Three life insurance model research--Family unite the insurance model

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Abstract: - Interest rate is assumed for the constant in some insurance references. They include the calculation of spouse allied scheme insurance premium and reserve and calculation of family combined insurance premium under the constant interest rate.Interest randomness in life insurance has received considerable attention in recent literatures on actuarial theory and its applications. In order to avoid the risk induced by interest randomness, we consider the force of interest function as a Wiener process. In this paper, we build an actuarial model for family combined insurance under fixed interest and random rate of interest differently. First, the family condition in the future is analyzed. In analyzing about muliti-life function, introducing the situation of three element, especially while considering the death order, consider a person dies before other two people (state of jointly surviving). Second, two kinds of association situations and two models are considered here. The first place is the couple and only children; the second place is couple own. At the last, the actuarial present value on the life insurance and annuity are calculated.

Key-Words: - life insurance, actuarial, survival function, random rate, multi-element life function，Wiener processs

1 Introduction

In a term life insurance policy, usually only insured one person, also possible for multi-person life insurance, such as couple, family, for elder parents, and etc. Usually we can divide by two groups by how many insured people every policy, individual policy and multi-person policy, also called connectable life insurance [1]. Usually annually fund is for individual policy. For this paper, we study for multi-person policy, and study target on future family. For life-insurance, expect for group policy, mostly are for ordinary person. It has a huge potential market for future family, though it is very necessary we need pay attention on it, and develop products for such demands.

2 An analysis for a future home

Let think about future family members, a husband, a wife, four elder parents, and one kid. The financial status, a couple has a decent income, enough for care about elder parents, growing a kid, and paying bills. Four elder parents have or partial has own endowment insurance, they basically do not need income from its child, financial dependently. And they have certain mount for medical insurance, expect from social security, they has lack information of commercial insurance. Young kid is still in study, financially not able to live independently, and need a quite spending on it. Another point, this couple has highly educated and has decent income; marriage age is elder than average from 20 century in China. Usually they married after 30, has own house and car, but instill financing. So they have quite a bit pressure, and their parents also have quite bit age.

Let study individual thoughts for each family member, for each elder person, they have been retired, quite old in age, and health condition is critical, health insurance and term life insurance are mainly focused, in health insurance we cover on critical sickness, hospitalization cover, and accident insurance. Based on social security, they already have some pension, and also can cover certain mount of medical expense, so they can offset these expenses in short term. For growing kids, education and growing are two main concerns. Once they left college, they will ability to independently. Although, these expense are pressure on the couple. In case, couple in accident, the kids education and growing will be major effected. They take care elders and grow and with educate kid, and paying bills for the couple, so they need more security.

We are put death condition of family in consideration and analyze that death has influence upon family [2]. To introduce death in the model, one is anyone in death by couple. Another case is both couple dead, on third case, the couple get old and kid dead in accidentally. We are not put these elder parents death in consideration. We analyze that three death conditions have influence on family. Two cases we are study with, one is husband in death second wife in death. These two cases are different. Let assume, the husband dead how effect the whole family financial situation. From income structure, psychological impact for whole family, and sexual difference, the husband death will be major impact for take care a family, such as take care elder parents and grow kid. If happen in wife, these problems are also effects. However, which one will be more effect than others? Let study psychological impact for wife or husband, due to variable research, lost wife is more psychological impact than lost husband. The husbands get more possibility become psychopathic because of lost loved one. So, in our model we have more insurance coverage for lost wife. Another case is both couple dead. In this case, the whole family suddenly down to terrible situation. Elder could death earlier because of lost loved kid, and young kid could live negatively because of loosing his/her parents. Thus, our proposal is elder parents could live normally, and young kid could still in normal education, and grow positively. For approach this goal, our coverage payment will be quite big. On third case, the couples get old and kid dead in accidentally. If kid still quite young, the possibility get another kid will be quite high, if young kid becomes adult, the possibility for another one will be quite low. We assume that exceeding 18 ages in death has more negative influence on family than under 18 age. In our life table and physiology, the possibility of a dead kid will be very low, even though younger than 5 years old will be quite high.

Then we create a allied insurance with husband, wife, and kid. We associated insurance with husband and wife. We set statement, any one of them dead, the state changed, they are able to claim the coverage. We give different mount base on the model we assumed. In second case, we set statement they only can claim only if couple are both dead, the payment coverage such as take care elders, grow kid and with education, and payment for after death. At last, we talk about whole family dead that mean, husband, wife, and kid.

3 Introducing multi-life function and actuarial model of life insurance

3.1 Multi-life function

From two chapters, we consider two conditions, husband, wife, kid and couple. Let assume three objects (x), (y), (z) means husband, wife and kid. For a allied insurance model, we set condition no change as long as all three in live, only condition change when one of them dead. For a couple policy the condition no change until both of them dead (husband and wife). For better understanding we give two definitions [1, 3-5].

Definition 1: For a condition has N life objects, No i object has x_i years old. The future life call $T(x_i)(i=1,2,...,N)$. If we set condition for the first object dead define a condition termination. And record this condition as $(x_1, x_2, \ldots, x_N)$, then the condition maintain time as :

$T(x_1, x_2, \ldots, x_N)=\min\{T(x_1),T(x_2),...,T(x_N)\}$

Then we called minimum life from, or union life form.

Definition 2: For a condition has N life objects, No i object has x_i years old. The future lifetime call $T(x_i)(i=1,2,\ldots,N)$. If we set condition for the last object dead define a condition termination. And record this condition as $(\overline{x_1, x_2, ..., x_N})$, then the condition maintain time as:

$$T(\overline{x_1, x_2, ..., x_N}) = \max\{T(x_1), T(x_2), ..., T(x_N)\}$$

Then we called maximum life form, or the last life form.

Thought, for union life form the condition remain when all life object remains, for the last life form, the condition only change when no life objects exist any more.

When we talk about T(x),T(y), T(z), we assume each person independently, but in fact, they are associated together. Anyone of them death could effect others life time.

