The Movement of Modular Walking Robot MERO in the Obstacles' Area

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Abstract: - In the area of level-walking gaits, gaits can be categorized into two types according to the terrain condition : gaits on perfectly smooth terrain and gaits on terrain which contains forbidden areas (areas which are not suitable for foot placement).The main characteristic of the modular walking robots is that they are able to move away on not arranged, horizontal and rough terrain. The performance of walking robots is closely related to the adopted gait. The movement of the walking robots can be divided in two modes: condition of the static stability; condition of the dynamic stability. During walking, the legs move according to the gait and two forelegs are adjusted to avoid forbidden areas. Different methods of leg adjustments and body adjustments are integrated into the strategy. In the work are analyzed the possibilities of determination of the limit conditions for the stable displacement of the walking robots. In the work are analyzed the possibilities of determination of the limit conditions for the stable displacement of the quadrupedal walking robots. Finally, this strategy is verified by using computer graphics simulations.

Key-Words: - Modular Walking Robot, Walk, Gait, Support phase,Transfer phase, Stability margin
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1 Introduction

The selection of the type of gait is a very complicated matter, especially in the real conditions of walking on the unarranged. Therefore, it is necessary that the terrain surface to be selected before the type of gait is chose.

The walking robot alternatively leans upon some of its legs and moves the others in a new position, ensuring to it self a stable support. To achieve and control a walking robot, one must know all its walking capabilities, as the choice of the number of legs and their structures depend a lot on the selected walking type. The selection of the walking gait type depends on a string of factors such as:

- shape and constituency of the ground the robot walks upon;
- the gait's stability;
- the way of guiding and controlling the movement of the shift system elements;
- attainment of the velocity and mobility that the motion requires.

It is quite sophisticated a job to choose the motion type, the more under the real field walking circumstances. The gait of a walking robot is a sequence of movements by its legs, coordinated to a sequel of movements of its body, whose final goal of the robot's moving to different places. In order to visualize the motion of a modular walking robot, models of the walking robot and the terrain must be established. Recently, the gaits for walking on rough terrain have drawn more attention from researchers. For example, some obstacle-crossing gaits of a hexapod and quadruped were studied in [4],[5]. A computer-generated free gait was developed in [12]. A discontinuous follow-the-leader gait was developed and successfully implemented to control a hexapod in walking over uneven terrain [4], [17] A continuous follow-the-leader gait was formulated and studied in [17],[11]. Two terrain-adaptable free gaits were developed to enable a quadruped to enable a quadruped to walk on a rough planar terrain [14].

Among the above-mentioned gaits, the gaits which have the follow-the-leader feature seem to be most suitable for rough terrain in walking. The discontinuous follow-the-leader gait developed in [17],[18] followed a special leg moving sequence and allowed only one leg to be lifted at a time so that good stability could be maintained. The body movements were inserted into two designed leg movements and the overall body motion was discontinuous [1],[19]. This type of walking mode provides good mobility on rough terrain.

This type of walking mode provides good mobility on rough terrain. However, it suffers a slow speed due to the discontinuous body motion. This may not be a problem for walking in severely rough terrain, but it certainly limits the performance of the walking machine in mildly rough terrain. Compared to the discontinuous follow-the-leader gait, a continuous follow-the-leader gait can reach a higher speed and maintain smoother body motion on mildly rough terrain due to its periodic nature[6], [17]. During walking, the walking machine has at least two legs in the air at one time and the stability is reduced.

As a result, the mobility of the continuous follow-the-leader gait is not as good as the discontinuous follow-the-leader gait.

2 Mathematical modeling of gait for modular walking robots. The walk as sequence of states.

A cycle of the movement of the leg of a modular walking robot has two phases: *the support phase* and *the transfer phase*. In the first phase, the leg's support part has a direct contact with the walking surface area. In the transfer phase, the leg of the robot is above the walking surface and is moving so that it realizes the stability state on the whole of the walking robot. The walk of the robot is characterized by the order *raise* and *seat* of the legs and by the trajectory form of the theoretical support point in comparison with the platform. To establish the walking order it is needed to number the legs. The state of the leg (i) at a given time [2], [14], [13], is described by a state's function $q^i(t)$, that has only two values, 0 and 1, as it follows:

$$q^i(t) = \begin{cases} 0 & \text{for the support phase;} \\ 1 & \text{for the transfer phase.} \end{cases} \qquad (1)$$

On the interval $[0, t_1]$, the leg is in the support phase. On the interval $[t_1, t_2]$, the leg is not leaning upon the support surface and it is in the transfer phase. On the interval $[t_2, t_3]$, the leg is on the support surface again etc. At a moment of time, the state of the walking robot with N legs is defined by a N-dimensional vector $\mathbf{q}$, named *the vector of the legs states*. The vector's components q^i, $i = \overline{1, N}$, are formed by the functions of the legs' states, ordered by their numbering:

$$\mathbf{q} = \left[q^1, q^2, ..., q^N\right]^T, \qquad (2)$$

so that the first component of the vector define the state of the leg 1, the second one is the state of the leg 2 etc.

It is assumed that in any finite interval of time there is a finit number of moments that defines the values of the functions $q^i(t)$. The $\mathbf{q}$ states, that appear at every change of the value of the function $q^i(t)$, are numbered chronologically as they are carried on. As a result, the walk of a walking robot is described by a succession of states ($\mathbf{q}_j$), j = 1, 2,.. An example of the succession of the states for a walking robot with 4 legs is:

$$\begin{aligned} &\mathbf{q}_1 = [0,0,0,0]^T, \mathbf{q}_2 = [1,0,0,0]^T, \mathbf{q}_3 = [0,0,0,0]^T, \\ &\mathbf{q}_4 = [0,1,0,0]^T, \mathbf{q}_5 = [0,0,0,0]^T, \mathbf{q}_6 = [0,0,1,0]^T, \\ &\mathbf{q}_7 = [0,0,0,0]^T, \mathbf{q}_8 = [0,0,0,1]^T, \mathbf{q}_9 = \mathbf{q}_1 \end{aligned} \qquad (3)$$

It is assumed that, at the initial moment, all the walking robot's legs are in the support phase. After this, the leg number 1 is raised and moved down, followed by the raise of the leg number 2 and its move down etc. The walk is cyclically if the succession of the states ($\mathbf{q}_i$) is periodical. The total amount of the realized states in a time period is named *the walk cycle*. In the above example, the walk is *cyclic* if after the state q_9, determined above, follows the state q_2, state q_3 etc, in that order.

3 Determination of stability conditions with the movement of the modular walking robot on rough terrain

An important feature of the walking robots is that the ground's configuration never influences or affects their movement too much. Such a feature makes this locomotion type turn into an attractive solution for many applications that requires movement on an unplanned land, namely having an uneven configuration.

As uneven land covers an endless variety it is difficult to match all the types and cases of walking on such grounds.

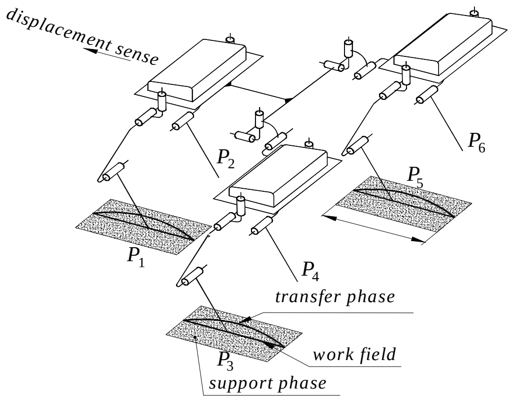

Fig.1 Model of walking modular robot

To study the issue we need a simplification of the geometrical features of the real terrain. For this, we introduce the notion *obstacle, hurdle*- The obstacle is *isolated* if it can be included in the area between a vertical plane and a perpendicular one on the walking direction; besides there are no forbidden area for the walking robot to step forward in the area of its movement..

