Static Output-Feedback Simultaneous Stabilization of Interval Time-Delay Systems

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Abstract: - This paper addresses the problem of simultaneous static output-feedback stabilization of a collection of interval time-delay systems. It is shown that this problem can be converted into a matrix measure assignment problem. Sufficient conditions for guaranteeing the robust stability for considered systems are derived in term of the matrix measures of the system matrices. By using matrix inequalities, we provide two cases of obtaining a static output-feedback controller that can stabilize the system, i.e., both $P = I$ and $P \neq I$ cases are considered where *I* is a identity matrix and *P* is a common positive definite matrix to guarantee the stability of the overall system. The sufficient condition with $P = I$ is formulated in the format of linear matrix inequalities (LMIs). When $P \neq I$ is considered, the sufficient condition becomes a nonlinear matrix inequality problem and a heuristic iterative algorithm based on the LMI technique is presented to solve the coupled matrix inequalities. Finally, an example is provided to illustrate the effectiveness of our approach.

Key-Words: - Matrix measure; Robust control; Output feedback; Interval systems; Linear Matrix Inequality

1. Introduction

The simultaneous stabilization problem was first introduced in Saeks and Murray [1] and Vidyasagar and Viswanadham [2]. This problem consists in answering the following questions: given *m* plants $G_1,\ldots,G_m$, does there exist a single feedback controller *C* so that the controller can stabilize all plants and all corresponding closed-loop systems have satisfactory performance? One scenario for this problem is the reliable stabilization problem, where $G_2,\ldots,G_m$ represent G_1 operating in various modes of failures (e.g., failure of sensor, severance of loops, software breakdown). If failures occur, the dynamics of the system will certainly change. Thus, in such a case, a controller is sought so that it can stabilize the system in all situations. Another application is the design of a fixed controller for a set of linear plants characterized by different modes of operations or for nonlinear plants linearized at several regions.

Lots of researchers have been devoted themselves in solving the simultaneous stabilization problem. Blondel and Gevers [10] studied the computational complexity of simultaneous stabilization and proved that the simultaneous stabilization for three linear systems is rationally non-decidable. From [15]-[16], it is possible to conclude that this problem is very difficult to solve due to its NP-hard nature. Paskota *et al.* [3] presented a computational technique for optimal simultaneous stabilization for linear systems via linear state feedback. Cao and Lam [7] dealt with simultaneous linear-quadratic (LQ) optimal control design for a set of linear time-invariant (LTI) systems via piecewise constant output feedback. Cao and Sun [8] proposed an iterative linear matrix inequality (LMI) algorithm to seek a state/output feedback controller for a set of MIMO plants. In [9], a numerical algorithm is introduced to solve the problem of simultaneous stabilization of a collection of MIMO plants via static output feedback using a set of coupled algebraic matrix inequalities (ARI's). Other design results on simultaneously stabilizing controller can also be found in [12]-[13].

In the above-mentioned approaches, these researchers employ the LQ control approach to solve the simultaneous stabilization problem for a collection of LTI systems without time delay and uncertainties. To our best knowledge, there are no general techniques for solving the problem of simultaneous stabilization for a collection of uncertain time-delay systems via static output feedback. For a single system with time delay and uncertainties, the robust stability analysis problem is quite complicated and recently, has been studied via several different techniques. The criteria for asymptotic stability of such systems can be classified as delay-independent, which are independent of the size of time-delay, for example [20,21], or delay-dependent, which include information on the size of delay, for example [22,23]. Meanwhile, some different stability criteria have also been proposed via the LMI approach [24]-[27].

In this paper, we focused on the problem of simultaneous stabilization for a collection of interval time-delay systems via a static output feedback controller. It will be shown that the considered problem is solvable if a corresponding matrix measure assignment problem is solvable. The matrix measure is widely applied in the analysis of stability properties of uncertain and/or time-delay systems [4, 5, 14]. Although it has been widely employed in the robustness analysis problem, nevertheless, few has investigated about the controller synthesis problem. Recently, linear matrix inequalities (LMI's) have emerged as a powerful formulation and design technique for a variety of linear control problems [6,17,18]. Software like Matlab's LMI Control Toolbox [18] is available to solve LMI's problems in a fast and user-friendly manner. In this paper, we shall show that the matrix measure assignment problem is equivalent to an LMI feasibility problem. Thus, a controller solving the matrix measure assignment problem and then solving the simultaneous stabilization problem for a collection of interval time delay systems can be obtained via solving an LMI problem.

The paper is organized as follows. Section 2 proposes the problem formulation and reviews the basic properties of the matrix measure. In Section 3, a collection of interval time-delay systems are discussed. In the section, the stability and robustness conditions for the considered system are also derived. It is shown that the problem of static output-feedback controller design for a collection of interval time-delay systems is solvable if a corresponding matrix measure assignment problem is solvable. Further, The sufficient condition with $P = I$ is formulated in the format of linear matrix inequalities (LMIs). When $P \neq I$ is considered, the sufficient condition becomes a nonlinear matrix inequality problem and a heuristic iterative algorithm based on the LMI technique is presented to solve the coupled matrix inequalities. Section 4 shows an illustrative example. Finally, conclusions are given in Section 5.

Notations:

In what follows, **O** is a zero matrix with an appropriate dimension, **I** is an identity matrix with an appropriate dimension, $\mathbf{M}^T$ denotes the transpose of the matrix **M**, $\mathbf{M}^*$ denotes the conjugate transpose of the matrix **M**, $\mathbf{M} > \mathbf{0}$ ($\mathbf{M} \geq \mathbf{0}$) means that the matrix **M** is positive definite (semidefinite), $\mathbf{M} < \mathbf{0}$ ($\mathbf{M} \leq \mathbf{0}$) means that the matrix **M** is negative definite (semidefinite).

2. Problem Formulation and Preliminaries

Consider a collection of interval time-delay systems:

$$\dot{\mathbf{x}}_i(t) = \hat{\mathbf{A}}_i \mathbf{x}_i(t) + \hat{\mathbf{D}}_i \mathbf{x}_i(t-h_i) + \mathbf{B}_i \mathbf{u}_i(t), \quad i = 1,2,\ldots,p \quad (1)$$

$$\mathbf{y}_i(t) = \mathbf{C}_i \mathbf{x}_i(t), \quad i = 1,2,\ldots,p \quad (2)$$

where $\mathbf{x}_i(t) \in \Re^n$ is the state, h_i is the time-delay of the system, $\mathbf{u}_i(t) \in \Re^m$ is the control input, and $\mathbf{y}_i(t) \in \Re^r$ is the controlled output. $\mathbf{B}_i \in \Re^{n\times m}$ and $\mathbf{C}_i \in \Re^{r\times n}$ are constant matrices. $\hat{\mathbf{A}}_i \in \Re^{n\times n}$ and $\hat{\mathbf{D}}_i \in \Re^{n\times n}$ are matrices whose elements vary in some prescribed ranges; e.g., $\hat{\mathbf{A}}_i$ and $\hat{\mathbf{D}}_i$ are such that

$$\begin{aligned} \hat{\mathbf{A}}_i &= [a^i_{jk}], \quad \underline{a}^i_{jk} \leq a^i_{jk} \leq \overline{a}^i_{jk} \\ \hat{\mathbf{D}}_i &= [d^i_{jk}], \quad \underline{d}^i_{jk} \leq d^i_{jk} \leq \overline{d}^i_{jk} \end{aligned} \quad i = 1,2,\ldots,p \quad (3)$$

where a^i_{jk} is the *jk*-th element of the matrix $\hat{A}_i$, $\underline{a}^i_{jk}$ and $\overline{a}^i_{jk}$ denote its low bound and upper bound, respectively, d^i_{jk} is the *jk*-th element of the matrix $\hat{D}_i$, and $\underline{d}^i_{jk}$ and $\overline{d}^i_{jk}$ denote its low bound and upper bound, respectively. Those bounds $\underline{a}^i_{jk}$, $\overline{a}^i_{jk}$, $\underline{d}^i_{jk}$, and $\overline{d}^i_{jk}$ are known real values.