3.1.1 The probably distribution of future life of three union life condition

T(xyz) [6~7] Distribution function:

$$\begin{aligned} F_{T(xyz)}(t) &= P\{T(xyz) \le t\} \\ &= P\{\min[T(x),T(y),T(z) \le t\} \\ &= 1-P\{T(x)>t\} \bullet P\{T(y)>t\} \bullet P\{T(z)>t\} \\ &= 1-{}_tp_x \bullet {}_tp_y \bullet {}_tp_z \end{aligned} \tag{1}$$

Then the probability of union live condition (xyz) at least t years:

$${}_tp_{xyz} = P\{T(xyz)>t\}=1-F_{T(xyz)}(t)={}_tp_x \bullet {}_tp_y \bullet {}_tp_z \tag{2}$$

T (xyz) [8]probability density function is:

$$\begin{aligned} f_{T(xyz)}(t) &= \frac{d}{dt}F_{T(xyz)}(t) \\ &= \frac{d}{dt}(1-{}_tp_x \bullet {}_tp_y \bullet {}_tp_z) \\ &= (-{}_tp_x u_{x+t})\,{}_tp_y \bullet {}_tp_z - (-{}_tp_y u_{y+t})\,{}_tp_x \bullet {}_tp_z - (-{}_tp_z u_{z+t})\,{}_tp_x \bullet {}_tp_y \\ &= {}_tp_x \bullet {}_tp_y \bullet {}_tp_z (u_{x+t} + u_{y+t} + u_{z+t}) \end{aligned} \tag{3}$$

3.1.2 The probably distribution of future life of two life condition

T { $\overline{xy}$ } distribution function[9-11] :

$$F_{T(\overline{xy})}(t)=P\{T(x)\le t\}\bullet P\{T(y)\le t\}$$
$$=F_{T(x)}(t)\bullet F_{T(y)}(t)$$
$$={}_tq_x\bullet {}_tq_y \quad (4)$$
$$=(1-{}_tp_x)(1-{}_tp_y)$$
$$=1-{}_tp_x-{}_tp_y+{}_tp_x\bullet {}_tp_y$$

Then the probability of union live condition $(\overline{xy})$ at least t years:

$${}_tp_{\overline{xy}}=P\{T(\overline{XY})\ge t\}$$
$$=1-F_{T(\overline{XY})}(t) \quad (5)$$
$$={}_tp_x+{}_tp_y-{}_tp_x\bullet {}_tp_y$$

T($\overline{XY}$) probability density function is:

$$f_{T(\overline{xy})}(t)=\frac{d}{dt}F_{T(\overline{xy})}(t)$$
$$=\frac{d}{dt}(1_t-p_x-{}_tp_y+p_x\bullet {}_tp_y) \quad (6)$$
$$={}_tp_x\bullet u_{x+t}+{}_tp_y\bullet u_{y+t}-(u_{x+t}+u_{y+t})\,{}_tp_x\bullet {}_tp_y$$

3.1.3 A probability of union conditions

Let us talk about three objects[12~13], that (x), (y),(z), any one of dead and others remain life. The probability presented by ${}_nq^1_{xyz}$ .Mean (x) is dead before (y),(z) and within n years. Then,

$$q^1_{xyz}=\int_0^n\int_0^\infty {}_sp_y\,{}_sp_z(u_{y+s}+u_{z+s})\bullet {}_tp_x u_{x+t}\,ds\,dt$$
$$=\int_0^n {}_tp_x u_{x+t}[\int_0^\infty {}_sp_y\,{}_sp_z(u_{y+s}+u_{z+s})ds]dt \quad (7)$$
$$=\int_0^n {}_tp_x u_{x+t}(1-{}_tp_y\,{}_tp_z)dt$$
$$=\int_0^n {}_tp_x u_{x+t}dt-\int_0^n {}_tp_x\,{}_tp_y\,{}_tp_z u_{x+t}dt$$

The probability presented by ${}_nq^1_{yxz}$. Mean (y) is dead before (x), (z) and within n years.Then,

$${}_nq^1_{yxz}=\int_0^n {}_tp_y u_{y+t}dt-\int_0^n {}_tp_y\,{}_tp_x\,{}_tp_z u_{y+t}dt \quad (8)$$

The probability presented by ${}_nq^1_{zxy}$. Mean (z) is dead before (x), (y) and within n years.Then,

$${}_nq^1_{zxy}=\int_0^n {}_tp_z u_{z+t}dt-\int_0^n {}_tp_z\,{}_tp_x\,{}_tp_y u_{z+t}dt \quad (9)$$

3.2 Discrete life insurance model

Insurance claims is paid when dying to happen immediately in insurance practising. But best information of residual life probably distribution from discrete life table is probably distribution of integral residual life.

Let assume age of insured is x when he(she) buys abandonment, and future integral residual life is k, insurance amount of k+1 year payment is b_{k+1}, v_{k+1} is discounted factor. Z_{k+1} is discounted factor of k+1 years.

Then, $Z_{k+1}= b_{k+1}.\ v_{k+1}$ (10)

So in discrete life insurance model, value of expectation of present value of random variable Z is E(Z), representation of E(Z) is

$$E(Z)=\sum_{k=0}^{\infty}v_{k+1}b_{k+1}\,{}_{k/}q_x \quad (11)$$

E(Z) is known as net single premium.

We explore a variety of actuarial present value of life insurance amount.

3.2.1 Mortality insurance

Mortality insurance consists of n years term insurance and whole life insurance.

(1) N years term insurance

For n years term insurance we set insurance amount as 1, actuarial present value of insurance amount is $A^1_{x:\overline{n}|}$, correlative functions is,

$$b_{k+1}=\begin{cases}1 & (0\leqslant k\leqslant n-1)\\ 0 & (k>n-1)\end{cases}$$

$$v_{k+1}=\begin{cases}v^{k+1} & (0\leqslant k\leqslant n-1)\\ 0 & (k>n-1)\end{cases}$$

$$Z=\begin{cases}v^{K+1} & (0\leqslant K\leqslant n-1)\\ 0 & (K>n-1)\end{cases}$$

Mortality probably of policy-holder for k+1 year is ${}_{k/}q_x$, according to (11),then,

$$A^1_{x\overline{n}|}=E(Z)=\sum_{k=0}^{n-1}v^{k+1}\,{}_{k\backslash}q_x=\sum_{k=0}^{n-1}v^{k+1}\,{}_kp_xq_{x+k} \quad (12)$$

(2) Whole life insurance

For discrete whole life insurance we set insurance amount as 1, present value is A_x .

$$A_x=\sum_{k=0}^{\infty}v_{k+1}\,{}_{k/}q_x=\sum_{k=0}^{\varpi-x-1}v_{k+1}\,{}_{k/}q_x \quad (13)$$

(ϖ is maximum Mortality age)

3.2.2 Endowment insurance

Endowment insurance of N years consists of pure endowment insurance of N years and term insurance of N years.No matter how (x) die befor x+n age and (x) die as x+n age , assurer pay 1 .