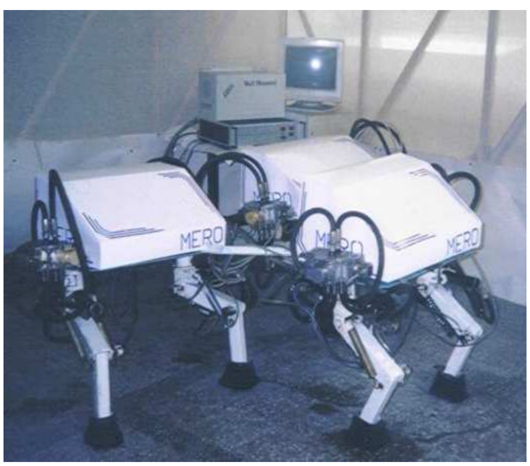

Fig.2. The walking modular robot MERO 2.

The support area separated through the mentioned planes is called the *obstacle's area* (*zone*), where it is forbidden to place the support points of the robot's legs and feet. The width of the obstacle's area depends on the ratio of the obstacle's dimensions, on the geometrical parameters and the technical features of the walking robot. The presence of the obstacle on the support area can lead to the change in the movement direction, in the height of the movement of the body of the walking robot or its orientation in space as well as to the appropriate redefining of the steps' order and sequence and in the regime of the legs' movement. Figure 1 shows the model of a modular hexapod robot MERO2 and figure 2 the walking modular robot MERO 2 was developed at University "Politehnica "of Bucharest .

3.1 Stepping over isolated obstacle while preserving static stability

The presence of the obstacles on the support surface may lead to changing the direction, the height of the body's movement and its orientation in space as well as the appropriate reorganization of the sequel of watching the regime of the legs' movement.

This paragraph will follow, in compliance with [2], the case when to prevent the obstacles, it is enough only to change the sequel of following and maybe the height of the body's center of gravity (mass point) preserving the same the other movement features of the robot. Like before we will assume that the legs' suspension points are symmetrically placed to the vertical plane including the body's center and the speed parallel to it. The paths followed for the right and left legs of the robot we assume to be rectilinear and parallel to the speed vector. We are given the body's mass point to be projected on the center line between the follow paths.

We will name the obstacle 'isolated' if we can include it in the field between two vertical planes, erect on the body's movement direction so that the follow paths beyond this field lack the points forbidden to advance. On the support area the above mentioned planes separate the area called the *obstacle's zone* where it is forbidden to place any follow points. Its ends are erect on the follow paths. The zone's width depends on the ratio of the obstacle's dimensions and the geometrical parameters characteristic to the robot as well as its control system. It occurs the problem of establishing the sequence of follow, which enables the robot to step over the area without disturbing its static stability.

3.2 Movement of walking robots on uneven surfaces

3.2.1. Waking robots step over obstacles with defined configuration. The obstacle's geometry is defined by one or two parameters.

Figure 3 shows the main types of obstacles such as a slope in figure 4a, a ditch in figure 4b, a step in figure 4c, and an insulated wall in figure 4d.

The walking robot moves on a slope. The main difference between walking on a slope and walking on a horizontal plane surface consists in the fact that the projection of the robot's center of gravity changes position against the sides of the support polygon, if the slope exceed a certain limit, the projection no longer lies inside the support polygon.

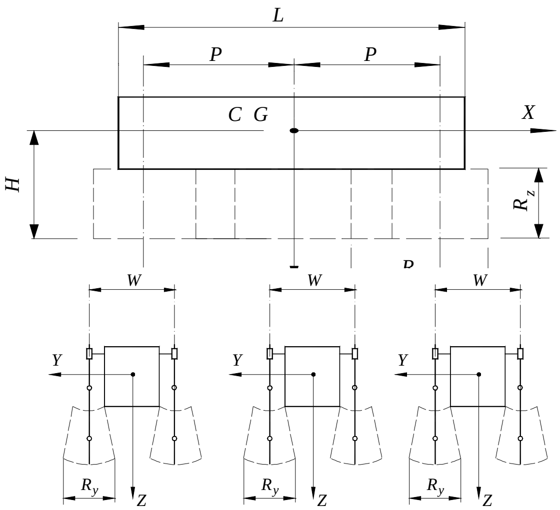

Fig 3 Geometric modular walking robot model

If the periodical walking on a plane, flat surface is symmetric to the longitudinal and lateral axes of the body, the limit of the. anterior (fore) longitudinal stability is equal to the limit of the posterior (back).longitudinal stability. When the robot walks on a slope, when it goes both up and down, the limits of the longitudinal fore and back stability diminish accordingly, due to the change in the position of the projection of the center of gravity. A sensor is used to emphasize the walking robot's stance a sensor that measures the platform's inclination in two planes, a sagittal and a frontal one.

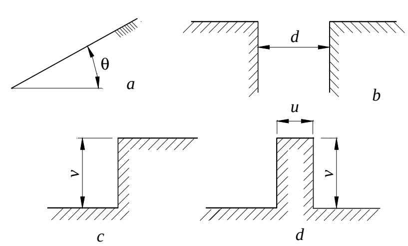

Fig. 4. Different shapes of obstacles

To improve the robot's stability when it moves along a slope, there are two strategies such as:

-its body's height is diminished and its position adjusted;

-the step's length is also curbed.

3.2.2. The walking robot moves on a slope, through adjusting the height it steps at and its body's position

The walking robot moves along a maximally inclined slope. First, it is analyzed the efficiency of diminishing the body's height when the robot walks along a slope. The movement happens when the body's longitudinal axis is parallel to the maximal slope's line. θ is the slope's angle (fig. 5 and H tanθ is the deviation of the projection of the center of gravity on the support surface. You can calculate the limit of longitudinal stability the longitudinal stability by:

$$S_l = S'_l \cos\theta \qquad (4)$$

Instability occurs when $S_l < 0$ at any moment of the full locomotion cycle. Instability occurs when $S_l < 0$ at any moment of the full locomotion cycle.

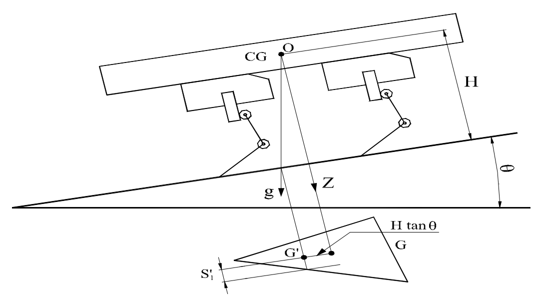

Fig. 5. The slope's inclination angle

Therefore we have to take in consideration only the minimal limit of the longitudinal stability.

No matter if the robot goes up or down the slope, its stability is similarly affected, because of the movement of the projection of the center of gravity, as undulated walk types are symmetrical to the body's side axis, at a similar inclination.S_O is the limit stability of the horizontal walk (slope angle θ =0). You can calculate stability limit at a slope angle θ, through the relation:

$$S = (S_0 - H\tan\theta)\cos\theta \qquad (5)$$

he maximal height the robot can move at, H_m can be calculated putting the condition that stability limit:

$$S = 0 \text{ and: } H_m = S_0 \cdot \tan\theta \qquad (6)$$

For a given height H of the body, the maximal inclination is given by the relation:

$$\theta_{max} = \theta_m = \arctan(S_0 / H). \quad (7)$$

As the body's minimal height is $H_0 - R_{Z0,}$ the maximal inclination is found by the relation:

$$\theta_m = \arctan(S_0 / (H_0 - R_{Z0})). \quad (8)$$

For the precise walk, the maximal stability is obtained lifting the robot's both back legs at an utmost height on the side where the legs go down, Then:

$$S_0 = P_0 + R_{Z0} / 2 \quad (9)$$

Taking into account the use factors $\beta = 3/4$ and $\beta = 11/12$, the limit stability of the undulated walking is determined by the equation:

$$S = ((n/2)-1)(P/ R)\beta+\beta - 3/4 \quad (10),$$

And the growth in the step's length leads to the rise in its stability. This is why the walking robot's stability limit is

$$S_0 = P/ 2 + (1 - 3/ (4\beta)) R \quad (11)$$

For walking on a slope it is advisable a use factor whose value is closer to the upper limit $\beta = 11/12$.

