Fuzzy Forecasting Applications on Supply Chains

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Abstract: - Demand forecasting; which sound basis for decision making process, is among the key activities that directly affect the supply chain performance. As the demand pattern varies from system to system, determination of the appropriate forecasting model that best fits the demand pattern is a hard decision in management of supply chains. The whiplash effect can be express as the variability of the demand information between stages of the supply chain and the increase of this variability as the information moves upstream through the chain. The usage of proper demand forecasting model that is adequate for the demand pattern is an important step for smoothing this undesirable variability. This paper evaluates the effects of fuzzy linear regression, fuzzy time series and fuzzy grey GM (1,1) forecasting models on supply chain performance quantifying the demand variability (i.e. whiplash) through the stages of a near beer game supply chain simulation model expanded with fuzzy parameters.

Key-Words: - Forecasting, Supply chain, Fuzzy regression, Fuzzy time series, Grey fuzzy

1 Introduction

Supply chain (Sc) models are dynamic complex systems including many sophisticated activities such as constant on time information, scheduling, production, distribution, services and decision making processes which are all required for satisfying the customer demand. A simple definition for these complex systems can be express as “the network of organizations that are involved, through upstream and downstream linkages, in the different process and activities that produce value in the form of products and services in the hand of ultimate customer” [1]. Although the information flow in Sc systems consists of cumulative data about costs parameters, production activities, inventory systems and levels, logistic activities and many other related processes; bethinking of the definition exposes that the performance of a successful Sc system concerns mostly with accurate and appropriate demand information as this vital data influences all decision making processes of Sc [2].

A well-known phenomenon of Sc systems is the variability of the demand information between the stages of the supply chain (i.e. the whiplash or bullwhip effect (WE).) This variability increases as the demand data moves upstream from the customer to the other stages of the SC system engendering undesirable excess inventory levels, defective labor force, cost increases, overload errors in production activities and etc. Forrester [3, 4] with a simple Sc simulation consisted of retailer, wholesaler, distributor and factory levels, discovered the existence of WE as ‘demand amplification’ in Sc model. He argued about the possible causes and suggested same ideas to control the WE. He emphasized on the decision making process in each phase of SC and betrayed that this process could be the main reason the demand amplification through the chain from the retailer to the factory level [2]. He also denoted that, time lags for clerical work, purchasing and transportation, lead times and specific factory capability could be other likely reasons of WE. Like Forrester, Sterman [5, 6] also focused on the existence and causes of WE. He used an experimental Sc simulation that simulate the beer distribution in a simple Sc consists of four echelons; retailer, wholesaler, distributor, factory which is then became a well-known Sc simulation model; “Beer Distribution Game”, widely used for teaching the behavior, concept and structure of Sc. The model was so simple but despite to its simplicity, it successfully showed the impact of the decision process in echelon on the demand variability. Main objective is to govern each echelon achieving the desired inventory and pipeline levels minimizing the total cost including inventory and shortage costs. Fig.1 illustrates the general system structure of a beer game [7]. Sterman apprised that inaccurate judgments made by the participants and the adoption of these judgments to the Sc system are the main reason of this phenomenon. Larsen *et.al* [8]; using a Sterman based beer game model, showed that the structure of a production-distribution chain produce broad variety of dynamic behaviors. In the study, different types of behaviors are summarized with Lypunov exponents. There were two important assumptions in the model; i) the parameters of the decision rule is constant, (i.e, remain the same through the entire solution), ii) decisions in each stage constituted under the information available in each stage. After showing the effect of inventory control policies on dynamics and costs, Larsen *et.al*. concluded that "a sophisticated management information system" and reducing time lags are important keys for reduction in demand amplification and costs. Also they claimed that "interlocking" common parameters in ordering policies might cause high cost and intricacy dynamics.

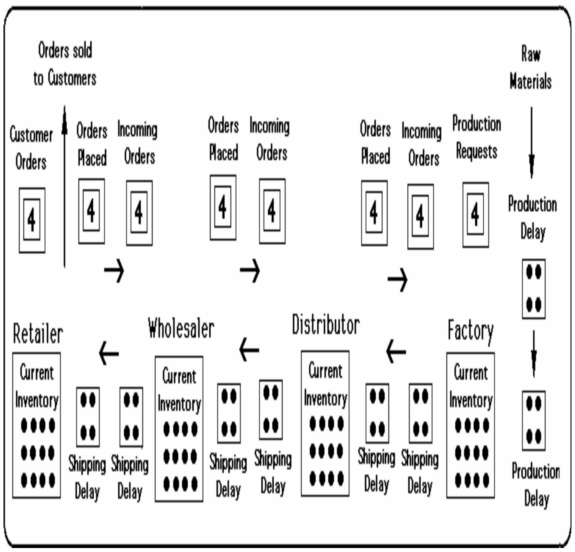

Fig.1. The general system structure of a beer game.

