A Multilevel Approach of Reliability Optimization in Complex Systems

GABRIELA TONȚ, DAN GEORGE TONȚ
Department of Electrical Engineering, Measurements and Electric Power Use,
Faculty of Electrical Engineering and Information Technology
University of Oradea
Universității st., no. 1, zip code 410087, Oradea,
ROMÂNIA,
dtont@uoradea.ro, gtont@uoradea.ro, http://www.uoradea.ro

Abstract: Reliability optimization of complex systems is aligning in the larger framework of solving business and technical issues by adopting solution and decision-making under the simultaneous multi-objective conditions. The optimization is a key problem in the conceptual design, the implementation and realization in context of use of system. The feasible directions of reliability improvement may be achieved by reducing system complexity, increasing reliability of components, using structural redundancy or adequate maintenance strategies.

The paper focuses on the opportunity of using continuous Markov chains, which allow modeling the behavior of complex systems under realistic time-dependent operational conditions, as an approach on managing and equipment stocks, a tool of reliability centered maintenance. The proposed simulation method is based on minimizing the stocking and failure costs of equipment under constraint of available acquisition and stocking sum. The case study, assessing two alternative situations of studied system, validates the effectiveness of method.

Key-Words: Reliability, Optimization, Minimal Costs, Configuration, Validation Results

1 Summary
Designing complex systems researchers faced a structural dilemma of achieving robust systems, insensitive to variation, yet flexible, highly available, safe and innovative. In addition to these aspects, a wide range of factors to be considered concern economic, social and environmental sustainability regarded as an economic state where the demands placed upon the environment by people and commerce can be met without reducing the capacity of the environment to provide for future generations [1]. Using multidisciplinary reliability optimization and cost modeling techniques paper bridged the disciplinary viewpoints towards supporting the optimum system in operation and maintenance with the aim of facilitating effective decision-making during the its task. In the design phase of a system/component the optimization is reported to the nominal conditions. During the operating stage appear functional situations different from nominal values, thus is necessary to set the conjunctural optimum parameters, which flexible adjust the system to the operating situation. The escalating demand to lower production costs has prompted engineers to look for optimization methods, to design and produce products both economically and efficiently. Optimization is a powerful tool of the trade for engineer in virtually every discipline. A rigorous, systematic method for rapidly zeroing in on the most innovative, cost-effective solutions to some of today's most challenging engineering design problems.

Reliability improvement requires establishing reliability feedback and feedforward methods for continuous improvement regarding effective corrective actions, life cycle management and planning, performance monitoring and cost-effective corrective maintenance focusing proactively on component criticality assessment. Due to the increasing complexity of systems, availability assumes implementation at each capability level. Concept development, determining product functionality is based upon customer requirements, technological capabilities, and economic realities.

Design development, is focusing on product and process performance issues necessary to fulfill the product and service requirements in manufacturing or delivery. Design optimization is seeking to minimize the impact of variation in production and use, creating a robust design.

Design verification is ensuring that the capability of the production system meets the appropriate level.
Failure of one component interrelated with others may not impact availability if the system is designed to support such a failure, while failure of another component may cause system downtime and hence degradation in availability. Performance of equipment depends on reliability and availability of the equipment used, operating environment, maintenance efficiency, operation process and technical expertise of operator. The implementation and realization of a product or service are depending on the context of use.

The expected return on investment is seen as being directly related to system capability, defined in terms of durability, performance, availability and reliability. A major part of any system operating costs is due to unplanned system stoppages for unscheduled repair of the entire system of components. Decreasing the impact of failure is a way to improve reliability and availability of a system.

2 Heuristic vs. Rigorous Formulations
The proposed method tries to solve a reliability optimization problem through analysis and modeling the influences of the critical operating parameters that can be associated with the optimal allocation of redundancy. In order to attempt this goal, the decomposed problem is coordinated in the following steps:
1. Identification of the different factors influencing the operation behavior of systems/components.
2. Development of a framework for classification of the optimization methods for reliability control or improvement and cost modeling.
3. Development of a reliability optimization model to arrive at the optimal maintenance policy for a system during its useful life.

The maintenance interval and, consequently, availability level of a complex system are directly dependent of the existence of elements in stock. Stock level can be optimizing under technical and economical criteria taking into account characteristics of components and system.

The factors of importance for components in the system architecture are determined by [2]:
- the position of the component in the system;
- intrinsic reliability of element;
- others elements reliability.