Replacing the values of the limit S_0 in equation (8) we find out the inclination's maximal value.

Adjusting the platform's position, the projection of the center of gravity does not move if the platform keeps horizontal and the same level. Thus, the stability limit is the same as that of waking on a plane and horizontal ground.

The body can be kept horizontally only if the slope's angle is smaller than the limit angle $\phi_{m.}$ Figure 6 shows that the limit angle is

$$\phi_m = \arctan(2R_{Z0} / \mathrm{L}) \quad (12)$$

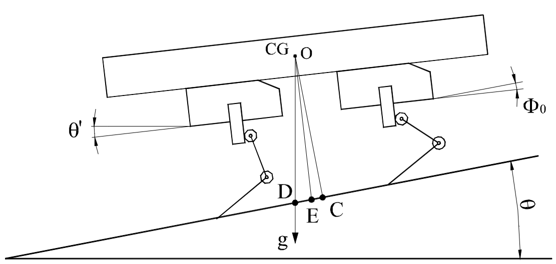

Fig. 6. The 6-legged modular robot walks on the slope and the body's inclination is diminished

So that all the six legs of the robot should have the same stroke, an horizontally placed body can cross an utmost inclination of the line running through the points A and B/ and the angle's size is:

$$\phi_0 = \arctan[R_{Z0} /(2P_0 + R_{Z0})] \quad (13)$$

For this robot, the authors built and tested, the angle ϕ_O that equals 15^0. The stability limit S_O' for a walk on a surface inclined at such an angle is

$$S_O' = S_O / \cos \phi_O. \quad (14)$$

The distance between the center of gravity and the hypotenuse AB is:

$$OC = (H_0 - R_{Z0} / 2) \cos\phi_0. \quad (15)$$

If the slope's angle θ is bigger than the angle $\phi_{0,}$ the robot's body cannot be perfectly horizontal (fig. 7). The body's angle is $\theta' = \theta - \phi_O$.

Point E where the axis OZ and the slope's surface intersect is called the geometrical center of the support area. As the supports are symmetrical to the body's lateral axis that runs through point E, a deviation of the center of gravity must be measured contingent to the position of point E.

Point D is the center of gravity's vertical projection (elevation) on the ground's sloped surface. The projection's deviation is DE: $DE = DC - EC$, or:

$$DE = OC(\tan \theta - \tan \phi_O) \quad (16)$$

In this case the limit stability on the slope is:

$$S' = S'_0 - DE \quad (17)$$

and the walking's limit stability becomes:

$$S = S' \cos \theta. \quad (18)$$

Replacing the equations (14), (16) and (17) in the equation (18) we get:

$$S=[S_0 /\cos \phi_0 - OC (\tan \theta - \tan \phi_0)]\cos \theta. \quad (19)$$

The slope reaches its maximal angle when S becomes zero. This is calculated through the equation:

$$\theta_m=\arctan[(S_0 / \cos \phi_0 + OC \tan \phi_0) / OC], \quad (20)$$

and for a walking robot it becomes

$$\theta_m = \arctan[(1.2\, S_0 + 12.46) / 51.21]. \quad (21)$$

Comparing the results of the previously used method where it has been diminished the height the body stands at, we notice that the previous method is more efficient. Sometimes we can combine the two methods specific to walking on a slope. First it is adjusted the body's position so that to reach the desired θ' angle.

Thus, the body's height is diminished till the upper front edge of the workspace touches the ground in this case the body's new height OC is:

$$OC = OE \cos (\theta - \theta') = \{H_0 [R_{Z0} - (P_0 + R_{X0}/ 2)\tan (\theta - \theta')]\} \cos (\theta - \theta') \quad (22)$$

The walk's geometrical center is the point E and the OE line and the line OC intersect at the angle $\theta - \theta'$. Replacing the difference $\phi_O = \theta - \theta'$ in the equation (23), the stability limit becomes:

$$S = (S_0 / \cos (\theta - \theta') - OC [\tan \theta - \tan (\theta - \theta')]) \cos\theta \quad (23)$$

3.2.3. Movement along a slope whose angle is zero. If a walking robot crosses a sloped area whose inclination angle is θ and if it moves along a line whose slope's angle is zero, and the walking robot's body keeps parallel to the slope's surface, the projection of the center of gravity runs laterally to the descending side, at a distance $H \tan\theta$ if the legs keep a normal position against the ground. summarizes such a situation.

For a precise walk, the maximal slope to be adjusted through such a strategy comes up when the deviation of the center of gravity equals W/2 and thus:

$$H \tan \theta = 0.5W. \quad (24)$$

If the body's height is maximally reduced, the equation (24) becomes

$$\theta_m = \arctan(0.5W/H). \quad (25)$$

If the legs stepping on the descending slope are fully extended, the maximal slope is

$$\theta_m = \arctan[(W + R_{YT})/2], \quad (26)$$

where R_{YT} is the stroke/haul of a lateral step for a maximal workspace value.

For the undulatory walk the stability limit for a movement on the slope is

$$S' = S'_0 - H \tan \theta/\tan \gamma, \text{ for } H \tan \theta \le W/2, \quad (27)$$

where γ is the angle for the minimal stability limit along the body's longitudinal axis.

The moment when the limit of the longitudinal stability for an undulatory walk has a minimal value is that where one of the back legs is risen [7], [4]. Here they are the positions for feet 3 and 6, for a walking robot, at the moment when foot 5 is risen

$$P(3) = R/2 - (2\beta - 1)\, R/\beta \quad (28)$$

$$P(6) = -P + R/2 - (\beta - 1/2)\, R/\beta \quad (29)$$

The size of the angle γ is:

$$\gamma = \arctan\{W/[P(3) - P(6)]\} = \arctan\{W/[P + (1/(2\beta) - 1)R]\} \quad (30)$$

where W is the platform's width measured between the positions where the legs hang up.

As the body's longitudinal axis is parallel to the horizontal plane $S' = S$ then *:*

$$S = S_0 - H \tan \theta / \tan \gamma \quad \text{for } H \tan \theta \le W/2. \quad (31)$$

For a slope having a given angle θ, the body's maximal height H_m is:

$$H_m = S_0 \tan\gamma / \tan\theta. \quad (32)$$

As the body's minimal height is $H_0 - R_{Z0}$, the maximal inclination in this case is the smallest of those resulting from the equation (27) and of the following one:

$$\theta_m = \arctan[S_0 \tan\gamma / (H_0 - R_{Z0})]. \quad (33)$$

For the second situation, the body's position is adjusted namely it is brought to level, through stretching the legs on one side and bending those on the opposite side

The projection of the center of gravity is kept on the central line so that it should not alter the body's stability limit. The slope's maximal inclination when the body can be completely brought to level is the following:

$$\alpha_O = \arctan(R_{Z0} / W). \quad (34)$$

If the slope's angle is bigger than α_0, the body cannot be fully abducted to the level