Lee *et.al*. [9, 10, 11, 12] declared causes of WE as: price fluctuations, rationing game, order batching, lead times and demand forecast updating; which is also the main scope of this paper. Baganha *et.al*. [13], Graves [14], Drezner *et.al* [15], Chen *et.al*.[16] and Li *et.al*. [17] also studied WE from the perspective of information sharing and demand forecasting / updating.

Accurate demand forecasting is one of the major minimization tools for WE but, finding the adequate model for the demand pattern is snarl [2]. Though past the studies about forecasting bring to light that, under relatively few data information and uncertainties (just like the situation in many Sc systems) the fuzzy forecasting models such as fuzzy time series (FTs) [18, 19, 20, 21, 22, 23], fuzzy linear regression (FR) [24, 25, 26], fuzzy grey GM(1,1) (FGG) [27] and other applied forecasting models [28, 29] performed successfully, not much attention is paid on these systematic for their usage in Sc management. This paper focuses on the effects of selected fuzzy forecasting models on the Sc performance via demand variability in the system.

The rest of this paper is organized as follow. In section 2, FR, FTs and FGG forecasting models are introduced. Section 3 deals with the quantification of WE for evaluation of Sc performance. In sections 4 and 5 proposed Sc simulation model is discussed and the effects of selected forecasting models on Sc performance are examined. Finally in section 6 research findings and conclusions are presented.

2 FR, FTs, FGG Forecasting Models

2.1 FR Forecasting Model

Linear regression; which shows the relation between response or dependent variable y and independent or explanatory variable x, can be formulate considering the relation of y to x as a linear function of parameters with $Y = f(x) = \theta X$ where θ is the vector of coefficients and X is the matrix of independent variable [2]. The application of linear regression model is suitable for the systems in which the data sets observed are distributed according to a statistical model (i.e. unobserved error term is mutually independent and identically distributed). But generally, fitting the demand pattern of a real Sc to a specific statistical distribution is not possible. The FR model introduced by Tanaka *et al.* [24, 25] in which "deviations reflect the vagueness of the system structure expressed by the fuzzy parameters of the regression model" (i.e. possibilistic) is suitable for the declared demand patterns and basically can be formulate as:

$$\bar{Y} = (c_0, s_0) + (c_1, s_1)x_1 + (c_2, s_2)x_2 + ... + (c_n, s_n)x_n \quad (1)$$

where c_k is the central value and s_k is the spread value, of the kth fuzzy coefficient; $\bar{A}_k = (c_k, s_k)$, usually presented as a triangular fuzzy number (TFN). And this representation is fact that relaxes the crisp linear regression model. Using fuzzy triangular membership function for A_k, the minimum fuzziness for $\bar{Y}_k$ can be maintained with the following linear programming (LP) model which minimizes the total spread for the fuzzy output parameter [2, 25, 30].

$$Z = Min\left\{ ms_0 - (1-h)\sum_{i=1}^{m}\sum_{k=0}^{n} s_k x_{ik} \right\}$$

st

$$\begin{aligned} &\sum\nolimits_{k=1}^{n} c_k x_{ki} - (1-h)\sum_{k=1}^{n} s_k x_{ki} \le y_i \\ &\sum\nolimits_{k=1}^{n} c_k x_{ki} - (1-h)\sum_{k=1}^{n} s_k x_{ki} \ge y_i \end{aligned} \quad (2)$$

where $x_{0i} = 1, \quad 0 \le h \le 1$ and $\forall_k = 1,2,3,...,n \quad \forall_i = 1,2,3,...,n$. Note that the constraints of the LP model satisfies the following equation which defines the degree of belonging of observations (that characterized by h) y_k to $\overline{Y}_k$.

$$\mu Y_i(y_i) \geq h, \quad i = 1,2,3...,n \tag{3}$$

2.2 FTs Forecasting Model

For dynamic systems like Scs, when historical demand data that will use to calculate the desirable forecast value are linguistic values and (or) are in small amounts, fuzzy time series model best fit the aspect [21, 22, 23]. Song *et.al.* [18, 19, 20]; fuzzifying the enrollments of the University of Alabama, used fuzzy time series in forecasting problems and proposed a first-order time-variant fuzzy time series with first-order time-invariant fuzzy time series for the solution of the forecasting problems. Later Song *et.al.* [20] introduced a new FTs model and betrayed that best results are held by applying neural network for defuzzifying data. Wang [21], Li *et.al.*[22] and Hwang *et.al.* [31] also successfully used FTs forecasting model.