$b_{k+1}=1$ （k=0,1,2, ...）

$$v_{k+1}=\begin{cases} v^{k+1} & (0\leqslant k\leqslant n-1) \\ v^n & (k>n-1) \end{cases}$$

$$Z=\begin{cases} v^{K+1} & (0\leqslant K\leqslant n-1) \\ v^n & (K>n-1) \end{cases}$$

Present value of insurance amount is,

$$A_{x:\overline{n}|}=\sum_{k=0}^{n-1} v^{k+1}\,{}_{k/}q_x + v^n\cdot{}_n p_x = A^1_{x:\overline{n}|}+A_{x:\overline{n}|}^{\ \ 1} \quad (14)$$

3.2.3 N years term insurance of H years deferred period

$$b_{k+1}=\begin{cases} 1 & (h\leqslant k\leqslant n-1) \\ 0 & (\text{other}) \end{cases}$$

$$v_{k+1}=\begin{cases} 1 & (h\leqslant k\leqslant h+n-1) \\ 0 & (\text{other}) \end{cases}$$

$$Z=\begin{cases} v^{K+1} & (h\leqslant K\leqslant h+n-1) \\ 0 & (\text{other}) \end{cases}$$

Present value of insurance amount is ${}_{h/}A^1_{x:\overline{n}|}$ or ${}_{h/n}A_x$, then

$$\begin{aligned} {}_{h/n}A_x &= \sum_{k=h}^{h+n-1} v^{k+1}\cdot{}_{k\backslash}q_x \\ &= \sum_{j=0}^{n-1} v^{j+h+1}\cdot{}_{j+m/}q_x \\ &= \sum_{j=0}^{n-1} v^{j+1}v^h\cdot{}_{j/}q_{x+h}\cdot{}_h p_x \quad (15) \\ &= v^{j+1}\,{}_h p_x\sum_{j=0}^{n-1} v^{j+1}\cdot{}_{j/}q_{x+h} \end{aligned}$$

if n to infinity, as whole life insurance of H years deferred period,then

$$\begin{aligned} {}_{h/}A_x &= \sum_{k=h}^{\infty} v^{k+1}\cdot{}_{k\backslash}q_x \\ &= \sum_{j=0}^{\infty} v^{j+h+1}\cdot{}_{j+h/}q_x \\ &= \sum_{j=0}^{\infty} v^{j+1}v^h\cdot{}_{j/}q_{x+h}\cdot{}_h p_x \quad (16) \\ &= v^{j+1}\,{}_h p_x\sum_{j=0}^{\infty} v^{j+1}\cdot{}_{j/}q_{x+h} \\ &= A_{x+h}A_{x:\overline{h}|}^{\ \ 1} = A_x - A_{x:\overline{h}|}^{\ \ 1} \end{aligned}$$

${}_{h/}A_x$ and ${}_{h/}A_{x:\overline{n}|}$ is present value of whole life insurance and endowment insurance separately.

$${}_{h/}A_x=\sum_{k=h}^{\infty} v^{k+1}\cdot{}_{k\backslash}q_x \quad (17)$$

$${}_{h/}A_{x:\overline{n}|}=\sum_{k=h}^{n+h-1} v^{k+1}\cdot{}_{k\backslash}q_x + v^{n+h}\cdot{}_{n+h}p_x \quad (18)$$

accordingly，

$${}_hA_x = A_x - A_{x:\overline{h}|}^{\ \ 1} = A^1_{x:\overline{h}|}.A_{h+x}$$

$${}_{h/}A_{x:\overline{n}|} = A_{x:\overline{h}|}^{\ \ 1}.A_{x+h:\overline{n}|}$$

3.3 Continuous life insurance model

Assurer pay insurance amount when future lifetime of polocyholder is T(x).

Let assume policyholder is x age, b_t is insurance amount at t, v_t is interest discounted factor,Z(T) is present value.Then,

$$\bar{A}=E(Z)=\int_0^{\infty}{}_t p_x\mu_{x+t}b_t v_t\,dt$$

3.3.1 Mortality insurance

（1） N years term life insurance

$$b_t=\begin{cases} 1 & (t\leq n) \\ 0 & (t>n) \end{cases}$$

$v_t=v^t \quad (t\geq 0)$

$$\mathbf{Z_T}=\begin{cases} v^T & (T\leq n) \\ 0 & (T>n) \end{cases}$$

Present value of insurance amount,

$$\bar{A}^1_{x:\overline{n}|}=E(Z)=\int_0^n v^t\cdot{}_t p_x\mu_{x+t}\,dt \quad (19)$$

（2） Whole life insurance

In (19), n to infinity,then

$$\bar{A}_x=\int_0^{\infty} v^t\cdot{}_t p_x\mu_{x+t}\,dt=\int_0^{\varpi-x} v^t\cdot{}_t p_x\mu_{x+t}\,dt \quad (20)$$

3.3.2 Endowment insurance

Random variable of present value,

$$\mathbf{Z_T} = \begin{cases} v^T & (T \le n) \\ v^n & (T > n) \end{cases}$$

Present value of insurance amount,

$$\bar{A}_{x:\overline{n|}} = E(Z) = \int_0^n v^t \cdot {}_t p_x \mu_{x+t} dt + v^n \cdot {}_n p_x = A^1_{x:\overline{n|}} + A_{x:\overline{n|}}^{\;\;1} \quad (21)$$

3.3.3 Deferred life insurance

Whole insurance of H years deferred period,

$${}_{h/}\bar{A}_x == \int_h^\infty v^t \cdot {}_t p_x \mu_{x+t} dt \quad (22)$$

N years term insurance of H years deferred period,

$${}_{h/n}\bar{A}_x == \int_h^{h+n} v^t \cdot {}_t p_x \mu_{x+t} dt \quad (23)$$

N years endowment insurance of H years deferred period,

$${}_{h/}A^1_{x:\overline{n|}} = {}_{h/n}A_x + {}_{h/}A_{x:\overline{n|}}^{\;\;1} \quad (24)$$

3.4 Discrete survival annuity

3.4.1 Survival annuity according annual pay one times

（1）Whole survival annuity

Annuity is payable at the beginning of the year of survival. Present value of insurance is $\ddot{a}_x$, then

$$\bar{Y} = \ddot{a}_{\overline{K+1|}}$$

$$\ddot{a}_x = E(\bar{Y}) = \sum_{k=0}^{\infty} \ddot{a}_{\overline{K+1|}} \cdot {}_{k\backslash} q_x = \sum_{k=0}^{\infty} v^k \cdot {}_k p_x \quad (25)$$

Annuity is payable at the end of the year of survival. Present value of insurance is a_x, then

$$a_x = E(Y) = \sum_{k=0}^{\infty} a_{\overline{k|}} \cdot {}_{k\backslash} q_x = \sum_{k=1}^{\infty} v^k \cdot {}_k p_x \quad (26)$$

（2）Survival annuity of N years term insurance

Annuity is payable at the beginning of the year of survival. Present value of insurance is $\ddot{a}_{x:\overline{n|}}$, then

$$Y = \begin{cases} \ddot{a}_{\overline{K+1|}} & (0 \le K \le n-1) \\ \ddot{a}_{\overline{n|}} & (K \ge n) \end{cases}$$

$$\ddot{a}_{x:\overline{n|}} = \sum_{k=0}^{n-1} v^k \cdot {}_k p_x = \sum_{k=0}^{n-1} \ddot{a}_{\overline{k+1|}} \cdot {}_{k\backslash} q_x + \ddot{a}_{\overline{n|}} \cdot {}_n p_x \quad (27)$$

