Electromagnetic Torque Analysis of a DFIG for Wind Turbines

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Abstract: Electromagnetic torque of doubly fed induction generator (DFIG) is a consequence of the rotor and stator supply. The stator voltage has a fixed amount and frequency. The rotor voltage of the DFIG as a part of a wind turbine has a variable amount and frequency. The aim of this paper is an investigation of the torque-speed characteristics of the DFIG depending on the amount of the stator and rotor voltage space vectors and angle δ between them. A mathematical model of DFIG for steady-state modes in a known double-axis *dq* reference frame concerned with stator voltage vector has been made. Expressions concerning DFIG power from both the rotor and the stator side have been derived. Expressions regarding torque values have been obtained from the generator active power and mechanical rotational speed. Torque consists of asynchronous and synchronous torque components, whereas DFIG has been observed as a hybrid between asynchronous and synchronous machine. The asynchronous torque and the synchronous torque depend on the amount of the stator and rotor voltage space vectors. The synchronous torque depends on the angle between stator and rotor space vectors δ, while asynchronous component do not depend on the angle δ.

Key-Words: DFIG, Electromagnetic torque, Wind turbine, Renewable energy

1 Introduction

Wind turbines connected to an electric grid represent a complex automated electrical-mechanical system. A variable speed DFIG in wind turbine consists of a turbine, a gearbox, a DFIG, and a control system dealing with the total active and reactive wind turbine power output. The control system consists of a back-to-back semiconductor converter, on one side being connected to the DFIG rotor and on the other to the electric grid. The basic integral parts of the variable speed DFIG connected to electric grid have been shown in Figure 1. [4]:

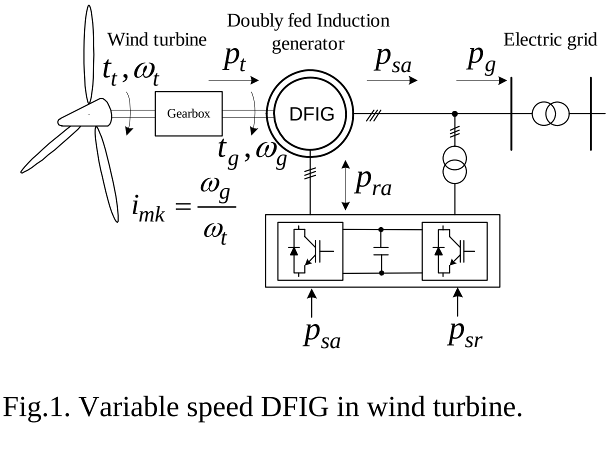

Fig.1. Variable speed DFIG in wind turbine.

The DFIG is the generator most widely used in variable speed wind turbines [3]. It is therefore indispensable to become better acquainted with both electrical and mechanical DFIG characteristics in stationary and dynamic modes of operation. One of the basic characteristics of the generator fed both from the stator and rotor sides is the electromagnetic torque as a function of rotational speed.

By controlling the rotor speed and electromagnetic torque, as well as active and reactive DFIG power connected to the electric grid, control of feed voltage on the rotor side can be affected.

In papers [1,2] methods of mathematical modelling of DFIG torque have been presented. In paper [1] equations expressing DFIG active and reactive power have been derived. The electromagnetic torque has been obtained by means of DFIG active power and the DFIG mechanical rotor speed. In this paper, analytical expression for the calculation of DFIG torque has been given and discussed. However, the DFIG torque simulation results have not been presented.

Paper [2] views the DFIG as a hybrid of an asynchronous and synchronous machine; as a synchronous generator since the DFIG torque depends upon the rotor voltage and angle δ between the rotor and stator feed voltage vector, and as an asynchronous machine due to the fact that DFIG torques depend upon the slip *s*. The torque equations have been derived with a negligible generator magnetizing inductance. In this paper mathematical expressions pertaining to both the DFIG asynchronous and synchronous torques have been derived.

DFIG active power is obtained as a sum (or difference) of powers on the stator and rotor terminals reduced by Joule's losses in the generator coils. The total DFIG electromagnetic torque is obtained as a quotient of DFIG active power and the mechanical rotational speed of the rotor. Steady-state modes have been analysed as a special case of machine status if transitional phenomena in the generator are to be neglected.

An analysis of the electromagnetic torque in the rotor feed voltage controlled generator has been done. Analytical expressions regarding four DFIG torque components, DFIG being perceived as a hybrid of an asynchronous and synchronous machine, have been derived. The components of asynchronous torque and the components of synchronous torque depend on the amount of the stator and rotor voltage space vectors. The synchronous components depend on the angle between stator and rotor space vectors δ, while asynchronous components do not depend on the angle δ.

2 Asynchronous Generator Mathematical Model

DFIG Voltage differential equations for stator and rotor coils written in vector form read as follows [4,5]:

$$\bar{u}_s = \bar{i}_s R_s + \frac{d\bar{\psi}_s}{dt},$$
$$\bar{u}_r = \bar{i}_r R_r + \frac{d\bar{\psi}_r}{dt}. \quad (1)$$

where $\bar{u}_s, \bar{u}_r$ - stator and rotor feed voltage, $\bar{i}_s, \bar{i}_r$ - stator and rotor current vectors, $\bar{\psi}_s$, $\bar{\psi}_r$ - stator and rotor magnetic flux vectors, and R_s, R_r - stator and rotor resistance.

In DFIG mathematical modelling *dq* coordinate systems connected to the stator magnetic flux vector or stator voltage vector are commonly used. Asynchronous vector voltage differential equations expressed in the *dq* coordinate system rotating at synchronous rotational speed read as follows:

$$\bar{u}_{sdq} = \bar{i}_{sdq} R_s + \frac{d\bar{\psi}_{sdq}}{dt} + j\omega_s \bar{\psi}_{sdq},$$
$$\bar{u}_{rdq} = \bar{i}_{rdq} R_r + \frac{d\bar{\psi}_{rdq}}{dt} + j(\omega_s - \omega)\bar{\psi}_{rdq}, \quad (2)$$

where ω_s - coordinate system electric rotational speed and stator voltage angular frequency, with ω - rotor mechanical rotational speed.