The design goal is to find a matrix **F** such that the static output feedback controller

$$\mathbf{u}_i(t) = \mathbf{F}\mathbf{y}_i(t) \quad , \quad i = 1, 2, \ldots, p \tag{4}$$

ensures all the closed-loop interval time-delay systems being asymptotically stable.

We now introduce several properties about matrix measure as follows. The matrix measure of a constant matrix **M** is defined as

$$\mu_v(\mathbf{M}) \equiv \lim_{\theta \to 0^+} \frac{\left(\| \mathbf{I} + \theta \mathbf{M} \|_v - 1\right)}{\theta} \tag{5}$$

where $\|\cdot\|_V$ is a suitable matrix norm (see [5]).

Lemma 2.1 [5]**:** The matrix measure has following properties.

(a). $\mu_V(\cdot)$ is convex; i. e.,

$$\mu_V\left(\sum_{j=1}^{k} \alpha_j \mathbf{M}_j\right) \le \sum_{j=1}^{k} \alpha_j \mu_V\left(\mathbf{M}_j\right) \quad \text{for all } \alpha_j \ge 0. \tag{6}$$

(b). For any norm and any constant matrix **M**

$$-\|\mathbf{M}\|_V \le -\mu_V(-\mathbf{M}) \le \operatorname{Re}\lambda(\mathbf{M}) \le \mu_V(\mathbf{M}) \le \|\mathbf{M}\|_V. \tag{7}$$

(c). Suppose m_{ij} is the *ij*-th element of **M**, then

$$\mu_1(\mathbf{M}) = \max_j \left[\operatorname{Re}(m_{jj}) + \sum_{i \ne j} |m_{ij}| \right], \tag{8}$$

$$\mu_2(\mathbf{M}) = \max_i \left[\lambda_i (\mathbf{M} + \mathbf{M}^*)/2 \right], \tag{9}$$

$$\mu_\infty(\mathbf{M}) = \max_i \left[\operatorname{Re}(m_{ii}) + \sum_{i \ne j} | m_{ij} | \right]. \tag{10}$$

3. Main Results

From (1), (2) and (4), the collection of closed-loop systems can be described as:

$$\begin{aligned} \dot{\mathbf{x}}_i(t) &= (\hat{\mathbf{A}}_i + \mathbf{B}_i \mathbf{F} \mathbf{C}_i)\mathbf{x}_i(t) + \hat{\mathbf{D}}_i \mathbf{x}_i(t - h_i) \quad , \quad i = 1, 2, \ldots, p \\ \mathbf{y}_i(t) &= \mathbf{C}_i \mathbf{x}_i(t), \quad i = 1, 2, \ldots, p \end{aligned}$$

Denote

$$\underline{\mathbf{A}}_i = [\underline{a}^i_{jk}], \quad \bar{\mathbf{A}}_i = [\bar{a}^i_{jk}], \quad i = 1, 2, \ldots, p \tag{11}$$

$$\underline{\mathbf{D}}_i = [\underline{d}^i_{jk}], \quad \bar{\mathbf{D}}_i = [\bar{d}^i_{jk}], \quad i = 1, 2, \ldots, p \tag{12}$$

and let

$$\mathbf{A}_i = \frac{1}{2}(\underline{\mathbf{A}}_i + \bar{\mathbf{A}}_i), \quad \mathbf{D}_i = \frac{1}{2}(\underline{\mathbf{D}}_i + \bar{\mathbf{D}}_i), \; i = 1, 2, \ldots, p \tag{13}$$

$$\mathbf{M}_i = \bar{\mathbf{A}}_i - \mathbf{A}_i, \quad \mathbf{N}_i = \bar{\mathbf{D}}_i - \mathbf{D}_i, \; i = 1, 2, \ldots, p \tag{14}$$

where $\mathbf{A}_i$ and $\mathbf{D}_i$ are the average matrices of $\underline{\mathbf{A}}_i$ and $\bar{\mathbf{A}}_i$, and of $\underline{\mathbf{D}}_i$ and $\bar{\mathbf{D}}_i$, respectively. Furthermore, $\mathbf{M}_i$ and $\mathbf{N}_i$ are the maximal bias matrices between $\hat{\mathbf{A}}_i$ and $\mathbf{A}_i$, and between $\hat{\mathbf{D}}_i$ and $\mathbf{D}_i$, respectively.

We first derive a new sufficient condition to ensure the stability of a collection of unforced time-delay systems without uncertainty

$$\begin{aligned} \dot{\mathbf{x}}_i(t) &= \mathbf{A}_i \mathbf{x}_i(t) + \mathbf{D}_i \mathbf{x}_i(t - h_i) \\ \mathbf{y}_i(t) &= \mathbf{C}_i \mathbf{x}_i(t) \end{aligned}, \quad i = 1, 2, \ldots, p. \tag{15}$$

3.1 LMI approach

Theorem 3.1: For any $\varepsilon > 0$, if

$$\mu_2(\mathbf{A}_i) < -\frac{1}{2\varepsilon} - \frac{1}{2}\varepsilon \left\| \mathbf{D}_i \right\|^2, \quad i = 1, 2, \ldots, p, \tag{16}$$

then the equilibrium of a collection of unforced system (15) is asymptotically stable.

Proof: Consider a Lyapunov function as

$$V(\mathbf{x}_i(t)) = \mathbf{x}_i^T(t)\mathbf{x}_i(t) + \frac{1}{\varepsilon}\int_{\mathrm{t-h_i}}^{\mathrm{t}} \mathbf{x}_i^T(s)\mathbf{x}_i(s)ds \quad ,$$

$i = 1, 2, \ldots, p$.

The time derivative of $V(\mathbf{x}_i(t))$ is

$$\begin{aligned} \dot{V}(\mathbf{x}_i(t)) &= 2\mathbf{x}_i^T(t)\dot{\mathbf{x}}_i(t) + \frac{1}{\varepsilon}\mathbf{x}_i^T(t)\mathbf{x}_i(t) \\ &\quad - \frac{1}{\varepsilon}\mathbf{x}_i^T(t - h_i)\mathbf{x}_i(t - h_i) \\ &= \mathbf{x}_i^T(t)(\mathbf{A}_i \mathbf{x}_i(t) + \mathbf{D}_i \mathbf{x}_i(t - h_i)) \\ &\quad + (\mathbf{A}_i \mathbf{x}_i(t) + \mathbf{D}_i \mathbf{x}_i(t - h_i))^T \mathbf{x}_i(t) \\ &\quad + \frac{1}{\varepsilon}\mathbf{x}_i^T(t)\mathbf{x}_i(t) - \frac{1}{\varepsilon}\mathbf{x}_i(t - h_i)^T \mathbf{x}_i(t - h_i) \\ &= \mathbf{x}_i^T(t)(\mathbf{A}_i^T + \mathbf{A}_i + \frac{1}{\varepsilon}\mathbf{I})\mathbf{x}_i(t) \\ &\quad + \mathbf{x}_i^T(t)\mathbf{D}_i \mathbf{x}_i(t - h_i) + \mathbf{x}_i^T(t - h_i)\mathbf{D}_i^T \mathbf{x}_i(t) \\ &\quad - \frac{1}{\varepsilon}(\mathbf{x}_i^T(t - h_i) \cdot \mathbf{x}_i(t - h_i)) \end{aligned}$$