Due of its importance in maintaining the performance (i.e. quality, reliability and safety) of systems/components an extensive literature is available on the optimizing reliability. The major part of heuristic methods have a commune feature that the solution is obtain by upgrading a variable with value 1, the incremented variable being selected based on a sensitivity factor [3].

[4] developed a heuristic method which requires minimal success paths of the system. On each iteration, a state is selected in two steps with the purpose to feasible increments the redundancy. In [5] genetic algorithms, as a part of metaheuristic algorithms are applied in solving reliability optimization identifying two options for each component of equivalent system diagram:
1. keep the existing structure;
2. changing structure with the improved reliability elements, implying supplementary costs.

The exact algorithms solution is applied in [6] through substitutive restriction method, to solve optimization problem when the objective function is separable. The study emphasizes that method performances are superiors to dynamical programming, leading to an exact solution. Heuristics method for optimal simultaneous coordination of intrinsic reliability–redundancy set presented by [7] is successful if the objective function and constrains are differentiable and monotone increasing. An approach for multi-objective optimization problems proposed by [8] maximize the reliability by minimizing resources cost.

3 Problem formulations
The stock equipment optimization represents a tool of putting into practice reliability centered maintenance, a component of optimization program of resources of systems. The proposed model allows stock optimization under available sum constrain for equipment acquiring and storing at system level.

System reliability optimization developed in two main directions [9]:
1. system reliability maximization:

$$R_S(x,r) = \sum_{j=1}^{N} f_j(x_1,r_1; x_2,r_2; \ldots; x_N,r_N)$$

In the following conditions:

$$\sum_{j=1}^{N} g_{ij}(x_j,r_j) \leq b_j, i = 1, m$$

where $$f_j(x_1,r_1; x_2,r_2; \ldots; x_N,r_N)$$ satisfy optimality principle conform to which in an optimal strategy regardless of initial state and initial decision, decisions on the states results from first decisions must have an optimal trajectory $$R_S(x,r)$$ - can be express as a sum of separable functions;
\( g_j(x_j, r_j) \) - quantity of resources \( i \) type consumed in subsystem \( j \)

\( b_j \) - constrain \( i \) type imposed to the system

\( x = (x_1, x_2, \ldots, x_N) \) - allocated redundancy vectors

\( r = (r_1, r_2, \ldots, r_N) \) - system reliability level

2. minimizing system cost:

\[
C_s(x, r) = \sum_{j=1}^{n} C_j(x_j, r_j) \quad \text{with the condition of assuring a minimal reliability level} \quad R_s(x, r) > R_{\text{sys}}
\]

where \( C_s(x, r) \) is the total cost of the system \( C_j(x_j, r_j) \) is the cost of subsystem \( j \) depending on the number of allocated components \( x_j \) and reliability level \( r_j \).

### 4 Optimization criteria

System design deals with reliability maximization and cost, volume or weight minimization, often conflicting. An approach for solving a multi-objective optimization problem supposes identification of a feasible set of solutions and applying the decisions based on.

Setting the optimum reliability follow the steps:

1. information acquisitions regarding the equipments behaviour;
2. setting the optimization criterion of reliability level;
3. objective function terms evaluation;
4. setting and applying the optimal solution.

Optimization reliability criteria valid for complex systems are presented in table 1.

**Table 1. Optimization reliability criteria for complex systems**

<table>
<thead>
<tr>
<th>Application Stage</th>
<th>Criterion of reliability optimization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design</td>
<td>Total updated cost (TUS); Economically justified effort, Economic reported effort</td>
</tr>
<tr>
<td>Operational</td>
<td>Damages minimization; Setting the optimal functioning states; Importance factors of elements; Choosing appropriate maintenance policies regarding: - strategy; - level of stocks; - maintenance support</td>
</tr>
</tbody>
</table>

In the design stage the optimization criteria of updated total cost, yearly costs, and economical justified effort are frequently applied. In the operating phase the effective criteria are damages minimization, setting the optimum states of running, components importance factors and setting adequate maintenance strategy regarding the stock levels and logistics.