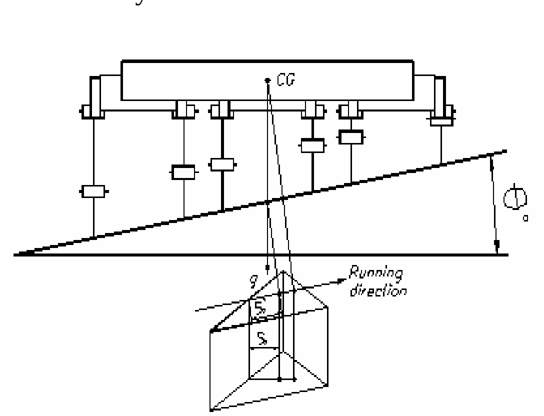

Fig. 7. Robot's movement on the slope to a direction whose inclination is null, through reducing the body's height

The distance between the extremities' positions, measured parallel to the slope's surface is:

$$W' = W / \cos \alpha_O. \quad (35)$$

The support's geometric center is in point *E*. The deviation of the projection of the center of gravity is:

$$DE = DC - E = OC\,(\tan\theta - \tan \alpha_O), \quad (36)$$

and:

$$OC = (H_0 - R_{Z0}/2)\cos \alpha_O. \quad (37)$$

For the precise walk the slope's angle is maximal when the deviation equals *W*/2 and therefore:

$$\theta_m = \arctan\,[0.5W' / OC + \tan \alpha_0]. \quad (38)$$

if the legs on the ascending side rise, and those on the descending side go down by the same distance R_{YT}, the projection of the center of gravity goes upwards by the distance $d = R_{YT} \cos\alpha_0$.

Then:

$$DE = OC(\tan \theta - \tan \alpha_O) - 0.5R_{YT} \cos\alpha_O; \quad (39)$$

$$OC = (H_0 - R_{Z0} / 2) \cos\alpha_O - 0.5R_{YT} \sin\alpha_O. \quad (40)$$

From equation (39) and using W', you can calculate maximal inclination angle θ_m:

$$\theta_m = \arctan\{[W'/2 + (0.5R_{YT} \cos \alpha_0] / OC + \tan \alpha_0)\}. \quad (41)$$

Undulatory walk's stability limit on a slope is:

$$S' = S'_0 - DE / \tan \gamma, \quad (42)$$

where angle γ defines the minimal limit S_l related to the body's longitudinal axis.

As the body's longitudinal axis runs parallel to the horizontal plane, $S = S'$ and $S_0 = S'_0$. The walk's stability limit in this case is:

$S = S_0 - DE / \tan \gamma$, for $DE \leq W' / 2$. (43)

Replacing the expression DE in the equation (34), in equation (40) we get:

$\theta_m = \arctan[(S_0 \tan \gamma + OC \tan \alpha_0) / OC]$. (44)

The slope's maximal angle is the minimal values that the equations (38) and (44) give.

Exemple. The Modular Mobil Walking Robot it is necessary to know all the walking possibilities, because the selection of the legs number and its structure depends on the selected type of the gait.[9], [15]. The selection of the type of gait is a very complicated matter, especially in the real conditions of walking on the rough terrain. The longitudinal stability margin, S_l is the shorter of the distances from the vertical projection of the center of gravity to the front and rear boundaries of the support pattern, as measured along the direction of motion (see figure 8). If certain obstacles occur on the walking surface, a special crossing gait must be used, after learning the dimensions of such obstacles. Depending on the type of the obstacle, its surpassing can be made by the precise arrangement of the legs in the permitted areas around the obstacle. In such a case, the a periodic gait, named "follow the leader" is highly recommended In case of walking on an unarranged terrain, due to the great diversity of the obstacle dimensions and forms, precise walk is not recommended.

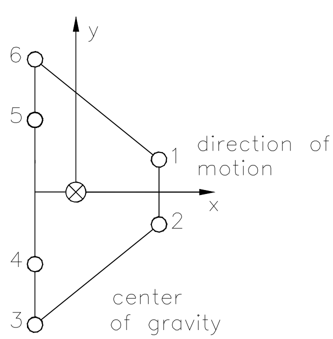

Fig. 8. The longitudinal stability margin

Figure 9 shows the model of a modular hexapod walking robot The body coordinate system x-y is attached to the body center and the x-axis is aligned with the body longitudinal axis. The center of gravity is coincident with the body center. Each leg is assigned a number as is shown in figure 8. Each leg is represented by a line segment which connects the foot point and hip point is considered unlimited (i.e., the workspace of each leg is unlimited).

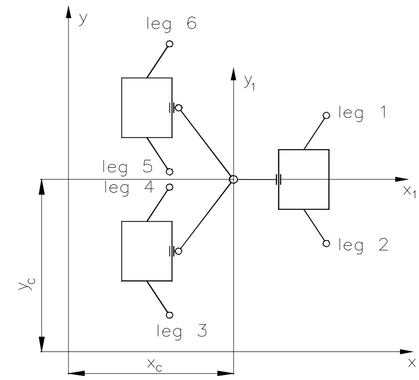

Fig.9 The model a modular walking robot

The terrain used in the study is two-dimensional, unarranged terrain. The terrain is divided into many cells and each cell is about the size of a footprint. The cells are classified into two types: a permitted cell and a forbidden cell. A forbidden cell is not suitable for a foot to tramp on it due to weakness of the soil structure, a ditch, or other reasons. A collection of many forbidden cells is a forbidden.

4.The movement of the walking robots

One of conditions imposed on the motion of walking machines is the stability. The movement of the legged robots can be divided in two modes:

- under condition of the static stability,
- under condition of the dynamical stability.

The main difference between robots which walked under the static stability and under the dynamical stability conditions originates from the fact that during statically walking, the vertical projection of the gravity center of the robot must lies into the supporting polygon, where as during the dynamical walking, this condition can be not satisfied. The problem of quasi-statical stability analysis in condition of arbitrary step when the accelerations of points of component elements are much smaller than gravity acceleration is identical with the problem of stability analysis when the robot does not walk. The inertial forces are neglected and the walking can be controlled in a kinematics way.

The investigation of statical stability is based on the notion namely *hardening configuration*.

Hardening configuration is a term used to indicate the rigidly structure of the robot, obtained through the shutting off the driving motors. The position of the walking robot is stable if the hardening configuration is in posture of stable equilibrium under the action of gravity forces.

The hardening configuration is statically stable if this accomplished the following conditions:

1. The vertical projection of the gravity center must be inside the *supporting polygon*. The supporting polygon is the minimum convex area which is obtained by connecting all support points. A body at rest in a gravitational field, subject to ideal connections, has the differential of the gravity center elevation equal with zero.

2. The tangential components of the reaction forces in the support points must be less than friction forces between feet and support surface.

4.1. The movement of the modular walking robots

A modular walking robot [10] (figure 10), which moved under dynamical stability condition can attain higher velocities and can take steps with a greater length and a greater height.

Fig.10 Quadrupedal modular walking robot

But, the central body of the robot cannot be maintained in the horizontal position because it tilted to the foot which is lifted off the ground area.The size of the maximum inclination angle depends to the forward speed of the robot. The stability problem is very important for the moving of the quadrupedal walking robots. When a foot is lifted off the ground, the other legs supporting the robot's body are in contact with the ground. If the vertical projection G' of the gravity center G of the legged robot is outside of the supporting polygon (triangle P^1 P^2 P^3) (Fig. 11), and the cruising speed is greater than a certain limit, the movement of the robot is realized under condition of the dynamical stability. When the leg (4) is lifted off the ground, the walking robot rotates around the straight line which passes through the support points P^1 and P^3.

