Redundant coverage for noise reduction in dynamic sensor networks

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Abstract: The paper deals with the problem of computing trajectories for a network of mobile sensors in order to maximize coverage within a fixed time interval avoiding collisions and maintaining communication network connectivity. A formulation of the field coverage problem is given in terms of optimal control and it is shown how the problem of a redundant coverage, useful for simultaneous multiple measurements and then noise reduction, can be handled within the same formulation. The redundancy is formally faced in terms of K-coverage, such that covering each point of the workspace with at least K different sensors. Global centralized solutions are computed after discretization and simulation results are reported to show the effects of the proposed approach.

Key–Words: Dynamic sensor network, noise reduction, heterogeneous sensors, K-coverage, optimal motion, network communications.

1 Introduction

Coverage represents a key measure of the quality of service provided by a sensor network. Considering static sensors, the coverage problem has usually been addressed in terms of optimal usage of a given set of sensors, randomly deployed, in order to assure full coverage and minimize energy consumption [2, 28, 25, 19, 20], or in terms of optimal sensors deployment on a given area as in [22, 23, 5, 18, 29].

The introduction of mobile sensors allow to configure networks in which sensors, starting from an initial random deployment, compute and move trough optimal locations ([21, 16, 8, 6, 24, 26]).

Once the sensors can move autonomously in the environment, the measurements can be performed also during the motion (*dynamic coverage*). Under the assumption, reasonable in many applications, that synchronous or asynchronous discrete time measures are acceptable instead of continuous ones, the number of sensors can be strongly reduced. Moreover, faults or critical situations can be faced and solved more efficiently, simply changing the paths of the working sensors dynamically. Clearly, coordinated motion of such *dynamic sensors network*, imposes additional requirements, such as avoiding collisions or preserving communication links between sensors. In order to better motivate why and when a mobile sensor network can be a more successful choice than a static one, some considerations are reported, even in an approximated way.

If a dynamic network is considered, the area covered by sensors is a function of time and, clearly, it does not decrease as time passes. A simplified discrete time model of the evolution of the area still uncovered, at (discrete) time $t = k+1$, by a dynamic sensors network moving with the strategy proposed in this paper, can be given by the following differences equation

$$A_u(k+1) = \left(1 - \frac{\dot{A}_N}{A_{tot}}\right) A_u(k) \qquad (1)$$

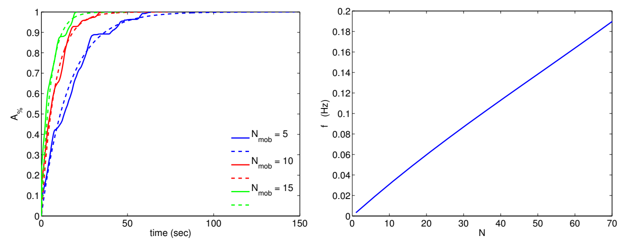
where

$$\dot{A}_N = \frac{v_{max}}{2\rho_S} A_{tot} \left(1 - \left(1 - \frac{\pi\rho_S^2}{A_{tot}}\right)^N\right)$$

represents the area covered in the time unit by a number N of mobile sensors subject to the maximum motion velocity v_{max}. Measurements are then modeled as obtained deploying randomly N static sensors on the workspace every $\frac{2\rho_S}{v_{max}}$ seconds.

Denoting by

$$A_u(0) = A_{tot} \left(1 - \frac{\pi\rho_S^2}{A_{tot}}\right)^N$$



(a) (b)

Figure 1: Comparison between coverage evolution obtained by the model (1) (*dashed*) and simulations of the proposed coverage strategy (*solid*) for different numbers of moving sensors

the area still uncovered at initial time, at each discrete time $t = k > 0$ the fraction of area covered is given by

$$A_{\%}(k) = 1 - \frac{A_u(k)}{A_{tot}} = 1 - \left[\frac{A_u(0)}{A_{tot}} \left(1 - \frac{\dot{A}_N}{A_{tot}} \right)^k \right] \quad (2)$$

The evolution computed using (2) with $N = 5$, $N = 10$ and $N = 15$ has been compared with the results of simulations where the approach described in the paper is applied. In Fig. 1 this comparison is reported, showing that (2) is a good model for describing the relationship between the area covered and the time using a dynamic solution with N sensors.

Then, referring to surveillance tasks, (2) can be used to evaluate the minimum number of sensors (with given ρ_S and v_{max}) required to cover a given fraction $\tilde{A}_{\%}$ of the area of interest according to a given measurement rate. In fact, it is possible to write the relation between the maximum rate at witch the network can cover the $\tilde{A}_{\%}$ fraction of A_{tot} and the number of moving sensors as

$$f = \frac{\log\left(1 - \frac{\dot{A}_N}{A_{tot}}\right)}{\log\left(1 - \tilde{A}_{\%}\right) - N \log\left(1 - \frac{\pi \rho_S^2}{A_{tot}}\right)} \quad (3)$$

Such a relationship between N and f is depicted in Fig. 1, showing, as intuitively expected, almost a proportionality between number of sensors and frequency of measurement at each point of the area A_{tot}.