Set $\bar{\mathbf{x}}_i(t) = [\mathbf{x}_i^T(t) \;\; \mathbf{x}_i^T(t - h_i)]^T$. We obtain

$$\dot{V}(\mathbf{x}_i(t)) = \bar{\mathbf{x}}_i^T(t) \begin{bmatrix} \mathbf{A}_i^T + \mathbf{A}_i + \frac{1}{\varepsilon}\mathbf{I} & \mathbf{D}_i \\ \mathbf{D}_i^T & -\frac{1}{\varepsilon}\mathbf{I} \end{bmatrix} \bar{\mathbf{x}}_i(t).$$

By Schur complement, if $\mathbf{A}_i^T + \mathbf{A}_i + \frac{1}{\varepsilon}\mathbf{I} + \varepsilon \mathbf{D}_i \mathbf{D}_i^T < 0$, then $\begin{bmatrix} \mathbf{A}_i^T + \mathbf{A}_i + \frac{1}{\varepsilon}\mathbf{I} & \mathbf{D}_i \\ \mathbf{D}_i^T & -\frac{1}{\varepsilon}\mathbf{I} \end{bmatrix}$ is negative definite.

From (16), we obtain

$$\begin{aligned} &\mu_2(\mathbf{A}_i^T + \mathbf{A}_i + \frac{1}{\varepsilon}\mathbf{I} + \varepsilon\mathbf{D}_i\mathbf{D}_i^T) \\ &\le \mu_2(\mathbf{A}_i^T + \mathbf{A}_i) + \mu_2(\frac{1}{\varepsilon}\mathbf{I}) + \mu_2(\varepsilon\mathbf{D}_i\mathbf{D}_i^T) \\ &\le 2\mu_2(\mathbf{A}_i) + \frac{1}{\varepsilon} + \varepsilon\|\mathbf{D}_i\|^2 \\ &< 0 \end{aligned}$$

This proves $\mathbf{A}_i^T + \mathbf{A}_i + \frac{1}{\varepsilon}\mathbf{I} + \varepsilon\mathbf{D}_i\mathbf{D}_i^T < 0$, and then

$$\begin{bmatrix} \mathbf{A}_i^T + \mathbf{A}_i + \frac{1}{\varepsilon}\mathbf{I} & \mathbf{D}_i \\ \mathbf{D}_i^T & -\frac{1}{\varepsilon}\mathbf{I} \end{bmatrix} < 0 .$$

So $\dot{V}(\mathbf{x}_i(t)) < 0$ for all $\mathbf{x}_i(t) \neq 0$. This completes the proof. □

Theorem 3.1 provides a simple method to verify the stability of a collection of unforced time-delay systems (15). In what follows, we shall consider the stability conditions of a collection of unforced time-delay systems *with uncertainty*.

Theorem 3.2: Consider a collection of interval time-delay systems

$$\begin{aligned} \dot{\mathbf{x}}_i(t) &= \hat{\mathbf{A}}_i\mathbf{x}_i(t) + \hat{\mathbf{D}}_i\mathbf{x}_i(t-h_i) \\ \mathbf{y}_i(t) &= \mathbf{C}_i\mathbf{x}_i(t) \end{aligned}, \quad i = 1, 2, \ldots, p \tag{17}$$

where $\hat{\mathbf{A}}_i$ and $\hat{\mathbf{D}}_i$ are defined in (3). If

$$\mu_2(\mathbf{A}_i) < -\frac{1}{2\varepsilon} - \|\mathbf{M}_i\| - \frac{1}{2}\varepsilon\|\mathbf{D}_i\|^2 - \varepsilon\|\mathbf{N}_i\|\cdot\|\mathbf{D}_i\| - \frac{1}{2}\varepsilon\|\mathbf{N}_i\|^2, \quad i = 1, 2, \ldots, p, \tag{18}$$

then (17) is robustly asymptotically stable.

Proof: Since

$$\begin{aligned} &2\mu_2(\hat{\mathbf{A}}_i) + \frac{1}{\varepsilon} + \varepsilon\|\hat{\mathbf{D}}_i\|^2 \\ &= 2\mu_2(\mathbf{A}_i + \Delta\mathbf{A}_i) + \frac{1}{\varepsilon} + \varepsilon\|\mathbf{D}_i + \Delta\mathbf{D}_i\|^2 \\ &= 2\mu_2(\mathbf{A}_i) + 2\mu_2(\Delta\mathbf{A}_i) + \frac{1}{\varepsilon} + \varepsilon\|\mathbf{D}_i\|^2 + 2\varepsilon\|\mathbf{D}_i\|\cdot\|\Delta\mathbf{D}_i\| + \varepsilon\|\Delta\mathbf{D}_i\|^2 \\ &< 2\mu_2(\mathbf{A}_i) + 2\|\mathbf{M}_i\| + \frac{1}{\varepsilon} + \varepsilon\|\mathbf{D}_i\|^2 + 2\varepsilon\|\mathbf{N}_i\|\cdot\|\mathbf{D}_i\| + \varepsilon\|\mathbf{N}_i\|^2 \\ &< 0 \end{aligned}$$

This completes the proof. □

With the above theorems, we have the following corollary.

Corollary 1: Suppose that static output feedback gain $\mathbf{F}$ satisfy the following conditions

$$\mu_2(\mathbf{A}_i + \mathbf{B}_i\mathbf{F}\mathbf{C}_i) < -\frac{1}{2\varepsilon} - \|\mathbf{M}_i\| - \frac{1}{2}\varepsilon\|\mathbf{D}_i\|^2 - \varepsilon\|\mathbf{N}_i\|\cdot\|\mathbf{D}_i\| - \frac{1}{2}\varepsilon\|\mathbf{N}_i\|^2, \quad i = 1, 2, \ldots, p, \tag{19}$$

then the equilibrium of the closed-loop time-delay system with norm-bounded uncertainties represented as

$$\begin{aligned} \dot{\mathbf{x}}_i(t) &= (\hat{\mathbf{A}}_i + \mathbf{B}_i\mathbf{F}\mathbf{C}_i)\mathbf{x}_i(t) + \hat{\mathbf{D}}_i\mathbf{x}_i(t-h_i) \\ \mathbf{y}_i(t) &= \mathbf{C}_i\mathbf{x}_i(t) \end{aligned}, i = 1, 2, \ldots, p \tag{20}$$

is robustly asymptotically stable.

Corollary 1 reveals that if the constant control gain satisfies (19), then a collection of interval time-delay systems are robustly asymptotically stable.

For simplicity of notation, define

$$\gamma_i = -\frac{1}{2\varepsilon} - \|\mathbf{M}_i\| - \frac{1}{2}\varepsilon\|\mathbf{D}_i\|^2 - \varepsilon\|\mathbf{N}_i\|\cdot\|\mathbf{D}_i\| - \frac{1}{2}\varepsilon\|\mathbf{N}_i\|^2,$$

$i = 1, 2, \ldots, p$.