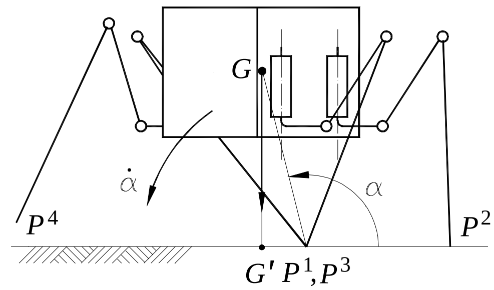

Fig. 11 The overturning movement of the walking robot

The magnitude of the forward speed did not influence the rotational motion of the robot around the straight line P^1P^3. This rotational motion can be investigated with the Lagrange's equation [13]:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}}\right)-\frac{\partial T}{\partial q}+\frac{\partial P}{\partial q}=Q \qquad (45)$$

The kinetic and potential energies of the hardening configuration of the robot have the forms

$$T=(m\,AG^2+I)\frac{\dot{\alpha}^2}{2}\,,\; P=m\,g\;AG(1-\sin\alpha)\,, \quad (46)$$

and the generalized force is

$$Q=-m\,g\;AG\cos\alpha\,. \qquad (47)$$

where m denotes the mass of the entire robot, I is the moment of inertia of the robot structure and AG is the distance between the gravity center G and the rotational axis P^1P^3 (Fig.11). Substituting the (46) and (47) into (45), it results:

$$\ddot{\alpha}=-\frac{2m\,g\;AG\cos\alpha}{I+m\,AG^2} \qquad (48)$$

Because the moment of inertia of a body is proportional with its mass, the angular acceleration $\ddot{\alpha}$ does not dependent on the mass m. The differential equation (48) may be integrated with the initial conditions. For example, if at $t = t_0 = 0$, $\alpha = \alpha_0 = 1.59185$, $I/m = 0.01125$ kg and $AG = 0.5$ m, results. [12],[16].

$t = 0.025$, $\alpha = 1.6246$;
$t = 0.050$, $\alpha = 1.7082$;
$t = 0.075$, $\alpha = 1.9192$.

The quadrupedal walking robot in question, which moved so that the step size is 0.2 m, with forward average speed equal to 3.63 m/s (13 km/h approximate) has the maximum inclination of the central body equal to 0.174533. This forward speed is very great for the usual applications of the walking robots. As a result, the movement of the legged robots is made under condition of the static stability. The conventional quadrupedal walking robots have rather sluggish gaits for walking, but are unable to move smoothly and quickly like animal beings.

4.2. The static stability of quadrupedal walking robot

The quadrupedal walking robot which walks, i.e. it leans upon three feet, is a statically determined system. When it leans upon four feet, it turns in a statically indeterminate system.[7],[8] For establishing the stable positions of a walking robot it is necessary to determine the forces distribution in the shifting mechanisms. In the case of a uniform and rectilinear movement of the walking robot on a plane and horizontal surface, the reaction forces do not have the tangential components, because the applied forces are the gravitational forces only.

Determination of the real forces distribution in the shifting mechanisms of a walking locomotion system which moves in rugged land at low speed is necessary for the analysis of stability. The position of a walking system depends on the following factors:

- the configuration of walking mechanisms;
- the masses of component elements and their position of gravity centers;
- the values of friction coefficients between terrain and feet;
- the stiffness of terrain;
- the shape of terrain surface.

The active surface of the foot is relatively small and it is considered that the reaction force is applied in the gravity center of this surface. The reaction force represents the resultant of the elementary forces, uniformly distributed on the foot sole surface. The gravity center of foot active surface is called *theoretical contact point.*To calculate the components of reaction forces, namely:

- normal component $\overline{N}$, perpendicular on the surface of terrain in the theoretical contact point;
- tangential component $\overline{T}$, or coulombian frictional force, situated in the tangent plane at terrain surface in the theoretical contact point,

it is necessary to determine the stable positions of walking robots.

The magnitude of $\overline{T}$ vector cannot be greater than the product of the magnitude of the normal component $\overline{N}$ by the frictional coefficient μ between foot sole and terrain. If this magnitude is greater than the friction force, then the foot slips along the support surface down to the stable position, where the magnitudes of this component decrease under the above-mentioned limit.

Therefore, the problem of determining the stable position of a walking robot upon some terrain has not a unique solution. For every foot is available a field which covers all contact points in which the condition $\mu T \leq N$ is true. The equal sign corresponds to the field's boundary.

4.3. The force distribution in the leg mechanisms

The system builds by the terrain on which to do the displacement and the walking robot, which has three legs in the support phase, is determinate static. The problem of determination of reaction force components is solved in simplifying assumption, namely the stiffness of the walking robot mechanical structure and terrain.[6],[10] The complex behavior of the earth may not be described than by an idealization of its properties. The surface of terrain which the robot walks on, is defined in respect to a fixed coordinates system $O_1\xi\eta\zeta$ annexed to the terrain, by the parametrical equations: $\xi = \xi(u, v)$; $\eta = \eta(u, v)$; $\zeta = \zeta(u, v)$, implicit equations: $F(\xi,\eta,\zeta) = 0$, or explicit equation: $\zeta = f(\xi,\eta)$.

These real, continuous and uniform functions with continuous first partial and ordinary derivative, established a biunique correspondence between the points of support surface and the ordered pairs (u, v), where $\{u, v\} \in \mathbf{R}$. Not all partial first order derivatives are null, and not all Jacobians

$$\frac{D(\xi,\eta)}{D(u,v)};\ \frac{D(\eta,\zeta)}{D(u,v)};\ \frac{D(\zeta,\xi)}{D(u,v)}, \tag{49}$$

are simultaneous null. On the entire surface of the terrain, the equation expressions may be unique or may be multiple, having the limited domains of validity.

The normal component $\overline{N}$ of the reaction force at the P^i contact point of the *i* leg with the terrain is positioned by the direction cosine (fig. 12):

$$\cos\alpha^i = \frac{A_i}{\sqrt{A_i^2 + B_i^2 + C_i^2}}; \tag{50}$$

$$\cos\beta^i = \frac{B_i}{\sqrt{A_i^2 + B_i^2 + C_i^2}}; \tag{51}$$

$$\cos\gamma^i = \frac{C_i}{\sqrt{A_i^2 + B_i^2 + C_i^2}}, \tag{52}$$

in respect to the fixed coordinate axes system, where:

$$A_i=\begin{vmatrix}\dfrac{\partial\eta^i}{\partial u_i} & \dfrac{\partial\zeta^i}{\partial u_i}\\ \dfrac{\partial\eta^i}{\partial v_i} & \dfrac{\partial\zeta^i}{\partial v_i}\end{vmatrix};\ B_i=\begin{vmatrix}\dfrac{\partial\zeta^i}{\partial u_i} & \dfrac{\partial\xi^i}{\partial u_i}\\ \dfrac{\partial\zeta^i}{\partial v_i} & \dfrac{\partial\xi^i}{\partial v_i}\end{vmatrix};\ C_i=\begin{vmatrix}\dfrac{\partial\xi^i}{\partial u_i} & \dfrac{\partial\eta^i}{\partial u_i}\\ \dfrac{\partial\xi^i}{\partial v_i} & \dfrac{\partial\eta^i}{\partial v_i}\end{vmatrix}. \tag{53}$$

The tangential component of reaction force, i.e. friction force, is comprised in the tangent plane at the support surface.