The relation between the magnetic flux vector and stator and rotor currents has been expressed by the following equations:

$$\bar{\psi}_{sdq} = L_s \bar{i}_{sdq} + L_m \bar{i}_{rdq},$$
$$\bar{\psi}_{rdq} = L_m \bar{i}_{sdq} + L_r \bar{i}_{rdq}, \quad (3)$$

where $L_s = L_{\sigma s} + L_m$, $L_r = L_{\sigma r} + L_m$ - total inductance of stator and rotor coils, while $L_{\sigma s}$, $L_{\sigma r}$, L_m - stator and rotor leakage inductance and magnetizing inductance.

Stator and rotor current vectors have been expressed as a function of magnetic flux and are as follows:

$$\bar{i}_{sdq} = \frac{1}{L_s'}\bar{\psi}_{sdq} - \frac{k_r}{L_s'}\bar{\psi}_{rdq},$$
$$\bar{i}_{rdq} = -\frac{k_s}{L_r'}\bar{\psi}_{sdq} + \frac{1}{L_r'}\bar{\psi}_{rdq}, \quad (4)$$

and the magnetizing current is:

$$\bar{i}_{mdq} = \bar{i}_{sdq} + \bar{i}_{rdq}.$$

If in the equations (2) stator and rotor current vectors should be expressed as a function of the correspondent magnetic flux, a system of vector voltage differential equations is obtained in which status variables are represented by the magnetic fluxes:

$$\bar{u}_{sdq} = \frac{d\bar{\psi}_{sdq}}{dt} + \left(\frac{1}{T_s'} + j\omega_s\right)\bar{\psi}_{sdq} - \frac{k_r}{T_s'}\bar{\psi}_{rdq},$$
$$\bar{u}_{rdq} = \frac{d\bar{\psi}_{rdq}}{dt} - \frac{k_s}{T_r'}\bar{\psi}_{sdq} + \left(\frac{1}{T_r'} + j(\omega_s - \omega)\right)\bar{\psi}_{rdq}. \quad (5)$$

The parameters appearing in equations (4) and (5) are:

$$L_s' = \sigma L_s,\ L_r' = \sigma L_r,\ \sigma = 1 - \frac{L_m^2}{L_s L_r},$$

$$k_s = \frac{L_m}{L_s},\ k_r = \frac{L_m}{L_r},\ T_s' = \frac{L_s'}{R_s},\ T_r' = \frac{L_r'}{R_r}$$

DFIG steady-state modes may be described by means of voltage differential equations (2), assuming that $d\bar{\psi}_{sdq}/dt = 0$ and $d\bar{\psi}_{rdq}/dt = 0$, and by substituting magnetic flux vectors in the equations (2) by means of stator and rotor currents from equation (3), thereby obtaining the following expression:

$$\begin{aligned} \bar{U}_s &= \bar{I}_s R_s + j\omega_s (L_s \bar{I}_s + L_m \bar{I}_r), \\ \bar{U}_r &= \bar{I}_r R_r + js\omega_s (L_m \bar{I}_s + L_r \bar{I}_r). \end{aligned} \tag{6}$$

or else:

$$\begin{aligned} \bar{U}_s &= \bar{I}_s \left(R_s + j\omega_s L_{s\sigma}\right) + j\omega_s L_m \bar{I}_m , \\ \frac{\bar{U}_r}{s} &= \bar{I}_r \left(\frac{R_r}{s} + j\omega_s L_{r\sigma}\right) + j\omega_s L_m \bar{I}_m . \end{aligned} \tag{7}$$

where $\omega_r = \omega_s - \omega = s\omega_s$ - rotor angular frequency and s – slip (for the sake of simplicity / more simply written as $\bar{U}_s = \bar{U}_{sdq}, \bar{U}_r = \bar{U}_{rdq}, \bar{I}_s = \bar{I}_{sdq}, \bar{I}_r = \bar{I}_{rdq}$).

By means of an equation system (7) a substitute DFIG scheme for steady-state mode is obtained as illustrated in Figure 2.

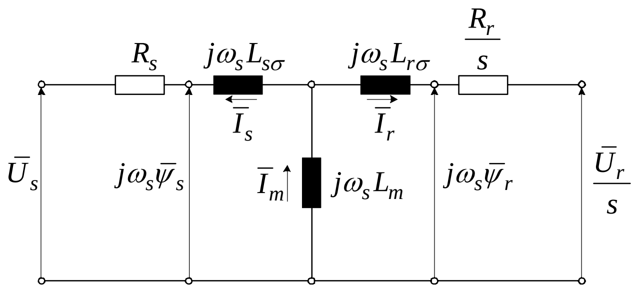

Fig.2. Equivalent circuit of DFIG for steady-state

By resolving the equations (6), stator and rotor current vectors pertaining to DFIG steady-state modes are obtained.

