A Plan of Lauding the Boxes for a Three Dimensional Bin Packing Model

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Abstract: We consider the rectangular three dimensional bin packing problem with knapsack, where the bin is packed with a set of rectangular boxes, without gaps or overlapping. Starting from a solution of the three dimensional bin packing model, our objective is to determine an order of the loading the boxes in the bin so that a box will be packed in the bin only if there are no empty spaces down to this box and the origin of the box is in a fixed position, determinated by the boxes situated in the West and North neighbourhood. By extension of the previous work regarding the two dimensional covering problem [12] and the three dimensional bin packing problem [15], we define three kind of adjacency relations between two boxes from a packing model, similarly with [13, 14]. Combining these relations we define an acyclic graph representation of the bin packing model. A plan for lauding of the boxes in the bin is obtained using a topological sorting algorithm of the vertices of this acyclic graph.

Key–Words: bin packing, topological sorting

1 Introduction

In computational complexity theory, the bin-packing optimization problem is a known NP-hard problem, concerns efficiently placing box-shaped objects of arbitrary size and number into a box-shaped. Such problems are also referred to as Cutting and Packing problems in [3] or Cutting and Covering in [5]. Since the advent of computer science, bin-packing remains one of the classic difficult problems today.

At this time, no optimal polynomial time algorithm is known for the bin-packing problem. In other words, finding a perfect solution to one non-trivial instance of the bin-packing problem with even the most powerful computer may take months or years.

Many kinds of bin-packing problems were considered, one dimensional, two dimensional and three dimensional with many kinds of constrains depending on technological restrictions [9].

The three-dimensional bin-packing problem retains the difficulty of lesser dimensional bin-packing problems, but holds unique and important applications. As one would expect, each object and bin exists in three dimensions: width, length, and height. The goal is to minimize the number of bins required to pack all the boxes. Like two dimensional bin-packing, each box must stay orthogonal, or maintains its orientation in the container. If two dimensional bin-packing is equivalent to rectangle-to-floor plan packing [5, 17], three dimensional bin-packing is equivalent to box-to-room packing.

The three dimensional bin-packing may involve a single bin or multiple bins. The singular bin-packing problem involves only one bin with either definite or infinite volume.

Bins with infinite volume are defined with finite length and width, but with height extending to infinity. This allows packing solutions to pack until the set of boxes are exhausted. Solutions dealing with infinitely sized bins aim to include a maximum number of objects.

Another way to approach this problem is by considering multiple boxes. Each bin has definite volume. In this way, if the volume of the objects exceeds the volume of the room, an algorithm must make choices of which boxes to include in the packing and which to throw away. This approach is good for deterministic approaches to the bin-packing problem.

These problems ask: ”Do the bins hold enough volume to fit these objects?”, and if it does: ”Can we arrange all objects in bins?”.

Like all bin-packing problems, extra constraints

may be added to the problem to match a certain real-world problem. In warehouses, gravity is a constraint meaning that all boxes must rest on the floor, box, or other platform (such as a pallet) below it.

Another constraint may be weight distribution. Real objects have mass and weight, so a packing may aim to obtain an equal weight distribution in the storage space. Placing all the weight in one location could cause a boat to tip or a truck to become unstable. Also, we may not want a large, heavy object on top of a smaller, lighter object-such as a 1 kg shoe box.

Time can be another constraint; for example, one might want a schedule that allows packing a room in a time-efficient manner.

Each of these bin-packing problems with constraints may be simplified.

Many models and algorithms exist for bin packing problem: formulation as a mixed integer program, which can solve the small sized instance to optimum [8], genetic algorithms [10], approximation algorithms [5, 7, 6, 20] or active learning method [11]. While an approximation algorithm is a method that attempts to place objects in the least amount of space and time, a mixed integer program gives solution as the position of the boxes in the bin. For this kind of situation we need a plan for packing the boxes in the bin. This is our objective, to determine the order of packing the boxes in the bin if we know the packing solution of the bin.

2 Problem formulation

Let $\mathcal{P}$ be a rectangular bin, characterized by length l, width w, height h. The bin $\mathcal{P}$ is filled with k rectangular boxes $C_1, C_2, \ldots C_k$ without gaps or overlapping. A box C_i is characterized by length l_i , width w_i, height h_i. We consider a coordinate system $xOyz$ so that the corner O is the origin of the coordinate system like in Figure 1.