In this paper Hwang's FTs forecasting models is selected and used for FTs demand forecasting. The model can be summarized as follow:

i. First, the variation between two continuous historical data is to be calculated and minimum / maximum increases (i.e. $D_{\min} / D_{\max}$) are to be determined,

ii. Next step is to define the universe discourse; U_d, with following equation using $D_{\min}$ and $D_{\max}$.

$$U_d = [D_{\min} - D_1 \, , \mathrm{D}_{\max} + D_2] \tag{4}$$

where D_1 and D_2 are positive values that fits for separating U_d into equal lengths.

iii. Then fuzzy sets on U_d are to be defined (i.e. defining fuzzy time series ($F(t)$) and variation data is to be fuzzified. Defining $F(t)$ as

$$F(t) = {p_{Z1}}/{u_1} + {p_{Z2}}/{u_2} + ... + {p_{Zm}}/{u_m} \tag{5}$$

where the memberships p_{zi} are $0 \leq p_{zi} \leq 1$. The fuzzy sets A of U then can be represented as;

$$A = \left\{ {p_{Z1}}/{u_1} + {p_{Z2}}/{u_2} + ... + {p_{Zm}}/{u_m} \right\} \tag{6}$$

Fuzzifications of variations are determined according to u_i that they fit.

iv. And the final step includes composing the relation matrix; $R(t)$, which is governed by operation and criterion matrixes (i.e. $O^w(t), Z(t)$) and defuzzifying the calculated variation [20,23] which will be used for estimating the forthcoming value using the relation of the chance value gathered from relation matrix . In this step the windows basis; w, have to be determined which shows the number of periods of variations that will be used for forecasting. For period t, $O^w(t), Z(t)$ and $R(t)$ is defined as follow [31]:

$$Z(t) = F(t-1) = [Z_1, \ \mathrm{Z}_2, \ Z_3, ...Z_n] \tag{7}$$

$$O^w(t) = \begin{bmatrix} F(t-2) \\ F(t-3) \\ . \\ . \\ \mathrm{F(t - w - 1)} \end{bmatrix} = \begin{bmatrix} O_{11} & O_{12} & ... & O_{1m} \\ O_{21} & O_{22} & ... & O_{2m} \\ . & . & . & . \\ . & . & . & . \\ O_{w1} & O_{w2} & ... & O_{wm} \end{bmatrix} \tag{8}$$

$$R(t) = \begin{bmatrix} O_{11}\mathrm{x}Z1 & O_{12}\mathrm{x}Z_{11} & ... & O_{1m}\mathrm{x}Z_m \\ O_{11}\mathrm{x}Z1 & O_{12}\mathrm{x}Z_{11} & ... & O_{1m}\mathrm{x}Z_m \\ . & . & . & . \\ . & . & . & . \\ O_{11}\mathrm{x}Z1 & O_{12}\mathrm{x}Z_{11} & ... & O_{1m}\mathrm{x}Z_m \end{bmatrix}$$

$$R(t) = \begin{bmatrix} R_{11} & R_{12} & ... & R_{1m} \\ R_{21} & R_{22} & ... & R_{2m} \\ . & . & . & . \\ . & . & . & . \\ R_{w1} & R_{w2} & ... & R_{wm} \end{bmatrix} \tag{9}$$

where $1 \leq j \leq m$ and $R_{ij} = O_{ij} \text{ x } Z_j, \quad 1 \leq i \leq w$. Then the estimated variation will be determined with the following equality.

$$F(t) = [r_1, r_2, ...r_m] \tag{10}$$

Where $r_j = Max(R_{ij})$; $i = 1, 2,...., w$ and $j = 1, 2, ..., m$. The forecast value for the period t is calculated by defuzzification of $F(t)$ and adding this value to the actual data of the period $t-1$ and this operation concludes the FTs forecasting method.

2.2 FGG Forecasting Model

The grey system theory introduced by Deng [32, 33] can simply be summarized as a methodology that concerns with the systems comprising uncertainties and lack of sufficient amount of information; in which, the term 'grey' indicates the system information that lies between the clearly and certainly known ones and the unknown part of the system [2].The discrete time sequence data is used to expose a regular differential equation with the accumulated generating operation (AGO); which establishes a regularity to on hand to data series, and inverse accumulated generic operation (IAGO). In grey differential model GM (n,m), terms n and m represents the order of ordinary differential equation and the number of grey variable respectively, defining the order of AGO and IAGO. Grey forecasting which is one of the most important part of the grey system theory, can basically be define as the use of past or current data of a system to develop a grey model for predicting the future trend of the system output (demand, time, ect.). As increases in n and m also increases the computation time exponentially causing likely correctness defects, most widely use model in grey system theory is GM (1,1) which has same important advantages those can be summarized as the usage for any kind of data distribution including small data sets and less requirement for computation [2]. The computation system structure of the Grey GM (1,1) model which also forms a base for FGG forecasting model can be summarized as follow [27].