Annuity is payable at the end of the year of Survival.Present value of insurance is $a_{x:\overline{n|}}$, then

$$a_{x:\overline{n|}} = \sum_{k=0}^{n-1} a_{\overline{k|}} \cdot {}_k q_x + a_{\overline{n|}} \cdot {}_n p_x = \sum_{k=1}^{n} v^k \cdot {}_k p_x \quad (28)$$

（3）Whole survival annuity and N years term survival annuity of H years deferred period

Annuity is payable at the beginning of the year of survival. Present value of insurance is ${}_{h\backslash}\ddot{a}_x$ and ${}_{h\backslash n}\ddot{a}_x$, then

$${}_{h\backslash}\ddot{a}_x = \sum_{k=h}^{\infty} v^k \cdot {}_k p_x = \sum_{k=h}^{\infty} (\ddot{a}_{\overline{k+1|}} - \ddot{a}_{\overline{h|}}) \cdot {}_{k\backslash} q_x \quad (29)$$

$${}_{h\backslash n}\ddot{a}_x = \sum_{k=h}^{h+n-1} v^k \cdot {}_k p_x = \sum_{k=h}^{h+n-1} (\ddot{a}_{\overline{k+1|}} - \ddot{a}_{\overline{h|}}) \cdot {}_{k\backslash} q_x + (\ddot{a}_{\overline{h+n|}} - \ddot{a}_{\overline{h|}})\, {}_{h+n} p_x \quad (30)$$

3.4.2 Survival annuity according annual pay M times

（1）. Whole survival annuity is paid the beginning of the period

Let assume annuity as 1, pay M times every year, Present value of insurance is $\ddot{a}_x^{(m)}$,then

$$\ddot{a}_x^{(m)} = \frac{1}{m} \sum_{k=0}^{\infty} v^{k/m} \cdot {}_{k/m} p_x \quad (31)$$

In general adopt similar formula of tradition,

$$\ddot{a}_x^{(m)} \approx \ddot{a}_x - \frac{m-1}{2m} \quad (32)$$

Then similar formula ,

$${}_{\frac{k}{m}}\ddot{a}_x \approx \ddot{a}_x - \frac{k}{m} (k = 1, 2, \ldots, (m-1)/m)$$

So,

$$\ddot{a}_x^{(m)} = \frac{1}{m}({}_{0/}\ddot{a}_x + {}_{\frac{1}{m}}\ddot{a}_x + {}_{\frac{2}{m}}\ddot{a}_x + \ldots + {}_{\frac{m-1}{m}}\ddot{a}_x)$$

$$\approx \frac{1}{m}[\ddot{a}_x + (\ddot{a}_x - \frac{1}{m}) + (\ddot{a}_x - \frac{2}{m}) + \ldots + (\ddot{a}_x - \frac{m-1}{m})]$$

$$= \ddot{a}_x - \frac{1+2+\ldots+(m-1)}{m^2}$$

$$= \ddot{a}_x - \frac{m-1}{2m} = a_x + \frac{m+1}{2m}$$

（2）Whole survival annuity is paid the end of the period

Present value of insurance is $a_x^{(m)}$, then

$$a_x^{(m)} = \ddot{a}_x^{(m)} - \frac{1}{m} \quad (33)$$

similar formula of tradition,

$$a_x^{(m)} = \ddot{a}_x - \frac{m+1}{2m} = a_x + \frac{m-1}{2m} \quad (34)$$

(3) N years term survival annuity is paid the beginning of the period

Present value of insurance is $\ddot{a}^{(m)}_{x:\overline{n}|}$, then

$$\ddot{a}^{(m)}_{x:\overline{n}|} = \ddot{a}^{(m)}_{x} - {}_{n\backslash}\ddot{a}^{(m)}_{x} = \ddot{a}^{(m)}_{x} - {}_{n}E_{x}.\ddot{a}^{(m)}_{x+n} \quad (35)$$

Under UDD hypothesis condition, convertted (35),

$$\ddot{a}^{(m)}_{x:\overline{n}|} = \alpha(m)\ddot{a}_{x:\overline{n}|} + \beta(m)(1 - {}_{n}E_{x}) \quad (36)$$

among the rest ,

$$\alpha(m) = id / i^{(m)}d^{(m)}$$

$$\beta(m) = (i - i^{(m)}) / i^{(m)}d^{(m)}$$

similar formula of tradition of $\ddot{a}^{(m)}_{x:\overline{n}|}$ is

$$\ddot{a}^{(m)}_{x:\overline{n}|} \approx \ddot{a}_{x:\overline{n}|} - \frac{m-1}{2m}(1 - {}_{n}E_{x}) \quad (37)$$

(4) N years term survival annuity is paid the end of the period

Present value of insurance is $a^{(m)}_{x:\overline{n}|}$,then

$$\begin{aligned} a^{(m)}_{x} &= a^{(m)}_{x} - {}_{n}E_{x}.a^{(m)}_{x+n} \\ &= (a_{x} - \frac{m-1}{2m}) - {}_{n}E_{x}(a_{x+n} - \frac{m-1}{2m}) \\ &= a_{x:\overline{n}|} - \frac{m-1}{2m}(1 - {}_{n}E_{x}) \\ &= \frac{1}{D_{x}}[N_{x+1} - N_{x+n+1} + \frac{m-1}{2m}(D_{x} - D_{x+n})] \end{aligned} \quad (38)$$

(5) Whole survival annuity of H years deferred period is paid the beginning of the period

Present value of insurance is ${}_{h/}\ddot{a}^{(m)}_{x}$,then

$${}_{h/}\ddot{a}^{(m)}_{x} = v^{h}\,{}_{h}p_{x}.\ddot{a}^{(m)}_{x+h} = {}_{h}E_{x}.\ddot{a}^{(m)}_{x+h} \quad (39)$$

Under UDD hypothesis condition, convertted (39),

$$\begin{aligned} {}_{h/}\ddot{a}^{(m)}_{x} &= {}_{h}E_{x}[\alpha(m)\ddot{a}_{x+h} + \beta(m)] \\ &= \alpha(m).{}_{h\backslash}\ddot{a}_{x} + \beta(m).{}_{h}E_{x} \end{aligned} \quad (40)$$

similar formula of tradition of is ${}_{h/}\ddot{a}^{(m)}_{x}$,

$${}_{h/}\ddot{a}^{(m)}_{x} = {}_{h\backslash}\ddot{a}_{x} - \frac{m-1}{2m}\,{}_{h}E_{x} \quad (41)$$

(6) Whole survival annuity of H years deferred period is paid the end of the period