The motivation and the support of the dynamic solution is evidenced by Fig. (1): lower is the refresh frequency of the measurements at each point (that is greater are the time intervals between measurements) and lower is the number of sensors required, once sensors motion is introduced.

Under the assumption of dynamic network, the area coverage problem is posed in terms of looking for optimal trajectories for the N moving sensors in presence of some constraints like communication connection preservation, motion limitations, energetic considerations and so on. In [26, 3] the dynamic coverage problem for multiple sensors is studied , with a variational approach, in the level set framework; obstacles occlusions are considered and suboptimal solutions are proposed also in three dimensional environments ([4]). A survey of coverage path planning algorithms for mobile robots moving on the plane is presented in [7]. In [1] the dynamic coverage problem for one mobile robot with finite range detectors is studied and an approach based on space decomposition and Voronoy graphs is proposed.

In [17], a distributed control law that guarantees to meet the coverage goal with multiple mobile sensors under the hypothesis of communication network connection is developed. Collisions avoidance is considered.

Various problems associated with optimal path planning for mobile observers such as mobile robots equipped with cameras to obtain maximum visual coverage in the three-dimensional Euclidean space are considered in [27]. Numerical algorithms for solving the corresponding approximate problems are proposed.

In [10] a general formulation of dynamic coverage is given by the authors, a sensor network model is proposed and a centralized optimal control formulation is given.

A distributed approach to the dynamic coverage has been proposed in [13, 12]. All constraints are considered including connectivity maintenance without imposing a fixed topology to the communication network.

In this paper dynamic k-coverage problem is considered. A point in the workspace is k-covered when it is *covered* at least by k different sensors. K is a configurable parameter, and larger value of K could be used to increase robustness to sensors failure ([14]), or to deal with noisy measurements ([11]).

Assuming, for example, that a give magnitude of interest ξ is time invariant (or slowly varying) and that sensors measurements $\tilde{\xi}$ are affected by independent Gaussian noises $\nu = \mathcal{N}(0, \sigma)$. The mean of K measures allow to reduce the estimation variance of a factor K. Solutions for the k-coverage problem in static sensor networks has been proposed in [29] and in [15]. In this paper the dynamic case is addressed. The formulation here introduced allows to face the problem of K-coverage, including dynamic, geometric and communication constraints. In this manner it is possible to increase the reliability of the sensors network. Assuming that m sensors allow to reach the desired coverage performances w.r.t. the magnitude of interest ξ, it is possible to reach K-coverage augmenting the number of sensors of a factor K and formulating opportunely the coverage problem. The problem is modeled as a particular case of a multiple magnitudes dynamic coverage problem. Suboptimal solution are computed by discretization. Sensors and actuators limits, geometric constraints, collisions avoidance, and communication network connectivity maintenance are considered.

In section 2 a general formulation of the problem is proposed. In section 3 some simplifying hypothesis are introduced . In section 5 a discretization is performed in order to reduce the computational complexity of the k-coverage nonlinear programming problem. Simulation results are proposed in 6. Some final considerations in 7 end the paper.

2 General Formulation

Let be W a compact subset of the real Euclidean space called the *workspace*. Let $\Xi = \{\xi_1, \xi_2, \dots\}$ be the set of magnitudes of interest defined on W. A dynamic sensor network Σ is composed by agents σ_j, called sensors or nodes, able to move, to perform measures on W and to communicate each other.

$$\Sigma = \{\sigma_1, \sigma_2, ..., \sigma_m\}$$

More formally, each *sensor* σ_j can be represented by

- a configuration space $\mathscr{C}^{(j)}$, that is the space of possible positions ($q^{(j)}$) that the sensor may attain;
- a dynamic model that describe the evolution of sensor configuration in time, according to a control input $u^{(j)}$ and that can be express by
$$f^{(j)}(\ddot{q}^{(j)}, \dot{q}^{(j)}, q^{(j)}, u^{(j)}) = 0$$
- a set $\Xi^{(j)} \subseteq \Xi$, that is the subset of magnitudes that the sensor can measure;
- for every magnitude $\xi \in \Xi^{(j)}$, an *active* set $M_\xi^{(j)} = M_\xi^{(j)}(q^{(j)}) \subseteq W$, that is the subset of W within the sensor in configuration $q^{(j)}$ can measure magnitude ξ.

Looking at the whole network it is possible to define generalized configuration $q = \{q^{(1)}, q^{(2)}, \dots, q^{(m)}\}$ and generalized input $u = \{u^{(1)}, u^{(2)}, \dots, u^{(m)}\}$.

Generalized dynamic model can written as:

$$F(\ddot{q}, \dot{q}, q, u) = 0$$

To describe communication between sensors a *communication function* must be defined for every couple of them:

$$\kappa^{(j,h)}\left(q^{(j)}, q^{(h)}\right) : \mathscr{C}^{(j)} \times \mathscr{C}^{(h)} \longrightarrow \mathbb{R}$$

Sensor σ_j can directly communicate with sensor σ_h if and only if $\kappa^{(j,h)}\left((q^{(j)}, q^{(h)})\right) > 0$.