The accurate assessment of objective function terms is essential for the validity of the solution that will be established. Regarding determinist components of the objective function (I, C, Vr and Wm), are necessary the following clarifications:

Investment (I) are categorized in actual (direct collateral, related), including funds for realizing the objectives and, respectively, investments of equivalence of variants, regarding production capacity, transport and power losses. Investments (I) enclose the found destined to the achievement of objective and, respectively, equivalent investments of variants regarding the production capacities, transport, power losses;

Investments by equivalence to cover the loss of power since year “\( t \)” are determined with equation:

\[
I_n = k_r \cdot \gamma \cdot \Delta P_t
\]

where:

- \( k_r \) - coefficient dependent on reserve power in the system,
- \( \gamma \) - the cost of installed power;
- \( \Delta P_t = P_t - P_{t-1} \) - the increase of power loss in the year “\( t \)”. 

- operational costs (C) during the running period of installations entail cost of preventive and corrective maintenance, retributions and losses;
- residual (Vr) or remanent (Wrm) values represents the equipments values, and components decontamination due of aging of system or inadequate running.

Applying a criterion aimed to minimize objective function "annual calculated cost ", with the following mathematical expression:

\[
CA = p_n \cdot I + C + D_{av} + D_{cal} + C_{tehs}
\]

Where:

- \( p_n \) - economic investment efficiency coefficient;
- I - investment cost;
- C - operating cost;
- \( D_{av} \) - damage caused by the interruption of power supply electricity;
- \( D_{cal} \) - damage caused irregularities quality of material resources;
- \( C_{tehs} \) - cost due to additional technical measures applied in the process for increasing safety and avoiding damages;

\( C, D_{av}, D_{cal} \) and \( C_{tehs} \) are determined with reference to a range of analysis TA = 1 year.
4.1 Justified economical effort criterion

[13] propose minimizing criterion of justified economical effort whose objective function is:

\[ \sum_{t=1}^{\mathcal{T}} G_t \cdot (1 + a)^{-t} \leq \sum_{t=1}^{\mathcal{T}} H_t \cdot (1 + a)^{-t} + W_{rT} \cdot (1 + a)^{-T} \]

where:

- \( G_t \) - represents the economic effort in the year \( t \);
- \( H_t \) - represents the economic effect in the year \( t \);
- \( W_{rT} \) - represents the remanent values of the objective at the end of the period of analysis \( T \);
- \( a \) - updating rate.

In general, the economic effort is depending on investment made at the beginning of the period of analysis \( (I) \) and operating costs \( (C_{eT}) \), spread over the entire period:

\[ \sum_{t=1}^{\mathcal{T}} G_t \cdot (1 + a)^{-t} = I_t + \sum_{t=1}^{\mathcal{T}} C_{eT} \cdot (1 + a)^{-t} \]

Economical effect is materialized in reducing damage:

\[ \sum_{t=1}^{\mathcal{T}} H_t \cdot (1 + a)^{-t} = \sum_{t=1}^{\mathcal{T}} \Delta D_t \cdot (1 + a)^{-t} \]

Based on these considerations, the objective function can be written under the form:

\[ I_t + \sum_{t=1}^{\mathcal{T}} C_{eT} \cdot (1 + a)^{-t} \leq \sum_{t=1}^{\mathcal{T}} \Delta D_t \cdot (1 + a)^{-t} + W_{rT} \cdot (1 + a)^{-T} \]

To prioritize the implementation of optimal variant, may be use the criterion of "economic effort minimum reported ", whose objective function is expressed mathematically as follows:

\[ g = \frac{\sum_{t=1}^{\mathcal{T}} G_t \cdot (1 + a)^{-t} + \sum_{t=1}^{\mathcal{T}} C_{eT} \cdot (1 + a)^{-t}}{\sum_{t=1}^{\mathcal{T}} H_t \cdot (1 + a)^{-t} + W_{rT} \cdot (1 + a)^{-T} + \sum_{t=1}^{\mathcal{T}} \Delta D_t \cdot (1 + a)^{-t}} \]

The solution is considered economically justified if \( g < 1 \).

The priority for implementing the proposed measures will be dictated by descending order of \( g \).