Thus, (19) becomes

$$\mu_2(\mathbf{A}_i + \mathbf{B}_i\mathbf{F}\mathbf{C}_i) < \gamma_i, \quad i = 1, 2, \ldots, p. \tag{21}$$

Define $\Im_i(\gamma_i) \equiv \{\mathbf{F} \in \Re^{m\times r} \mid \mu_2(\mathbf{A}_i + \mathbf{B}_i\mathbf{F}\mathbf{C}_i) < \gamma_i\}$, for $i = 1, 2, \ldots, p$. The admissible solution set is $\Im \equiv \Im_1(\gamma_1)\bigcap\Im_2(\gamma_2)\bigcap\cdots\bigcap\Im_p(\gamma_p)$. Then, we can have the following theorem.

Theorem 3.4: The admissible solution set $\Im$ is convex.

Proof: Since the intersection of convex sets is convex, we only need to prove that $\Im_i(\gamma_i)$ is convex for each i. Assume $\mathbf{F}_1 \in \Im_i(\gamma_i)$ and $\mathbf{F}_2 \in \Im_i(\gamma_i)$, which means $\mu_2(\mathbf{A}_i + \mathbf{B}_i\mathbf{F}_1\mathbf{C}_i) < \gamma_i$ and $\mu_2(\mathbf{A}_i + \mathbf{B}_i\mathbf{F}_2\mathbf{C}_i) < \gamma_i$. Then, to prove that $\Im_i(\gamma_i)$ is convex is same as to prove $\alpha\mathbf{F}_1 + (1-\alpha)\mathbf{F}_2 \in \Im_i(\gamma_i)$, or equivalently to prove $\mu_2(\mathbf{A}_i + \mathbf{B}_i(\alpha\mathbf{F}_1 + (1-\alpha)\mathbf{F}_2)\mathbf{C}_i) < \gamma_i$, for all $0 \le \alpha \le 1$. Note that

$$\mu_2(\mathbf{A}_i + \mathbf{B}_i(\alpha\mathbf{F}_1 + (1-\alpha)\mathbf{F}_2)\mathbf{C}_i)$$
$$= \mu_2(\alpha\mathbf{A}_i + (1-\alpha)\mathbf{A}_i + \alpha\mathbf{B}_i\mathbf{F}_1\mathbf{C}_i + (1-\alpha)\mathbf{B}_i\mathbf{F}_2\mathbf{C}_i)$$
$$= \mu_2(\alpha(\mathbf{A}_i + \mathbf{B}_i\mathbf{F}_1\mathbf{C}_i) + (1-\alpha)(\mathbf{A}_i + \mathbf{B}_i\mathbf{F}_2\mathbf{C}_i))$$
$$\le \alpha\mu_2(\mathbf{A}_i + \mathbf{B}_i\mathbf{F}_1\mathbf{C}_i) + (1-\alpha)\mu_2(\mathbf{A}_i + \mathbf{B}_i\mathbf{F}_2\mathbf{C}_i)$$
$$< \gamma$$

This completes the proof. □

From the above discussions, it is concluded that the matrix measure assignment problem can be considered as a convex feasibility problem. Thus, we now turn our attention to reduce the matrix measure assignment problem to an LMI feasibility problem.

For a matrix **U**, define $\mathbf{U}_\perp$ as a matrix whose columns form bases of the null bases of **U**. Then, we can have the following theorem, which is the main result of this paper.

Theorem 3.5:
(1). The matrix F satisfies

$$\mu_2(\mathbf{A}_i + \mathbf{B}_i\mathbf{F}\mathbf{C}_i) < \gamma_i,\ \ i = 1,2,\ldots,p. \quad (22)$$

if and only if F satisfies LMIs

$$(\mathbf{A}_i + \mathbf{A}_i^T - 2\gamma_i\mathbf{I}) + \mathbf{B}_i\mathbf{F}\mathbf{C}_i + \mathbf{C}_i^T\mathbf{F}^T\mathbf{B}_i^T < \mathbf{0},$$
$$i = 1,2,\ldots,p. \quad (23)$$

(2). There exists F satisfies (23) if and only if

$$(\mathbf{B}_i)_\perp(\mathbf{A}_i + \mathbf{A}_i^T - 2\gamma_i\mathbf{I})(\mathbf{B}_i^T)_\perp < \mathbf{0},\ \ i = 1,2,\ldots,p \quad (24)$$

and

$$(\mathbf{C}_i^T)_\perp(\mathbf{A}_i + \mathbf{A}_i^T - 2\gamma_i\mathbf{I})(\mathbf{C}_i)_\perp < \mathbf{0},\ \ i = 1,2,\ldots,p \quad (25)$$

Proof: We first prove part (1). From (9), it can be shown that $\mu_2(\mathbf{A}_i + \mathbf{B}_i\mathbf{F}\mathbf{C}_i) < \gamma_i$, $i = 1,2,\ldots,p$, are equivalent to

$$(\mathbf{A}_i + \mathbf{B}_i\mathbf{F}\mathbf{C}_i) + (\mathbf{A}_i + \mathbf{B}_i\mathbf{F}\mathbf{C}_i)^* - 2\gamma_i\mathbf{I} < \mathbf{0},\ \ i = 1,2,\ldots,p. \quad (26)$$

which are equivalent to

$$(\mathbf{A}_i + \mathbf{A}_i^T - 2\gamma_i\mathbf{I}) + \mathbf{B}_i\mathbf{F}\mathbf{C}_i + \mathbf{C}_i^T\mathbf{F}^T\mathbf{B}_i^T < \mathbf{0},\ \ i = 1,2,\ldots,p.$$

This completes the proof of part (1). For the part (2), recall the result in [19]. Given a symmetric matrix $\mathbf{\Psi} \in \Re^{n\times n}$ and two matrices U and V both with a column dimension n, there exists a matrix $\mathbf{\Theta}$ of a compatible dimension such that $\mathbf{\Psi} + \mathbf{U}^T\mathbf{\Theta}^T\mathbf{V} + \mathbf{V}^T\mathbf{\Theta}\mathbf{U} < \mathbf{0}$ if and only if $\mathbf{U}_\perp^T\mathbf{\Psi}\mathbf{U}_\perp < \mathbf{0}$ and $\mathbf{V}_\perp^T\mathbf{\Psi}\mathbf{V}_\perp < \mathbf{0}$. Letting $\mathbf{\Psi}_i = (\mathbf{A}_i + \mathbf{A}_i^T - 2\gamma_i\mathbf{I})$, $\mathbf{V}_i = \mathbf{B}_i^T$, $\mathbf{U}_i = \mathbf{C}_i$, and $\mathbf{\Theta} = \mathbf{F}$, the part (2) is obvious.

Theorem 3.3 tells us that if (24) and (25) hold, then there exists a matrix F that satisfies LMIs (23). In fact, such an F also solves (22). This means that if (24) and (25) hold, then the admissible solution set $\Im$ is not empty. Note that a matrix F satisfying LMIs (23) can easily be obtained by using Matlab's LMI Control Toolbox if $\Im$ is not empty. The obtained F then can also solve the considered problem.

Remark: The approach described above can be applied to solve the simultaneous output feedback stabilization problem for a collection of uncertain systems:

$$\dot{\mathbf{x}}_i(t) = (\mathbf{A}_i + \Delta\mathbf{A}_i)\mathbf{x}_i(t) + \mathbf{B}_i\mathbf{u}_i(t),\ \ i = 1,2,\ldots,p$$
$$\mathbf{y}_i(t) = \mathbf{C}_i\mathbf{x}_i(t),\ \ i = 1,2,\ldots,p$$
$$\|\Delta\mathbf{A}_i\| \le \rho_i,\ \ i = 1,2,\ldots,p$$

where $\mathbf{x}_i \in \Re^n$ is the state, $\mathbf{u}_i \in \Re^m$ is the control input, and $\mathbf{y}_i \in \Re^r$ is the output; and $\mathbf{A}_i$, $\mathbf{B}_i$, and $\mathbf{C}_i$ are constant matrices of appropriate dimensions. The design goal is to find a matrix F such that the static output feedback controller

$$\mathbf{u}_i(t) = \mathbf{F}\mathbf{y}_i(t),\ \ i = 1,2,\ldots,p$$

can stabilize all the closed loop systems in the presence of uncertainty $\Delta\mathbf{A}_i$.