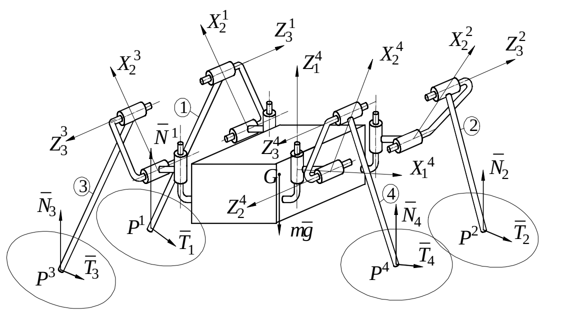

Fig.12. The Hartenberg – Denavit coordinate systems and the reaction force components

The equation of the tangent plane in the P^i (ξ_{Pi}, η_{Pi}, ζ_{Pi}) point is

$$\begin{vmatrix}\xi^i-\xi_{Pi} & \eta^i-\eta_{Pi} & \zeta^i-\zeta_{Pi}\\ \dfrac{\partial\xi^i}{\partial u_i} & \dfrac{\partial\eta^i}{\partial u_i} & \dfrac{\partial\zeta^i}{\partial u_i}\\ \dfrac{\partial\xi^i}{\partial v_i} & \dfrac{\partial\eta^i}{\partial v_i} & \dfrac{\partial\zeta^i}{\partial v_i}\end{vmatrix}=0, \tag{54}$$

or:

$$\xi^i A_i + \eta^i B_i + \zeta^i C_i - \xi_{Pi} A_i - \eta_{Pi} B_i - \zeta_{Pi} C_i = 0. \tag{48}$$

The straight-line support of the friction force is included in the tangent plane:

$$\frac{\xi^i-\xi_{Pi}}{l_i}=\frac{\eta^i-\eta_{Pi}}{m_i}=\frac{\zeta^i-\zeta_{Pi}}{n_i}, \tag{55}$$

therefore:

$$A_i l_i + B_i m_i + C_i n_i = 0. \tag{56}$$

If the surface over which the robot walked is plane, it is possible that the robot may slip to the direction of the maximum slope. Generally, the sliding result is a rotational motion superposed on a translational one. The instantaneous axis has an unknown position. Let:

$$\frac{\xi-U}{\cos\alpha_r}=\frac{\eta-V}{\cos\beta_r}=\frac{\zeta}{\cos\gamma_r}, \tag{57}$$

the equation of instantaneous axis under canonical form, in respect to the fixed coordinate axes system. The components of speed of the P^i point, on the fixed coordinate axes system with OZ axis identical with the instantaneous axis, are:

$$\bar V_X=-\omega Y\bar i;\ \bar V_Y=\omega X\bar j;\ \bar V_Z=\bar V_0, \tag{58}$$

The projections of the P^i point speed on the axes of fixed system $O_1\xi\eta\zeta$ are:

$$\begin{Vmatrix}V_\xi\\ V_\eta\\ V_\varsigma\end{Vmatrix}=\mathbf{R}\begin{Vmatrix}V_X\\ V_Y\\ V_Z\end{Vmatrix} \tag{59}$$

where $\mathbf{R}$ is the matrix of rotation in space.

The carrier straight line of P^i point speed, i.e. of the tangential component of reaction force, has the equations

$$\frac{\xi-\xi_{Pi}}{V_\xi}=\frac{\eta-\eta_{Pi}}{V_\eta}=\frac{\zeta}{V_\zeta}, \tag{60}$$

and is contained in the tangent plane to the terrain surface in the point P^i:

$$V_\xi\, l + V_\eta\, m + V_\zeta\, n = 0 \tag{61}$$

To determine the stable position of the walking robot which leans upon n legs, on some shape terrain, it is necessary to solve a nonlinear system, which is formed by:

- the transformation matrix equation

$$\begin{Vmatrix}1\\ X_{0Pi}\\ Y_{0Pi}\\ Z_{0Pi}\end{Vmatrix}=\mathbf{A}\mathbf{A}_0\mathbf{A}_1^i\mathbf{A}_2^i\mathbf{A}_3^i\begin{Vmatrix}1\\ X_{4Pi}\\ Y_{4Pi}\\ Z_{4Pi}\end{Vmatrix}, \tag{62}$$

where: $\mathbf{A}$ is the transformation matrix of coordinate of a point from the system $O_0X_0Y_0Z_0$ of the robot platform to the system $O_1\xi\eta\zeta$;
$\mathbf{A}_i$ is the Hartenberg – Denavit [3] transformation matrix of coordinates of a point from the system $O_{i+1}X_{i+1}Y_{i+1}Z_{i+1}$ of the element $\boldsymbol{i}$ to the system $O_iX_iY_iZ_i$ of the element ($\boldsymbol{i}$**-1**) [3],

- the balance equations

$$\sum_{i=1}^{n}\bar R_i+\bar F=0;\ \sum_{i=1}^{n}\bar M_{(R_i)}+\bar M=0, \tag{63}$$

which expressed the equilibrium of the forces and moments system which acted on the elements of walking robot.

The $\bar F$ and $\bar M$ are the wrench components of the forces and moments which represent the robot load, including the own weight. The unknowns of this system are:

- the coordinates X_T, Y_T, Z_T and direction cosines $\cos\alpha_T$, $\cos\beta_T$, $\cos\gamma_T$ which define the platform position in respect to the terrain:
- the normal $\bar N_i$ and the tangential $\bar T_i$, $i=\overline{1,n}$, components of the reaction forces;
- the direction numbers l_i, m_i, n_i, $i=\overline{1,n}$, of the tangential components;

- the position parameters U, V, $\cos\alpha$, $\cos\beta$, $\cos\gamma$ of the instantaneous axis;
- the magnitude of V_0/ω ratio, where V_0 is the translational instantaneous speed of the hardening structure.

The system is compatible for n = 3 support points. If the number of feet which are simultaneous in the support phase is bigger than three, the system is undetermined static and is necessary to take into consideration the deformations of the mechanical structure of the walking robot and terrain. In case of a quadrupedal walking robot, the hardening configuration is a six-fold hyperstatical structure (Fig 13). To determinate the force distribution, one must use a specific method for indeterminate static systems. The canonical equations in stress method [8], [12] are:

$$\delta_{11} x_1 + \delta_{12} x_2 + \ \ldots + \delta_{16} x_6 = -\delta_{10}; \qquad (64)$$

$$\delta_{21} x_1 + \delta_{22} x_2 + \ \ldots + \delta_{26} x_6 = -\delta_{20}; \qquad (65)$$

$$\cdots\cdots$$

$$\delta_{61} x_1 + \delta_{62} x_2 + \ \ldots + \delta_{66} x_6 = -\delta_{60}; \qquad (66)$$

where:

- δ_{ij} is the displacement along the X_i direction of stress owing to unit load which acts on the direction and in application point of the X_j;

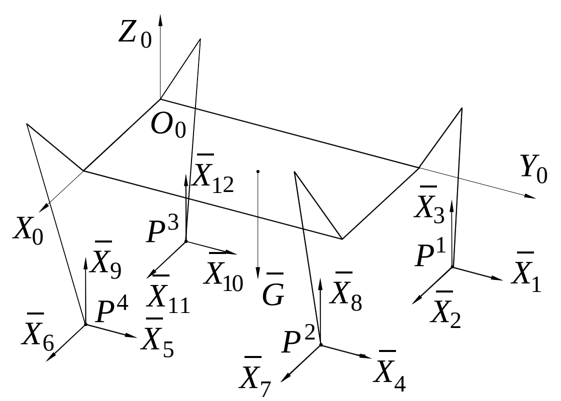

Fig. 13 The six fold hyperstatical structure

- δ_{i0} is the displacement along the X_i direction of stress owing to the external load when $X_j = 0$, $i = \overline{1,6}$:

$$\delta_{i0} = \sum_{p=1}^{4}\int \frac{M_{y0} m_{yi}}{EI_{y1}} \mathrm{d}x + \sum_{q=1}^{4}\int \frac{M_{y0} m_{yi}}{EI_{y2}} \mathrm{d}x, i = \overline{1,6}; \qquad (67)$$

- GI_{xi}, $i = \overline{1,3}$, are the torsion stiffness of the legs elements lowers, middles and uppers respectively;
- EI_{yi} and EI_{zi}, $i = \overline{1,3}$, are the bend stiffness of the legs elements lowers, middles and uppers respectively;

$$\delta_{ij} = \sum_{p=1}^{4}\int \frac{M_{xi} m_{xi}}{GI_{x1}} \mathrm{d}x + \sum_{q=1}^{4}\int \frac{M_{xi} m_{xi}}{GI_{x2}} \mathrm{d}x + \sum_{q=1}^{4}\int \frac{M_{xi} m_{xi}}{GI_{x3}} \mathrm{d}x + \sum_{p=1}^{4}\int \frac{M_{yi} m_{yi}}{GI_{y1}} \mathrm{d}x + \sum_{q=1}^{4}\int \frac{M_{yi} m_{yi}}{GI_{y2}} \mathrm{d}x + \sum_{q=1}^{4}\int \frac{M_{yi} m_{yi}}{GI_{y3}} \mathrm{d}x + \sum_{p=1}^{4}\int \frac{M_{zi} m_{zi}}{GI_{z1}} \mathrm{d}x + \sum_{q=1}^{4}\int \frac{M_{zi} m_{zi}}{GI_{z2}} \mathrm{d}x + \sum_{q=1}^{4}\int \frac{M_{zi} m_{zi}}{GI_{z3}} \mathrm{d}x, \quad i = \overline{1,6},\ j = \overline{1,6}; \qquad (68)$$

- M are the bending moments in basic system which is loaded with basic charge;
- m are the bending moments in the basic system loaded with the unit charge.