We will use the following notations:

- $ABCO$ is the bottom face of the bin
- $GDEF$ is the top face of the bin
- $OADG$ is the West face of the bin
- $OCFG$ is the North face of the bin
- $EBCF$ is the East face of the bin
- $ABDE$ is the South face of the bin
- $O - C_i$ is the O-corner of the box C_i of coordinates (x_i, y_i, z_i)

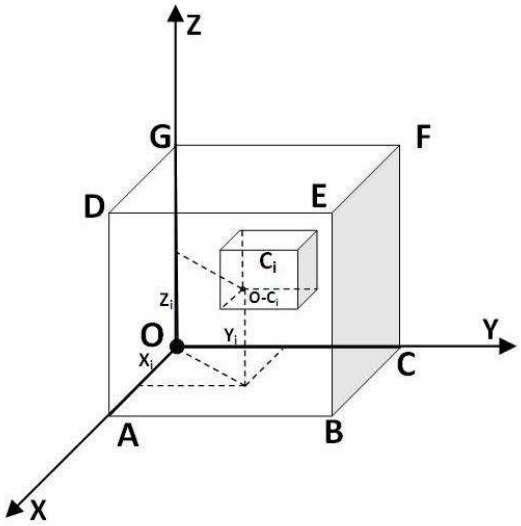

Figure 1: The position of box C_i in the bin

By extending the results from [13, 14] for two dimensional covering model to the three dimensional bin packing model, we define three kinds of adjacency relations between the boxes C_i and C_j from the bin, adjacency in the direction of Ox, Oy and Oz.

Definition 1 *The box C_i is adjacent in Ox direction with the box C_j in the bin $\mathcal{P}$ (Figure 2) if the South face of C_i and the North face of C_j have at lest three non-collinear common points, one of which is $O - C_j$.*

Remark 2 *From Definition 1 it follows that if C_i is adjacent with C_j in the direction of Ox we will have:*

$$x_i < x_j$$
$$y_i \leq y_j$$
$$z_i \leq z_j$$

In Figure 3 is one situation when the two boxes are not adjacent in the direction of Ox because $O - C_j$ is not a common point of the boxes C_i and C_j.

Definition 3 *The box C_i is adjacent in Oy direction with box C_j in the bin $\mathcal{P}$ (Figure 4) if the East face of C_i and the West face of C_j have at lest three non-collinear common points, one of which is $O - C_j$.*

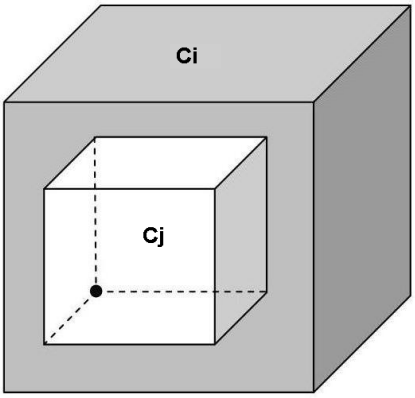

Figure 2: Adjacency in Ox direction

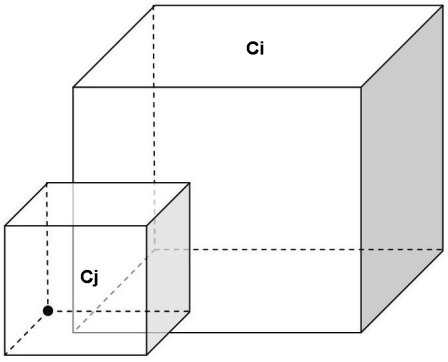

Figure 3: Non adjacency in Ox direction

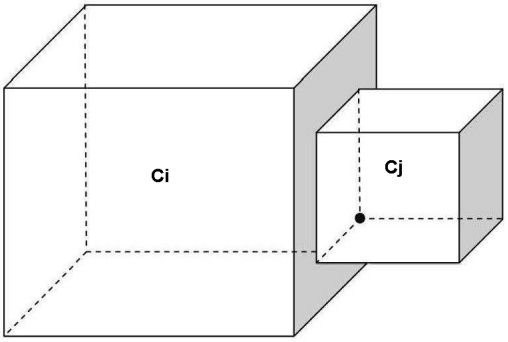

Figure 4: Adjacency in Oy direction

Remark 4 *From Definition 3 it follows that if C_i is adjacent with C_j in Oy direction we will have:*

$$y_i < y_j$$
$$x_i \leq x_j$$
$$z_i \leq z_j$$

In Figure 5 is one situation when the two boxes are not adjacent in the direction of Oy because $O - C_j$ is not a common point of the boxes C_i and C_j.