Sensors can then be regarded as nodes of the communication network that con be represented by the graph

$$\mathscr{G}_\Sigma(t) = < V_\Sigma, E_\Sigma(q) >$$

where

$$V_\Sigma = \{1, 2, \dots, m\}$$

indicate the vertexes set and

$$E_\Sigma(q) = \{(j,h) \in V_\Sigma \times V_\Sigma \,|\, \kappa^{(j,h)}(q^{(j)}, q^{(h)}) > 0\}$$

indicates the edges set. The edges set depends from the network generalized configuration.

While sensors move, network configuration changes in time, so the communication graph, and in particular its edges set, is time varying.

2.1 Coverage and K-Coverage

Denote with $q(\Theta)$ the evolution of the generalized network configuration during a given time interval Θ. It is possible to define the subset of W on witch the magnitude ξ has been measured as

$$M_\xi(q(\Theta)) = \bigcup_{j \,|\, \xi \in \Xi^{(j)}} M_\xi^{(j)}(q^{(j)}(\Theta)) \tag{4}$$

Looking at the whole magnitudes set, the subset of W on witch all magnitudes $\xi \in \Xi$ have been measured by the network can be defined as

$$M_\Xi(q(\Theta)) = \bigcap_{\xi \in \Xi} M_\xi(q(\Theta)) \tag{5}$$

The area covered by the sensor network with respect to the whole magnitudes set during Θ is then the measure of $M(q(\Theta))$

$$A_\Xi(\Theta) = \mu(M_\Xi(q(\Theta))) \tag{6}$$

K-coverage is a particular case and can be addressed modifying the magnitudes set (Ξ) and the sets of the magnitudes that can be measured by each sensor ($\Xi^{(j)}$). For every magnitude ξ that must be k-covered (measured by k different sensors), let us modify the magnitudes set Ξ replacing ξ with k virtual magnitudes $\{\hat{\xi}^{(1)}, \hat{\xi}^{(2)}, \dots, \hat{\xi}^{(K)}\}$. Every virtual magnitude can be measured within the same radius ρ_ξ.

Consider a partition of the set of the sensors that are able to measure ξ ($\Sigma^{\xi} = \{\sigma_j \mid \xi \in \Xi^{(j)}$) composed by k sets $\{\Sigma_1^{\xi}, \Sigma_2^{\xi}, \ldots, \Sigma_k^{\xi}\}$. For every sensor $\sigma_j \in \Sigma_i^{\xi}$ let us modify the set $\Xi^{(j)}$, that is the set of the magnitudes that it can measure, replacing ξ with the virtual magnitude $\hat{\xi}^{(i)}$

$$\hat{\xi}^{(i)} \in \Xi^{(j)} \iff \sigma_j \in \Sigma_i^{\xi} \quad \forall i$$

After this manipulation, the objective function (6) can be used to evaluate the k-coverage.

3 The Dynamic Sensor Network Model

In this section some hypothesis are introduced on the elements of the general model presented in section 2. The so obtained simplified model for a dynamic sensor network allows to face in a more easy way the k-coverage problem.

3.1 Sensors Dynamics

Sensors are modeled, from the dynamic point of view, as material points moving on the Euclidean plane, so

$$C^{(j)} = \mathbb{R}^2 \quad \forall j$$

. For sake of simplicity, all sensors are assumed to have unitary mass. This hypothesis could be easily relaxed. According to the classical simple formulation, the dynamic model is given by

$$f^{(j)}(\ddot{q}^{(j)}, \dot{q}^{(j)}, q^{(j)}, u^{(j)}) = \ddot{q}^{(j)} - u^{(j)} = 0 \qquad (7)$$

The linearity of (7) allows one to write the dynamics in the form

$$\begin{array}{l} \dot{x}^{(j)}(t) == A_j x^{(j)}(t) + B_j u^{(j)}(t) \\ q^{(j)}(t) = C_j x^{(j)}(t) \end{array} \qquad (8)$$

where

$$x^{(j)} = \left(\begin{array}{cccc} \dot{q}_1^{(j)} & q_1^{(j)} & \dot{q}_2^{(j)} & q_2^{(j)} \end{array} \right)$$

represent the $j-th$ sensor state. The evolutions of state and position (output) depend, as well known, on the input and on the starting conditions, that is

$$x^{(j)}(t) = \phi_j(x^{(j)}(0), u^{(j)}(t)) \qquad (9)$$

and

$$q^{(j)}(t) = \psi_j(x^{(j)}(0), u^{(j)}(t)) \qquad (10)$$

Once the whole network is considered,

$$x(t) = \left(x^{(1)}(t) \quad x^{(2)}(t) \quad \cdots \quad x^{(m)}(t) \right)^T$$

can be defined to denote the generalized state, and the vector

$$u(t) = \left(u^{(1)}(t) \quad u^{(2)}(t) \quad \cdots \quad u^{(m)}(t) \right)^T$$

to denote the generalized input.