The definite integrals

$$I = \int_a^b Mm\mathrm{d}x \qquad (69)$$

are calculated by the Simpson method:

$$I = \frac{b-a}{6}[(M_a + M_b)(m_a + m_b) + M_a m_a + M_b m_b] \qquad (70)$$

To calculate the m_{xi}, m_{yi}, m_{zi}, M_{xi}, M_{yi}, M_{zi}, $i = \overline{1,6}$, seven systems are used (Fig. 12), namely:

- the system S_0, where the single load is G, and $X_i = 0$, $i = \overline{1,6}$;
- the systems S_i, where the single load is $X_i = 1$, $i = \overline{1,6}$.

The remaining unknowns, namely x_i, $i = \overline{7,\ 12}$, are calculated from the equations (53). The normal and tangential components of the reaction forces are calculated as function of the positions of tangent planes on the terrain surface at the support points. The following hypothesis are considered as true:

- the stiffnesses of legs are much less than the robot's platform stiffness;
- the four legs are identically.
- the cross sections of the leg's elements are constant;

5. Experimental measurement of the reaction forces in the contact points where the robot's feet reach the ground

We carried out several experiments simulating several walking situations, in order to measure the values of the forces at the point where the robot touches the ground. both in the sagittal and the front plane. Subsequently we found the values of the reaction forces in pre-set circumstances, when the four-legged robot is walking across an even horizontal ground.

Fig.14 General view of the MERO1 modular walking robot

The force cells placed on the *MERO1* (Fig.14) walking robot's feet and the increment conduct sensor enable the robot to control its direction by adjusting its slant, during the tests the robot carries only its own weight namely 84.8 kg. The values of the reaction forces are graphically shown in (Fig. 15.a).

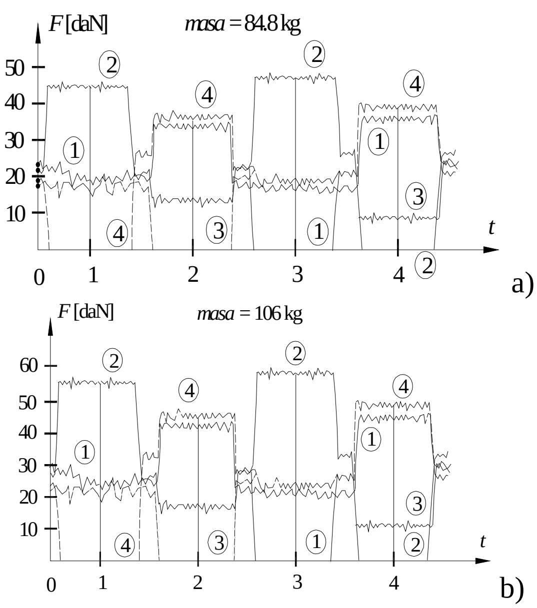

Fig. 15 Reaction forces between the ground and the robot's legs: a) m=84.4kg; b) m=106 kg

The tests were resumed with an additional load weighing 21.2 kg and thus the full load reached 106 kg, (Fig.15.b).

Using the walking robot as a transportation means we can change a few parameters defining its dynamic properties, at an enough large extent For instance, an additional load placed on the platform, changes the weight, the barycenter's position and the inertia moments of the robot's body.

We can apply on the walking robot several forces such as, the resultant of the wind's action, whose influence can hardly be anticipated. The contact cells also ensure the protection of the force transducers. The force transducers we have made use of, turn the variation of a mechanical value (such as linear or angular movement achieved by distorting an elastic element) into the variation of an electric value such as voltage current by means of the electroresisitive transducers. Each module of the MERO1 walking robot is equipped with two identical sensors placed at the ends of the robot's feet (Fig.14). The robot's guidance and control system collect and process the data these sensors supply.

6. Conclusions

The MERO2, MERO1, modular walking robots (figure2 and figure 14) was developed at University ”Politehnica “of Bucharest. Such modul robot has two/four/six legs with three degrees of freedom each. The body of modular walking robot carries a gyroscopic attitude sensor to measure the pitch and roll angles of the body.

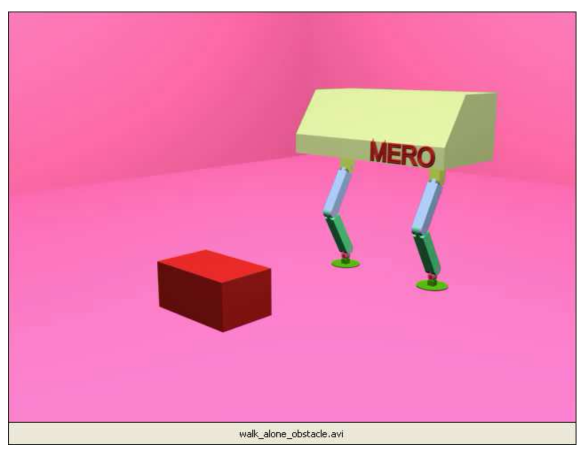

Fig.16.a. Computer graphics simulation for the biped gait

The legs are powered by hydraulical drives and are equipped with joint angle potentiometer transducers. Each leg has three degree of freedom and a tactile sensor to measure the contact which consists of lower and upper levels.

The computer graphics simulation of the movement of modular walking robot MERO in the obstacles'area is presented in the figures 16.a - 16.f

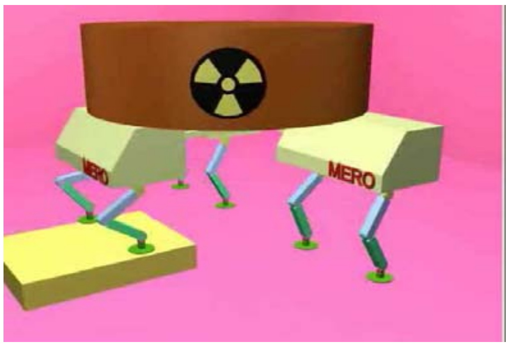

Fig.16.b. Computer graphics simulation for . obstacle's area gait

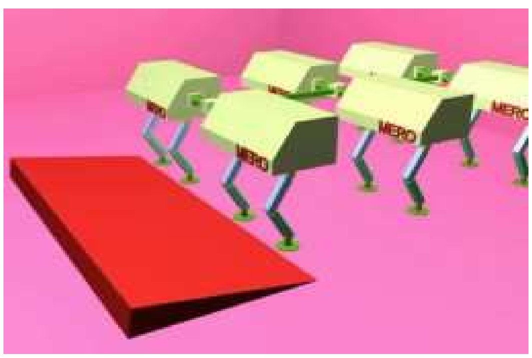

Fig.16.c. Computer graphics simulation for the slope gait

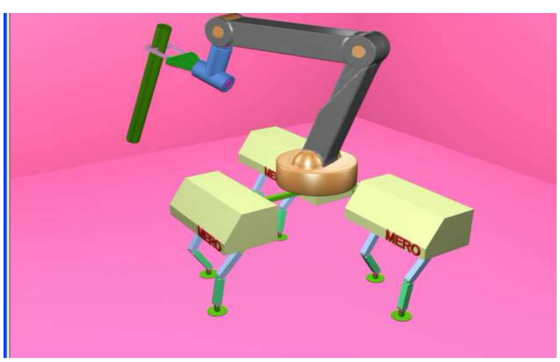

Fig.16.d. Computer graphics simulation for the tripod gait for the movement of the technological.equipments

The MERO type tansducers used in walking robots offer both force control and robot protection. Each of the feet is equipped with stain gauged force sensing device optimized by finite element analysis. Each of the rotational pairs are closed-loop controlled by software servocontrolled by an externel computer. The movement of the quatrupedal walking robot may by easy realized under conditions of quasi-static stability. This conditions are applied only if the forward speed of walking robot is less than a certain limit.