Let D^0 show on hand data collected from the system as;

$$D^0 = (D_1^0, D_2^0, D_3^0, ..., D_n^0) \tag{11}$$

where n represents the number of data. The generated AGO series of D^0 ; D^1, then can be denoted as

$$D^1 = (D_1^1, D_2^1, D_3^1, ..., D_n^1) \tag{12}$$

where $\underset{k}{\overset{1}{D}} = \sum_{i=1}^{k} D_i^0$, $\forall_i = 1,2,...,n$. Composing a differential equation for D^1 as in equation (13) and depicting differential equation as a discrete time series with one unit sampling interval the relation described in equation (14) can be constructed.

$$\frac{D^1}{dt} + aD^1 = b \tag{13}$$

$$\frac{D^1}{dt} = D_{t+1}^1 - D_t^1 \qquad , \forall t \geq 1 \tag{14}$$

where a and b denotes the unknown developed coefficient and the unknown grey control variable respectively. For $\forall t \geq 1$ equation (14) is equal to D_{t+1}^0 and for $t = 1, 2, 3,....n$ equation (13) can be redesign in a matrix form as in equation (16) by setting the second part of the first order grey model to $D_{average}^1$;

$$D_{average(t+1)}^1 = 1/2(D_t^1 + D_{t+1}^1) \tag{15}$$

$$\begin{bmatrix} D_2^0 \\ D_3^0 \\ . \\ . \\ D_n^0 \end{bmatrix} = \begin{bmatrix} -D_{average(1)}^1 & 1 \\ -D_{average(2)}^1 & 1 \\ . & \\ . & \\ -D_{average(n)}^1 & 1 \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix} \tag{16}$$

After applying least square method, solutions of a and b can be obtained and using this two parameters equation (13) can also be solved and output forecast value can be determined with the following equations.

$$D_{t+1}'^1 = \left(D_1^0 - (b/a)\right) - e^{-at} + (b/a), \tag{17}$$

$$D_{t+1}'^0 = D_{t+1}'^1 - D_t'^1 \tag{18}$$

where $D_{t+1}'^1$ is the estimated cumulated value of D_{t+1}^1 and $D_{t+1}'^0$ is the forecast value for $\forall t \geq 1$.

In addition to the facts defined for the usage of grey systems and grey forecasting model above; if the data sets collected from the system are linguistic, FGG forecasting models may perform successfully. The FGG forecasting model introduced by Tsaur [27]; which assumes data series collected from the system are symmetrical triangular fuzzy numbers (TFN), is very similar to the crisp one and can be explained with the following equations. The original data series collected from the model is:

$$\hat{D}^{0'} = \left(\hat{D}_1^{\,0'} + \hat{D}_2^{\,0'} + \hat{D}_3^{\,0'} + ... + \hat{D}_n^{\,0'}\right) \tag{19}$$

where n is the number data collected from the system. And $\hat{D}^{1'}$; the new fuzzy data sequence generated with AGO, can be shown with the following equation as:

$$\hat{D}^{1'} = \left(\hat{D}_1^{\,1'} + \hat{D}_2^{\,1'} + \hat{D}_3^{\,1'} + ... + \hat{D}_n^{1\,1}\right) \tag{20}$$

where $\hat{D}_k^{\,1}$; $\forall k = 1,2,...,n$, is a symmetrical TFN with central and spread values $\sum_{i=1}^{k} D_i^{\,0}$, $\sum_{i=1}^{k} s_i^0$ $\forall \mathrm{i} = 1,2,...,\mathrm{k}$ respectively. And the fuzzy GM(1,1) model is denoted as;

$$d\hat{D}^{1'} / dk + a\hat{D}^{1'} = \hat{b} \tag{21}$$

where a is the developing coefficient and STFN $\hat{b}$ denotes the fuzzy grey input with the central value b and the spread value b_1 and the membership function for $\hat{b}$ is constructed as follow.

$$\mu\hat{b}(\alpha) = \begin{cases} 1 - \dfrac{|\alpha - b|}{b_1} & \mathrm{b - b_1} \le \alpha \le b + b_1 \\ 1 & \text{otherwise} \end{cases} \tag{22}$$

