940914, 1994.

[14] C.E. Strother, R.L. Woolley, and M.B. James: A comparison between NHTSA crash test data an CRASH3 frontal stiffness coefficients, *SAE*, 900101, 1990.

[15] R.L. Woolley, C.E. Strother, and M.B. James: Rear stiffness coefficients derived from barrier test data, *SAE*, 910120, 1991.

[16] C.E. Strother, R.W. Kent, and C.Y. Warner: Estimating vehicle deformation energy for vehicles struck in the side, *SAE*, 980215, 1998.

[17] K.J. Welsh: Crush energy and structural characterization, *SAE*, 1999-01-0099, March 1999.

[18] G. Belforte, F. Dabene, P. Gay: LPV approximation of distributed parameter systems in environmental modelling, *Int. J. of Environmental Modelling & Software*, Vol.20., 2005, pp. 1063-1070.

[19] D. Coca, S.A. Billings: Identification of finite dimensional models of infinite dimensional dynamical systems, *Automatica*, Vol.38., 2002, pp. 1851-1865.

[20] L. Szeidl, P. Várlaki, HOSVD Based Canonical Form for Polytopic Models of Dynamic Systems, *Accepted paper, Journal of Advanced Computational Intelligence and Intelligent Informatics*, Under publication.

[21] Szeidl, L., Baranyi, P., Petres, Z. Várlaki, P., Numerical Reconstruction of the HOSVD Based Canonical Form of Polytopic Dynamic Models, *Proc. of International Symposium on Computational Intelligence and Intelligent Informatics (ISCIII 2007)*, Agadir, ISBN: 1-4244-1158-02007, 2007, pp. 111-116.

[22] L.D. Lathauwer, B.D. Moor, J. Vandewalle: A multilinear singular value decomposition, *SIAM Journal on Matrix Analysis and Applications*, Vol.21., No.4, April 2000, pp. 1253-1278.

[23] Szeidl, L., Rudas, I., Rövid, A., Várlaki, P., HOSVD Based Method for Surface Data Approximation and Compression, *Proc. of International Conference on Intelligent Engineering Systems (INES 2008)*, Miami, ISBN: 978-1-4244-2082-7, 2008, pp. 197-202.

[24] K. Tanaka, H.O. Wang: *Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach*, John Wiley & Sons, Inc., 2001

[25] Mastorakis, N.E., General Fuzzy Systems as extensions of the Takagi-Sugeno methodology, *WSEAS Transactions on Systems*, 2004, pp. 795–800.

[26] Mastorakis, N.E., Modeling Dynamical Systems via the Takagi-Sugeno Fuzzy Model, *WSEAS Transactions on Systems*, Vol. 3, Nr. 2, 2004, pp. 668–676.

[27] Sz. Nagy, Z. Petres, P. Baranyi: *TP Toolbox homepage, url: http://tptool.sztaki.hu*, visited on 2008. Sept.