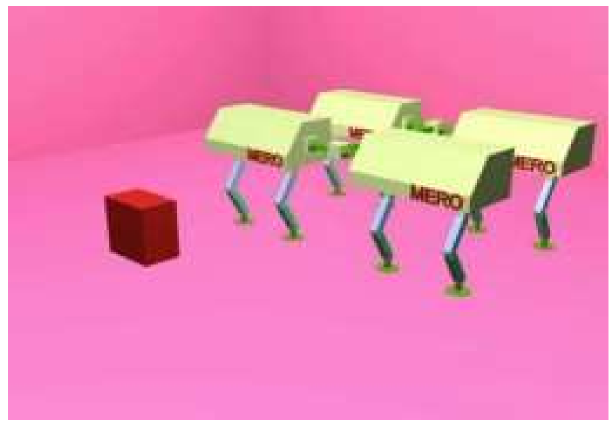

Fig.16.e. Computer graphics simulation for the obstacle gait

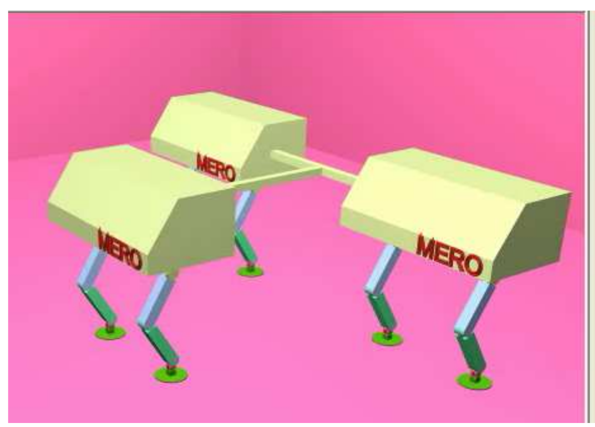

Fig.16.f. Computer graphics simulation for the tripod gait

The quasi-static movement is slight to achieved. A study of the dynamic motion of a legged walking robot indicates that while the robot body moves at a substantially constant speed, the legs are repeatedly accelerated and decelerated, to a great degree in each cycle of movement. The physical model contains more kinematic chains, and when formulating the equations of motion, the principle of mechanics having the form of

Lagrange's equations should be employed. The dynamic motion problem of a walking robot is considerably harder, both theoretical and practical point of view.

References

[1] Bagheri, M. E. Felezi, P. N. Mousavi , Adaptive Control and Simulation of a Seven-Link Biped Robot for the Combined Trajectory Motion and Stability Investigations, *WSEAS Transation on Systems, Issue 5, Volume 5,* May 2006, pg.1214-1223, ISSN 1109-2777

[2] Bessonov, A.P., Umnov, N.V., (1973) The Analysis of Gaits in six-legged Vehicles according to their Static Stability, *Proc. Symp.Theory and Practice of Robots and Manipulators*, Udine, Italy,

[3] Denavit, J., Hartenberg, R.S., (1955), A kinematics notation for lower-pair mechanisms based on matrices, In: *Journal of Applied Mechanics*, no.22,

[4] Figliolini G., Rea P.; (2006) Mchanics and Simulation of six –Legged Walking Robots, 9thInternational, *Conference on Climbing and Walking Robots,and Associated Technologies, Royal Military Academy*, Brussels Belgium, Sept 11-14, 2006

[5] Hirose, S. , (1991) Three Basic Types of Locomotion in Mobile Robots, *IEEE 1991*, pp. 12-17

[6] Tommaso Raimondi, Francesco Maria Raimondi, Salvatore Pennacchio, Giuseppe Bosco, A New Controller for Mobile Robots Based on Fuzzy Logic and Genetic Algorithms, *WSEAS Transaction on Circuits and Systems, Issue 9, Volume 3,* November 2004, pg.1871-1875, ISSN 1109-2734

[7] Kumar, V., Waldron, K.J., (1988) Force Distribution in Walking Vehicle, In: *Proc. 20thASME Mechanism Conference,* Orlando, Florida, 1988, Vol. DE-3, pp. 473-480.

[8] Ion, I., Stefanescu, D.M., (1999) Force Distribution in the MERO Four-Legged Walking Robot, *ISMCR'99 - Topical Workshop on Virtual Reality and Advanced Human-Robot Systems, vol. X,* Tokyo, Japan, June10 – 11,

[9] Ion I., Simionescu I., Curaj A. (2002). MERO modular walking robot having applications in farming and forestry *The 5thInternational Conference on Climbing and Walking Robots,* September 25-27, -Paris –France

[10] Ion I., Ungureanu M., Simionescu I. (2000). Control System of MERO modular Walking Robot, *Proceedings of the International Conference on Manufacturing Systems,* Bucharest, October 10-11, Romanien Academy Publishing House, pp 351-354 ISBN 973-27-0932-4.

[11] Ion I., Simionescu I., Curaj A. *(*2002). Mobil Mechatronic System With Applications in Agriculture and Sylviculture. *The 2thIFAC International Conference on Mechatronic Systems*, December 8-12, -Berkeley University – USA

[12] Ion I**.,** Simionescu I., Curaj A., Ungureanu M (2004). MERO Modular Walking Robot support of Technological Equipments *International Conference of Robots, Timisoara,* 0ct. 14 -17, Romania

[13] McGhee, R.B., Frank, A.A., (1968) On the Stability Properties of Quadruped Creeping Gait, In: *Mathematical Biosciences, Vol. 3*, No.2, pp. 331-351, 1968.

[14] Mc Ghee, R.B., Orin, D.E., A(1973) Mathematical Programming Approach to Control of Joint Positions and Torques in Legged Locomotion *Systems Proc. 2nd CISM-IFToMM Conference for the Theory and Practice of Robot Manipulators* (ROMANSY), Warsaw, Poland, 1973

[15] Pedro Ponce, Ricardo Fernandez, et.all, Control of a Robot Based on Intelligent Systems, *WSEAS Transation on Systems, Issue 2, Volume 6, February 2007,* pg.251-256, ISSN 1109-2777

[16] Simionescui.,IonI.,CurajA,MarinA.,(2006), Feet for Modular Walking Robots, *The 9ThInternational Conference on Climbing andWalking Robots, and Associated Technologies*, Royal Military Academy, Brussels Belgium,Sept 11-14, 2006

[17] Song S.M., Waldron K.J. (1989). Machines that Walk, *Massachusetts Institute of Technology Press*, 1989,

[18] Song S.M.,Waldorn,K.,J., (1987) Geometric design of a walking machines for a optimal mobility, *ASME Journal of Mechanism, Transmissions and Automation in Design*, 109,21-28

[19] Waldron, K.J., (1991) Force and Motion Management in Legged Locomotion, *Proceedings of the 24th IEEE Conference on Decision and Control*, Fort Lauderdale, pp.12-17