4.2 Updated total cost criterion

Applying this criterion aiming minimizing objective function "updated total cost" \( (CTA) \) with the following general expression:

\[ CTA = \sum_{t=1}^{\mathcal{T}} \left( I_t \cdot (1 + a)^{-t} + \sum_{i=r}^{\mathcal{T}} (C_i + D_i) \cdot \left( \frac{1 + r}{1 + a} \right)^{t-i} - \sum_{i=r}^{\mathcal{T}} V_{iT} \cdot (1 + a)^{-t} - W_{rT} \cdot (1 + a)^{-T} \right) \]

where:

- \( a \) - update rate;
- \( r \) - the inflation rate;
- \( t \) - the current year;
- \( t^{*} \) - the period of execution;
- \( \mathcal{T} \) - the duration of analysis (in energy is working with \( T = 20 \) years);
- \( T \) - duration of service;
- \( I_t \) - investments (\( i \) § effective equivalence) since \( "t" \);
- \( C_t \) - cost (operating equivalence) since \( "t" \);
- \( D_t \) - probable damage in \( "t" \);
- \( V_{iT} \) - the values of residual components disabled in \( "t" \);
- \( W_{rT} \) - the remanent value of the disabled objective year after the "T".

Fig.1 Cost-reliability dependency

Graphic expression of cost-reliability dependency, based on the criterion CTA is illustrated in Fig.1.

4.3 Updating rate

Determining the value updating rate \( (a) \) is one of the difficult problems. The magnitude of updating rate depends on a number of technical elements and economic such as the nature of investment, the degree of development of the economy, socio-economic situation, and competition with other projects. Updating rate used in Romanian economy until 1990, was 8% for energy field, lower than the rate used in most of the other branches of the economy (10%). Using the lowest rate for investments in energy reflects an economic policy aiming at developing energetic base of the economy. In general, the updating rate is inverse proportionally to the degree of economic development in the country (as the economy is more developed, the updating rate can be lower). The basic principle accepted in the calculations of economic upgrade begin from the premise that the value of money is variable in time because of the possibility existing as an alternative, to deposit the amount in the bank, increasing its value over time due to the accumulation of interest. Accepting the idea that upgrade involves the assumption that initials investments \( (I) \) brings in each year an net income equal to \( a \), which is fully reinvested with a efficiency equal to the initial fund, arriving that the total amount to be distributed at first year ending is:
For a period of \( n \) years, the amount initially \( I \) becomes \( I_n \):

\[ I_n = I \cdot (1 + a)^n \]

The updating technique is based on this relation, which admit that the annual income \( a \) has brought the amount invested \( I \), which is fully reinvested, with an efficiency at least equal with the initial fund. The updating technique is used to bring to one present referential year future cost, establishing what amount should represent today a sum \( I \), required to be available over \( n \) years. In this case is taking into account an updating factor, \( a \):

\[ I = \frac{I_n}{(1 + a)^n} \]

Calculation upgrade is important when have to be compare incomes or cost that occur in different periods of time, those amounts can be compared only after expressions of their value have been brought to the same year of reference, typically the initial year.

The value of an updated cost (or benefit) in future is higher to smaller updating rates of current from becoming neglecting for a period of analysis over 50 years, if updated rate is over 10%. The future values of a current cost (or a benefit obtained in the present) increase along with of updating rate and period of calculation, highlighting the economic disadvantages of investments immobilization. In the field of efficiency analysis a role has duration of the execution of the works. For the duration of the works execution over 3-4 years, including in the economic calculations and the effect of investments immobilization and gain importance. The yearning to reduce up to minimum investment efforts from the previous period of putting in function of the objective, instructions of operations in the calculations of effectiveness has the economic requirement that the weight center of investment to be in the second half of the actual execution (closer to the moment when investment become productive). For example fig. 2 presents the dynamics of the investment for the development of the power plant with four groups of 330 MW.

**5 Mathematical model**

The aimed goal is to determine the optimal level \((s_{\text{opt}})\) of equipment stock corresponding to operational updated minimal cost. The operational cost in average updated for a stock of volume \( s \) is:

\[ C(s) = s \cdot C_{\text{av}} + D(s) \cdot \sum_{i=1}^{T} (1 + a)^{-1} \cdot [UM] \]  

(3)

where: \( C_{\text{av}} \) is acquisition and storing cost afferent to a single component of analyzed equipment:

\[ D(s) \] is the yearly damages due to the unavailability of the observed equipment.

The reliability indices for damages assessment (number of defects \( n_d \) and defect duration \( T_d \) ) were computed using Markov chains assuming following hypothesis:

- the system contain \( n \) identical equipments;
- stochastic variables distribution functions, mean time between failure (MTBF) and time corrective maintenance (TCM) are exponential,
- failure rate \( (\lambda) \) and mean time to failure (MTTF) \( \mu \) have constant values;
- when a equipment fails is replaced with a reserve from the stock, replacement time being \( \tau \);
- when a equipment fails, no reserve in the stock is repair with repair rate \( (\mu) \);
- the stock equipment are controlled periodically and the failure rate is zero in stocking interval.