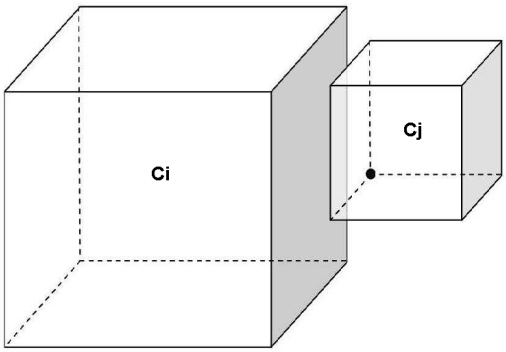

Figure 5: Non adjacency in Oy direction

Definition 5 *The box C_i is adjacent in Oz direction with box C_j in the bin $\mathcal{P}$ (Figure 6) if the North face of C_i and the South face of C_j have at lest three non-collinear common points.*

Remark 6 *In Definition 5 it is not necessary for C_i and C_j to have $O - C_j$ as a common point because our purpose is to give an order of packing and from this point of view we will pack the box C_j only if all the boxes situated downward C_j were already packed. For this reason if C_i is adjacent in Oz direction with C_j conclude that $z_i < z_j$ and there are no more conditions.*

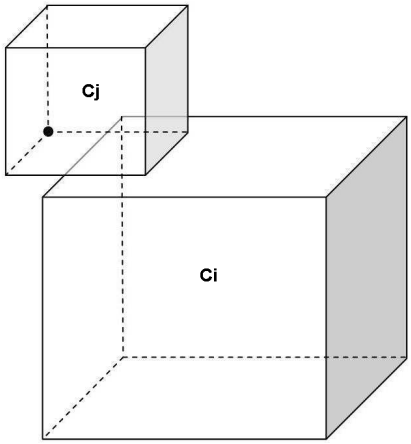

Figure 6: Adjacency in Oz direction

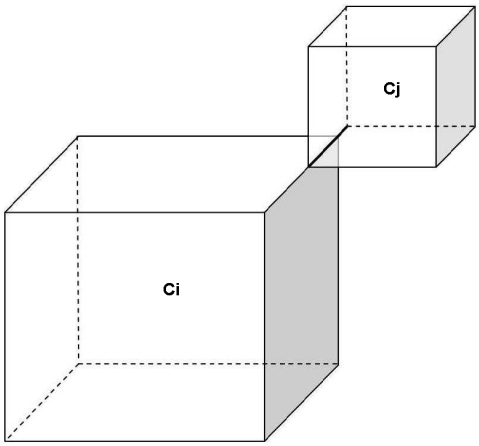

Figure 7: Non adjacency in Oz and Oy directions

Similarly, in Figure 7 is one situation when the two boxes are not adjacent in the direction of Oz and Oy. That means we can pack C_j in the bin before or after packing C_i.

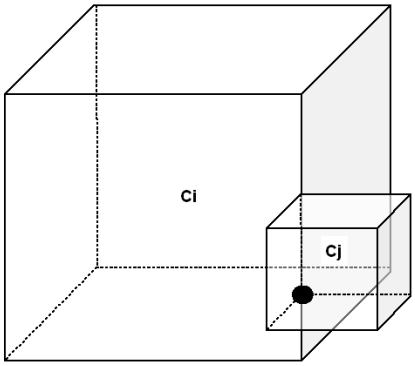

Figure 8: Non adjacency in Ox and Oy directions

We consider that C_i and C_j from Figure 8 are not adjacency in Ox nor Oy direction because they have only collinear common points.

Starting from these three kinds of adjacency we define three kind of graphs :

- G_{Ox}- the graph of adjacency in direction Ox
- G_{Oy}- the graph of adjacency in direction Oy
- G_{Oz}- the graph of adjacency in direction Oz

Definition 7 *The graph of adjacency in Ox direction is $G_{Ox} = (C \cup x, \Gamma_{Ox})$. where the vertices are the boxes from $C = (C_1, C_2, ...C_k)$ and x represents the face $GOCF$ situated on the yOz plane, and*

$$\begin{cases} \Gamma_{Ox}(C_i) \ni C_j \text{ if } C_i \text{ is adjacent in} \\ \qquad\qquad \text{direction } Ox \text{ with } C_j \\ \Gamma_{Ox}(X) \ni C_i \text{ if the North face of } C_i \\ \qquad\qquad \text{touches the } yOz \text{ plan} \end{cases}$$