Generalized dynamics for the whole network can be written as

$$\begin{array}{l} \dot{x}(t) = Ax(t) + Bu(t) \\ q(t) = Cx(t) \end{array}$$

According to (9) and (10), generalized state and output are related to the generalized input by

$$x(t) = \Phi(x(0), u(t)) = \begin{pmatrix} \phi_1(x^{(1)}(0), u^{(1)}(t)) \\ \phi_2(x^{(2)}(0), u^{(2)}(t)) \\ \vdots \\ \phi_m(x^{(m)}(0), u^{(m)}(t)) \end{pmatrix} \qquad (11)$$

and

$$q(t) = \Psi(x(0), u(t)) = \begin{pmatrix} \psi_1(x^{(1)}(0), u^{(1)}(t)) \\ \psi_2(x^{(2)}(0), u^{(2)}(t)) \\ \vdots \\ \psi_m(x^{(m)}(0), u^{(m)}(t)) \end{pmatrix} \qquad (12)$$

.

3.2 Proximity Measure Model

In the present formulation, it is assumed that at any time t sensor σ_j can take measures on magnitude $\xi \in \Xi^{(j)}$ in a circular area of radius ρ_{ξ} around its current position $x^{(j)}(t)$.

The sensor field of measure respect to ξ is then a disk of center $x^{(j)}(t)$ and radius ρ_{ξ}

$$M_{\xi}(q^{(j)}) = \{p \in W : \|x^{(j)} - x_p\| \leq \rho_{\xi} \quad \xi \in \Xi^{(j)}\} \qquad (13)$$

To simplify the notation, the assumption that the radius ρ_{ξ} is the same for all sensors that are able to measure ξ has been done. This assumption can be relaxed considering different sensing radii $\rho_{\xi}^{(j)}$.

As seen in section 2, starting from $M_{\xi}^{(j)}(q^{(j)})$ it is possible to define the area k-covered with respect to a single magnitude ξ or to the whole magnitude set Ξ during a given time interval Θ.

3.3 Proximity Communication Model

For the communication among sensors the well known proximity model is assumed, that is two sensors can communicate directly if they are *near enough*. The communication function, for every couple of sensors, is given by

$$\kappa^{(j,h)}(q^{(j)},q^{(h)}) = \rho_C - \|q^{(j)} - q^{(h)}\| \tag{14}$$

The communication network can then be modeled as an Euclidean graph. Two mobile sensors at time t are assumed to communicate each other if the distance between them is lower than a given communication radius ρ_C. The structure of the communication function, and in particular the fact that the communication radius is the same for all the network nodes, makes the communication network graph $\mathscr{G}_\Sigma$ undirected. In fact

$$\kappa^{(j,h)}(q^{(j)},q^{(h)}) = \kappa^{(h,j)}(q^{(h)},q^{(j)}) \quad \forall j,h$$

This assumption is very useful to simplify the problem of constraining nodes motion in order to maintain the communication network connectivity.

4 K-Coverage Problem Formulation

According to the model introduced in section 3, it is possible to formulate the k-coverage problem, that is the problem of maximizing the area simultaneously covered by K different sensors, as an optimal control problem. The idea is to maximize the area k-covered by sensors in a fixed time interval according to the given constraints.

4.1 Objective functional

The objective functional in terms of (6) is a quantity very hard to be computed, even for the simple measure set model introduced in subsection 3.2. It would be better to use an alternative performance measure.

Defining the distance between a point p of the workspace and a generalized trajectory $q(\Theta)$, within a time interval Θ, as

$$d_\xi(q(\Theta),p) = \min_{t\in\Theta, j\in\{1,2,\ldots m\}} \left\| p - q^{(j)}(t) \right\| \qquad \xi \in \Xi^{(j)} \tag{15}$$

and making use of the function

$$\mathrm{pos}(\chi) = \begin{cases} \chi & \text{if} \quad \chi > 0 \\ 0 & \text{if} \quad \chi \leq 0 \end{cases} \tag{16}$$

that fixes to zero any non positive value, the function

$$\hat{d}_\xi(q(\Theta),p,\rho_\xi) = \mathrm{pos}\left(d_\xi(q(\Theta),p) - \rho_\xi\right) \geq 0$$

can be defined. Then, a measure of how much the generalized trajectory $q(\Theta)$ produces a good coverage of the workspace with respect to magnitude ξ can be given by

$$J_\xi(q(\Theta)) = \int_{p\in W} \hat{d}_\xi(q(\Theta),p,\rho_\xi) \tag{17}$$

In order to evaluate how much a given generalized trajectory $x(\Theta)$ covers (or k-covers) the set of interest W with respect to the whole set of magnitudes Ξ, the following function can be considered

$$J_\Xi(q(\Theta)) = \sum_{\xi\in\Xi} J_\xi(q(\Theta)) \tag{18}$$

The minimization if $J_\Xi(x(\Theta)$ corresponds to the k-coverage maximization of W.