In the situation without stock the damage evaluation is made applying the general solution method. The probability state for the running method is:

\[ P_s = \frac{1}{1 + \sum_{k=1}^{\infty} C_k \cdot \left( \frac{\lambda}{\mu} \right)^k} \]  

(4)
the probability state of \( k \) defect equipments is

\[
P_k = C_n^k \cdot \left( \frac{\lambda}{\mu} \right)^k \cdot P_0
\]

(5)

The number of yearly passing of system in the state characterized by \( k \) failure of equipments and mean time occupancy of this estate can be written as follows:

\[
\begin{align*}
n_{dk} &= P_{k-1} \cdot (n-k+1) \cdot \lambda \cdot 8760 \\
T_{dk} &= \frac{P_k \cdot 8760}{\frac{1}{n_{dk}}} = \frac{1}{k \cdot \mu}
\end{align*}
\]

(6)

The yearly damages due to equipment unavailability in the situation without stocks were evaluated with relation:

\[
D(0) = \sum_{k=1}^{n} n_{dk} \cdot (d_i \cdot T_{dk} + d_n)
\]

(7)

where \( d_i \) is specific damage proportional with defect duration [MU/h];

\( d_n \) is specific damage proportional with defect number [MU/defect].

The probability of states are:

\[
\begin{align*}
\Phi &= \lambda \cdot \mu \\
\Psi &= n \cdot (\phi + \chi) \cdot \sum_{j=0}^{S-1} \sum_{i=0}^{n} (n \cdot \phi)^j
\end{align*}
\]

(8)

\[
\begin{align*}
\psi^* &= (n \cdot \phi)^s \cdot \sum B(k,n) \cdot n^k \cdot (\phi + \chi)^k \\
B(n,k) &= \frac{n!}{(n-k)^k n^k}
\end{align*}
\]

(9)

In the situation of existing stocks, based on states graphs are obtain the solutions of Markov equation [10] assigned to the \( 2s + n \) states of system shown in table 2.

Table 2 Systems states with \( n \) components and \( s \) elements stock

<table>
<thead>
<tr>
<th>System states</th>
<th>Number of elements</th>
<th>Number of reserves</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0</td>
<td>n 0</td>
<td>S 0</td>
</tr>
<tr>
<td>1</td>
<td>n-1 1</td>
<td>S 0</td>
</tr>
<tr>
<td>2</td>
<td>n 0 s-1</td>
<td>1 j</td>
</tr>
<tr>
<td>2j+1</td>
<td>n-1 1 s-j</td>
<td>j j</td>
</tr>
<tr>
<td>2s+1</td>
<td>n-1 1 s</td>
<td>0 s</td>
</tr>
<tr>
<td>2s+n</td>
<td>0 n 0</td>
<td>s s</td>
</tr>
</tbody>
</table>

Fig.3 Graph of the states system

The reliability indices for damages evaluation are stated as follows:

\[
\begin{align*}
n_{d2j+1} &= P_{2j} \cdot n \cdot \lambda \cdot 8760 = \frac{(n \cdot \lambda)^j}{\mu^j} \cdot P_0 \cdot 8760 \\
n_{2s+n} &= P_{2s+k} \cdot (n-k+1) \cdot \lambda \cdot 8760 = \frac{n^s \cdot \phi^{s+k-1} \cdot \lambda \cdot P_0}{(n-k)^k} \cdot 8760 \\
T_{2s+n} &= \frac{P_{2s+k} \cdot 8760}{\frac{1}{n_{2s+n}}} = \frac{1}{k \cdot \mu}
\end{align*}
\]

(10)

The annual damages due to unavailability, in the situation of a \( s \) equipments stock are evaluated with relation:

\[
D(s) = \sum_{j=0}^{S-1} n_{d2j+1} \cdot (d_i \cdot \tau + d_n) + \sum_{k=1}^{n} n_{d2s+k} \cdot (d_i \cdot T_{2s+n} + d_n)
\]

(11)

The optimization model bellow allows equipments stock optimization without cost constrains.