Since $\mu_2(\Delta\mathbf{A}_i) \le \|\Delta\mathbf{A}_i\|$, it is known that if we can find a feedback matrix F such that

$$\mu_2(\mathbf{A}_i + \mathbf{B}_i\mathbf{F}\mathbf{C}_i) < -\rho_i,\ \ i = 1,2,\ldots,p \quad (27)$$

then all the closed-loop systems are asymptotically stable. This problem can be easily solved via our approach.

3.2 Iterative LMI approach

Theorem 3.6: For any $\varepsilon > 0$, if exists a symmetric and positive definite matrix $\mathbf{P} \in \Re^{n\times n}$ such that the following inequalities are satisfied

$$\mu_2(\mathbf{P}\mathbf{A}_i) < -\frac{1}{2\varepsilon} - \frac{\varepsilon}{2}\|\mathbf{P}\mathbf{D}_i\|^2,\ \ i = 1,2,\ldots,p, \quad (28)$$

then the equilibrium of a collection of unforced system (15) is asymptotically stable.

Proof: Consider a Lyapunov function as

$$V(\mathbf{x}_i(t)) = \mathbf{x}_i^T(t)\mathbf{P}\mathbf{x}_i(t) + \frac{1}{\varepsilon}\int_{t-h_i}^{t}\mathbf{x}_i^T(s)\mathbf{x}_i(s)ds\ ,$$

$i = 1,2,\ldots,p$.

The time derivative of $V(\mathbf{x}_i(t))$ is

$$\dot{V}(\mathbf{x}_i(t)) = \mathbf{x}_i^T(t)\mathbf{P}\dot{\mathbf{x}}_i(t) + \dot{\mathbf{x}}_i^T(t)\mathbf{P}\mathbf{x}_i(t) + \frac{1}{\varepsilon}\mathbf{x}_i^T(t)\mathbf{x}_i(t)$$
$$-\frac{1}{\varepsilon}\mathbf{x}_i^T(t-h_i)\mathbf{x}_i(t-h_i)$$

$$= \mathbf{x}_i^T(t)\mathbf{P}(\mathbf{A}_i\mathbf{x}_i(t) + \mathbf{D}_i\mathbf{x}_i(t-h_i))$$
$$+ (\mathbf{A}_i\mathbf{x}_i(t) + \mathbf{D}_i\mathbf{x}_i(t-h_i))^T\mathbf{P}\mathbf{x}_i(t)$$
$$+ \frac{1}{\varepsilon}\mathbf{x}_i^T(t)\mathbf{x}_i(t) - \frac{1}{\varepsilon}\mathbf{x}_i(t-h_i)^T\mathbf{x}_i(t-h_i)$$
$$= \mathbf{x}_i^T(t)(\mathbf{A}_i^T\mathbf{P} + \mathbf{P}\mathbf{A}_i + \frac{1}{\varepsilon}\mathbf{I})\mathbf{x}_i(t)$$
$$+ \mathbf{x}_i^T(t)\mathbf{P}\mathbf{D}_i\mathbf{x}_i(t-h_i) + \mathbf{x}_i^T(t-h_i)\mathbf{D}_i^T\mathbf{P}\mathbf{x}_i(t)$$
$$- \frac{1}{\varepsilon}(\mathbf{x}_i^T(t-h_i)\cdot\mathbf{x}_i(t-h_i))$$

Set $\overline{\mathbf{x}}_i(t) = [\mathbf{x}_i^T(t) \;\; \mathbf{x}_i^T(t-h_i)]^T$. We obtain

$$\dot{V}(\mathbf{x}_i(t)) = \overline{\mathbf{x}}_i^T(t)\begin{bmatrix} \mathbf{A}_i^T\mathbf{P} + \mathbf{P}\mathbf{A}_i + \frac{1}{\varepsilon}\mathbf{I} & \mathbf{P}\mathbf{D}_i \\ \mathbf{D}_i^T\mathbf{P} & -\frac{1}{\varepsilon}\mathbf{I} \end{bmatrix}\overline{\mathbf{x}}_i(t).$$

By Schur complement, if $\mathbf{A}_i^T\mathbf{P} + \mathbf{P}\mathbf{A}_i + \frac{1}{\varepsilon}\mathbf{I} + \varepsilon\mathbf{P}\mathbf{D}_i\mathbf{D}_i^T\mathbf{P} < 0$, then $\begin{bmatrix} \mathbf{A}_i^T\mathbf{P} + \mathbf{P}\mathbf{A}_i + \frac{1}{\varepsilon}\mathbf{I} & \mathbf{P}\mathbf{D}_i \\ \mathbf{D}_i^T\mathbf{P} & -\frac{1}{\varepsilon}\mathbf{I} \end{bmatrix}$ is negative definite. From (16), we obtain

$$\mu_2(\mathbf{A}_i^T\mathbf{P} + \mathbf{P}\mathbf{A}_i + \frac{1}{\varepsilon}\mathbf{I} + \varepsilon\mathbf{P}\mathbf{D}_i\mathbf{D}_i^T\mathbf{P})$$
$$\leq \mu_2(\mathbf{A}_i^T\mathbf{P} + \mathbf{P}\mathbf{A}_i) + \mu_2(\frac{1}{\varepsilon}\mathbf{I}) + \mu_2(\varepsilon\mathbf{P}\mathbf{D}_i\mathbf{D}_i^T\mathbf{P})$$
$$\leq 2\mu_2(\mathbf{P}\mathbf{A}_i) + \frac{1}{\varepsilon} + \varepsilon\left\|\mathbf{P}\mathbf{D}_i\right\|^2$$
$$< 0$$

This proves $\mu_2(\mathbf{A}_i^T\mathbf{P} + \mathbf{P}\mathbf{A}_i + \frac{1}{\varepsilon}\mathbf{I} + \varepsilon\mathbf{P}\mathbf{D}_i\mathbf{D}_i^T\mathbf{P}) < 0$ $\Rightarrow$ $\mathbf{A}_i^T\mathbf{P} + \mathbf{P}\mathbf{A}_i + \frac{1}{\varepsilon}\mathbf{I} + \varepsilon\mathbf{P}\mathbf{D}_i\mathbf{D}_i^T\mathbf{P} < 0$, and then

$$\begin{bmatrix} \mathbf{A}_i^T\mathbf{P} + \mathbf{P}\mathbf{A}_i + \frac{1}{\varepsilon}\mathbf{I} & \mathbf{P}\mathbf{D}_i \\ \mathbf{D}_i^T\mathbf{P} & -\frac{1}{\varepsilon}\mathbf{I} \end{bmatrix} < 0.$$

So $\dot{V}(\mathbf{x}_i(t)) < 0$ for all $\mathbf{x}_i(t) \neq 0$. This completes the proof. □

In what follows, we shall consider the stability conditions of a collection of unforced time-delay systems *with uncertainty*.