By setting the sampling interval one unit as in crisp model $\hat{D}^{1'} / dk$ can be rewritten as

$$d\hat{D}^{1'} / dk = \hat{D}_k^{\,1'} + \hat{D}_{k-1}^{1'} = \hat{D}_k^{0'} \;;\; \forall \mathrm{k} = 2,3,...,\mathrm{n} \tag{23}$$

or

$$d\hat{D}^{1'} / dk = \hat{D}_k^{\,1'} + \hat{D}_{k+1}^{1'} = \hat{D}_k^{0'} \,,\; \forall \mathrm{k} = 1,2,...,\mathrm{n} \tag{24}$$

where $\hat{D}_k^{0'}$ is a fuzzy number with central and spread values; $\hat{D}_k^0$ and , c_k^0 $\forall k = 2,3,...,n$. Let the average of $\hat{D}_k^{0'}$ and $\hat{D}_{k+1}^{0'}$, $\hat{Q}_{k+1}^1$ (which is a STFN with the central value Q_{k+1} and the spread value p_{k+1}) be the second part of equation (21) as ;

$$\hat{\mathrm{D}}_{\mathrm{k}}^0 = -a\hat{Q}_{k+1}^1 + \hat{b}\,, \tag{25}$$

where $\hat{Q}_{k+1}^1 = 1/2\left(\hat{D}_k^{1'} + \hat{D}_{k+1}^{1'}\right)$ with the central value

$$Q_{k+1} = 1/2\left(\sum_{i=1}^{k} D_i^0 + \sum_{i=1}^{k+1} D_i^0\right) \tag{26}$$

and the spread value

$$p_{k+1} = 1/2\left(\sum_{i=1}^{k} s_i^{\,0} + \sum_{i=1}^{k+1} s_i^{\,0}\right) \tag{27}$$

As the spread determines fuzziness; values of unknown variables a, b and b_1 can be obtained from the solution of the following LP model with the objective function that minimizes the spread value of STFN $\hat{b}$.

$$\begin{aligned} & Min\ z = b_1 \\ & st \\ & (b - aQ_k) + (1-h)(b_1 - ap_k) \ge D_k^0 + (1-h)s_k^0 \\ & (b - aQ_k) + (1-h)(b_1 - ap_k) \le D_k^0 + (1-h)s_k^0 \\ & \qquad 0 \le h \le 1; \qquad a, b, b_1 \in R \end{aligned} \tag{28}$$

After solving the LP problem; similar to the crisp grey GM(1,1) model, Tsaur suggested that estimated fuzzy number $\overline{\hat{D}}_k^1$ with lower bound $\hat{D}_k^{1,low}$ and upper bound $\hat{D}_k^{1,upr}$; $\overline{\hat{D}}_k^1 = \left(\hat{D}_k^{1,low}, \hat{D}_k^{1,upr}\right)$, could be obtained. Finally the fuzzy forecast value for period k+1; $\overline{\hat{D}}_k^0 = \left(\hat{D}_k^{0,low}, \hat{D}_k^{0,upr}\right)$, could be determined as follow;

$$\hat{D}_k^{0,g} = \hat{D}_k^{1,g} - \hat{D}_{k-1}^{1,g} \quad \text{for }, k \ge 2 \;\; g = low, up \tag{29}$$

3 The Measuring Sc Performance with Quantified WE

This study evaluates the Sc performance by quantifying the demand variability (i.e. WE) in various stages of the proposed Sc simulation defining WE as the ratio of demand variances of two consequent stages [16]. The smaller the WE the better the Sc performance will be.

Chen *et al.*[16]; assuming the customer demand in period t to the retailer (D_t) as random variables; defined D_t with the following equation.

$$D_t = \mu + D_{t-1}\rho + \varepsilon_{t'} \tag{30}$$

where μ and ρ denotes a non negativity constant and correlation parameters $(|p| < 1)$ respectively (as ρ indicates the relationship between demands $\rho = 0$ betrays the independent identically distributed (i.d.d.) demand). The variance of D_t is emerged as

$$Var(D_t) = \frac{\delta^2}{1 - \rho^2} \tag{31}$$

Assuming the inventory system in the retailer is order-up-to policy with fixed lead time and the forecasting technique used is simple moving average, the orders placed by the retailer at period t; q_t (which is an important variable owing to the fact that WE is quantified as $Var(q)$ relative to $Var(D)$) is denoted as:

$$q_t = y_t - y_{t-1} + D_{t-1} \qquad (32)$$

As q_t at period t can be negative (equation (32)) an important assumption is made as the "excess inventory is returned without cost" in Chen *et.al.* model. Using the estimates of lead time demand and standard deviation of the forecast in the period, Equation (32) can be rewritten as follow;

$$q_t = \hat{D}_t^L - \hat{D}_{t-1}^L + z(\hat{\sigma}_{et}^L - \hat{\sigma}_{et}^L) + D_{t-1} \qquad (33)$$

where $\hat{D}_t^L$ and $\hat{\sigma}_t^L$ represents the estimate of mean lead time and the estimate of standard deviations of L period forecast error respectively and e_t is the one period forecast error.. Using variance and covariance Dt $Var(q_t)$ can be determined with the following equation.