Due to the increasing cost limitations in a competitive market is required to establish the optimum level of electrical stocks under multiple cost constraints. A solution may be represented as follows:

\[
\begin{align*}
\text{FOB} &= \sum_{j=1}^{m} C_{si} (s_j) = \sum_{j=1}^{m} s_j \cdot C_{si} + D_i (s_j) \cdot (1 + a)^r \\
\text{min} \sum_{j=1}^{m} s_j \cdot C_{si} \leq C_{si \text{ max}}
\end{align*}
\]

(12)

is where: \( m \) is number of equipments consider in the analysis and \( C_{si \text{ max}} \) is the available sum at system level for acquire and storing the equipments [11].
Optimal allocation which fulfills the constraint is modeled by means of a heuristic algorithm based on direct search procedure of possible solutions without enumeration.

6 Case study

Electrical equipments (switches and isolating switches) are in the structure medium voltage (MV) of auxiliaries and high voltage (HV) cells in power plant.

The corresponding flow chart of algorithm is shown in figure 2.

Basic characteristics of analyzed equipment are presented in table 3.

Table 3 Characteristics of commutation equipment

<table>
<thead>
<tr>
<th>Equipment</th>
<th>λ</th>
<th>µ</th>
<th>τ</th>
<th>Cₚₐₜ</th>
</tr>
</thead>
<tbody>
<tr>
<td>MV Switch</td>
<td>0.040</td>
<td>580</td>
<td>1</td>
<td>1000</td>
</tr>
<tr>
<td>MV Isolating Switch</td>
<td>0.004</td>
<td>400</td>
<td>1</td>
<td>500</td>
</tr>
<tr>
<td>HV Switch</td>
<td>0.060</td>
<td>500</td>
<td>1</td>
<td>5,000</td>
</tr>
<tr>
<td>HV Isolating Switch</td>
<td>0.007</td>
<td>400</td>
<td>1</td>
<td>1,000</td>
</tr>
</tbody>
</table>

The values of specific damages were calculated as mean values for n analyzed equipments depending on damages in the bus-bars of auxiliaries.

Taking into account all analyzed equipments values of updated operational, costs are minimal for 1 = s, (table 4).

Table 4 Annually damage values, respectively updated operational cost depending on stock lot level

<table>
<thead>
<tr>
<th>s</th>
<th>D(s), C(s) [USD]</th>
<th>Switch MV</th>
<th>Separator s MV</th>
<th>Switch HV</th>
<th>Separators HV</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>D(0), C(0)</td>
<td>5.793,8</td>
<td>1032,0</td>
<td>22.680,0</td>
<td>31.226,0</td>
</tr>
<tr>
<td>1</td>
<td>D(1), C(1)</td>
<td>1372,4</td>
<td>1775</td>
<td>1181,7</td>
<td>1399,3</td>
</tr>
<tr>
<td>2</td>
<td>D(2), C(2)</td>
<td>1599,3</td>
<td>1770</td>
<td>1135,5</td>
<td>1251,0</td>
</tr>
<tr>
<td>3</td>
<td>D(3), C(3)</td>
<td>1571,3</td>
<td>2507,2</td>
<td>19664,6</td>
<td>12650,1</td>
</tr>
<tr>
<td>4</td>
<td>D(4), C(4)</td>
<td>1599,3</td>
<td>1770</td>
<td>1135,2</td>
<td>1250,8</td>
</tr>
</tbody>
</table>

Fig.4 Flow chart of stock level optimization

The optimal level stock is for an element in stock (sₜₜₒₜᵦₗ=1) for all categories of commutation equipments. In terms of objective function we note that stock level depend on replacement time (τ) and cost of acquisition and storing (Cₚₐₜ). Analyzing the variances of updated operational costs for different values of replacement time we identify that the optimal solution is not influenced by τ. The values of Cₚₚₜ affects stocks optimal level, but for realistic evaluation of Cₚₚₜ the optimum remain sₜₜₒₜᵦₗ=1. The dependency replacement time- update operating cost are presented in figure 5.