Theorem 3.7: If

$$\mu_2(\mathbf{P}\mathbf{A}_i) < -\frac{1}{2\varepsilon} - \left\|\mathbf{P}\right\|\left\|\mathbf{M}_i\right\| - \frac{\varepsilon}{2}\left\|\mathbf{P}\right\|^2\left\|\mathbf{D}_i\right\|^2 - \varepsilon\left\|\mathbf{P}\right\|^2\left\|\mathbf{N}_i\right\|\cdot\left\|\mathbf{D}_i\right\|$$
$$-\frac{1}{2}\varepsilon\left\|\mathbf{P}\right\|^2\left\|\mathbf{N}_i\right\|^2, \; i = 1, 2, \ldots, p \quad (29)$$

then (17) is robustly asymptotically stable.

Proof: Since

$$2\mu_2(\mathbf{P}\hat{\mathbf{A}}_i) + \frac{1}{\varepsilon} + \varepsilon\left\|\mathbf{P}\hat{\mathbf{D}}_i\right\|^2$$
$$= 2\mu_2(\mathbf{P}\mathbf{A}_i + \mathbf{P}\Delta\mathbf{A}_i) + \frac{1}{\varepsilon} + \varepsilon\left\|\mathbf{P}\mathbf{D}_i + \mathbf{P}\Delta\mathbf{D}_i\right\|^2$$
$$= 2\mu_2(\mathbf{P}\mathbf{A}_i) + 2\mu_2(\mathbf{P}\Delta\mathbf{A}_i) + \frac{1}{\varepsilon} + \varepsilon\left\|\mathbf{P}\mathbf{D}_i\right\|^2 + 2\varepsilon\left\|\mathbf{P}\mathbf{D}_i\right\|\cdot\left\|\mathbf{P}\Delta\mathbf{D}_i\right\|$$
$$+\varepsilon\left\|\mathbf{P}\Delta\mathbf{D}_i\right\|^2$$
$$< 2\mu_2(\mathbf{P}\mathbf{A}_i) + 2\left\|\mathbf{P}\right\|\left\|\mathbf{M}_i\right\| + \frac{1}{\varepsilon} + \varepsilon\left\|\mathbf{P}\right\|^2\left\|\mathbf{D}_i\right\|^2 + 2\varepsilon\left\|\mathbf{P}\right\|^2\left\|\mathbf{N}_i\right\|\cdot\left\|\mathbf{D}_i\right\|$$
$$+\varepsilon\left\|\mathbf{P}\right\|^2\left\|\mathbf{N}_i\right\|^2$$
$$< 0$$

This completes the proof. □

With the above theorems, we have the following corollary.

Corollary 2: Suppose that static output feedback gain $\mathbf{F}$ satisfy the following conditions

$$\mu_2(\mathbf{P}(\mathbf{A}_i + \mathbf{B}_i\mathbf{F}\mathbf{C}_i)) < -\frac{1}{2\varepsilon} - \left\|\mathbf{P}\right\|\left\|\mathbf{M}_i\right\| - \frac{1}{2}\varepsilon\left\|\mathbf{P}\right\|^2\left\|\mathbf{D}_i\right\|^2$$
$$-\varepsilon\left\|\mathbf{P}\right\|^2\left\|\mathbf{N}_i\right\|\cdot\left\|\mathbf{D}_i\right\| - \frac{1}{2}\varepsilon\left\|\mathbf{P}\right\|^2\left\|\mathbf{N}_i\right\|^2, \quad i = 1, 2, \ldots, p, \quad (30)$$

then the equilibrium of the closed-loop time-delay system with norm-bounded uncertainties represented as (20) is robustly asymptotically stable.

For simplicity of notation, define

$$\eta_i = -\frac{1}{2\varepsilon} - \left\|\mathbf{P}\right\|\left\|\mathbf{M}_i\right\| - \frac{1}{2}\varepsilon\left\|\mathbf{P}\right\|^2\left\|\mathbf{D}_i\right\|^2 - \varepsilon\left\|\mathbf{P}\right\|^2\left\|\mathbf{N}_i\right\|\cdot\left\|\mathbf{D}_i\right\|$$
$$-\frac{1}{2}\varepsilon\left\|\mathbf{P}\right\|^2\left\|\mathbf{N}_i\right\|^2, \quad i = 1, 2, \ldots, p.$$

Thus, (30) becomes

$$\mu_2(\mathbf{P}(\mathbf{A}_i + \mathbf{B}_i\mathbf{F}\mathbf{C}_i)) < \eta_i, \quad i = 1, 2, \ldots, p. \quad (31)$$

The matrix inequality (31) leads to nonlinear matrix inequality optimization, a non-convex programming problem. Non-convexity implies the existence of local minima and the nonlinear matrix inequality problems are NP-hard. However, we shall reduce the matrix measure assignment problem (31) to a matrix inequality problem from the following theorem.

Theorem 3.8: The static output feedback gains **F** satisfy the following conditions

$$\mu_2(\mathbf{P}(\mathbf{A}_i+\mathbf{B}_i\mathbf{F}\mathbf{C}_i))<\eta_i,\ \ i=1,2,\ldots,p. \tag{32}$$

if and only if **F** satisfy the following matrix inequality

$$(\mathbf{P}\mathbf{A}_i+\mathbf{A}_i^T\mathbf{P}-2\eta_i\mathbf{I})+\mathbf{P}\mathbf{B}_i\mathbf{F}\mathbf{C}_i+\mathbf{C}_i^T\mathbf{F}\mathbf{B}_i^T\mathbf{P}<0,\quad i=1,2,\ldots,p. \tag{33}$$

Proof: From (9), it can be shown that $\mu_2(\mathbf{P}(\mathbf{A}_i+\mathbf{B}_i\mathbf{F}\mathbf{C}_i))<\eta_i$, $i=1,2,\ldots,p$, are equivalent to $\mathbf{P}(\mathbf{A}_i+\mathbf{B}_i\mathbf{F}\mathbf{C}_i)+(\mathbf{P}(\mathbf{A}_i+\mathbf{B}_i\mathbf{F}\mathbf{C}_i))^T-2\eta_i\mathbf{I}<0$, $i=1,2,\ldots,p$, which are also equivalent to $(\mathbf{P}\mathbf{A}_i+\mathbf{A}_i^T\mathbf{P}-2\eta_i\mathbf{I})+\mathbf{P}\mathbf{B}_i\mathbf{F}\mathbf{C}_i+\mathbf{C}_i^T\mathbf{F}\mathbf{B}_i^T\mathbf{P}<0$, $i=1,2,\ldots,p$. This completes the proof. □

In fact, the above matrix inequality (33) problem is generally very difficult for which to obtain solutions or to determine feasibility. However, if we can derive an iterative form for its feasibility, we may construct an iterative algorithm based on the LMI technique [28].

If P is fixed in (33), then it reduces to an LMI problem in the unknowns **F** The LMI problem is convex and can be solved if a feasible solution exists. If we simply perturb (33) by $-\beta P$, then we obtain a necessary condition for static output feedback stabilizability, i.e.,

$$\mathbf{P}\mathbf{A}_i+\mathbf{A}_i^T\mathbf{P}-2\eta_i\mathbf{I}-\beta\mathbf{P}+\mathbf{P}\mathbf{B}_i\mathbf{F}\mathbf{C}_i+\mathbf{C}_i^T\mathbf{F}^T\mathbf{B}_i^T\mathbf{P}<0,\quad i=1,2,\ldots,p.$$

Consequently, the closed-loop system matrices $\mathbf{A}_i+\mathbf{B}_i\mathbf{F}\mathbf{C}_i$ have eigenvalues on the left-hand side of the line $\Re(s)=\beta/2$ in the complex s-plane.