$$\begin{aligned} \bar{I}_s = & \frac{\left(\omega_r - j\dfrac{\sigma}{T_r^{'}}\right)}{\omega_s L_s^{'}\left[\dfrac{1}{T_r^{'}} + \dfrac{\omega_r}{\omega_s T_s^{'}} + j\left(\omega_r - \dfrac{\sigma}{\omega_s T_s^{'} T_r^{'}}\right)\right]}\bar{U}_s - \\ & - \frac{\omega_s k_r}{\omega_s L_s^{'}\left[\dfrac{1}{T_r^{'}} + \dfrac{\omega_r}{\omega_s T_s^{'}} + j\left(\omega_r - \dfrac{\sigma}{\omega_s T_s^{'} T_r^{'}}\right)\right]}\bar{U}_r , \end{aligned} \tag{8}$$

$$\begin{aligned} \bar{I}_r = & \frac{\left(\omega_s - j\dfrac{\sigma}{T_s^{'}}\right)}{\omega_s L_r^{'}\left[\dfrac{1}{T_r^{'}} + \dfrac{\omega_r}{\omega_s T_s^{'}} + j\left(\omega_r - \dfrac{\sigma}{\omega_s T_s^{'} T_r^{'}}\right)\right]}\bar{U}_r - \\ & - \frac{\omega_r k_s}{\omega_s L_r^{'}\left[\dfrac{1}{T_r^{'}} + \dfrac{\omega_r}{\omega_s T_s^{'}} + j\left(\omega_r - \dfrac{\sigma}{\omega_s T_s^{'} T_r^{'}}\right)\right]}\bar{U}_s . \end{aligned}$$

Multiplying the above expressions representing stator and rotor current vectors by a conjugated complex denominator the following equations are obtained for stator and rotor current vector expressed as a function of stator and rotor feed voltage:

$$\begin{aligned} \bar{I}_s &= \bar{Y}_{11}\bar{U}_s + \bar{Y}_{12}\bar{U}_r , \\ \bar{I}_r &= \bar{Y}_{21}\bar{U}_s + \bar{Y}_{22}\bar{U}_r , \end{aligned} \tag{9}$$

and space vector of the magnetizing current is:

$$\bar{I}_m = \bar{I}_s + \bar{I}_r \tag{10}$$

Complex coefficients in the equations (9) (presented in appendix) written in the *dq* coordinate system can be divided into the real and imaginary parts:

$$\begin{aligned} \bar{Y}_{11} &= Y_{11d} + jY_{11q} , & \bar{Y}_{21} &= Y_{21d} + jY_{21q} , \\ \bar{Y}_{12} &= Y_{12d} + jY_{12q} , & \bar{Y}_{22} &= Y_{22d} + jY_{22q} . \end{aligned} \tag{11}$$

On the basis of known stator and rotor voltage and current vectors the DFIG power and electromagnetic torque can be calculated.

3 DFIG Power

Active and reactive power at the DFIG stator and rotor terminals are as follows [1]:

$$\bar{S} = \bar{u}_s \bar{i}_s^{*} + \bar{u}_r \bar{i}_r^{*} . \tag{12}$$

Stator and rotor feed voltage vectors can be expressed by means of current vectors and stator and rotor magnetic flux vectors (equations (2)):

$$\begin{aligned} \bar{S} &= \bar{i}_s^{*}\left(R_s \bar{i}_s + \frac{d\bar{\psi}_s}{dt} + j\omega_s\bar{\psi}_s\right) + \bar{i}_r^{*}\left(R_r \bar{i}_r + \frac{d\bar{\psi}_r}{dt} + j\omega_r\bar{\psi}_r\right) = \\ &= R_s\left|\bar{i}_s\right|^2 + R_r\left|\bar{i}_r\right|^2 + j\omega_s\left(\bar{\psi}_s\bar{i}_s^{*} + \bar{\psi}_r\bar{i}_r^{*}\right) - \\ &- j\omega\bar{\psi}_r\bar{i}_r^{*} + \left\{\frac{d\bar{\psi}_s}{dt}\bar{i}_s^{*} + \frac{d\bar{\psi}_r}{dt}\bar{i}_r^{*}\right\}. \end{aligned} \tag{13}$$

Changes in magnetic flux as shown by equation (13) in the stationary mode can be neglected ($d\overline{\psi}_s / dt = 0$ and $d\overline{\psi}_r / dt = 0$). If stator and rotor currents should be substituted for stator and rotor magnetic flux by means of the stator and rotor equation (3), the following result will be obtained:

$$\begin{aligned}\overline{S} &= R_s\left|\overline{I}_s\right|^2 + R_r\left|\overline{I}_r\right|^2 + \\ &+ j\omega_s\left[\left(L_s\overline{I}_s + L_m\overline{I}_r\right)\overline{I}_s^* + \left(L_m\overline{I}_s + L_r\overline{I}_r\right)\overline{I}_r^*\right] - \\ &- j\omega\left(L_m\overline{I}_s + L_r\overline{I}_r\right)\overline{I}_r^*.\end{aligned} \quad (14)$$

After equation (14) has been resolved, and slip value substituted, as shown in the following expression, $s = \omega_r / \omega_s$, a vector representing total power at the DFIG stator and rotor terminals in the stationary mode is obtained:

$$\begin{aligned}\overline{S} &= \underbrace{\left(R_s + j\omega_s L_s\right)\left|\overline{I}_s\right|^2 + \left(R_r + js\omega_s L_r\right)\left|\overline{I}_r\right|^2}_{P_{Cu} + jQ_L} + \\ &+ \underbrace{js\omega_s L_m\overline{I}_s \cdot \overline{I}_r^* + j\omega_s L_m\overline{I}_r \cdot \overline{I}_s^*}_{P_g + jQ_g},\end{aligned} \quad (15)$$

where $P_g + jQ_g$ describes active and reactive generator power, and $P_{Cu} + jQ_L$ represents Joule's losses and the DFIG reactive power consumption.