Present value of insurance is ${}_{h/}a^{(m)}_{x}$,then

$$\begin{aligned} {}_{h/}a^{(m)}_{x} &= v^{h}\,{}_{h}p_{x}.\ddot{a}^{(m)}_{x+h} \\ &= v^{h}\,{}_{h}p_{x}(a_{x+h} + \frac{m-1}{2m}) \\ &= {}_{h/}a_{x} - \frac{m-1}{2m}\,{}_{h}E_{x} \end{aligned} \quad (42)$$

3.5 Continuous survival annuity

3.5.1 Whole survival annuity

Let assume annuity as 1, Whole survival annuity of continuous pay, Present value of insurance is $\bar{a}_{x}$,future lifetime of (x) is T(x), density function is $f_{T}(t) = {}_{t}p_{x}\mu_{x+t}$

Present value of annuity is $\bar{Y}$,then

$$\bar{Y} = \bar{a}_{\overline{T}|} = \int_{0}^{T} v^{t}dt$$

Present value of insurance is

$$\begin{aligned} \bar{a}_{x} &= \int_{0}^{\infty} \bar{a}_{\overline{t}|}\,{}_{t}p_{x}\mu_{x+t}dt \\ &= \int_{0}^{\infty} \frac{1-v^{t}}{\delta}\,{}_{t}p_{x}\mu_{x+t}dt \\ &= \int_{0}^{\infty} \frac{d(1-{}_{t}p_{x})}{dt}\frac{1-v^{t}}{\delta}dt({}_{t}p_{x}\mu_{x+t} = -\frac{d\,{}_{t}p_{x}}{dt}) \\ &= \int_{0}^{\infty} \frac{1-v^{t}}{\delta}d(-{}_{t}p_{x}) \\ &= \int_{0}^{\infty} \frac{1}{\delta}\,{}_{t}p_{x}de^{-t\delta}(v = e^{-\delta}) \\ &= \int_{0}^{\infty} {}_{t}p_{x}v^{t}dt \end{aligned} \quad (43)$$

3.5.2 N years term survival annuity

Present value of insurance is $\bar{a}_{x:\overline{n}|}$,then

$$\bar{a}_{x:\overline{n}|} = \int_{0}^{n} {}_{t}p_{x}v^{t}dt \quad (44)$$

3.5.3 Whole survival annuity of H years deferred period

Present value of insurance is ${}_{h\backslash}\bar{a}_{x}$,then

$$\begin{aligned} {}_{h\backslash}\bar{a}_{x} &= \int_{0}^{\infty} {}_{t}p_{x}v^{t}dt \\ &= \bar{a}_{x} - \bar{a}_{x:\overline{h}|} \\ &= \int_{0}^{\infty} {}_{s+h}p_{x}v^{h+s}ds \\ &= v^{h}.{}_{h}p_{x}.\int_{0}^{\infty} {}_{s}p_{x+h}v^{s}ds \\ &= {}_{h}E_{x}.\bar{a}_{x+h} \end{aligned} \quad (45)$$

3.5.3 N years term survival annuity of H years deferred period is paid the end of the period

Present value of insurance is ${}_{h\backslash n}\bar{a}_{x}$,then

$$\begin{aligned} {}_{h\backslash n}\bar{a}_{x} &= \int_{0}^{n+h} {}_{t}p_{x}v^{t}dt \\ &= \bar{a}_{x:\overline{h+n}|} - \bar{a}_{x:\overline{h}|} \\ &= \int_{0}^{n} {}_{s+h}p_{x}v^{h+s}ds \\ &= v^{h}.{}_{h}p_{x}.\int_{0}^{n} {}_{s}p_{x+h}v^{s}ds \\ &= {}_{h}E_{x}.\bar{a}_{x+h:\overline{n}|} \end{aligned} \quad (46)$$

4 Analysis of Policy Requirement

4.1 Insurance target

A family, male 30 years or older, Female 28 years or older, good health condition, a couple are able to work and has a 5 years old or older child. Also accept a couple without child[14~15].

4.2 Responsibility[16]

For Term-life insurance: husband, wife and kid, any of them dead, we pay for lost. The payment ratio base on we talked about before. M1,M2, M3, and M2>M1>M3.

For couple both dead, we pay for M4 and M4>(M1+M2+M3), that we can take care good for elder parents and child for couple both lost.

For annuity: kid able to get payment after 10, and every year get M5, that enough for pay education expense. After 18, he/she get M6 annually. M6>M5. That moment, kids should be in college, the education expense will higher than before, and payment stops at 25 years old. At that moment, he should in master degree or in work. For couple, anyone is more than 60 years old, they can get the payment until they dead.

4.3 Exception

These conditions are not in our coverage insured person dead by drug.Insured person dead by suicide. Insured person dead by drunk drive or driving without LC .Insured person dead by involve rebellions.

5 Valuation of Premium with Fixed Interest

5.1 Long-life insurance

For easier calculation we set payment as 1, then in three life union condition (xyz) terminates, pay off is 1;couple last live condition finishes,Payment set 1;in three life union condition $(\overset{1}{x}\,yz),(\overset{1}{y}\,xz),(\overset{1}{z}\,yx)$ finishes,Payment set 1.Then, these three single premium and on-level premium is $\bar{A}_{xyz}$, ${}_h p_1$, $\bar{A}_{\overline{xy}}$, ${}_h p_2$, $\bar{A}^1_{xyz}$, ${}_h p_{3x}$, $\bar{A}^1_{yxz}$, ${}_h p_{3y}$, $\bar{A}^1_{zxy}$, ${}_h p_{3z}$, then[17-18],

$$\bar{A}_{xyz} = E(v^{T(xyz)}) = \int_0^\infty v^t \bullet {}_t p_x\, {}_t p_y\, {}_t p_z (\mu_{x+t} + \mu_{y+t} + \mu_{z+t}) dt \quad (47)$$

$$\bar{A}_{\overline{xy}} = \int_0^\infty v^t [{}_t p_x \mu_{x+t} + {}_t p_y\, {}_t\mu_{y+t} - {}_t p_x\, {}_t p_y (\mu_{x+t} + \mu_{y+t})] dt$$

$$= \bar{A}_{\bar{x}} + \bar{A}_{\bar{y}} - \bar{A}_{xy} \quad (48)$$

$$\bar{A}^1_{zxy} = \int_0^\infty v^t \bullet {}_t p_x \mu_{x+t} (1 - {}_t p_y\, {}_t p_z) dt$$

$$= \int_0^\infty v^t \bullet {}_t p_x \mu_{x+t} dt - \int_0^\infty v^t \bullet {}_t p_x\, {}_t p_y\, {}_t p_z \mu_{x+t} dt \quad (49)$$

$$= \bar{A}_x - \int_0^\infty v^t \bullet {}_t p_x\, {}_t p_y\, {}_t p_z \mu_{x+t} dt$$

$$\bar{A}^1_{yxz} = \int_0^\infty v^t \bullet {}_t p_y \mu_{y+t} (1 - {}_t p_x\, {}_t p_z) dt$$

$$= \int_0^\infty v^t \bullet {}_t p_y \mu_{y+t} dt - \int_0^\infty v^t \bullet {}_t p_y\, {}_t p_x\, {}_t p_z \mu_{y+t} dt \quad (50)$$

$$= \bar{A}_y - \int_0^\infty v^t \bullet {}_t p_y\, {}_t p_x\, {}_t p_z \mu_{y+t} dt$$

$$\bar{A}^1_{zxy} = \int_0^\infty v^t \bullet {}_t p_z \mu_{z+t} (1 - {}_t p_x\, {}_t p_y) dt$$

$$= \int_0^\infty v^t \bullet {}_t p_z \mu_{z+t} dt - \int_0^\infty v^t \bullet {}_t p_z\, {}_t p_x\, {}_t p_y \mu_{z+t} dt \quad (51)$$

$$= \bar{A}_z - \int_0^\infty v^t \bullet {}_t p_z\, {}_t p_x\, {}_t p_y \mu_{z+t} dt$$

On-level premium is in h year when (xyz)set as initial mount. No K（0≤K≤h）year payment is 1 the today value is v^k , and actuarial present value is $\ddot{a}_{xyz:\overline{h}|}$, then,

$$\ddot{a}_{xyz:\overline{h}|} = \sum_{k=0}^{h-1} v^k \bullet {}_k p_{xyz} = \sum_{k=0}^{h-1} v^k \bullet {}_k p_x\, {}_k p_y\, {}_k p_z \quad (52)$$

Based on level premium, actuarial present value of premium is equal to payment after death precision calculated value, then,

$$\bar{A}_{xyz} = {}_h p_1 \bullet \ddot{a}_{xyz:\overline{h}|} \quad (53)$$

$$\bar{A}_{\overline{xy}} = {}_h p_2 \bullet \ddot{a}_{xyz:\overline{h}|} \quad (54)$$

$$\bar{A}^1_{xyz} = {}_h p_{3x} \bullet \ddot{a}_{xyz:\overline{h}|} \quad (55)$$

$$\bar{A}^1_{yxz} = {}_h p_{3y} \bullet \ddot{a}_{xyz:\overline{h}|} \quad (56)$$

$$\bar{A}^1_{zxy} = {}_h p_{3z} \bullet \ddot{a}_{xyz:\overline{h}|} \quad (57)$$

Then,

$${}_h p_1 = \bar{A}_{xyz} / \ddot{a}_{xyz:\overline{h}|} \quad (58)$$

$${}_h p_2 = \bar{A}_{\overline{xy}} / \ddot{a}_{xyz:\overline{h}|} \quad (59)$$

$${}_h p_{3x} = \bar{A}^1_{xyz} / \ddot{a}_{xyz:\overline{h}|} \quad (60)$$

$${}_h p_{3y} = \overline{A}^{1}_{yzx} / \ddot{a}_{xyz:\overline{h}|} \quad (61)$$

$${}_h p_{3z} = \overline{A}^{1}_{zxy} / \ddot{a}_{xyz:\overline{h}|} \quad (62)$$

For n years both sides policies, the single premiums is,

$$\overline{A}_{u:\overline{n}|} = \int_0^n v^t \, {}_t p_u \mu_{x+t} dt + v^n \, {}_n p_x \quad (63)$$

(u) means common condition, could be (xyz),also could be $(\overline{xy})$.

5.2 Survive annuity

5.2.1 Life time survive annuity

when the condition continues, every policy initially pay 1 yuan for life time annual payment $Y = \ddot{a}_{\overline{K+1}|}$,the precise value as ,

$$\begin{aligned} \ddot{a}_{xyz} &= E(Y) \\ &= \sum_{k=0}^{\infty} \ddot{a}_{\overline{K+1}|} \, {}_{k\backslash} q_{xyz} \\ &= \sum_{k=0}^{\infty} v^k \, {}_k p_{xyz} \\ &= \sum_{k=0}^{\infty} v^k \, {}_k p_x \, {}_k p_y \, {}_k p_z \end{aligned} \quad (64)$$

$$\ddot{a}_{\overline{xy}} = \sum_{k=0}^{\infty} v^k \, {}_k p_{\overline{xy}} = \sum_{k=0}^{\infty} v^k ({}_k p_x + {}_k p_y - {}_k p_{xy}) \quad (65)$$

5.2.2 N years fixed survive annuity

$$\begin{aligned} \ddot{a}_{xyz:\overline{n}|} &= E(Y) \\ &= \sum_{k=0}^{n-1} \ddot{a}_{\overline{K+1}|} \, {}_{k\backslash} q_{xyz} + \ddot{a}_{\overline{n}|} \, {}_n p_u \\ &= \sum_{k=0}^{n-1} v^k \, {}_k p_{xyz} \\ &= \sum_{k=0}^{n-1} v^k \, {}_k p_x \, {}_k p_y \, {}_k p_z \end{aligned} \quad (66)$$

$$\ddot{a}_{\overline{xy}:\overline{n}|} = \sum_{k=0}^{n-1} v^k \, {}_k p_{\overline{xy}} = \sum_{k=0}^{n-1} v^k ({}_k p_x + {}_k p_y - {}_k p_{xy}) \quad (67)$$

5.2.3 Life time survive annuity of N years deferred period

$$\begin{aligned} {}_{n\backslash} \ddot{a}_{xyz} &= E(Y) \\ &= \sum_{k=n}^{\infty} \ddot{a}_{\overline{K+1}|} \, {}_{k\backslash} q_{xyz} \\ &= \sum_{k=n}^{\infty} v^k \, {}_k p_{xyz} \\ &= \sum_{k=0}^{\infty} v^{k+n} \, {}_{k+n} p_x \, {}_{k+n} p_y \, {}_{k+n} p_z \end{aligned} \quad (68)$$

$$\begin{aligned} {}_{n\backslash} \ddot{a}_{\overline{xy}} &= \sum_{k=n}^{\infty} v^k [{}_k p_x (1 - {}_k p_y) + {}_k p_y (1 - {}_k p_x)] \\ &= \sum_{k=0}^{\infty} v^{k+n} [{}_{k+n} p_x (1 - {}_{k+n} p_y) + {}_{k+n} p_y (1 - {}_{k+n} p_x)] \end{aligned} \quad (69)$$

5.2.4 N-M years survive annuity of M years deferred period

$${}_{m/} \ddot{a}_{xyz:\overline{n-m}|} = E(Y) = \sum_{k=m}^{n-1} v^k \, {}_k p_{xyz} = \sum_{k=m}^{n-1} v^k \, {}_k p_x \, {}_k p_y \, {}_k p_z \quad (70)$$

$${}_{m/} \ddot{a}_{\overline{xy}:\overline{n-m}|} = \sum_{k=m}^{n-1} v^k \, {}_k p_{\overline{xy}} = \sum_{k=m}^{n-1} v^k ({}_k p_x + {}_k p_y - {}_k p_{xy}) \quad (71)$$

6 Valuation of Premium with Random Rate of Interest

Establish model[19] aim at cumulation function of interest power ,

y(t) =δt+βW(t)

W(t) is a standard Wiener process, β is real number, δ is real constant, and we assume y(t) and T(x),T(y), T(z) independently.

6.1 Long-life insurance

For easier calculation we set payment as 1, then in three life union condition (xyz) terminates, pay off is 1;couple last live condition finishes,Payment set 1;in three life union condition $(\overset{1}{x}yz), (\overset{1}{y}xz), (\overset{1}{z}yx)$ finishes,Payment set 1.Then, these three single premium and on-level premium is $\overline{A}_{xyz}$, ${}_h p_1$, $\overline{A}_{\overline{xy}}$, ${}_h p_2$, $\overline{A}^{1}_{xyz}$, ${}_h p_{3x}$, $\overline{A}^{1}_{yxz}$, ${}_h p_{3y}$, $\overline{A}^{1}_{zxy}$, ${}_h p_{3z}$, then,

$$\begin{aligned} \overline{A}_{xyz} &= E(e^{-y(T(xyz))}) \\ &= \int_0^{\infty} e^{-(\delta - \frac{\beta^2}{2})t} \bullet {}_t p_x \, {}_t p_y \, {}_t p_z (\mu_{x+t} + \mu_{y+t} + \mu_{z+t}) dt \end{aligned} \quad (72)$$

$$\bar{A}_{\overline{xy}} = E(e^{-y(T(\overline{xy}))})$$
$$= \int_0^\infty e^{-(\delta-\frac{\beta^2}{2})t}[{}_tp_x\mu_{x+t} + {}_tp_y\mu_{y+t} - {}_tp_x\,{}_tp_y(\mu_{x+t}+\mu_{y+t})]dt \quad (73)$$

$$\bar{A}^1_{xy} = \int_0^\infty e^{-(\delta-\frac{\beta^2}{2})t} \bullet {}_tp_x\mu_{x+t}(1-{}_tp_y\,{}_tp_z)dt$$
$$= \int_0^\infty e^{-(\delta-\frac{\beta^2}{2})t} \bullet {}_tp_x\mu_{x+t}dt - \int_0^\infty e^{-(\delta-\frac{\beta^2}{2})t} \bullet {}_tp_x\,{}_tp_y\,{}_tp_z\mu_{x+t}dt \quad (74)$$

$$\bar{A}^1_{yxz} = \int_0^\infty e^{-(\delta-\frac{\beta^2}{2})t} \bullet {}_tp_y\mu_{y+t}(1-{}_tp_x\,{}_tp_z)dt$$
$$= \int_0^\infty e^{-(\delta-\frac{\beta^2}{2})t} \bullet {}_tp_y\mu_{y+t}dt - \int_0^\infty e^{-(\delta-\frac{\beta^2}{2})t} \bullet {}_tp_y\,{}_tp_x\,{}_tp_z\mu_{y+t}dt \quad (75)$$

$$\bar{A}^1_{xy} = \int_0^\infty e^{-(\delta-\frac{\beta^2}{2})t} \bullet {}_tp_z\mu_{z+t}(1-{}_tp_x\,{}_tp_y)dt$$
$$= \int_0^\infty e^{-(\delta-\frac{\beta^2}{2})t} \bullet {}_tp_z\mu_{z+t}dt - \int_0^\infty e^{-(\delta-\frac{\beta^2}{2})t} \bullet {}_tp_z\,{}_tp_x\,{}_tp_y\mu_{z+t}d \quad (76)$$

On-level premium is in h year when (xyz) set as initial mount. No K（0≤K≤h）year payment is 1 the today value is $e^{-(\delta-\frac{\beta^2}{2})k}$, and actuarial present value is $\ddot{a}_{xyz:\overline{h}|}$, then,

$$\ddot{a}_{xyz:\overline{h}|} = \sum_{k=0}^{h-1} e^{-(\delta-\frac{\beta^2}{2})k} \bullet {}_kp_{xyz} = \sum_{k=0}^{h-1} e^{-(\delta-\frac{\beta^2}{2})k} \bullet {}_kp_x\,{}_kp_y\,{}_kp_z \quad (77)$$

Based on level premium, actuarial present value of premium is equal to payment after death precision calculated value, then,

$$\bar{A}_{xyz} = {}_hp_1 \bullet \ddot{a}_{xyz:\overline{h}|} \quad (78)$$

$$\bar{A}_{\overline{xy}} = {}_hp_2 \bullet \ddot{a}_{xyz:\overline{h}|} \quad (79)$$

$$\bar{A}^1_{xyz} = {}_hp_{3x} \bullet \ddot{a}_{xyz:\overline{h}|} \quad (80)$$

$$\bar{A}^1_{yxz} = {}_hp_{3y} \bullet \ddot{a}_{xyz:\overline{h}|} \quad (81)$$

$$\bar{A}^1_{zxy} = {}_hp_{3z} \bullet \ddot{a}_{xyz:\overline{h}|} \quad (82)$$

Then,

$${}_hp_1 = \bar{A}_{xyz} / \ddot{a}_{xyz:\overline{h}|} \quad (83)$$

$${}_hp_2 = \bar{A}_{\overline{xy}} / \ddot{a}_{xyz:\overline{h}|} \quad (84)$$

$${}_hp_{3x} = \bar{A}^1_{xyz} / \ddot{a}_{xyz:\overline{h}|} \quad (85)$$

$${}_hp_{3y} = \bar{A}^1_{yzx} / \ddot{a}_{xyz:\overline{h}|} \quad (86)$$

$${}_hp_{3z} = \bar{A}^1_{zxy} / \ddot{a}_{xyz:\overline{h}|} \quad (87)$$

For n years both sides policies, the single premiums is,

$$\bar{A}_{u:\overline{n}|} = \int_0^n e^{-(\delta-\frac{\beta^2}{2})t}\,{}_tp_u\mu_{x+t}dt + e^{-(\delta-\frac{\beta^2}{2})n}\,{}_np_x \quad (88)$$

(u) means common condition, could be (xyz),also could be $(\overline{xy})$.