Iterative Linear Matrix Inequality Algorithm:

Step 1) Set m=1, select **S**>0. Solve the following ARE:

$$\mathbf{A}_i^T\mathbf{P}+\mathbf{P}\mathbf{A}_i-\mathbf{P}\mathbf{B}_i\mathbf{B}_i^T\mathbf{P}+\mathbf{S}=0$$

and set $\mathbf{F}=\mathbf{P}$, $i=1,2,\ldots,p$.

Step 2) Solve the following optimization problem for $\mathbf{P}_m$ and β_m.

OP1: Minimize β_m subject to the LMI constraints shown in (34)-(35).

$$\mathbf{P}_m\mathbf{A}_i+\mathbf{A}_i^T\mathbf{P}_m-2\gamma_i\mathbf{I}-\beta_m\mathbf{P}_m+\mathbf{P}_m\mathbf{B}_i\mathbf{F}\mathbf{C}_i+\mathbf{C}_i^T\mathbf{F}^T\mathbf{B}_i^T\mathbf{P}_m<0 \tag{34}$$

$$\mathbf{P}_m=\mathbf{P}_m^T>0 \tag{35}$$

Step 3) If $\beta_m\le 0$, $\mathbf{P}_m$ is a feasible solution. STOP.

Step 4) Solve the following optimization problem for $\mathbf{P}_m$.

OP2: Minimize trace($\mathbf{P}_m$) subject to the LMI constraints shown in (34)-(35).

Step 5) If $\|\mathbf{F}-\mathbf{P}_m\|<\delta$, a predetermined tolerance, go to Step 6); else set $\mathbf{F}=\mathbf{P}$ and m=m+1, then go to Step 2).

Step 6) This algorithm cannot get a feasible solution. STOP.

In Step 2) is viewed as a generalized eigenvalue minimization problem. This step ensures that the poles of the global closed-loop system move to the left half-plane gradually. Numerical experiences denoted that β may converge slowly in some cases. The algorithm is terminated when $\beta_{m-1}-\beta_m$ is smaller than a prescribed tolerance for a fixed number of successive iterations. In Step 3), we set $\beta_m=0$ and let the algorithm continue iterating to make the difference of **F** and **P** as small as possible if a feasible solution is obtained and the feedback gain is too large. The condition (34) guarantees the existence of a solution of optimization problem OP2. The solution $\mathbf{P}_m$ implies that the sequence trace($\mathbf{P}_m$) is bounded below. If β_m is fixed for m>q and q is a positive constant, it is not difficult to find that the solution sequence trace($\mathbf{P}_m$) is a monotonic decreasing sequence. OP2 may be infeasible due to the effect of numerical errors in Step 2). In such a case, one may set $\beta_m=\beta_m+\Delta\beta_m$ for some small positive number $\Delta\beta_m$, and solve OP2 again.

4. Illustrative Examples

Example: Consider two interval time-delay systems described by (1)-(3) with the same dimension:

System 1:

$$\underline{\mathbf{A}}_1 = \begin{bmatrix} -103.8 & -105.6 & -93.5 \\ -134.3 & -79.1 & -119.2 \\ -89.4 & -133.6 & -136.5 \end{bmatrix},$$

$$\overline{\mathbf{A}}_1 = \begin{bmatrix} -101.7 & -103.9 & -91.6 \\ -131.5 & -77.2 & -116.6 \\ -86.2 & -129.1 & -113.4 \end{bmatrix},$$

$$\underline{\mathbf{D}}_1 = \begin{bmatrix} -0.4 & 0.7 & -0.8 \\ 0.3 & -3.2 & -2.1 \\ -3.3 & -0.5 & -2.3 \end{bmatrix},$$

$$\overline{\mathbf{D}}_1 = \begin{bmatrix} -0.2 & 1.0 & -0.3 \\ 0.8 & -1.8 & 0.1 \\ -0.7 & 0.5 & -1.2 \end{bmatrix},$$

$$\mathbf{B}_1 = \begin{bmatrix} 1 & -7 \\ 5 & -3 \\ 3 & -4 \end{bmatrix}, \qquad \mathbf{C}_1 = \begin{bmatrix} -7 & 8 & 1 \\ 2 & -5 & -3 \end{bmatrix}.$$

System 2:

$$\underline{\mathbf{A}}_2 = \begin{bmatrix} 102.1 & 111.3 & 90.7 \\ 91.2 & 86.1 & 113.7 \\ 130.8 & 120.5 & 77.2 \end{bmatrix},$$

$$\overline{\mathbf{A}}_2 = \begin{bmatrix} 104.6 & 114.2 & 94.5 \\ 93.7 & 89.9 & 116.8 \\ 134.6 & 124.4 & 80.5 \end{bmatrix},$$

$$\underline{\mathbf{D}}_2 = \begin{bmatrix} -0.7 & 0.2 & -1.1 \\ 0.9 & -5.0 & -4.0 \\ -2.3 & -0.9 & -4.2 \end{bmatrix},$$

$$\overline{\mathbf{D}}_2 = \begin{bmatrix} -0.3 & 0.8 & -0.7 \\ 1.3 & -3.0 & -1.0 \\ -1.1 & 0.2 & -3.4 \end{bmatrix},$$

$$\mathbf{B}_2 = \begin{bmatrix} 1 & -7 \\ 5 & -3 \\ 3 & -4 \end{bmatrix}, \quad \mathbf{C}_2 = \begin{bmatrix} -13.37 & -10.51 & -8.59 \\ 11.49 & 9.67 & 8.2 \end{bmatrix}.$$

The delay times $h_1 = 1$ and $h_2 = 1$. The problem is to find $\mathbf{F}$ such that $\mu_2(\mathbf{A}_i + \mathbf{B}_i\mathbf{F}\mathbf{C}_i) < \gamma_i$ for $i = 1, 2$, where

$$\mathbf{A}_i = \frac{1}{2}(\underline{\mathbf{A}}_i + \overline{\mathbf{A}}_i),\ \mathbf{D}_i = \frac{1}{2}(\underline{\mathbf{D}}_i + \overline{\mathbf{D}}_i),\ i = 1, 2,$$
$$\mathbf{M}_i = \overline{\mathbf{A}}_i - \mathbf{A}_i,\ \mathbf{N}_i = \overline{\mathbf{D}}_i - \mathbf{D}_i,\ i = 1, 2,$$

$$\gamma_i = -\frac{1}{2\varepsilon} - \|\mathbf{M}_i\| - \frac{1}{2}\varepsilon\|\mathbf{D}_i\|^2 - \varepsilon\|\mathbf{N}_i\|\cdot\|\mathbf{D}_i\| - \frac{1}{2}\varepsilon\|\mathbf{N}_i\|^2 ,$$
$i = 1, 2$.

We can obtain $\gamma_1 = -10.4484$ and $\gamma_2 = -14.9262$. Then we can easily compute a solution $\mathbf{F}$ from the following LMIs using Matlab's LMI Control Toolbox.

$$(\mathbf{A}_1 + \mathbf{A}_1^T - 2\gamma_1\mathbf{I}) + \mathbf{B}_1\mathbf{F}\mathbf{C}_1 + \mathbf{C}_1^T\mathbf{F}^T\mathbf{B}_1^T < \mathbf{0}$$
$$(\mathbf{A}_2 + \mathbf{A}_2^T - 2\gamma_2\mathbf{I}) + \mathbf{B}_2\mathbf{F}\mathbf{C}_2 + \mathbf{C}_2^T\mathbf{F}^T\mathbf{B}_2^T < \mathbf{0}$$

A solution is obtained as:

$$\mathbf{F} = \begin{bmatrix} -10.4616 & -13.3610 \\ -1.9983 & -0.3117 \end{bmatrix}.$$

It is easy to check that $\mu_2(\mathbf{A}_1 + \mathbf{B}_1\mathbf{F}\mathbf{C}_1) = -12.3590$, which is less than $\gamma_1 = -10.4484$. Similarly, $\mu_2(\mathbf{A}_2 + \mathbf{B}_2\mathbf{F}\mathbf{C}_2) =$ -26.5851 < $\gamma_2 = -14.9262$. It then can be inferred that the collection of systems $\dot{x}(t) = (\hat{\mathbf{A}}_i + \mathbf{B}_i\mathbf{F}\mathbf{C}_i)x_i(t) + \hat{\mathbf{D}}_i x_i(t - h_i)$, for $i = 1, 2$, are all robustly stable.