$$Var(q_t) = \left((1 + L/p)^2 Var(D_{t-1}) - 2(L/p)(1 + L/p)\right)$$

$$\text{x } Cov(D_{t-1}, D_{t-p-1}) + (L/p)^2\ Var((D_{t-p-1}) \qquad (34)$$

$$+ z^2 Var(\hat{\sigma}_{et}^L - \hat{\sigma}_{et-1}^L) + 2z(1 + 2L/p)xCov(D_{t-1}, \hat{\sigma}_{et}^L)$$

and $Var(q)/Var(D)$ have the following relation:

$$\frac{Var(q)}{Var(D)} \geq 1 + \left(\frac{2L}{p} + \frac{2L^2}{p^2}\right)(1 - \rho^p) \qquad (35)$$

where p is the number of observations.

Makui *et.al.* [34] proposed another approach for quantifying WE using Lyapunov exponent (LPE). Authors stated that LPE; which may use for quantification of the irregularities of non-linear system dynamics, may also be use for quantifying BWE for Cdi and nonCdi if LPE is sensed as a factor for expanding an error term of a system. The relation between $Var(q^s)$ and $Var(D)$ can be stated as ;

$$Var(q^s) \Big/ Var(D) = 1 + \left(\left[(2\sum_{j=1}^{s} j)/n\right] + \left[2(\sum_{j=1}^{s} Lt_j)^2/n\right]\right) \text{x } (1 - \rho^n) \qquad (36)$$

where n denotes the number of observations for the calculation of forecast value of demand average and Lt_s is the lead time at stage s

4 The Simulation Model

In this study a near beer distribution game model is used; which is extended from the Paik's [35] model with predetermined cost items (holding, setup / production), inventory restrictions, production restrictions and delay functions. The model proposed here, is simply a two staged Sc system consists of a retailer and a factory. Due to its common use and successful primed fuzzy functions; MatLab is the adjudicated simulation tool. Demand information in each period can be either crisp or fuzzy depending on the forecasting model that will be analyzed; similar to our previous model [2]. The model evaluates the Sc performance by computing the ratio of demand variances of consequent stages; i.e. $VarD_{S_i}/VarD_{S_{i+1}}$, where D_{S_i} denotes the demand from stage S to upstream stage S_{i+1} (i.e $i = c, r, f$ where c, r, f represent customer, retailer, factory respectively). The cost, delay and factory production capacity parameters are variable and their values depend on the analyzer. The generic decision rule in each time period t can be summarized with the following equation [2, 6].

$$\begin{aligned} Upstream\ Order\ \ Quantity = \big[&Forecat\ \text{Value} \\ &+ Correction\ of\ \ Invetory \\ &+ Correction\ for\ Supply\ Line\big] \end{aligned} \qquad (37)$$

The ordering decision system of the beer game model is illustrated in Fig.2 [6, 35]. Simple exponential smoothing (EXS) model is used as a crisp forecasting technique for comparison. The formulation of EXS is as follow;

$$F_{(t)} = \alpha D_{t-1} + F_{t-1}(1 - \alpha) \qquad (38)$$

where F_t is the forecast value for period t, D_{t-1} is observation of demand in period $t-1$, F_{t-1} is the calculated forecast value of the previous period $t-1$ and α is the smoothing constant; $0 < \alpha \leq 1$.