6.2 Survive annuity

6.2.1 Life time survive annuity

when the condition continues, every policy initially pay 1 yuan for life time annual payment $Y = \ddot{a}_{\overline{K+1}|}$,the precise value as ,

$$\ddot{a}_{xyz} = \sum_{k=0}^{\infty} e^{-(\delta-\frac{\beta^2}{2})k}\,{}_kp_{xyz} = \sum_{k=0}^{\infty} e^{-(\delta-\frac{\beta^2}{2})k}\,{}_kp_x\,{}_kp_y\,{}_kp_z \quad (89)$$

$$\ddot{a}_{\overline{xy}} = \sum_{k=0}^{\infty} e^{-(\delta-\frac{\beta^2}{2})k}\,{}_kp_{\overline{xy}} = \sum_{k=0}^{\infty} e^{-(\delta-\frac{\beta^2}{2})k}({}_kp_x + {}_kp_y - {}_kp_{xy}) \quad (90)$$

6.2.2 N years fixed survive annuity

$$\ddot{a}_{xyz:\overline{n}|} = \sum_{k=0}^{n-1} e^{-(\delta-\frac{\beta^2}{2})k}\,{}_kp_{xyz} = \sum_{k=0}^{n-1} e^{-(\delta-\frac{\beta^2}{2})k}\,{}_kp_x\,{}_kp_y\,{}_kp_z \quad (91)$$

$$\ddot{a}_{\overline{xy}:\overline{n}|} = \sum_{k=0}^{n-1} e^{-(\delta-\frac{\beta^2}{2})k}\,{}_kp_{\overline{xy}} = \sum_{k=0}^{n-1} e^{-(\delta-\frac{\beta^2}{2})k}({}_kp_x + {}_kp_y - {}_kp_{xy}) \quad (92)$$

6.2.3 Life time survive annuity of N years deferred period

$${}_{n\backslash}\ddot{a}_{xyz} = \sum_{k=n}^{\infty} e^{-(\delta-\frac{\beta^2}{2})k}\,{}_kp_{xyz}$$
$$= \sum_{k=0}^{\infty} e^{-(\delta-\frac{\beta^2}{2})(k+n)}\,{}_{k+n}p_x\,{}_{k+n}p_y\,{}_{k+n}p_z \quad (93)$$

$${}_{n\backslash}\ddot{a}_{\overline{xy}} = \sum_{k=n}^{\infty} e^{-(\delta-\frac{\beta^2}{2})k}[{}_kp_x(1-{}_kp_y) + {}_kp_y(1-{}_kp_x)]$$
$$= \sum_{k=0}^{\infty} e^{-(\delta-\frac{\beta^2}{2})(k+n)}[{}_{k+n}p_x(1-{}_{k+n}p_y) + {}_{k+n}p_y(1-{}_{k+n}p_x)] \quad (94)$$

6.2.4 N-M years survive annuity of M years deferred period

$$_{m/}\ddot{a}_{xyz:\overline{n-m}|}=\sum_{k=m}^{n-1}e^{-(\delta-\frac{\beta^2}{2})k}\,{}_kp_{xyz}=\sum_{k=m}^{n-1}e^{-(\delta-\frac{\beta^2}{2})k}\,{}_kp_x\,{}_kp_y\,{}_kp_z \quad (95)$$

$$_{m/}\ddot{a}_{\overline{xy}:\overline{n-m}|}=\sum_{k=m}^{n-1}e^{-(\delta-\frac{\beta^2}{2})k}\,{}_kp_{\overline{xy}}=\sum_{k=m}^{n-1}e^{-(\delta-\frac{\beta^2}{2})k}\left({}_kp_x+{}_kp_y-{}_kp_{xy}\right) \quad (96)$$

7 Conclusion

For this paper, we study for multi-person policy, and study target on future family. First, the family condition in the future is analyzed. In analyzing about muliti-life function, introducing the situation of three elements, especially while considering the death order, consider a person dies before other two people, but these two people are a kind of state of jointly surviving. Second, two kinds of association situations and two models are considered here. The first place is the couple and only children; the second place is couple own. At the last, the actuarial present value on the life insurance and annuity are calculated.

References:

[1] Zhizhong Shen, Fusheng Xiong:*Life Actuarial Science.*Wu Han University Press: Wu Han，2003.11，58～143

[2]Subjective life expectancy in the US: conrrespondence to actuarial estimates by age,sex and race[J]. *Social Science &Medicine.49(1999),967~979*

[3] Technologies of trust:actuarial theory,insurance sciences,and the establishment of the welfare state in Germany and Switzerland around 1900[J]. *Information and Organization, 13(2003):131~150*

[4] A class of bicariate stochastic orderings,with applications in actuarial sciences[J].*Insurance: Mathematics and Economics.24(1999):31~50*

[5]Vojevodina, Dovile,Kulvietis, Genadijus: Change and exceptions in business processes: Handling differences[J].WSEAS Transactions on Systems.*4(2005):1668-1675*

[6] Xiufang Li,Anping Fu,Jing Li: *Life Actuarial*, the People's University of China Press: Peking, 2004，70～188

[7] On a Functional Equation in Actuarial Mathematics [J].*Journal of Mathematical Analysis and Applications,253(2001),16~34*

[8] Wang.S,V.R.Yong.Ordering of risk:Utility Theory versus Yaari's Dual theory of risk[J]. *Insurance:Mathematics and Economics, 22(1998),145~161*

[9]Liyan Wang:Study on Actuarial Theory and Method of Life Insurance under Random Rates of Interest［thesis of doctoral degree］.*Liaoning：Da Lian Polytechnic University，2004*

[10]Georgescu, Vasile: Capturing nonlinearities and randomness by forward-looking models under learning and heterogeneity［J］WSEAS Transactions on Systems.*4（2005）：2459-2466*

[11]Wang.S,V.R.Yong:Ordering of risks:Utility Theory versus Yaari's Dual theory of risk. *Insurance: Mathematics and Economics, 22(1998),145~161*

[12] Yaohua Wu,Xinzhong Cai,Zhiqiang Wu:Calculation of the Insurance for a Couple Against Pension Annuity (Including Death Insurance), *Transaction of University of Science and Technology of China，1998，28（4）：439～445*

[13] Black,F and Scholes,M.The pricing of options and corporate liabilities[J].Journal of Political Econimy.*81(1983),637~654*

[14] Box,G.E.P and Jenkins,G.M. Times series Analysis[M].San Francisco,Hold en-day,1976

[15]Svitek, Miroslav. Switched probabilistic multi-models[J].WSEAS Transactions on Systems.*4(2005),2066-2072*

[16]Jetton,M.F. Interest Rates Scenarios [J].Transactions of the Society of Actuaries, *(40)1988,423~437*

[17] London D.Survival Models and Their Estimation [M].ACTEX Publications,1988

[18]Clancy,R.P.Option on Books Applications to Produce Pricing[J].Transactions of the society of Actuaries,37(1985),97~130

[19] Bo Zhang: *Applied Stochastic Process* , the People's University of China Press: Peking, 2000, 50～78