4.2 Geometric Constraints

It is possible to constrain sensors to move inside a box subset of $\mathbb{R}^2$

$$q_{min} \leq q^{(j)}(t) \leq q_{max}$$

If needed it is possible to set the staring and/or the final state (positions and/or speeds)

$$x(0) = q_{start}$$
$$x(t_f) = q_{end}$$

A particular case is the periodic trajectories constraint, useful in tasks in which measures have to be repeated continuously

$$x^{(j)}(0) = x^{(j)}(t_f)$$

It is also necessary to avoid collisions between sensors. Then at any time t

$$\|q^{(j)}(t) - q^{(h)}(t)\| \geq \rho_B$$

for $j \neq h$

4.3 Dynamic Constrains

Physical limits on the actuators (for the motion) and/or on the sensors (in terms of velocity in the measure acquisition) suggest the introduction of the additional constraints

$$|\dot{q}(t)| \leq v_{max}$$
$$|u(t)| \leq u_{max}$$

4.4 Communication connectivity constraints

Communication aspects are very important, since the mobile units constitute also the communication network used for data exchange and transmission (measurement data transfer, sensor localization, coordination and commands communications). In order to assure communication between sensors, a full connectivity of the sensors network is required. This can be imposed introducing some motion constraints.

As said in subsection 3.3, the communication graph $\mathscr{G}_\Sigma$ is undirected. An undirected graph is connected if it contains a spanning tree. Assuming $\mathscr{G}_\Sigma$ to be connected at time $t = 0$, it is possible to maintain network connectivity just maintaining links that belong to a spanning tree. Weighting edges of $\mathscr{G}_\Sigma$ with the Euclidean distance between their vertex

$$w(j,h) = \|q^{(j)} - q^{(h)}\|$$

it seems natural to choose the *Minimum Spanning Tree* (figure 2), that in this case is said to be *Euclidean* (EMST), that is the spanning tree with minimum weight and then the less constraining one for nodes motion. The EMST can be easily and efficiently computed by standard well known algorithms (for example Kruskal's algorithm, Prim's algorithm, etc. [9]). Denoting the EMST with $\mathscr{T}_\Sigma(t)$, in order to maintain communication network connectivity the following constraints must be imposed to sensors relative positions $\forall t$

$$\|q^{(j)}(t) - q^{(h)}(t)\| \leq \rho_C \quad \forall (j,h) \in E_{\mathscr{T}_\Sigma}(t) \tag{19}$$

This approach can be used only in a centralized control architecture, in which a central computer on the basis of global informations, evaluates inputs for all the network nodes.

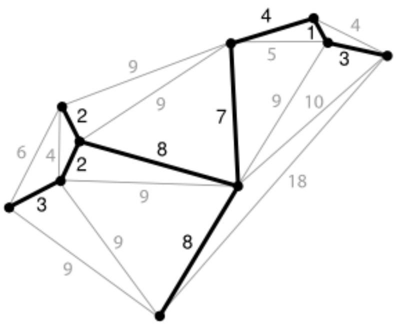


Figure 2: Minimum Spanning Tree for a planar weighted undirected graph

4.5 Optimal control problem formulation

On the basis of the previous considerations, the optimal control formulation of the k-coverage problem is expressed as

$$\min_{x(0),u(\Theta)} J_\Xi(q(\Theta))$$

$$\begin{gathered} q_{min} \leq q(t))) \leq q_{max} \quad \forall t \in \Theta \\ (x(0) = x_{start}) \\ (x(t_f) = x_{end}) \end{gathered}$$

$$\begin{gathered} v_{min} \leq \dot{q}(t) \leq v_{max} \quad \forall t \in \Theta \\ u_{min} \leq |u(t)| \leq u_{max} \quad \forall t \in \Theta \end{gathered}$$

$$\|q^{(j)}(t) - q^{(h)}(t)\| \leq \rho_C \quad \forall (j,h) \in E_{\mathscr{T}_\Sigma}(t)$$

5 Discretized Model

In order to overcome the difficulty of the problem defined in the previous subsection, a discretization is performed, both with respect to space W and with respect to time t in all the time dependent expressions. The workspace W is then divided into square cells, with resolution (size) l_{res}, obtaining a grid in witch each point c_{rs} is the center of a cell, and the trajectories are discretized with sample time T_s. This allows to represent the k-coverage problem as a solvable *Nonlinear Programming Problem*.