If stator and rotor current vectors should be expressed in an equation (15) by means of *d* i *q* components, an equation representing active and reactive DFIG power will be obtained as is illustrated below

$$\begin{aligned}P_g + jQ_g &= \frac{3}{2} j\omega_s L_m\left(I_{sd} - jI_{sq}\right)\left(I_{rd} + jI_{rq}\right) + \\ &+ \frac{3}{2} js\omega_s L_m\left(I_{sd} + jI_{sq}\right)\left(I_{rd} - jI_{rq}\right),\end{aligned} \quad (16)$$

which, after being resolved, reads as follows:

$$\begin{aligned}P_g + jQ_g &= \frac{3}{2} j\omega_s L_m\Big\{\Big[\left(I_{rd}I_{sd} + I_{rq}I_{sq}\right) + \\ &+ j\left(I_{rq}I_{sd} - I_{rd}I_{sq}\right)\Big] + s\Big[\left(I_{sd}I_{rd} + I_{sq}I_{rq}\right) + \\ &+ j\left(I_{sq}I_{rd} - I_{sd}I_{rq}\right)\Big]\Big\}.\end{aligned} \quad (17)$$

After the equation has been resolved, the DFIG active and reactive power is

$$\begin{aligned}P_g &= \frac{3}{2}\omega_s L_m\left(1 - s\right)\left(I_{rd}I_{sq} - I_{rq}I_{sd}\right), \\ Q_g &= \frac{3}{2}\omega_s L_m\left(1 + s\right)\left(I_{rd}I_{sd} + I_{rq}I_{sq}\right).\end{aligned} \quad (18)$$

An expression describing electromagnetic torque is obtained on the basis of DFIG active power.

4 DFIG Electromagnetic Torque

The electromagnetic torque of the DFIG is derived from eq. (18).

$$T = p\frac{P_g}{\omega}. \quad (19)$$

The equation used for obtaining the generator torque in the stationary mode with synchronous rotational speed reads as follows:

$$T = \frac{3}{2} pL_m\left(I_{rd}I_{sq} - I_{rq}I_{sd}\right), \quad (20)$$

Consequently, for obtaining the generator torque in the steady-state mode stator and rotor current vector components should be known. Here, the rotor voltage space vector is a function of the slip s and the angle between stator and rotor voltage vector δ.

Equation of the rotor voltage space vector is

$$\overline{U}_r = s\left|\overline{U}_s\right|\left(\cos\delta + j\sin\delta\right). \quad (21)$$

Figure 3 shows the stator and rotor voltage space vectors in the *dq* reference frame. The direct axes of the *dq* reference frame is aligned to the space vector of the stator voltage ($\overline{U}_s = U_{sd} + j0.0$), whereas the rotor voltage vectors can be generally taken as reading $\overline{U}_r = U_{rd} + jU_{rq}$.

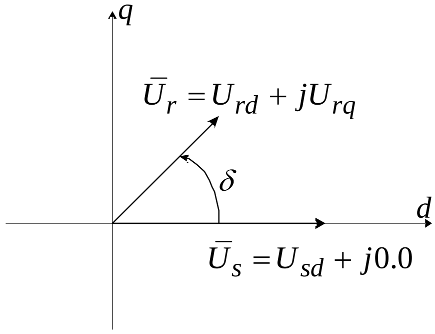

Fig. 3. Stator and rotor feed voltage vector represented in *dq* reference frame

Figure 4 shows the total electromagnetic torque determined by eq. (20). The components of the stator current space vectors (I_{sd} and I_{sq}) and components of the rotor current space vectors (I_{rd} and I_{rq}) are determined by eq. (9). The amounts of the space vectors of the stator I_s, rotor I_r, and magnetizing current I_m are determined by (9) and (10) and shown in Figures 5, 6 and 7. The electromagnetic torque and the currents versus the rotor voltage and the angle between the stator and rotor voltage space vectors are obtained for slip s = 0.4. The rotor voltage is the range from 0.0 to $0.5U_b$ ($U_b = 563[V]$), and angle δ is the range from 0° to 360°. It is evident that the currents and the electromagnetic torque of the DFIG significantly depend on the amount and space position of the rotor space vector.

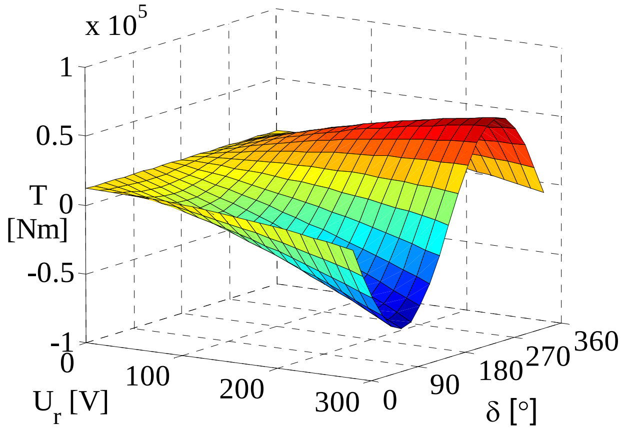

Fig. 4. Electromagnetic torque T versus rotor voltage U_r and angle δ; slip s=0.4

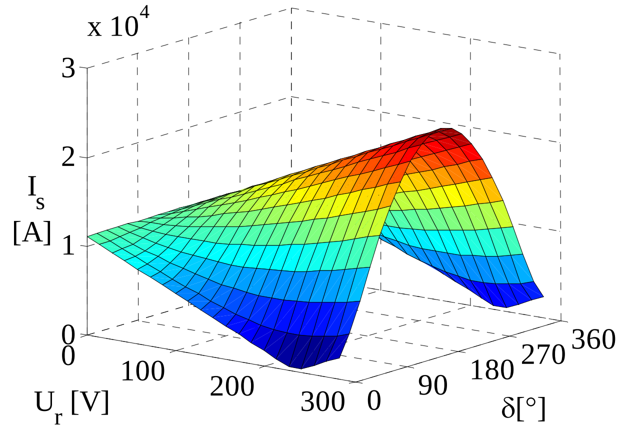

Fig. 5. Stator current I_s versus rotor voltage U_r and angle δ; slip s=0.4

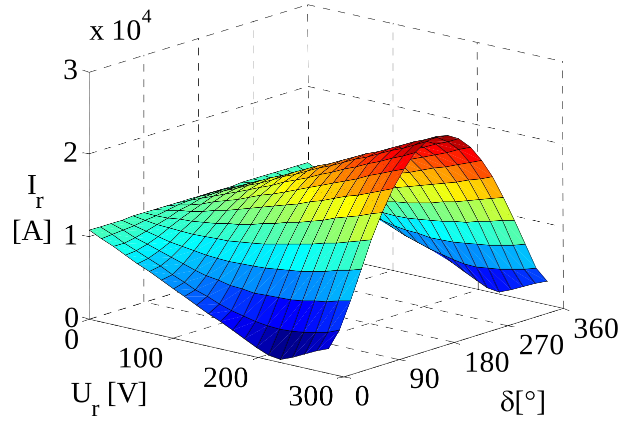

Fig. 6. Rotor current I_r versus rotor voltage U_r and angle δ; slip s=0.4

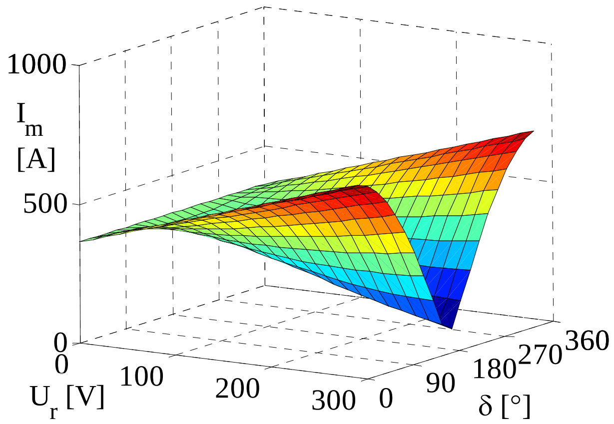

Fig. 7. Magnetizing current I_m versus rotor voltage U_r and angle δ; slip s=0.4

5 Components of asynchronous and synchronous torque DFIG

For a proper analysis of DFIG electromagnetic torque asynchronous torque components due to feed on the stator and rotor side, as well as synchronous torque components due to generator excitation with U_{rd} and U_{rq} rotor voltage vector respectively need to be explored.

Substituting equation (11) into (9) yields:

$$\bar{I}_s = (Y_{11d} + jY_{11q})(U_{sd} + j0) + (Y_{12d} + jY_{12q})(U_{rd} + jU_{rq})$$
$$\bar{I}_r = (Y_{21d} + jY_{21q})(U_{sd} + j0) + (Y_{22d} + jY_{22q})(U_{rd} + jU_{rq}) \quad (22)$$

or

$$\begin{aligned} \bar{I}_s &= \left(Y_{11d}U_{sd} + Y_{12d}U_{rd} - Y_{12q}U_{rq}\right) + \\ &+ j\left(Y_{11q}U_{sd} + Y_{12d}U_{rq} + Y_{12q}U_{rd}\right), \\ \bar{I}_r &= \left(Y_{21d}U_{sd} + Y_{22d}U_{rd} - Y_{22q}U_{rq}\right) + \\ &+ j\left(Y_{21q}U_{sd} + Y_{22d}U_{rq} + Y_{22q}U_{rd}\right). \end{aligned} \tag{23}$$

If in the equation representing electromagnetic torque (20) expressions describing stator and rotor current vector components are substituted, the following result will be obtained:

$$\begin{aligned} T &= \frac{3}{2}pL_m\left(Y_{21d}U_{sd} + Y_{22d}U_{rd} - Y_{22q}U_{rq}\right)\left(Y_{11q}U_{sd} + Y_{12q}U_{rd} + Y_{12d}U_{rq}\right) \\ &-\frac{3}{2}pL_m\left(Y_{21q}U_{sd} + Y_{22q}U_{rd} + Y_{22d}U_{rq}\right)\left(Y_{11d}U_{sd} + Y_{12d}U_{rd} - Y_{12q}U_{rq}\right). \end{aligned} \tag{24}$$

Upon resolving the equation (24) the total electromagnetic torque can be expressed by means of the following four components:

$$T_g = T_s + T_r + T_{sr\cos} + T_{sr\sin}, \tag{25}$$

where T_s - asynchronous torque due to feed on the stator side, T_r - torque components due to feed on the rotor side, $T_{sr\cos}$ and $T_{sr\sin}$ - synchronous torques caused by the excitation of rotor voltage vector components.

Asynchronous Torque Components of DFIG

The asynchronous torque of a DFIG is caused by feed from the stator and rotor side.
Asynchronous torque components caused by feed on the stator side reads as follows:

$$T_s = \frac{3}{2}pL_mU_s^2\left(Y_{21d}Y_{11q} - Y_{21q}Y_{11d}\right), \tag{26}$$

Torque component caused by feed on the rotor side:

$$T_r = \frac{3}{2}pL_mU_r^2\left(Y_{22d}Y_{12q} - Y_{22q}Y_{12d}\right), \tag{27}$$

where $U_r^2 = U_{rd}^2 + U_{rq}^2$.

Therefore, the component of the torque T_s and T_r depends only of the stator and rotor voltage vector amount.

The torque between the voltage components d and q equals zero.

The DFIG torque characteristics have been calculated for the rotor voltage vector obtained by following expressions:

$$U_r = \sqrt{U_{rd}^2 + U_{rq}^2} = s\left|\bar{U}_s\right|,$$

where

$$\begin{aligned} U_{rd} &= s\left|\bar{U}_s\right|\cos\delta, \\ U_{rq} &= s\left|\bar{U}_s\right|\sin\delta \text{ i } \delta = \frac{\pi}{4}. \end{aligned} \tag{28}$$

Mechanical rotational speed ω varies from $0.5\frac{\omega_b}{p}$ to $1.5\frac{\omega_b}{p}$ ($\omega_b = 314[1/s]$). Generator parameters are given in the appendix.

The DFIG asynchronous torque depending on the rotational speed ω due to feed on the stator side T_s and the rotor side T_r have been presented (Fig. 8.).

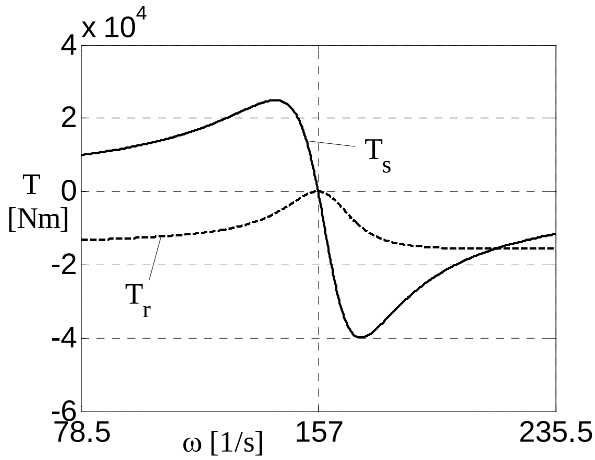

Fig.8. DFIG asynchronous torque component caused by feed on the stator $T_s(\omega)$ and rotor $T_r(\omega)$ sides.

The torque-speed characteristic T_s is well known as the asynchronous machine torque characteristic. It is obtained at the constant voltage $U_s = 690[V]$ and the frequency $\omega = 314[1/s]$. The torque-speed characteristic of the asynchronous torque T_r is obtained for the rotor voltage $U_r = s\left|\bar{U}_s\right| = s$ and the rotor frequency. The asynchronous torque components T_s and T_r are not depend on the angle δ.

Synchronous Torque Components of DFIG

DFIG synchronous torque is caused by rotor excitation voltage. Torque characteristics due to voltage vector component U_{rd} and those caused by excitation rotor voltage component U_{rq} can be separately analysed.

Torque component due to rotor voltage vector component U_{rd} reads as follows:

$$T_{sr\cos} = \frac{3}{2} pL_m U_{sd} U_{rd} \left(Y_{21d} Y_{12q} + Y_{22d} Y_{11q} - Y_{21q} Y_{12d} - Y_{22q} Y_{11d} \right) \quad (29)$$

where as torque component due to rotor voltage vector U_{rq} is:

$$T_{sr\sin} = \frac{3}{2} pL_m U_{sd} U_{rq} \left(Y_{21d} Y_{12d} - Y_{22q} Y_{11q} + Y_{21q} Y_{12q} - Y_{22d} Y_{11d} \right) \quad (30)$$

In Figure 9 DFIG torque components caused by the rotor feed voltage *d* and the stator feed voltage $T_{sr\cos}$, as well as rotor and stator supply voltage components, *q* and $T_{sr\sin}$ respectively, have been illustrated.

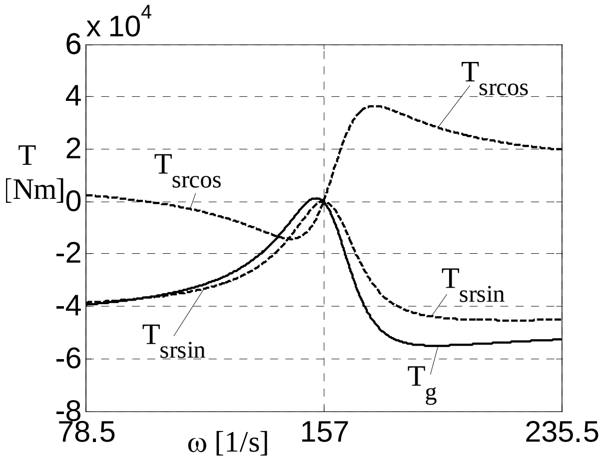

Fig.9. DFIG synchronous torque components due to components U_{rd} of rotor voltage vector $T_{sr\cos}(\omega)$, component U_{rq} of rotor voltage vector component $T_{sr\sin}(\omega)$ and total electromagnetic torque $T_g(\omega)$.

Total Generator Torque

Total DFIG electromagnetic torque T_g depending on rotor rotational speed ω has been shown in Figure 9. The torque has been obtained by means of equation (24) as a sum of the generator asynchronous and synchronous torques.

By analysing the total DFIG electromagnetic torque under observation, as well as the torque per each particular component, the following conclusion may be drawn: the sum of the asynchronous torques T_s and T_r is approximately equal in amount with the synchronous torque $T_{sr\cos}$, but have opposite signs. Consequently, the amount and the sign of the total generator torque is mainly determined by the synchronous torque component $T_{sr\sin}$.

6 Torque-speed curves of DFIG

In this section it will be investigated the components of the electromagnetic torque obtained over a wide range of the angle δ. Here, the considered rotor speed changes from $0.5\frac{\omega_b}{p}$ to $1.5\frac{\omega_b}{p}$. As mentioned above, the asynchronous components of the electromagnetic torque T_s and T_r do not depend on the angle δ.

Figure 10 shows the asynchronous torque components T_s and T_r, the synchronous torque components $T_{sr\sin}$, $T_{sr\cos}$ and the total DFIG torque T_g depending on the angle δ between the stator and rotor voltage vectors for slip value s = 0.2.

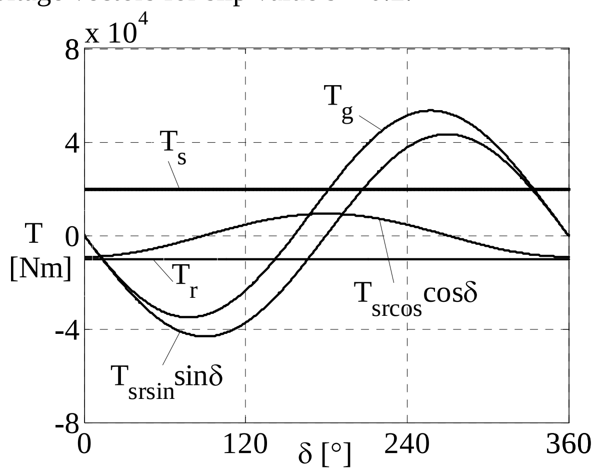

Fig.10. Asynchronous and synchronous electromagnetic torque and total DFIG torque for s = 0.2 and $\delta = 0^\circ - 360^\circ$.

Asynchronous torque component depending upon the stator voltage T_s and that depending upon the rotor voltage T_r represent constant values and are different in sign. Synchronous torque component, on the other hand, is variable in character, substantially determining the total generator torque. Total DFIG electromagnetic torque is obtained by adding all the components constituting the generator torque.

Figures 11, 12, 13 and 14 show the components of the synchronous torque $T_{sr\sin}(\omega), T_{sr\cos}(\omega)$ and the total electromagnetic torque $T_g(\omega)$ obtained for the angle $\delta = 0^\circ, 90^\circ, 180^\circ, 270^\circ$ respectively.

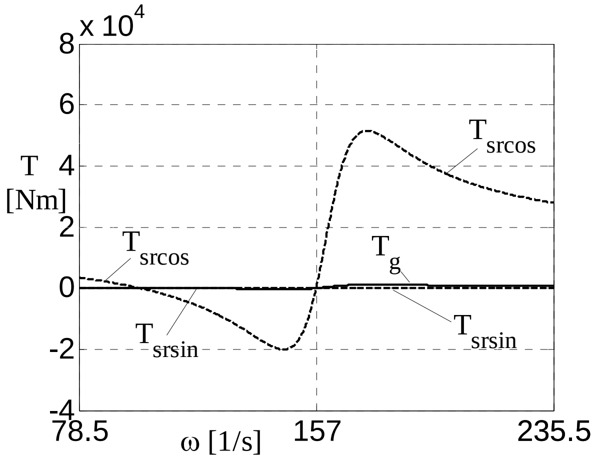

Fig. 11. Components of synchronous torque $T_{sr\sin}(\omega)$, $T_{sr\cos}(\omega)$ and total electromagnetic torque $T_g(\omega)$; $\delta = 0^\circ$.

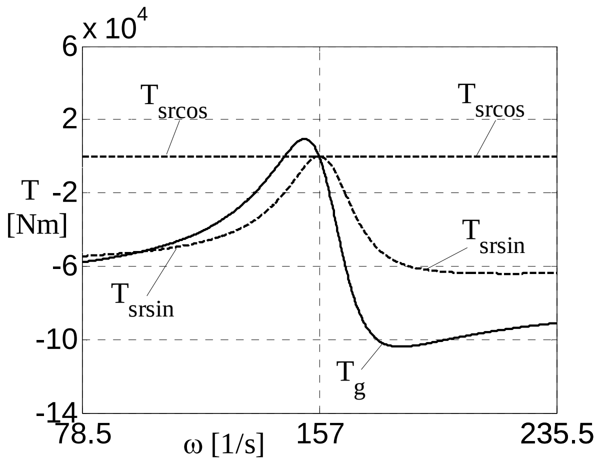

Fig.12. Components of synchronous torque $T_{sr\sin}(\omega)$, $T_{sr\cos}(\omega)$ and total electromagnetic torque $T_g(\omega)$; $\delta = 90^\circ$.

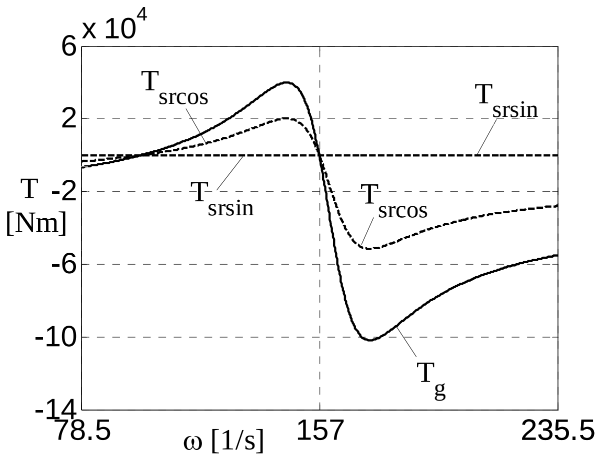

Fig.13. Components of synchronous torque $T_{sr\sin}(\omega)$, $T_{sr\cos}(\omega)$ and total electromagnetic torque $T_g(\omega)$; $\delta = 180^\circ$

In Figure 13 the total electromagnetic torque T_g have the identical sign as the synchronous component $T_{sr\cos}$.

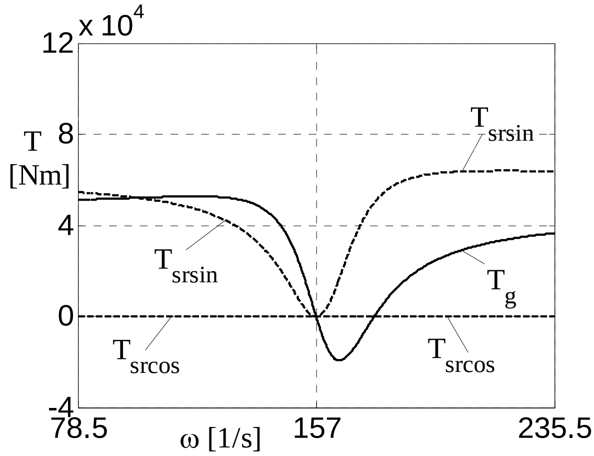

Fig. 14. Components of synchronous torque $T_{sr\sin}(\omega)$, $T_{sr\cos}(\omega)$ and total electromagnetic torque $T_g(\omega)$; $\delta = 270^\circ$.