Definition 8 *The graph of adjacency in Oy direction is $G_{Oy} = (C \cup y, \Gamma_{Oy})$. where the vertices are the boxes from $C = (C_1, C_2, ...C_k)$ and y represents the face $DAOG$ situated on the xOz plane, and*

$$\begin{cases} \Gamma_{Oy}(C_i) \ni C_j \text{ if } C_i \text{ is adjacent in} \\ \qquad\qquad \text{direction } Oy \text{ with } C_j \\ \Gamma_{Oy}(Y) \ni C_i \text{ if the West face of } C_i \\ \qquad\qquad \text{touches the } xOz \text{ plan} \end{cases}$$

Definition 9 *The graph of adjacency in Oz direction is $G_{Oz} = (C \cup z, \Gamma_{Oz})$. where the vertices are the boxes from $C = (C_1, C_2, ...C_k)$ and z represents the face $ABCO$ situated on the xOy plane, and*

$$\begin{cases} \Gamma_{Oz}(C_i) \ni C_j \text{ if } C_i \text{ is adjacent in} \\ \qquad\qquad \text{direction } Oz \text{ with } C_j \\ \Gamma_{Oz}(Z) \ni C_i \text{ if the bottom face} \\ \qquad\qquad \text{of } C_i \text{ touches the } xOy \text{ plan} \end{cases}$$

Example 1.
Let us have the packing model from Figures 9, 10.

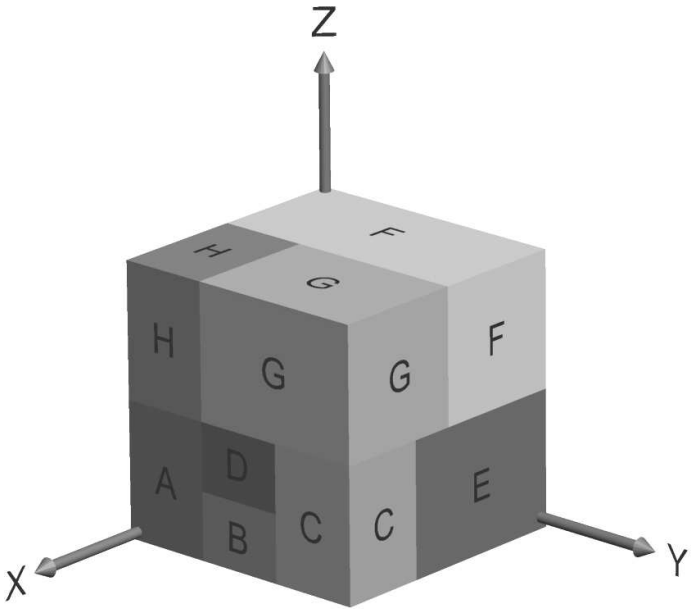

Figure 9: The view of the Packing model from the Southeastern-top point

Then the G_{Ox}, G_{Oy} and G_{Oz} are the graphs from Figures 11, 12, 13.

The graphs G_{Ox}, G_{Oy} and G_{Oz} have important properties:

Theorem 10 *The graphs G_{Ox}, G_{Oy} and G_{Oz} for a packing model are acyclic.*

Proof: We will prove the theorem for G_{Ox} graph because for G_{Oy} or G_{Oz} the result is similar.
Let the graph G_{Ox} for a packing model of a bin $\mathcal{P}$ with the boxes $C_1, C_2, ...C_k$.

Assume that the graph G_{Ox} is cyclic. That means there is a simple path in G_{Ox} which leaves from an element C_{i_1} and returns to C_{i_1}.
Let this path be $\mu = [C_{i_1}, C_{i_2}, \ldots, C_{i_1}]$.

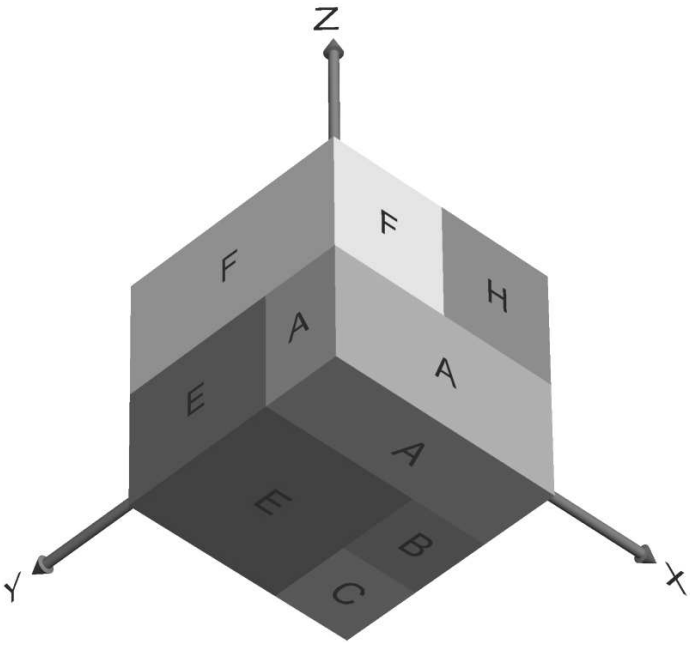

Figure 10: The view of the Packing model from the Northweastern-bottom point

From *Remark 2* it follows that:

$$x_{i_1} < x_{i_2} < \ldots < x_{i_\mu} < x_{i_1} \Rightarrow x_{i_1} < x_{i_1}$$

This is impossible, so the presumption G_{Ox} is cyclic, is false. □

Theorem 11 *Assume we have the graphs G_{Ox}, G_{Oy} and G_{Oz} for a packing model of the bin $\mathcal{P}$ with boxes $C_1, C_2, \ldots C_k$. We have the following properties:*

- *if $C_j \in \Gamma_{Ox}(C_i)$ then $C_j \notin \Gamma_{Oy}(C_i) \cup \Gamma_{Oz}(C_i)$*
- *if $C_j \in \Gamma_{Oy}(C_i)$ then $C_j \notin \Gamma_{Ox}(C_i) \cup \Gamma_{Oz}(C_i)$*
- *if $C_j \in \Gamma_{Oz}(C_i)$ then $C_j \notin \Gamma_{Ox}(C_i) \cup \Gamma_{Oy}(C_i)$*

Proof: Assume that $C_j \in \Gamma_{Ox}(C_i)$. It follows that C_i is adjacent with C_j in the Ox direction. From *Definition 1* it follows that the South face of C_i and the North face of C_j have at least three non-collinear points. Now we can use *Remark 7* and it follows is impossible that the East face of C_i and the West face of C_j to have also three non-collinear points. It is impossible also that the top face of C_i and bottom face of C_j to have three non-collinear points.