5.1 Sensors Discretized Dynamics

The discrete time sensors dynamic is well described by the following equations

$$\begin{aligned} x^{(j)}((k+1)T_s) &= A_{dj}x^{(j)}(kT_s) + B_{dj}u^{(j)}(kT_s) \\ q^{(j)}(kT_s) &= C_j x^{(j)}(kT_s) \end{aligned} \tag{20}$$

where

$$A_{dj} = e^{A_j T_s} \quad B_{dj} = \int_0^{T_s} e^{A_j \tau} B_j d\tau$$

Representing the $j-th$ sensor input sequence from time $t = 0$ to time $t = NT_s$ as

$$u_N^{(j)} = \begin{pmatrix} u^{(j)}(0) \\ u^{(j)}(T_s) \\ \vdots \\ u^{(j)}((N-1)T_s) \end{pmatrix}$$

and defining the following vectors

$$v_N^{(j)} = \begin{pmatrix} q^{(j)}(0) \\ u_N^{(j)} \end{pmatrix} \qquad H_n^{(j)} = \begin{pmatrix} A_{dj}^n \\ A_{dj}^{n-1}B_{dj} \\ \vdots \\ B_{dj} \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

it is possible to write state and output values at time $nT_s \leq NT_s$ as

$$x^{(j)}(nT_s) = {H_n^{(j)}}^T v_N^{(j)} \tag{21}$$

and

$$q^{(j)}(nT_s) = C_j x^{(j)}(nT_s) = C_j {H_n^{(j)}}^T v_N^{(j)} \tag{22}$$

State and output sequences, from time $t = 0$ to time $t = NT_s$, can be represented by the following vectors

$$x_N^{(j)} = \begin{bmatrix} x^{(j)}(0) \\ x^{(j)}(T_s) \\ \vdots \\ x^{(j)}(NT_s) \end{bmatrix} \qquad q_N^{(j)} = \begin{bmatrix} q^{(j)}(0) \\ q^{(j)}(T_s) \\ \vdots \\ q^{(j)}(NT_s) \end{bmatrix}$$

According to (21) and (22) the relations between these sequences and the input ones are described by

$$x_N^{(j)} = H^{(j)} v_N^{(j)} = \begin{pmatrix} {H_0^{(j)}}^T \\ {H_1^{(j)}}^T \\ \vdots \\ {H_N^{(j)}}^T \end{pmatrix} v_N^{(i)} \tag{23}$$

and

$$x_N^{(j)} = C^{(j)} H^{(j)} v_N^{(j)} \tag{24}$$

where

$$C^{(j)} = \begin{pmatrix} C_j & 0 & \cdots & 0 \\ 0 & C_j & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & C_j \end{pmatrix}$$

As performed in subsection 3.1, it is possible to define generalized input, state and output sequences of the whole system

$$v_N = \begin{bmatrix} v_N^{(1)} \\ v_N^{(2)} \\ \vdots \\ v_N^{(m)} \end{bmatrix}$$

$$x_N^{(i)} = \begin{bmatrix} x_N^{(1)} \\ x_N^{(2)} \\ \vdots \\ x_N^{(m)} \end{bmatrix} \qquad q_N^{(i)} = \begin{bmatrix} q_N^{(1)} \\ q_N^{(2)} \\ \vdots \\ q_N^{(m)} \end{bmatrix}$$

related by means of the relationships

$$x_N = H v_N = \begin{pmatrix} H^{(1)} & 0 & \cdots & 0 \\ 0 & H^{(2)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & H^{(m)} \end{pmatrix} v_N \tag{25}$$

and

$$q_N = C H v_N \tag{26}$$

where

$$C = \begin{pmatrix} C^{(1)} & 0 & \cdots & 0 \\ 0 & C^{(2)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & C^{(m)} \end{pmatrix}$$

5.2 Objective Function

The objective functional defined in subsection 4.1 becomes, after the discretization, a function of the vector v_N

$$J_\Xi(v_N) = \sum_\xi \sum_r \sum_s \hat{d}_\xi (CHv_N, c_{rs}, \rho_\xi) \tag{27}$$

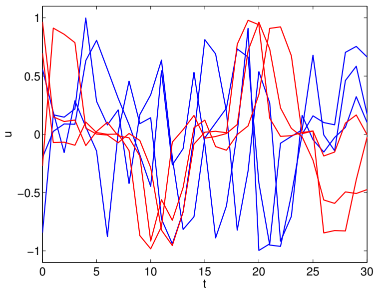
5.3 Nonlinear Programming Problem

Defining geometric, dynamic and communication constraints as in section 4, it is possible to write the coverage problem for a dynamic sensors network as the constrained optimization problem

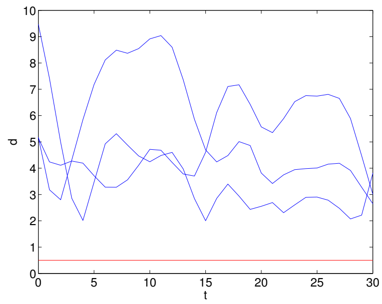
$$\min_{v_N} J_\Xi(v_N) = \sum_\xi \sum_r \sum_s \hat{d}_\xi (C_N H_N v_N, c_{rs}, \rho_\xi)$$

$$q_{min} \leq C{H_n^{(j)}}^T v_N^{(j)} \leq q_{max} \; \forall j,$$
$$\|C{H_n^{(j)}}^T v_N^{(j)} - C{H_n^{(h)}}^T v_N^{(h)}\| \geq \rho_B \; j \neq h,$$
$$({H_0^{(j)}}^T V_N^{(j)} = x_{start}^{(j)} \; \forall j)$$
$$({H_N^{(j)}}^T V_N^{(j)} = x_{end}^{(j)} \; \forall j)$$

$$|B^T {H_n^{(j)}}^T v_N^{(j)}| \leq v_{max} |u^{(j)}(nT_s)| \leq u_{max}$$
$$\|C_j {H_n^{(j)}}^T v_N^{(j)} - C_h {H_n^{(h)}}^T v_N^{(h)}\| \leq \rho_C \; \forall (j,h) \in E_{\mathscr{T}_\Sigma}$$



(a) Control Evolution



(b) Collisions Avoidance

Figure 3: Simulation 1 .(a) x (blue) and y(red) input components trajectories. (b) Distances between nodes is maintained over the safe threshold.

Suboptimal solutions can be computed using numerical methods. In the simulations performed, the SQP (Sequential Quadratic Programming) method has been applied.

6 Simulations

In this section simulations results are presented to show the effectiveness of the proposed methodology. At first, a sensor network composed by three dynamic nodes is considered. The network is asked to cover (or *1-cover*) a box set, on witch a single magnitude ξ is defined, within a time interval of 15 seconds.

$$W = \{(x,y)| \quad -6 \geq x \leq 6,\ -6 \geq y \leq 6\}$$

Sensors can get measures within a radius

$$\rho = 1.5$$

around their position. Sensors dynamic constraints are

$$|\dot{x}^{(j)}(t)| \leq 1\,\forall j$$
$$|u^{(j)}(t)| \leq 1.5\,\forall j$$

Safe distance for collisions avoidance has been fixed to

$$\rho_B = 0.5$$

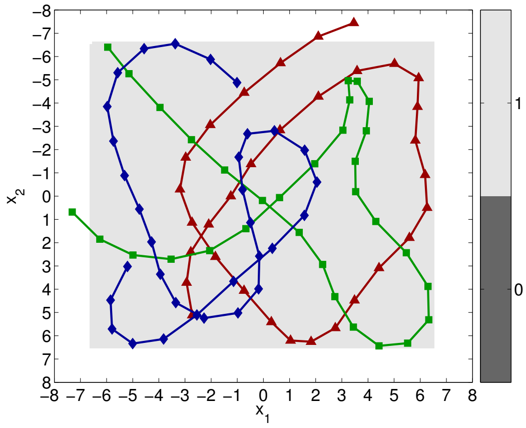


Figure 4: Simulation 1. Coverage of box shaped workspace by a three nodes dynamic sensor network. Sensors trajectories and coverage status of the workspace (1 means covered, 0 means uncovered)

The maximum distance for reliable communication is

$$\rho_C = 5.5$$

Sensors are constrained to maintain communication network connectivity. In figure 3 input trajectories computed with the proposed method are displayed together with the evolution of distances between sensors that show the effectiveness of the collisions avoidance constraints. Sensors trajectories and the coverage status of the workspace are shown in figure 4

In the second simulation the same sensor network (same parameters and constraints) of the first one is asked to *k-cover* (k=3) the workspace W. Then, following what presented in subsection 4.1, three *virtual* magnitudes $\{\hat{\xi}^{(1)}, \hat{\xi}^{(2)}, \hat{\xi}^{(3)}\}$ that can be measured within radius $\rho = 1.5$ have been defined. Each virtual magnitude is associated to one sensor

$$\Xi_1 = \{\hat{\xi}^{(1)}\} \qquad \Xi_2 = \{\hat{\xi}^{(2)}\} \qquad \Xi_3 = \{\hat{\xi}^{(3)}\}$$

The solution computed with the proposed method is shown in figures 5,6.

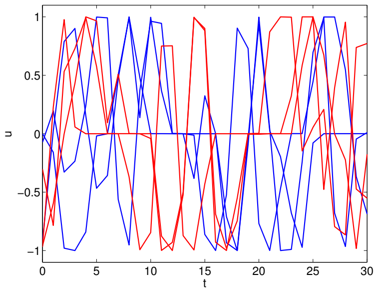
In the last simulation a network of four nodes, equal to the ones of the first two simulations, is considered. The workspace W is generic shaped and two magnitudes are defined on it ($\Xi = \{\xi_1, \xi_2\}$, $\rho_{\xi_1} = 2$, $\rho_{\xi_2} = 1$).

Network nodes sensing capabilities are:

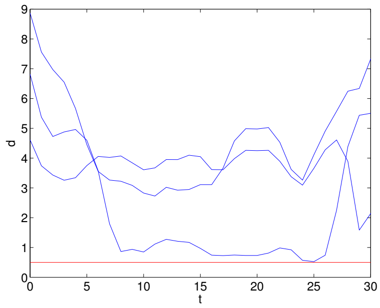
$$\Xi^{(1)} = \Xi^{(2)} = \{\xi_1\}$$
$$\Xi^{(3)} = \Xi^{(4)} = \{\xi_2\}$$

Two virtual magnitudes are then considered, each one associated to a single node.

$$\Xi_1 = \{\hat{\xi}_1^{(1)}\} \quad \Xi_2 = \{\hat{\xi}_1^{(2)}\} \quad \Xi_3 = \{\hat{\xi}_2^{(1)}\} \quad \Xi_4 = \{\hat{\xi}_2^{(2)}\}$$



(a) Control Evolution



(b) Collisions Avoidance

Figure 5: Simulation 2 .(a) x (blue) and y(red) input components trajectories. (b) Distances between nodes is maintained over the safe threshold.

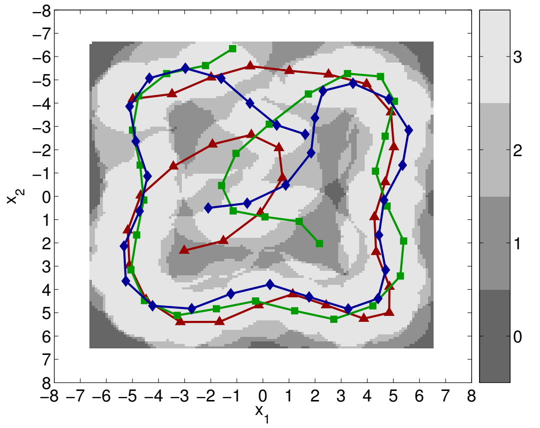


Figure 6: Simulation 2. K-Coverage of box shaped workspace by a three nodes dynamic sensor network. Sensors trajectories and coverage status of the workspace (colors correspond to the number of measurements)

Results are shown in figures 7,8,9.

7 Conclusions

In this paper the case of heterogeneous mobile sensor networks has been considered. The mobility of the sensors is introduced in order to allow a reduced number of sensors to cover the workspace. A general formulation of the field coverage problem has been introduced in terms of optimal control techniques. Constraints introduced by kinematic and dynamic limits on mobility of the moving elements as well as by communications limits (network connectivity) have been considered. The same formulation has been used to model the problem of K-coverage useful, for example, to increase measurements reliability. A global approach has been followed making use of time and space discretization. The effectiveness of the proposed solution has been shown reporting some simulation results.

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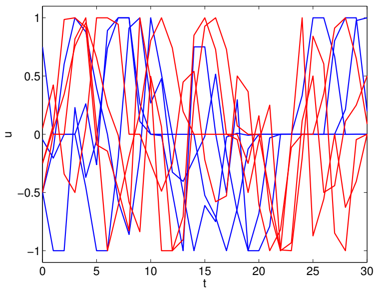
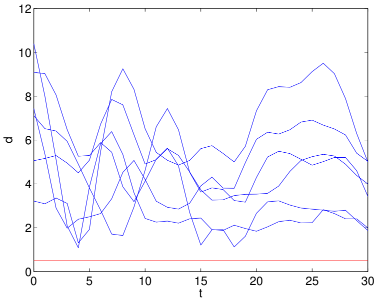


(a) Control Evolution



(b) Collisions Avoidance

Figure 7: Simulation 3 .(a) x (blue) and y(red) input components trajectories. (b) Distances between nodes is maintained over the safe threshold.

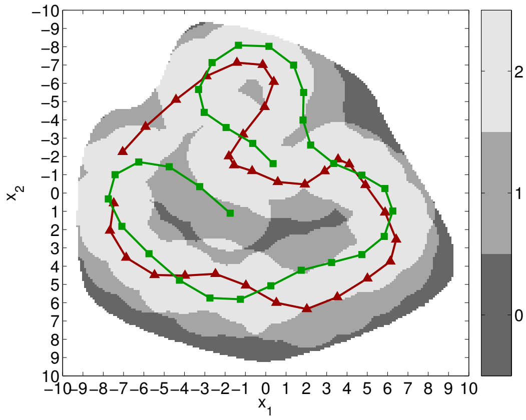


Figure 8: Simulation 3. K-Coverage of generic shaped workspace by a four nodes dynamic sensor network. Sensors trajectories and coverage status of the workspace w.r.t magnitude ξ_1 (colors correspond to the number of measurements)

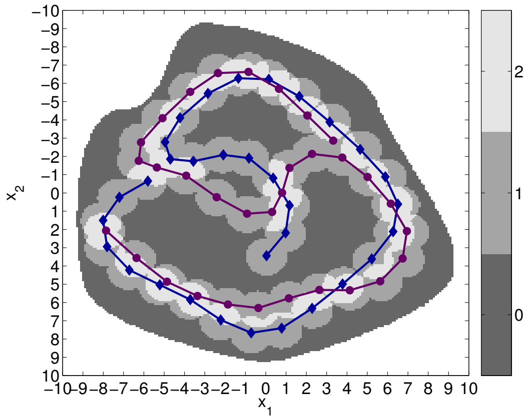


Figure 9: Simulation 3. K-Coverage of generic shaped workspace by a four nodes dynamic sensor network. Sensors trajectories and coverage status of the workspace w.r.t magnitude ξ_2 (colors correspond to the number of measurements)

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