5. Conclusions

In this paper, finding an admissible solution to the matrix measure assignment problem can solve the problem of simultaneously stabilizing controller design via static output feedback for a collection of interval time-delay systems. We presented an LMI approach to solve the matrix measure assignment problem. It was shown that the admissible solution set of the matrix measure assignment problem is convex. It is also shown that the matrix measure assignment problem is equivalent to an LMI feasibility problem. A necessary and sufficient condition for the existence of output feedback controllers to the matrix measure assignment problem is obtained. Finally, an illustrative example is given to show the correctness of the proposed approach. Our approach does not need to find a common positive definite matrix and the verification of stability is very easy. Simulation results have verified and confirmed the effectiveness of the new approach in the simultaneous stabilization of a collection of interval time-delay systems.

References:

[1] SAKES, R., and MURRAY, J., "Fractional representation algebraic geometry and the simultaneous stabilization problem," *IEEE Trans. Automatic Control*, vol. 27, 1982, pp. 895-903.

[2] VIDYASAGAR, M., and VISWANADHAM, N., "Algebraic design techniques for reliable stabilization," *IEEE Trans. Automatic Control*, vol. 27, 1982, pp. 1085-1095.

[3] PASKOTA, M., SREERAM, V., TEO, K. L., and MEERS, A. I., "Optimal simultaneous stabilization of linear single-input systems via linear state feedback," *Int. J. Control*, vol. 60, no. 4, 1994, pp. 483-498.

[4] FANG, Y., LOPARO, K. A., and FENG, X., "A sufficient condition for stability of a polytope of matrices," *Systems & Control Letters*, vol. 23, 1994, pp. 237-245.

[5] FANG, Y., LOPARO, K. A., and FENG, X., "Sufficient condition for the stability of interval matrices," *Int. J. Control*, vol. 58, no. 4, 1993, pp. 969-977.

[6] BOYD, S., EL GHAOUI, L., FERON, E., and BALAKRISHNAN, V., *Linear Matrix Inequalities in System and Control Theory*, SIAM, Philadelphia, 1994.

[7] CAO, Y. Y., and Lam, J., "A computational method for simultaneous LQ optimal control design via piecewise constant output feedback," *IEEE Trans. Systems, Man, and Cybernetics-Part B: Cybernetics*, vol. 318, no. 5, 2001, pp. 836-842.

[8] CAO, Y. Y., and SUN, Y. X, "Static output feedback simultaneous stabilization: ILMI approach," *Int. J. Control*, vol. 70, no. 5, 1998, pp. 803-814.

[9] CAO, Y. Y., SUN, Y. X, and LAM, J. "Simultaneous stabilization via static output feedback and state feedback," *IEEE Trans. Automatic Control*, vol. 44, 1999, pp. 1277-1282.

[10] Blondel, V., and Gevers, M., "Simultaneous stabilizability of three linear system is rational undecidable," *Math. Contr. Sig. Syst.*, vol. 6, 1993, pp. 135-145.

[11] MORI, T., NOLDUS, E., and KUWAHARA, M., "A way to stabilize linear systems with delayed state," *Automatica*, vol. 19, no. 5, 1983, pp. 571-573.

[12] MILLER, D. E., and ROSSI, M., "Simultaneous stabilization with near optimal LQR performace," *IEEE Trans. on Automatic Control*, vol. 46, no. 10, 2001, pp. 1543-1555.

[13] SAIF, A.-W., GU, D.-W., KAVRANOGLU, D., and POSTLETHWAITE, I., "Simultaneous stabilization of MIMO systems via robustly stabilizing a central plant," *IEEE Trans. on Automatic Control*, vol. 47, no. 2, 2002, pp. 363-369.

[14] TISSIR, E., and HMAMED, A., "Stability tests of interval time delay systems," *Systems & Control Letter*s, vol. 23, 1994, pp. 263-270.

[15] TOKER, O., and Ozbay, H., "On the NP-hardness of solving bilinear matrix inequalities and simultaneous stabilization with static output feedback," *Proc. American Contr. Conf.*, Seattle, WA, 1995.

[16] SYRMOS, V. L., ABDALLAH, C., DORATO, P., and GRIGORIADIS, K., "Static output feedback: A survey," *Automatica*, vol. 33, no. 2, 1997, pp. 125-137.

[17] NEMIROVSKII, A., and GAHINET, P., "The projective method for solving linear matrix inequalities," *Proc. American Control Conference*, 1994, pp. 840-844.

[18] GAHINET, P., NEMIROVSKII, A., LAUB, A. J., and CHILALI, M., *LMI Control Toolbox*, The Math Works, Inc., 1994.

[19] GAHNET, P., and APKARAIN, P., "A linear matrix inequality approach to H_∞ control," *Int. J. Robust Nonlinear Contr.*, vol. 4, 1994, pp. 421-448.

[20] KIM, J. H., "Robust stability of linear systems with delayed perturbations," *IEEE Trans. on Automatic Control*, vol. 41, 1996, pp. 1820-1822.

[21] TRINH, H., and ALDEEN, M., "On the stability of linear systems with delayed perturbations," *IEEE Trans. on Automatic Control*, vol. 39, 1994, pp. 1948-1951.

[22] SHYU, K. K., and YAN, J. J., "Robust stability of uncertain time-delay systems and its stabilization by variable structure control," *Int. J. Control*, vol. 57, 1993, pp. 237-247.

[23] NICULESCU, S. I., DE SOUZA, C. E.,DUGARD, L., and DION, J. M., "Robust exponential stability of uncertain systems with time-varying delays," *IEEE Trans. on Automatic Control*, vol. 43, 1998, pp. 743-748.

[24] CRO, Y. Y., SUN, Y. X., and CHENG, C., "Delay-dependent robust stabilization of uncertain systems with multiple state delays," *IEEE Trans. on Automatic Control*, vol. 43, 1998, pp. 1608-1612.

[25] LI, X., and DE SOUZA, C. E., "Criteria for robust stability and stabilization of uncertain

linear systems with state delay," *Automatica*, vol. 33, 1997, pp. 1657-1662.
[26] KOLMANOVSKII, V. B., NICULESCU, S. I., and RICHARD, J. P., "On the Lyapunov-Krasovskii functions for stability analysis of linear delay systems," *Int. J. Control*, vol. 72,1999, pp. 374-384.
[27] SU, T.-J., LU, C.-Y., and TSAI, J. S.-H, "LMI approach to delay-dependent robust stability for uncertain time-delay system," *IEE Proc. Control Theory Appl.*, vol. 148, 2001, pp. 209-212.
[28] Y. Y. Cao, Y. X. Sun, and J. Lam, Simultaneous stabilization via static output feedback and state feedback, *IEEE Trans. on Automatic Control,* vol. 44, no. 6, 1999, pp. 1277-1282.