The proof for the graphs G_{Oy} and G_{Oz} are similar. □

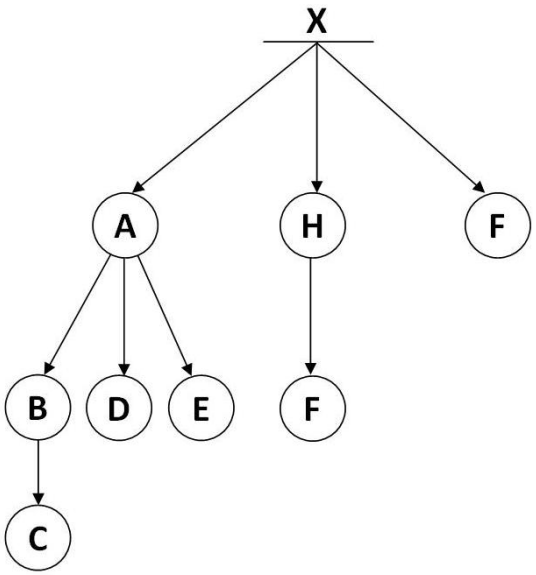

Figure 11: Graph G_{Ox}

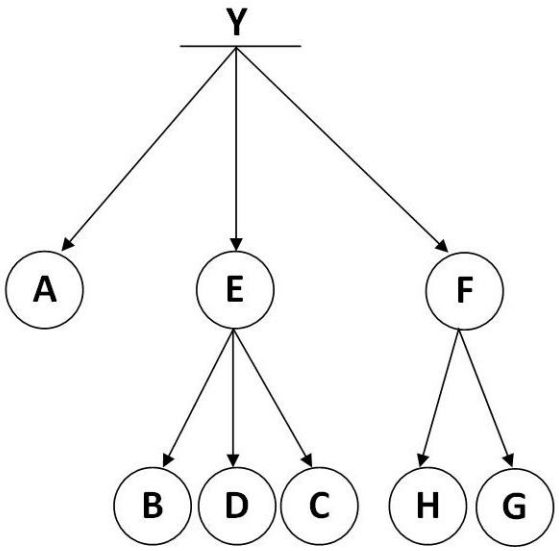

Figure 12: Graph G_{Oy}

3 The Compound Graph for the Packing problem

Similarly with [14] due the *Theorem 11* it is possible to represent simultaneously the graphs of adjacency in the directions of Ox, Oy or Oz by single adjacency matrix T, a matrix with elements from the set 0,1,2,3. So we will use 1 for G_{Ox}, 2 for G_{Oy}, 3 for G_{Oz} and 0 if there is no adjacency.

The matrix T is defined like:

$$T_{ij} = \begin{cases} 1, \text{ if } \Gamma_{Ox}(C_i) \ni C_j \\ 2, \text{ if } \Gamma_{Oy}(C_i) \ni C_j \\ 3, \text{ if } \Gamma_{Oz}(C_i) \ni C_j \\ 0, \text{ the other cases} \end{cases}$$

From the *Theorem 11* it follows that the matrix T is correctly defined because the box C_j has at most one relation with the box C_j in the bin packing model.

Definition 12 *For any packing model we define a network, a graph of compound adjacency $G_c = (C, \Gamma_c)$ where $\Gamma_c(C_i) \ni C_j$ if $T_{ij} \neq 0$. Additionally, the value of the arch (C_i, C_j) is T_{ij}, if $T_{ij} \neq 0$.*

Exemple 2.

For the packing model from the Example 1, illustrated in the Figures 9 and 10, the matrix T is:

$$T = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \end{array} \begin{array}{c} \begin{array}{cccccccc} A & B & C & D & E & F & G & H \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 3 & 0 & 3 \\ 0 & 0 & 1 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 2 & 2 & 2 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

and the graph of compound adjacency is represented like in Figure 14.

In [12] it is shown that the adjacency relation in one plan which is parallel with xOy define an acyclic compound graph.

Let's see the situation when these relations are 3D-dimensional.

Theorem 13 *The graph G_c for a packing model of the boxes $C_1, C_2, \ldots C_k$ in the bin $\mathcal{P}$ is acyclic.*

Proof: Assume that the graph G_c for a packing model of the boxes $C_1, C_2, \ldots C_k$ in the bin $\mathcal{P}$ is cyclic and let examine two situations:

(i) Assume that we have a cycle $C_{i_1}, C_{i_2}, \ldots, C_{i_n}$ in the compound graph G_c and this cycle is composed only from arches with value 1 and 2. It means that we consider only the arches from G_{Ox} and G_{Oy}.

From the *Remark 2* and the *Remark 4* it follows that:

$$x_{i_1} \leq x_{i_2} \leq \ldots \leq x_{i_n}$$

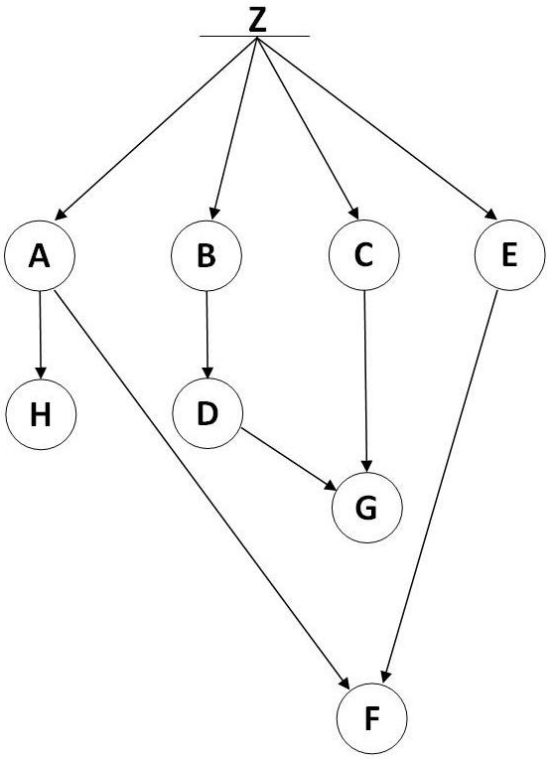

Figure 13: Graph G_{Oz}

and

$$y_{i_1} \leq y_{i_2} \leq \ldots \leq y_{i_n}$$

and there is at least one index i_k so that $x_{i_k} < x_{i_{k+1}}$ or $y_{i_k} < y_{i_{k+1}}$.

It follows that $x_{i_1} < x_{i_n}$ or $y_{i_1} < y_{i_n}$ that means $C_{i_1} \neq C_{i_n}$ and is impossible to have such a kind of cycle in the graph G_c.

(ii) Suppose that we have a cycle $C_{i_1}, C_{i_2}, \ldots, C_{i_n}$ with arches from G_{Ox} and from G_{Oy} and there is at least one arch $(C_{i_k}, C_{i_{k+1}})$ from G_{Oz}. From the *Remarks 2, 4 and 6* we have that:

$$z_{i_1} \leq z_{i_2} \leq \ldots \leq z_{i_k} < z_{i_{k+1}} \leq \ldots \leq z_{i_n}$$

It folows that:

$$z_{i_1} < z_{i_n}$$

and so $C_{i_1} \neq C_{i_n}$ and the path $C_{i_1}, C_{i_2}, \ldots, C_{i_n}$ is not a cycle.

From (i) and (ii) it follows that the graph G_c for a packing model of the boxes $C_1, C_2, \ldots C_k$ in the bin $\mathcal{P}$ is acyclic. □

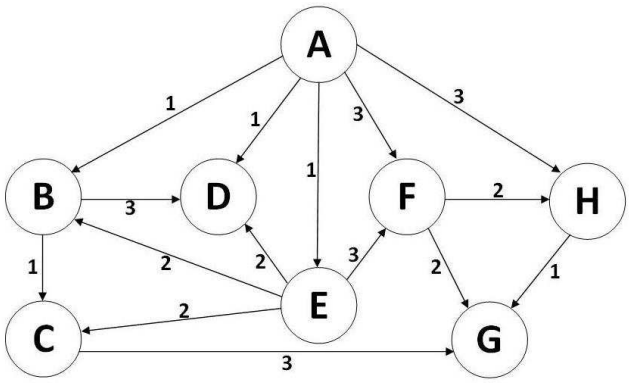

Figure 14: The compound graph G_c

Definition 14 *[1] A topological sorting of a directed acyclic graph G = (C, Γ) is a linear ordering of all its vertices so that, if G contains an arch (C_i, C_j) then C_i appears in the order before C_j.*

Theorem 15 *There is a topological order of the vertices from the set C in the compound G_c.*

Proof: The proof results directly from *Theorem 13* and from [1] because the compound graph G_c is acyclic. □

From the definition of the compound graph G_c, a topological order of the vertices from the set C means that if there is an arch from C_i to C_j in G_c (i.e. if C_i is adjacent with C_j in direction of Ox, Oy or Oz) then C_i must appear before C_j in the topological order.

Overview the significance of the compound graph G_c for the packing model, it follows that a box C_j is packed only if the corner $O - C_j$ is in a fixed position (with one neighbor box on the West and one neighbor box on the North side) and all the boxes situated bellow C_j, which are adjacent with C_j in Oz direction, were already packed. This is a reasonable condition from a practical point of view.

Theorem 16 *The compound graph for a rectangular covering model G_c is a particular network, where there is a single vertex without ascendants.*

Proof: Let $S_1, S_2, \ldots, S_k$ the topological order of the vertex from the set C. Let C_i so that $O - C_i$ is O, that means C_i is situated in the corner O of box P. From the definition of graphs G_{Ox}, G_{Oy} and G_{Oz} it follows that

$$C_i \in \Gamma_{Ox}(X) \cap \Gamma_{Oy}(Y) \cap \Gamma_{Oz}(Z)$$

.

If we eliminate the vertices X, Y, Z from G_c it follows that C_i is a vertex without ascendants in the graph G_c. It is evident that $S_1 = C_i$, the first in the topological order.

More, for every $S_i \in C$ there is a path from S_1 to S_i. □

4 Topological Sorting Algorithm

To determine a topological order of the boxes $C_1, C_2, \ldots, C_k$ we can use a topological sorting algorithm from [2] or a new algorithm, OVERDIAG-3D which is an extension of previous algorithm from [12]. This algorithm is based on the particularity of the compound graph G_c, respectively on the form of the matrix T, attached to the graph G_c.