To set the global optimal level stocks for commutation equipments with an certain available sum at system level (Cₚₚₜ max) we write the optimal allocation as follows:

\[
\text{AOPT} = [s_{1opt}, s_{2opt}, s_{3opt}, s_{4opt}]
\]

where:   
\(s_{1opt}\) stocks optimal level of MV switch;  
\(s_{2opt}\) stocks optimal level of MV isolating switch;  
\(s_{3opt}\) stocks optimal level of HV switch;  
\(s_{4opt}\) stocks optimal level of HV switch;  

\[\text{(13)}\]
The manner of algorithm implementation for $C_{as\ max} = 7.500\text{USD}$ is shown in table 4. Running the program for other values of constrain ($C_{as\ max}$) is obtain the optimal allocations for stocks level values of objective function presented under graphical form in figure 6.

Table 5 Algorithm implementation

<table>
<thead>
<tr>
<th>Adopted Allocation</th>
<th>Elementary Allocation</th>
<th>Tested Allocation</th>
<th>$C\text{as}_{\ max}$ [USD]</th>
<th>FOB [USD]</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0,0,0,0]</td>
<td>[0,0,0]</td>
<td>0</td>
<td>517.010</td>
<td></td>
</tr>
<tr>
<td>[1,0,0,0]</td>
<td>[1,0,0]</td>
<td>1.000</td>
<td>480.371</td>
<td></td>
</tr>
<tr>
<td>[0,1,0,0]</td>
<td>[0,1,0]</td>
<td>500</td>
<td>510.236</td>
<td></td>
</tr>
<tr>
<td>[0,0,1,0]</td>
<td>[0,0,1]</td>
<td>5.000</td>
<td>338.995</td>
<td></td>
</tr>
<tr>
<td>[0,0,0,1]</td>
<td>[0,0,0]</td>
<td>1.000</td>
<td>263.589</td>
<td></td>
</tr>
<tr>
<td>[1,0,0,1]</td>
<td>[1,0,1]</td>
<td>2.000</td>
<td>226.950</td>
<td></td>
</tr>
<tr>
<td>[0,1,0,1]</td>
<td>[0,1,1]</td>
<td>1.500</td>
<td>256.815</td>
<td></td>
</tr>
<tr>
<td>[0,0,1,1]</td>
<td>[0,0,2]</td>
<td>6.000</td>
<td>85.574</td>
<td></td>
</tr>
<tr>
<td>[0,0,0,2]</td>
<td></td>
<td>2.000</td>
<td>263.833</td>
<td></td>
</tr>
<tr>
<td>[0,0,1,2]</td>
<td></td>
<td>85.574</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig.6 The optimum stocks and values of FOB depending on the constrain $C_{as\ max}$

Table 5 Algorithm implementation
We note that FOB attain the minimum value (42.161 USD) if the available sum for system is $C_{as max} \geq 7.500 USD$, situation in which the optimum is \([1, 1, 1, 1]\), solution obtained also without constrain optimization.

All identified solutions representing the optimal level stocks at different constrain values are viable (Profit of investment, PI > 1).

\[
PI = \frac{\sum_{i=1}^{m} \Delta D_i \left( s_{i\text{opt}} \right) \sum_{t=0}^{T} (1 + a)^{-t} \times s_{i\text{opt}} \times C_{as i}}{\sum_{i=1}^{m} s_{i\text{opt}} \times C_{as i}}
\]

(14)

To form a feasibility image of obtained solutions we calculate the profitability indices of stocks level. Profitability indices values are presented under graphical form in figure 7.

The equipment stock optimization is a component of resources planning and a technique to implement reliability centered maintenance. The proposed model, based on minimizing afferent cost of stoking and costs of the equipments failure is simple to use requiring a reduced quantity of input data. The model permit stock optimization under acquiring and stoking costs constrain of equipment, depending on the available maximum sum ($C_{as max}$), stock are establish for every analyzed equipment.

## Conclusions

The optimal stock level is not influenced by time replacement ($\tau$) situation explainable because ($\tau$) is lesser than repair time ($1/\mu$), but is affected by the acquiring and storing costs. The obtained optimum level are realistic taking into account the volume of equipments in installations and the reliability behaviour of commutation equipments. The maintenance in operational stage obliges to costs which are inverse proportionally with the system reliability level. In most of the safety-critical process the accurate prediction of failures and other reliability parameters are essential to make effective maintenance. Therefore, a systematic approach requires defined levels of performance of the system/component during its operational and maintenance phase. The aim is to integrate equipment aging management, preventive, predictive, and corrective maintenance with economic planning and other activities in order to: minimize equipment reliability issues, manage the material condition of the plant, optimize the remaining operating life of the plant, and maximize plant value.

## References:


