Three life insurance model research--Family unite the insurance model

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Abstract: - Interest rate is assumed for the constant in some insurance references. They include the calculation of spouse allied scheme insurance premium and reserve and calculation of family combined insurance premium under the constant interest rate. Interest randomness in life insurance has received considerable attention in recent literatures on actuarial theory and its applications. In order to avoid the risk induced by interest randomness, we consider the force of interest function as a Wiener process. In this paper, we build an actuarial model for family combined insurance under fixed interest and random rate of interest differently. First, the family condition in the future is analyzed. In analyzing about multi-life function, introducing the situation of three element, especially while considering the death order, consider a person dies before other two people (state of jointly surviving). Second, two kinds of association situations and two models are considered here. The first place is the couple and only children; the second place is couple own. At the last, the actuarial present value on the life insurance and annuity are calculated.

Key-Words: - life insurance, actuarial, survival function, random rate, multi-element life function, Wiener process

1 Introduction
In a term life insurance policy, usually only insured one person, also possible for multi-person life insurance, such as couple, family, for elder parents, and etc. Usually we can divide by two groups by how many insured people every policy, individual policy and multi-person policy, also called connectable life insurance [1]. Usually annually fund is for individual policy. For this paper, we study for multi-person policy, and study target on future family. For life-insurance, expect for group policy, mostly are for ordinary person. It has a huge potential market for future family, though it is very necessary we need pay attention on it, and develop products for such demands.

2 An analysis for a future home
Let think about future family members, a husband, a wife, four elder parents, and one kid. The financial status, a couple has a decent income, enough for care about elder parents, growing a kid, and paying bills. Four elder parents have or partial has own endowment insurance, they basically do not need income from its child, financial dependently. And they have certain mount for medical insurance, expect from social security, they has lack information of commercial insurance. Young kid is still in study, financially not able to live independently, and need a quite spending on it. Another point, this couple has highly educated and has decent income; marriage age is elder than average from 20 century in China. Usually they married after 30, has own house and car, but instill financing. So they have quite a bit pressure, and their parents also have quite bit age.

Let study individual thoughts for each family member, for each elder person, they have been retired, quite old in age, and health condition is critical, health insurance and term life insurance are mainly focused, in health insurance we cover on critical sickness, hospitalization cover, and accident insurance. Based on social security, they already have some pension, and also can cover certain mount of medical expense, so they can offset these expenses in short term. For growing kids, education and growing are two main concerns. Once they left college, they will ability to independently. Although, these expense are pressure on the couple. In case, couple in accident, the kids education and growing will be major effected. They take care elders and grow and with educate kid, and paying bills for the couple, so they need more security.

We are put death condition of family in consideration and analyze that death has influence upon family [2]. To introduce death in the model, one is anyone in death by couple. Another case is both couple dead, on third case, the couple get old and kid dead in accidentally. We are not put these elder
parents death in consideration. We analyze that three death conditions have influence on family. Two cases we are study with, one is husband in death second wife in death. These two cases are different. Let assume, the husband dead how effect the whole family financial situation. From income structure, psychological impact for whole family, and sexual difference, the husband death will be major impact for take care a family, such as take care elder parents and grow kid. If happen in wife, these problems are also effects. However, which one will be more effect than others? Let study psychological impact for wife or husband, due to variable research, lost wife is more psychological impact than lost husband. The husbands get more possibility become psychopathic because of lost loved one. So, in our model we have more insurance coverage for lost wife. Another case is both couple dead. In this case, the whole family suddenly down to terrible situation. Elder could death earlier because of lost loved kid, and young kid could live negatively because of losing his/her parents. Thus, our proposal is elder parents could live normally, and young kid could still in normal education, and grow positively. For approach this goal, our coverage payment will be quite big. On third case, the couple get old and kid dead in accidentally. If kid still quite young, the possibility get another kid will be quite high, if young kid becomes adult, the possibility for another one will be quite low. We assume that exceeding 18 ages in death has more negative influence on family than under 18 age. In our life table and physiology, the possibility of a dead kid will be very low, even though younger than 5 years old will be quite high.

Then we create a allied insurance with husband, wife, and kid. We associated insurance with husband and wife. We set statement, any one of them dead, the state changed, they are able to claim the coverage. We give different mount base on the model we assumed. In second case, we set statement they only can claim only if couple are both dead, the payment coverage such as take care elders, grow kid and with education, and payment for after death. At last, we talk about whole family dead that mean, husband, wife, and kid.

3 Introducing multi-life function and actuarial model of life insurance

3.1 Multi-life function
From two chapters, we consider two conditions, husband, wife, kid and couple. Let assume three objects (x), (y), (z) means husband, wife and kid. For a allied insurance model, we set condition no change as long as all three in live, only condition change when one of them dead. For a couple policy the condition no change until both of them dead (husband and wife). For better understanding we give two definitions [1, 3, 5].

Definition 1: For a condition has N life objects, No i object has x_i years old. The future life call T(x_i)(i=1,2,...,N). If we set condition for the first object dead define a condition termination. And record this condition as (x_1, x_2, ..., x_N), then the condition maintain time as:

\[
T(x_1, x_2, ..., x_N) = \min\{T(x_1), T(x_2), ..., T(x_N)\}
\]

Then we called minimum life from, or union life form.

Definition 2: For a condition has N life objects, No i object has x_i years old. The future lifetime call T(x_i)(i=1,2,...,N). If we set condition for the last object dead define a condition termination. And record this condition as (x_1, x_2, ..., x_N), then the condition maintain time as:

\[
T(x_1, x_2, ..., x_N) = \max\{T(x_1), T(x_2), ..., T(x_N)\}
\]

Then we called maximum life form, or the last life form.

Thought, for union life form the condition remain when all life object remains, for the last life form, the condition only change when no life objects exist any more.

When we talk about T(x),T(y), T(z), we assume each person independently, but in fact, they are associated together. Anyone of them death could effect others life time.

3.1.1 The probably distribution of future life of three union life condition
T(xyz) [6-7] Distribution function:

\[
F_{T(xyz)}(t) = P\{T(xyz) \leq t\}
\]

\[
= P\{\min\{T(x), T(y), T(z)\} \leq t\}
\]

= 1 - P\{T(x)>t\} * P\{T(y)>t\} * P\{T(z)>t\}

= 1 - p_x * p_y * p_z

Then the probability of union live condition (xyz) at least t years:

\[
t_{T(xyz)} = P\{T(xyz)>t\} = 1 - F_{T(xyz)}(t) = 1 - p_x * p_y * p_z
\]

Then the probability density function (xyz) [8] is:

\[
f_{T(xyz)}(t) = \frac{d}{dt} \left(1 - F_{T(xyz)}(t)\right)
\]

\[
= -\left(-u_x u_y u_z\right) = \left(u_x u_y u_z\right)
\]

Where, u_x, u_y, u_z are respectively 1 - p_x, 1 - p_y, 1 - p_z.
3.1.2 The probably distribution of future life of two life condition

\[ T \{ x, y \} \text{ distribution function}^{[9-11]}: \]

\[ F_{T(x, y)}(t) = P\{T(x) \leq t\} \times P\{T(y) \leq t\} \]

\[ = t \cdot g_x \cdot t \cdot g_y \]

\[ = (1 - p_x \cdot t) \cdot (1 - p_y \cdot t) \]

\[ = 1 - p_x \cdot t \cdot p_y + p_x \cdot t \cdot p_y \]

Then the probability of union live condition \( T(xy) \) at least \( t \) years:

\[ t \cdot t \cdot p_y \cdot p_y \]

\[ = 1 - F(t) \]

\[ = 1 - p_x - p_y + p_x \cdot p_y \]

(4)

3.1.3 A probability of union conditions

Let us talk about three objects \([12~13]\), that (x), (y), (z), any one of dead and others remain life. The probability presented by \( n \cdot q_{xyz} \). Mean (x) is dead before (y), (z) and within \( n \) years. Then,

\[ q_{xy} = \int_0^n t \cdot p_y \cdot u_{y+t} \cdot p_x \cdot u_{x+t} \cdot dt \]

\[ = \int_0^n t \cdot P_x \cdot u_{x+t} \cdot (1 - t \cdot p_y \cdot p_x) \cdot dt \]

\[ = \int_0^n t \cdot p_x \cdot u_{x+t} \cdot (1 - t \cdot p_y \cdot p_z) \cdot dt \]

The probability presented by \( n \cdot q_{xy} \). Mean (y) is dead before (x), (z) and within \( n \) years. Then,

\[ q_{yx} = \int_0^n t \cdot p_y \cdot u_{y+t} \cdot dt - \int_0^n t \cdot p_y \cdot p_x \cdot u_{x+t} \cdot dt \]

3.2 Discrete life insurance model

Insurance claims is paid when dying to happen immediately in insurance practising. But best information of residual life probably distribution from discrete life table is probably distribution of integral residual life.

Let assume age of insured is \( x \) when he/she buys abandonment, and future integral residual life is \( k \), insurance amount of \( k+1 \) year payment is \( b_{k+1} \), \( v_{k+1} \) is discounted factor. \( Z_{k+1} \) is discounted factor of \( k+1 \) years.

Then, \( Z_{k+1} = b_{k+1} \cdot v_{k+1} \) (10)

So in discrete life insurance model, value of expectation of present value of random variable \( Z \) is \( E(Z) \), representation of \( E(Z) \) is

\[ E(Z) = \sum_{k=0}^{\infty} v_{k+1} b_{k+1} \]

(11)

\( E(Z) \) is known as net single premium.

We explore a variety of actuarial present value of life insurance amount.

3.2.1 Mortality insurance

Mortality insurance consists of \( n \) years term insurance and whole life insurance.

(1) N years term insurance

For \( n \) years term insurance we set insurance amount as 1, actuarial present value of insurance amount is \( A_x \), correlative functions is,

\[ b_{k+1} = \begin{cases} 1 & (0 \leq k \leq n-1) \\ 0 & (k > n-1) \end{cases} \]

\[ v_{k+1} = \begin{cases} v^{k+1} & (0 \leq k \leq n-1) \\ 0 & (k > n-1) \end{cases} \]

\[ Z = \begin{cases} 0 & (k > n-1) \end{cases} \]

Mortality probably of policy-holder for \( k+1 \) year is \( k / q_x \), according to (11), then,

\[ A_x = E(Z) = \sum_{k=0}^{n-1} v^{k+1} b_{k+1} = \sum_{k=0}^{n-1} v^{k+1} b_{k+1} \]

(12)

(\( \omega \) is maximum Mortality age)

(2) Whole life insurance

For discrete whole life insurance we set insurance amount as 1, present value is \( A_x \).

\[ A_x = \sum_{k=0}^{\infty} v_{k+1} b_{k+1} = \sum_{k=0}^{\infty} v_{k+1} b_{k+1} \]

(13)

(\( \sigma \) is maximum Mortality age)

3.2.2 Endowment insurance
Endowment insurance of N years consists of pure endowment insurance of N years and term insurance of N years. No matter how \( x \) die before \( x+n \) age and \( x \) die as \( x+n \) age, assurer pay 1.

\[ b_{k+1} = 1 \quad (k=0,1,2, \ldots) \]

\[ v_{k+1} = \begin{cases} v^{k+1} & (0 \leq k \leq n-1) \\ v^n & (k > n-1) \end{cases} \]

\[ Z = \begin{cases} v^n & (K > n-1) \\ v_{K+1} & (0 \leq K \leq n-1) \end{cases} \]

Present value of insurance amount is,

\[ A_{x\mid n} = \sum_{k=0}^{n-1} v^{k+1} \cdot q_x + v^n \cdot P_x = A_{x\mid n}^1 + A_{x\mid n}^1 \quad (14) \]

### 3.2.3 N years term insurance of H years deferred period

\[ b_{k+1} = \begin{cases} 1 & (h \leq k \leq n-1) \\ 0 & (\text{other}) \end{cases} \]

\[ v_{k+1} = \begin{cases} 1 & (h \leq k \leq h+n-1) \\ 0 & (\text{other}) \end{cases} \]

\[ Z = \begin{cases} v^n & (h \leq K \leq h+n-1) \\ 0 & (\text{other}) \end{cases} \]

Present value of insurance amount is \( h/A_{x\mid n}^1 \) or \( h/A_{x\mid n} \), then

\[ h/A_{x\mid n} = \sum_{k=h}^{h+n-1} v^{k+1} \cdot q_x \]

\[ h/A_{x\mid n} = \sum_{k=h}^{n-h} v^{k+1} \cdot q_x + v^n \cdot P_x = A_{x\mid n}^1 + A_{x\mid n}^1 \quad (17) \]

3.3 Continuous life insurance model

Assurer pay insurance amount when future lifetime of policyholder is \( T(x) \).

Let assume policyholder is \( x \) age, \( b_t \) is insurance amount at \( t \), \( v_t \) is interest discounted factor, \( Z(T) \) is present value. Then,

\[ \bar{A} = E(Z) = \int_0^\infty P_x \mu_{x+t} b_t v_t \cdot dt \]

3.3.1 Mortality insurance

(1) N years term life insurance

\[ b_t = \begin{cases} 1 & (t \leq n) \\ 0 & (t > n) \end{cases} \]

\[ v_t = v^t \quad (t \geq 0) \]

\[ Z_t = \begin{cases} v^t & (T \leq t) \\ 0 & (T > t) \end{cases} \]

Present value of insurance amount,

\[ \bar{A}_{x\mid n} = E(Z) = \int_0^n v^t \cdot P_x \mu_{x+t} dt \quad (19) \]

(2) Whole life insurance

In (19), \( n \) to infinity, then

\[ \bar{A}_x = \int_0^\infty v^t \cdot P_x \mu_{x+t} dt = \int_0^\infty v^t \cdot P_x \mu_{x+t} dt \quad (20) \]

3.3.2 Endowment insurance
Random variable of present value,
\[ Z_t = \begin{cases} v^T (T \leq n) \\ v^n (T > n) \end{cases} \]
Present value of insurance amount,
\[ \overline{A}_{x:n} = E (Z) \]
\[ = \int_0^n v^t \cdot p_x \mu_{x+t} dt + v^n \cdot \overline{a}_x \]
\[ = \overline{A}_{x:n} + A_{x:n} \quad (21) \]

3.3.3 Deferred life insurance

Whole insurance of H years deferred period,
\[ h \cdot \overline{A}_x = \int_h^n v^t \cdot p_x \mu_{x+t} dt \quad (22) \]
N years term insurance of H years deferred period,
\[ h \cdot \overline{A}_{x:n} = \int_h^n v^t \cdot p_x \mu_{x+t} dt \quad (23) \]
N years endowment insurance of H years deferred period,
\[ h \cdot A_{x:n} = h \cdot \overline{A}_x + h \cdot A_{x:n} \quad (24) \]

3.4 Discrete survival annuity

3.4.1 Survival annuity according annual pay one times

(1) Whole survival annuity

Annuity is payable at the beginning of the year of survival. Present value of insurance is \( \ddot{a}_x \), then
\[ \ddot{a}_x = E(\bar{Y}) = \sum_{k=0}^n \ddot{a}_{x:k} q_x + \sum_{k=0}^{n-1} v^k p_x \quad (25) \]
Annuity is payable at the end of the year of survival. Present value of insurance is \( a_x \), then
\[ a_x = E(Y) = \sum_{k=0}^n a_{x:k} q_x + \sum_{k=0}^{n-1} v^k p_x \quad (26) \]

(2) Survival annuity of N years term insurance

Annuity is payable at the beginning of the year of survival. Present value of insurance is \( \ddot{a}_{x:n} \), then
\[ Y = \begin{cases} \ddot{a}_{x:k} & (0 \leq K \leq n-1) \\ \ddot{a}_{x:n} & (K \geq n) \end{cases} \]
\[ \ddot{a}_{x:n} = \sum_{k=0}^{n-1} v^k p_x = \sum_{k=0}^{n-1} \ddot{a}_{x:k} q_x + \ddot{a}_{x:n} \quad (27) \]
Annuity is payable at the end of the year of survival. Present value of insurance is \( a_{x:n} \), then
\[ a_{x:n} = \sum_{k=0}^{n-1} a_{x:k} q_x + a_{x:n} = \sum_{k=0}^n v^k p_x \quad (28) \]

3. Whole survival annuity and N years term survival annuity of H years deferred period

Annuity is payable at the beginning of the year of survival. Present value of insurance is \( \ddot{a}_x \) and \( \ddot{a}_{x:n} \), then
\[ h \cdot \ddot{a}_x = \sum_{k=0}^{h-1} v^k p_x = \sum_{k=0}^{h-1} (\ddot{a}_{x:k} - \ddot{a}_x) q_x \quad (29) \]
\[ h \cdot \ddot{a}_{x:n} = \sum_{k=0}^{h-1} v^k p_x \]
\[ = \sum_{k=0}^{h-1} (\ddot{a}_{x:k} - \ddot{a}_x) q_x + (\ddot{a}_{x:n} - \ddot{a}_n) \quad (30) \]

3.4.2 Survival annuity according annual pay M times

(1) Whole survival annuity is paid the beginning of the period

Let assume annuity as 1, pay M times every year, present value of insurance is \( \ddot{a}_x^{(m)} \), then
\[ \ddot{a}_x^{(m)} = \sum_{k=0}^M \frac{v^k}{m} \cdot \frac{1}{m} p_x \quad (31) \]
In general adopt similar formula of tradition,
\[ \ddot{a}_x^{(m)} \approx \ddot{a}_x - \frac{m-1}{2m} \quad (32) \]
Then similar formula,
\[ \ddot{a}_x^{(m)} = \frac{1}{m} \left( \ddot{a}_x + 1 \ddot{a}_x + 2 \ddot{a}_x + \ldots \ddot{a}_x \right) \]
\[ \approx \frac{1}{m} \left[ \ddot{a}_x + (\ddot{a}_x - \frac{1}{m}) + (\ddot{a}_x - \frac{2}{m}) + \ldots + (\ddot{a}_x - \frac{m-1}{m}) \right] \]
\[ = \ddot{a}_x - \frac{1 + 2 + \ldots + (m-1)}{m^2} \]
\[ = \ddot{a}_x - \frac{m-1}{2m} = a_x + \frac{m+1}{2m} \quad (33) \]

(2) Whole survival annuity is paid the end of the period

Present value of insurance is \( a_x^{(m)} \), then
\[ a_x^{(m)} = \ddot{a}_x^{(m)} - \frac{1}{m} \quad (34) \]
similar formula of tradition,
\[ a_x^{(m)} = \ddot{a}_x - \frac{m+1}{2m} = a_x + \frac{m-1}{2m} \quad (35) \]
(3) N years term survival annuity is paid the beginning of the period
Present value of insurance is \( \overline{a}_{x:n}^{(m)} \), then
\[
\overline{a}_{x:n}^{(m)} = \overline{a}_x - n \overline{a}_x = \overline{a}_x - n E_x \overline{a}_{x+n}^{(m)} \tag{35}
\]
Under UDD hypothesis condition, converted (35),
\[
\overline{a}_{x:n}^{(m)} = \alpha (m) \overline{a}_{x:n} + \beta (m) (1 - n E_x) \tag{36}
\]
among the rest,
\[
\alpha (m) = i d / i^{(m)} d^{(m)}
\]
\[
\beta (m) = (i - i^{(m)}) / i^{(m)} d^{(m)}
\]
similar formula of tradition of is \( \overline{a}_{x:n}^{(m)} \) is
\[
\overline{a}_{x:n}^{(m)} \approx \overline{a}_x - \frac{m - 1}{2m} (1 - n E_x) \tag{37}
\]
(4) N years term survival annuity is paid the end of the period
Present value of insurance is \( a_{x:n}^{(m)} \), then
\[
a_{x:n}^{(m)} = a_x^{(m)} - s E_x a_{x+n}^{(m)} = (a_x - \frac{m - 1}{2m}) - s E_x (a_{x+n} - \frac{m - 1}{2m}) \tag{38}
\]
\[
= a_x - \frac{m - 1}{2m} (1 - s E_x)
\]
\[
= \frac{1}{D_x} [N_{x+n} - N_{x+n+1} + \frac{m - 1}{2m} (D_x - D_{x+n})]
\]
(5) Whole survival annuity of H years deferred period is paid the beginning of the period
Present value of insurance is \( h \overline{a}_x^{(m)} \), then
\[
h \overline{a}_x^{(m)} = v^h h p_x \overline{a}_{x+h}^{(m)} = h E_x \overline{a}_x^{(m)} \tag{39}
\]
Under UDD hypothesis condition, converted (39),
\[
h \overline{a}_x^{(m)} = h E_x [\alpha (m) \overline{a}_{x+h} + \beta (m) ] \tag{40}
\]
similar formula of tradition of is \( h \overline{a}_x^{(m)} \),
\[
h \overline{a}_x^{(m)} = h \overline{a}_x - \frac{m - 1}{2m} h E_x \tag{41}
\]
(6) Whole survival annuity of H years deferred period is paid the end of the period
Present value of insurance is \( h a_x^{(m)} \), then
\[
h a_x^{(m)} = v^h h p_x a_{x+h}^{(m)} = v^h h p_x (a_{x+h} + \frac{m - 1}{2m}) \tag{42}
\]
\[
= h a_x - \frac{m - 1}{2m} h E_x
\]
3.5 Continuous survival annuity
3.5.1 Whole survival annuity

Let assume annuity as 1, Whole survival annuity of continuous pay, Present value of insurance is \( a_x \), future lifetime of (x) is \( T(x) \), density function is
\[
f_T(t) = p_x \mu_{x+t}
\]
Present value of annuity is \( \overline{Y} \), then
\[
\overline{Y} = a_{\overline{1}|T} = \int_0^T v^t dt
\]
Present value of insurance is
\[
\overline{a}_x = \int_0^\infty a_x p_x v^t dt = \int_0^\infty \frac{1}{\delta} d(t, p_x) dt
\]
where
\[
\frac{1}{\delta} = \delta(p_x, \mu_{x+t}) = \frac{d t}{d t}
\]
3.5.2 N years term survival annuity
Present value of insurance is \( \overline{a}_{x:n} \), then
\[
\overline{a}_{x:n} = \int_0^n p_x v^t dt
\]
3.5.3 Whole survival annuity of H years deferred period
Present value of insurance is \( h \overline{a}_x \), then
\[
h \overline{a}_x = \int_0^\infty p_x v^t dt = \int_0^\infty v^t dt
\]
3.5.3 N years term survival annuity of H years deferred period is paid the end of the period
Present value of insurance is \( h n \overline{a}_x \), then
\[
h n \overline{a}_x = \int_0^{n+h} p_x v^t dt = \int_0^{n+h} v^t dt
\]
4 Analysis of Policy Requirement

4.1 Insurance target
A family, male 30 years or older, Female 28 years or older, good health condition, a couple are able to work and has a 5 years old or older child. Also accept a couple without child\[14~15\].

4.2 Responsibility\[16\]
For Term-life insurance: husband, wife and kid, any of them dead, we pay for lost. The payment ratio base on we talked about before. M1, M2, M3, and M2>M1>M3.
For couple both dead, we pay for M4 and M4>(M1+M2+M3), that we can take care good for elder parents and child for couple both lost.
For annuity: kid able to get payment after 10, and every year get M5, that enough for pay education expense. After 18, he/she get M6 annually. M6>M5. That moment, kids should in college, the education expense will higher than before, and payment stops at 25 years old. At that moment, he should in master degree or in work. For couple, anyone is more than 60 years old, they can get the payment until they dead.

4.3 Exception
These conditions are not in our coverage insured person dead by drug. Insured person dead by suicide. Insured person dead by drunk drive or driving without LC. Insured person dead by involve rebellions.

5 Valuation of Premium with Fixed Interest

5.1 Long-life insurance
For easier calculation we set payment as 1, then in three life union condition (xyz) terminates, pay off is 1; couple last live condition finishes, Payment set 1; in three life union condition (x yz), (y xz), (z xy) finishes, Payment set 1. Then, these three single premium and on-level premium is \(\overline{A}_{xyz} = h p_1 \overline{\ddot{a}}_{xyz} \), \(\overline{A}_{xy} = h p_2 \overline{\ddot{a}}_{xy} \), \(\overline{A}_{xz} = h p_3 x \overline{\ddot{a}}_{xzy} \), \(\overline{A}_{yz} = h p_3 y \overline{\ddot{a}}_{zyx} \), \(\overline{A}_{zy} = h p_3 y \overline{\ddot{a}}_{zyx} \), \(h p_3 z \), then \[17-18\],

\[
\overline{A}_{xyz} = \int_0^\infty \overline{\ddot{a}}_{xyz} (1 - p_x p_y p_z) \mu_{x+y+z} dt
\]

\[
\overline{A}_{xy} = \int_0^\infty \overline{\ddot{a}}_{xy} (1 - p_x p_y p_z) dt
\]

\[
\overline{A}_{xz} = \int_0^\infty \overline{\ddot{a}}_{xz} (1 - p_x p_y p_z) dt
\]

\[
\overline{A}_{yz} = \int_0^\infty \overline{\ddot{a}}_{yz} (1 - p_x p_y p_z) dt
\]

\[
\overline{A}_{zy} = \int_0^\infty \overline{\ddot{a}}_{zy} (1 - p_x p_y p_z) dt
\]

On-level premium is in h year when (xyz) set as initial mount. No k (0 \leq k \leq h) year payment is 1 the today value is \(v_k\), and actuarial present value is \(\overline{\ddot{a}}_{xyz} \), then,

\[
\overline{\ddot{a}}_{xyz} = \sum_{k=0}^{h-1} v_k p_{xyz} = \sum_{k=0}^{h-1} v_k p_k p_y p_z \] (52)

Based on level premium, actuarial present value of premium is equal to payment after death precision calculated value, then,

\[
\overline{A}_{xyz} = h p_1 \overline{\ddot{a}}_{xyz} \] (53)

\[
\overline{A}_{xy} = h p_2 \overline{\ddot{a}}_{xy} \] (54)

\[
\overline{A}_{xz} = h p_3 x \overline{\ddot{a}}_{xzy} \] (55)

\[
\overline{A}_{yz} = h p_3 y \overline{\ddot{a}}_{zyx} \] (56)

\[
\overline{A}_{zy} = h p_3 y \overline{\ddot{a}}_{zyx} \] (57)

Then,

\[
h p_1 = \overline{A}_{xyz} / \overline{\ddot{a}}_{xyz} \] (58)

\[
h p_2 = \overline{A}_{xy} / \overline{\ddot{a}}_{xy} \] (59)

\[
h p_3 x = \overline{A}_{xz} / \overline{\ddot{a}}_{xzy} \] (60)
\[ kP_{3y} = \hat{A}_{3xy}/\hat{a}_{xyz} \]  
(61)

\[ kP_{3z} = \hat{A}_{3xy}/\hat{a}_{xyz} \]  
(62)

For n years both sides policies, the single premiums is,

\[ \overline{A}_{n} = \sum_{k=0}^{n} v^k P_{x+k} + v^k P_{y+k} + v^k P_{z+k} \]  
(63)

\( u \) means common condition, could be (xyz), also could be \((xy)\).

5.2 Survive annuity

5.2.1 Life time survive annuity
when the condition continues, every policy initially pay 1 yuan for life time annual payment

\[ Y = \hat{a}_{xyz} \]  
the precise value as,

\[ \hat{a}_{xyz} = E(Y) \]

\[ = \sum_{k=0}^{\infty} \hat{a}_{k+1} \cdot q_{xyz} \]  
\[ = \sum_{k=0}^{\infty} v^k \cdot P_{xyz} \]  
\[ = \sum_{k=0}^{\infty} v^k \cdot P_{x+k} \cdot P_{y+k} \cdot P_{z+k} \]  
(64)

5.2.2 N years fixed survive annuity

\[ \hat{a}_{xyz} = E(Y) \]

\[ = \sum_{k=0}^{\infty} \hat{a}_{k+1} + \hat{a}_{n} \cdot P_{n} \]  
\[ = \sum_{k=0}^{\infty} v^k \cdot P_{xyz} \]  
\[ = \sum_{k=0}^{\infty} v^k \cdot P_{x+k} \cdot P_{y+k} \cdot P_{z+k} \]  
(65)

5.2.3 Life time survive annuity of N years deferred period

\[ \tilde{a}_{xy} = \sum_{k=0}^{n-1} v^k P_{xy} = \sum_{k=0}^{n-1} v^k \left( kP_x + kP_y - kP_z \right) \]  
(66)

5.2.4 N-M years survive annuity of M years deferred period

\[ \tilde{a}_{xyz-m} = E(Y) = \sum_{k=m}^{n-1} v^k P_{xyz} = \sum_{k=m}^{n-1} v^k \left(kP_x + kP_y + kP_z\right) \]  
(70)

\[ \tilde{a}_{xy} = \sum_{k=m}^{n-1} v^k P_{xy} = \sum_{k=m}^{n-1} (kP_x + kP_y + kP_z) \]  
(71)

6 Valuation of Premium with Random Rate of Interest

Establish model\(^{(19)}\) aim at cumulation function of interest power,

\[ y(t) = \delta t + \beta W(t) \]

\( W(t) \) is a standard Wiener process, \( \beta \) is real number, \( \delta \) is real constant, and we assume \( y(t) \) and \( T(x), T(y), T(z) \) independently.

6.1 Long-life insurance
For easier calculation we set payment as 1, then in three life union condition \((xyz)\) terminates, pay out is 1; couple last live condition finishes, Payment set 1; in three life union condition \((1 \ yz), (1 \ xz), (1 \ yx)\) finishes, Payment set 1. Then, these three single premium and on-level premium is \( \overline{A}_{xyz}, \overline{A}_{x}, \overline{A}_{y}, \overline{A}_{z}, \overline{A}_{x+y}, \overline{A}_{x+z}, \overline{A}_{y+z}, \overline{A}_{x+y+z}\), then,

\[ \overline{A}_{xyz} = E(e^{-\delta(T_{xyz})}) \]

\[ = \int_{0}^{\infty} e^{-\delta \beta t^2} \cdot tP_{1} + P_{x+y+z} (\mu_{x} + \mu_{y} + \mu_{z}) dt \]  
(72)
\[ A_{xy} = B e^{(\alpha - \beta) t} \] 

(73)

\[ \bar{A}_{xy} = \int_0^\infty e^{-\frac{t}{2}} \left[ P_{1} \mu_{x} + P_{2} \mu_{y} - i P_{1} P_{2} (\mu_{x} + \mu_{y}) \right] dt \]

(74)

\[ \bar{A}_{xy} = \int_0^\infty e^{-\frac{t}{2} \beta} \cdot P_{1} \mu_{x} dt - \int_0^\infty e^{-\frac{t}{2} \beta} \cdot i P_{1} P_{2} \mu_{y} dt \]

(75)

\[ \bar{A}_{xy} = \int_0^\infty e^{-\frac{t}{2} \beta} \cdot P_{2} \mu_{y} dt - \int_0^\infty e^{-\frac{t}{2} \beta} \cdot i P_{1} P_{2} \mu_{x} dt \]

(76)

On-level premium is in h year when \((xyz)\) set as initial mount. No \(K (0 \leq K \leq h)\) year payment is 1 the today value is \(e^{-\frac{t}{2} \beta} \), and actuarial present value is \(\ddot{a}_{xy} \) then,

\[ \ddot{a}_{xy} = \sum_{k=0}^{h-1} e^{-\frac{k}{2} \beta} \cdot k P_{3xy} \]

(77)

Based on level premium, actuarial present value of premium is equal to payment after death precision calculated value, then,

\[ \bar{A}_{xy} = k P_{1} \ddot{a}_{xy} \]

(78)

\[ \bar{A}_{xy} = k P_{2} \ddot{a}_{xy} \]

(79)

\[ \bar{A}_{xy} = k P_{3x} \ddot{a}_{xy} \]

(80)

\[ \bar{A}_{xy} = k P_{3y} \ddot{a}_{xy} \]

(81)

\[ \bar{A}_{xy} = k P_{3z} \ddot{a}_{xy} \]

(82)

Then,

\[ k P_{1} = \bar{A}_{xy} / \ddot{a}_{xy} \]

(83)

\[ k P_{2} = \bar{A}_{xy} / \ddot{a}_{xy} \]

(84)

\[ k P_{3x} = \bar{A}_{xy} / \ddot{a}_{xy} \]

(85)

\[ k P_{3y} = \bar{A}_{xy} / \ddot{a}_{xy} \]

(86)

\[ h P_{3z} = \bar{A}_{xy} / \ddot{a}_{xy} \]

(87)

For \(n\) years both sides policies, the single premiums is,

\[ \bar{A}_{xy} = \sum_{k=0}^{n} e^{-\frac{k}{2} \beta} \cdot k P_{3xy} + e^{-\frac{k}{2} \beta} \cdot n P_{3z} \]

(88)

\((u)\) means common condition, could be \((xyz)\), also could be \((\bar{xy})\).

6.2 Survive annuity

6.2.1 Life time survive annuity

when the condition continues, every policy initially pay 1 yuan for life time annual payment

\[ Y = \ddot{a}_{xy} \]

the precise value as ,

\[ \ddot{a}_{xy} = \sum_{k=0}^{\infty} e^{-\frac{k}{2} \beta} \cdot k P_{3xy} = \sum_{k=0}^{\infty} e^{-\frac{k}{2} \beta} \cdot k P_{3xy} \]

(89)

\[ \ddot{a}_{xy} = \sum_{k=0}^{\infty} e^{-\frac{k}{2} \beta} \cdot k P_{3xy} = \sum_{k=0}^{\infty} e^{-\frac{k}{2} \beta} \cdot k P_{3xy} \]

(90)

6.2.2 N years fixed survive annuity

\[ \ddot{a}_{xy} = \sum_{k=0}^{n-1} e^{-\frac{k}{2} \beta} \cdot k P_{3xy} = \sum_{k=0}^{n-1} e^{-\frac{k}{2} \beta} \cdot k P_{3xy} \]

(91)

\[ \ddot{a}_{xy} = \sum_{k=0}^{n-1} e^{-\frac{k}{2} \beta} \cdot k P_{3xy} = \sum_{k=0}^{n-1} e^{-\frac{k}{2} \beta} \cdot k P_{3xy} \]

(92)

6.2.3 Life time survive annuity of N years deferred period

\[ n \ddot{a}_{xy} = \sum_{k=n}^{\infty} e^{-\frac{k}{2} \beta} \cdot k P_{3xy} + \sum_{k=n}^{\infty} e^{-\frac{k}{2} \beta} \cdot k P_{3xy} \]

(93)

\[ n \ddot{a}_{xy} = \sum_{k=n}^{\infty} e^{-\frac{k}{2} \beta} \cdot k P_{3xy} + \sum_{k=n}^{\infty} e^{-\frac{k}{2} \beta} \cdot k P_{3xy} \]

(94)
6.2.4 N-M years survive annuity of M years deferred period

\[
m_{\gamma_{x+y}} = \sum_{k=m}^{n} e^{\frac{-\delta_{x+y}}{2}} kP_{X+k}P_{Y+k}P_{z}\]

7 Conclusion

For this paper, we study for multi-person policy, and study target on future family. First, the family condition in the future is analyzed. In analyzing about multi-life function, introducing the situation of three elements, especially while considering the death order, consider a person dies before other two people, but these two people are a kind of state of jointly surviving. Second, two kinds of association situations and two models are considered here. The first place is the couple and only children; the second place is couple own. At the last, the actuarial present value on the life insurance and annuity are calculated.

References:


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