4.1 OVERDIAG-3D Algorithm

From *Theorem 15* it follows that there is a topological order in G_c. Then the matrix *T*, attached to the compound graph G_c, is an over diagonal matrix with the main diagonal equal to 0, when the vertices are in topological order. We will base our algorithm on two observations:

1. By changing lines and columns in the adjacency matrix, the number of elements equal to zero remains unchanged ;

2. We can always find a column with the necessary number of zeroes.

For finding the topological order of the set C we extend our matrix $T(k\times k)$ to the matrix $A(k\times(k+1))$ by attaching a new column, $k+1$, to the columns of T, which preserve the original number of rows. We will transform the matrix *A* in an overdiagonal matrix, by changing the lines and the columns.
Finally the new column, $k+1$, of the matrix A represents the topological order of the set C.

4.1.1 Correctness and Complexity

Applying the OVERDIAG-3D algorithm we change the order of the vertices C_i so that the matrix *T* for the compound graph G_c became an over diagonal matrix. It follows that T_{ij} = 0 for all $i \geq j$ and it is possible to have $T_{ij} \neq 0$ only if $i < j$.

For the compound graph G_c that means there is an arch from C_i to C_j only if $i < j$ so C_i appears before C_j in the ordered set *C*. It follows that *C* is topologically sorted.

Remark that the OVERDIAG-3D algorithm is of linear complexity in k^2, the maximal number of vertices in the compound graph G_c.

for *i=1* **to** *n-1* **do**
  $jc = 0;\ jc3 = 0; jl = 0; jl3 = 0; j = 1;$
  repeat
    if $A(i,j) = 0$ **then** $j = j + 1$;
  until $A(i,j) \neq 0$ **or** $j = n + 1$;
  $k = i + 1$;
  repeat
    if $A(k,j) = 0$ **then** $k = k + 1$
    /* test if the column is 0 */ ;
  until $k = n + 1$ **or** $A(k,j) \neq 0$;
  if $k = n + 1$ **then**
    if $A(i,j) = 3$ **then** $jc3 = j$
    else if $A(i,j) \neq 0$ **then** $jc = j$
  end
  repeat
    if $A(j,i) = 0$ **then** $j = j + 1$
    /* looking for nonzero elements */
  until $A(j,i) \neq 0$ *or* $j = n$;
  $k = i + 1$
  repeat
    if $A(j,k) = 0$ **then** $k = k + 1$
    /* test if the row is 0 */
  until $k = n + 1$ **or** $A(j,k) \neq 0$;
  if $k = n + 1$ **then**
    if $A(j,i) = 3$ **then** $jl3 = j$
    else if $A(j,i) \neq 0$ **then** $jl = j$
  end
  if $jc \neq 0$ **then** $Changecol(i, jc)$;
  /* interchange the columns i and jc */
  else if $jl \neq 0$ **then** $Changelin(i, jc)$;
  /* interchanage the rows i and jc */
  if $jc3 \neq 0$ **then** $Changecol(i, jc3)$
  else $Changelin(i, jc3)$ /* from Theoreme 11 it follows that there is at least one nonzero element */
end

5 Conclusions

The two dimensional and the three dimensional bin-packing problems hold importance to many fields. The two dimensional bin packing problem is used in the paper industry or glas industry or in the field of pattern recognition [4]. Set coverig problem is applied in instance of the set reduction for machine learning in [18] or to approximate cover an area by antennas.

Shipping and moving industries, architecture, engineering, and design are all areas where three dimensional bin-packing could apply.

Industry uses bin-packing for everything from scheduling television programming to stacking cargo in a semi-truck to designing automobiles and airplanes.

Another application of the three dimensional bin packing problem is for cutting in the wood industry [16].

Recently, the Institute for Algorithms and Scientific Computing in Germany used three dimensional packing for research in molecular biology and chemistry and also with automobile design for manufacturer Mercedes-Benz [21].

In New Zeeland a manufacturer of electrical cable requested a way to efficiently pack drums and pallets of cable into standard shipping containers for transport overseas.

A problem here, after the determination of a packing model, is to determine the order in which the boxes will be packed in the bin, the plan of packing.

This kind of order can be the topological order given by us, where the placement begins with the northwestern-bottom corner of the bin and it ends with a box situated on the top of the bin.

We intend to apply these results and previous results [12] in the field of the Renewable Energy for determine the optimal placement of the set of photovoltaic cells in a field of cells.

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