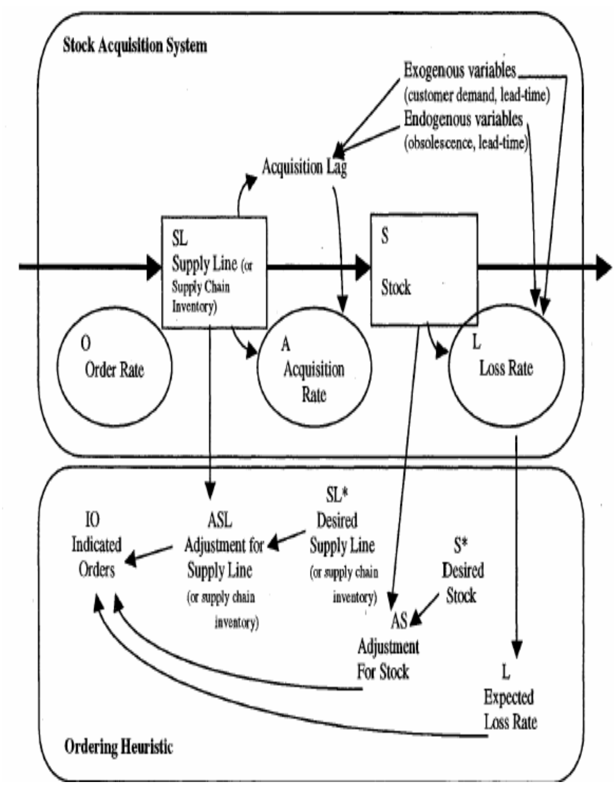

Fig.2. The ordering decision system of the beer game

The customer order received from retailer in period t is taken as a base for forecasting the forthcoming demand, and each time the order received, forecasting function update its structure according to the new demand information. After estimating the forthcoming demand; simulation model, using the decision rule in (37) and other parameters (i.e. cost, lead time, availability, demand pattern etc.), makes an ordering decision to upper echelon of Sc. And the ratio of variability between the customer orders, retailer orders and manufacturing decision of factory shows the performance of Sc system based on the selected forecasting model. The value of adjustment parameters used for correction of inventory and supply line is the same as Sterman's and Paik's model.

The formulation of system structure in each stage is as follow;

$$\begin{aligned} OD_{i,t} &= FW_t + \alpha(DINV_{i,t} - INV_{i,t}) \\ &\quad + \alpha\beta(SD_{i,t} - SA_{i,t}) \end{aligned} \tag{39}$$

$$OB_{i,t} = OB_{i,t-1} + IO_{i,t-1} - OS_{i,t-1} \tag{40}$$

$$OS_{i,t} = \begin{cases} OB_{i,t} & \text{if } INV_{i,t} \geq OB_{i,t} \\ INV_{i,t} & \text{else} \end{cases} \tag{41}$$

$$INV_{i,t} = INV_{i,t-1} + (IS_{i,t-1} - OS_{i,t-1}) \tag{42}$$

$$DINV_{i,t} = SC_i \text{x} FW_{i,t} \tag{43}$$

$$SD_{i,\text{t}} = FW_{i,\text{t}} \text{x} DL_{\text{i}} \tag{44}$$

$$SA_{i,t} = BD_{i,t} + OM_{i,t} + MA_{i,t} + OB_{i,t} \tag{45}$$

$$BD_{i,t} = BD_{i,t-1} + (OD_{i,t} + IO_{i+1,t}) \tag{46}$$

$$OM_{i,t} = OM_{i,t-1} + (DS_{i,t} - IO_{i+1,t}) \tag{47}$$

$$MA_{i,t} = MA_{i,\,t-1} + (OS_{i+1,t} - IS_{i,t}) \tag{48}$$

where, OD is the order decision, FW is the forecast value determined from the selected forecasting model, INV and $DINV$ are inventory and desired inventory, SA and SD are the supply line and desired supply line, OB is the orders backlogged, IO is the incoming orders, OS and IS are outgoing and incoming shipments, SC is the safety constant, DL is the total delay in stage i at period t. α and β represents the adjustment parameters for inventory and supply line respectively.

5 Experiment

The following figures in the next page illustrate randomly generated D_c (the same for all simulation runs), and calculated D_r and D_f values derived from the simulations using selected forecasting methods for a time horizon of 30 periods. And to reflect the response of Sc performance to the selected forecasting model, the calculated standard deviation values are given Table-1 (variance values can also obtain using Table-1). In the simulations, the production capacity of factory is taken as 100 units per period and on hand inventory in each echelon at time zero is set to again 100 units. The set of D_c values for 30 periods; D_{set}, are given below..

D_{set}={66.2297; 62.7269; 76.2016; 56.5420; 36.5401; 47.0135; 1.0196 ; 59.4657; 52.3389; 38.1779; 36.9058; 28.3868; 49.0454; 57.5869; 43.3928; 40.0020; 49.2804; 46.5048; 30.8547; 75.8510; 58.8182; 75.6188; 40.0454; 27.6257; 66.1530; 50.8240; 34.8758; 48.2174; 9.8230; 71.6784}.

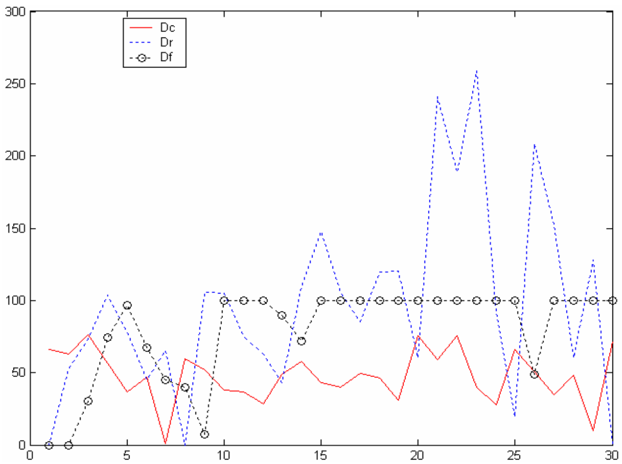

Fig.3. The demand response to EXS forecasting model (i.e. W.E.)

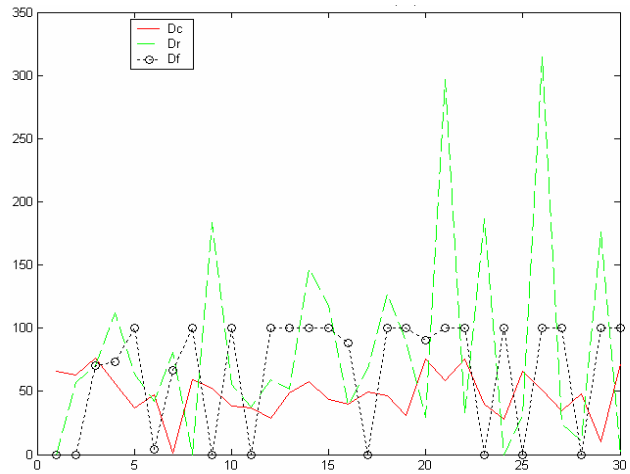

Fig.3. Demand response to FR forecasting model

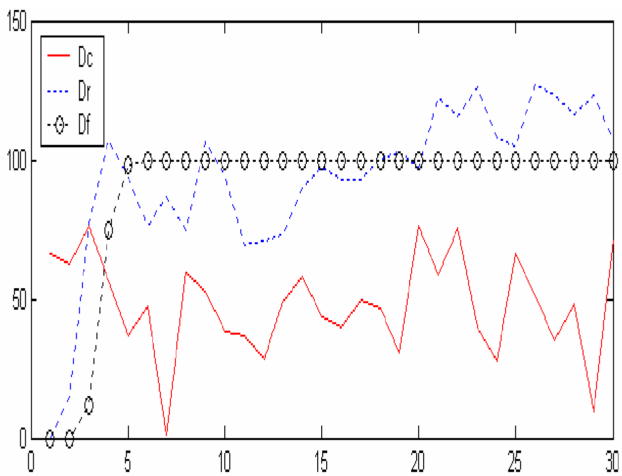

Fig.2. Demand response to FR forecasting model

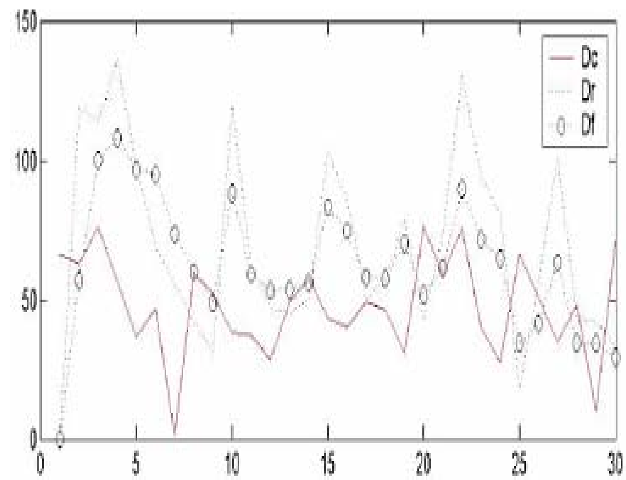

Fig.2. Demand response to FR forecasting model

	Forecasting Model			
	EXS	**FR**	**FTs**	**FGG**
Sd_c	18.24	18.24	18.24	18.24
Sd_r	65.39	28.86	34.58	81.13
Sd_f	33.28	29.42	23.8	44

Table-1. Standard deviation values

6 Research Findings and Conclusion

In this study the effects of selected fuzzy forecasting models; FR, FTs and FGG, on Sc performance are analyzed using computed demand variability as a ratio of variances of consequent stages (i.e.WE). For comparing the obtained results of the simulation using fuzzy forecasting, EXS forecasting model is chosen as a base crisp forecasting model. A simple numerical example is made using random generated demand data. The results exposed that the fuzzy forecasting models used in the study; except FGG, quickly captured the demand pattern and considerably increased the performance of proposed Sc system decreasing demand variability and through the chain. Further researches can be made using fuzzy lead times as to more adapt the model to complex real-world Sc systems.

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