The synchronous rotor speed ω_s of the DFIG is equal 157 [1/s].

The following conclusion may be reached:

- the synchronous component of the torque $T_{sr\cos}$ obtained for angles δ = 90° and 270° is equal to zero,
- the synchronous component of the torque $T_{sr\sin}$ obtained for angles δ = 0° and 180° is equal to zero,
- for angles δ = 90° and 270° the amount and the sign of the total torque is determined by the synchronous component $T_{sr\sin}$,
- for angles δ = 0° i 180° the amount and the sign of the total torque is determined by the synchronous component $T_{sr\cos}$.

The total electromagnetic torque T_g is always negative for angle $\delta = 90^\circ$ and for angle $\delta = 270^\circ$ always positive.

7 Conclusion

This paper deals with torque characteristics of DFIG, the generator most commonly employed in variable speed wind turbines. The system of DFIG voltage equations for steady-state modes has been expressed in the *dq* reference frame with respect to the stator voltage vector. From the expression for total generator power a mathematical expression stating the generator active power is obtained. Generator active power yields an equation used for calculating the generator asynchronous and synchronous torque(s) by means of stator and rotor current vector components. Stator and rotor current vectors are expressed as a function of stator and rotor voltage vectors, the relation between them being represented by complex coefficients determined by the parameters of the generator itself. By substituting expressions for stator and rotor current vectors into the torque equation the DFIG torque components are obtained, represented as functions of stator and rotor feed voltage vectors. By a further analysis of the total DFIG electromagnetic torque, as well as separate consideration of its particular components, the following conclusion may be reached:

- the synchronous component of the torque $T_{sr\cos}$ obtained for angles $\delta = 90^\circ$ and 270° is equal to zero,
- the synchronous component of the torque $T_{sr\sin}$ obtained for angles $\delta = 0^\circ$ and 180° is equal to zero,
- for angles $\delta = 90^\circ$ and 270° the amount and the sign of the total torque is determined by the synchronous component $T_{sr\sin}$
- for angles $\delta = 0^\circ$ i 180° the amount and the sign of the total torque is determined by the synchronous component $T_{sr\cos}$.
- the total electromagnetic torque T_g is always negative for angle $\delta = 90^\circ$ and for angle $\delta = 270^\circ$ always positive.

Appendix

A. Generator Data

Rated power	P_{gn}=2 [*MW*]
Nominal voltage	U_n=690 [*V*]
Rated frequency	f_n=50 [*Hz*]
Number of pole-pairs	*p*=2
Stator resistance	R_s=0,0114 [Ω]
Rotor resistance	R_r=0,0043 [Ω]
Stator inductance	L_s=2,925 [mH]
Rotor inductance	L_r=2,959 [mH]
Magnetizing inductance	L_m=2,868 [mH]

B. Complex coefficients

$$\bar{Y}_{11} = \frac{\left[\omega_r\left(\frac{1}{T_r^{'}} + \frac{\omega_r}{\omega_s T_s^{'}}\right) - \frac{\sigma}{T_r^{'}}\left(\omega_r - \frac{\sigma}{\omega_s T_s^{'} T_r^{'}}\right)\right]}{\omega_s L_s^{'} N} - j\frac{\left[\frac{\sigma}{T_r^{'}}\left(\frac{1}{T_r^{'}} + \frac{\omega_r}{\omega_s T_s^{'}}\right) + \omega_r\left(\omega_r - \frac{\sigma}{\omega_s T_s^{'} T_r^{'}}\right)\right]}{\omega_s L_s^{'} N},$$

$$\bar{Y}_{12} = -\frac{\omega_s k_r\left(\frac{1}{T_r^{'}} + \frac{\omega_r}{\omega_s T_s^{'}}\right)}{\omega_s L_s^{'} N} + j\frac{\omega_s k_r\left(\omega_r - \frac{\sigma}{\omega_s T_s^{'} T_r^{'}}\right)}{\omega_s L_s^{'} N},$$

$$\bar{Y}_{21} = -\frac{\omega_r k_s\left(\frac{1}{T_r^{'}} + \frac{\omega_r}{\omega_s T_s^{'}}\right)}{\omega_s L_r^{'} N} + j\frac{\omega_r k_s\left(\omega_r - \frac{\sigma}{\omega_s T_s^{'} T_r^{'}}\right)}{\omega_s L_r^{'} N},$$

$$\bar{Y}_{22} = \frac{\left[\omega_s\left(\frac{1}{T_r^{'}} + \frac{\omega_r}{\omega_s T_s^{'}}\right) - \frac{\sigma}{T_s^{'}}\left(\omega_r - \frac{\sigma}{\omega_s T_s^{'} T_r^{'}}\right)\right]}{\omega_s L_r^{'} N} - j\frac{\left[\frac{\sigma}{T_s^{'}}\left(\frac{1}{T_r^{'}} + \frac{\omega_r}{\omega_s T_s^{'}}\right) + \omega_s\left(\omega_r - \frac{\sigma}{\omega_s T_s^{'} T_r^{'}}\right)\right]}{\omega_s L_r^{'} N},$$

where:

$$N = \left(\frac{1}{T_r^{'}} + \frac{\omega_r}{\omega_s T_s^{'}}\right)^2 + \left(\omega_r - \frac{\sigma}{\omega_s T_s^{'} T_r^{'}}\right)^2.$$

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