Extracting Fatigue Damage Features Using STFT and CWT

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Abstract: - The fatigue feature extraction using the Short-Time Fourier Transform (STFT) and wavelet transform approaches are presented in this paper. The transformation of the time domain signal into time-frequency domain computationally implemented using the STFT and Morlet wavelet methods provided the signal energy distribution display with respect to the particular time and frequency information. In this study, cycles with lower energy content were eliminated, and these selections were based on the signal energy distribution in the time representation. The simulation results showed that the Morlet wavelet was found to be a better approach for fatigue feature extraction. The wavelet-based analysis obtained a 59 second edited signal with the retention of at least 94 % of the original fatigue damage. The edited signal was 65 seconds (52 %) shorter than length of the edited signal that was found using the STFT approach. Hence, this fatigue data summarising algorithm can be used for accelerating the simulation works related to fatigue durability testing.

Key-Words: - Fatigue strain signal, fatigue damage, STFT, Morlet wavelet, edited signal.

1 Introduction

Occasionally, fatigue signals measured from critical automotive parts have variable amplitude patterns with mean value of the data that change with time (each pattern has different statistical value). They contain a large percentage of small amplitude cycles and the fatigue damage for these cycles can be small. For this reason, in many cases, the signal was edited by removing these cycles in order to produce representative and meaningful yet economical testing [1-2].

In a fatigue life assessment, fatigue signal extraction is described as a method for fatigue data editing which leads to summarising a fatigue signal. The method is performed by segment identification and extraction of those that contribute to the more fatigue damaging events of a metallic material. On the other hand, segments containing lower amplitude cycles are omitted, since these data type theoretically gave minimal or no fatigue damage. This process generates a new shortened signal, for which this signal type can be used to reduce the testing time and costs for fatigue testing [3]. Two key factors are suggested for achieving an efficient design and modification processes to ensure adequate fatigue life assessment, i.e.: the signal statistical parameters and the fatigue damage should be as accurate as possible and the component durability tests should be as short as possible.

Several fatigue data editing approaches have been introduced in various domains: time, peak and valley, frequency, cycles, damage, and histogram [4]. The most commonly applied procedures in the research literature have been based on time and frequency domains. One of the new approaches that was developed for the fatigue signal extraction is the one in time-frequency domain. Previously, the time-frequency approach had been applied to the problem of fatigue signal extraction, but only for the purpose of spike removal and de-noising [5]. Among the time-frequency domain analyses that have been used widely in engineering problems are Short-time Fourier Transform (STFT) and wavelet transform (WT)

The STFT or windowed Fourier transform is one of the methods for transforming the time domain signal into the time-frequency domain [6]. In addition, the STFT adopted the Fourier transform to analyze only a small section of the signal at one specific time. Finally, the STFT provides information on when and at what frequency a signal occurs. However, this information is only obtained with limited precision determined by the size of the window. Many signals require a more flexible approach, in order to determine more accurately either time or frequency [7].

With the advances in digital signal processing research, there has been an increasingly strong interest in the related application for fatigue life assessment of automotive components. During the last decade, an improved signal processing technique, called the WT, has been frequently used in the field of vibrational diagnostics and also in fault detection. In addition, the wavelet coefficient analysis has also have been applied to detect fatigue transverse cracks in rotors. Its peak absolute value is highly sensitive to the depth of crack and even a very shallow crack can be detected. The rotor is not required to stop and the detection process is applied for a rotating shaft makes the methodology more versatile, convenient and unambiguous [8].

In order to obtain the appropriate technique for the fatigue strain signal extraction, the STFT and WT approaches were utilized to transform the time domain signal into time-frequency domain and trace the lower energy cycles contained in the original signal. Those segments were then removed from the original signal in order to gain a new edited signal containing the higher energy cycles. Segments which have been removed have minimal or no fatigue damaging potential. Therefore, the original fatigue damage can be retained in the edited signal produced at the end of the process. The effectiveness of these techniques was validated based on the fatigue damaging retention in the shortened signals.

This STFT-based fatigue feature extraction algorithm was previously developed by Abdullah et al [9]. Since the WT has been found to be theoretically better than the STFT in the time-frequency localisation, it gave a motivation to the authors for developing a similar data extraction approach in the WT. Therefore, a new algorithm for fatigue feature extraction using the Morlet wavelet was developed. The WT results were compared to the findings using the STFT extraction approach in order to see the suitability approach in fatigue history editing.

2 Literature Background

2.1 The STFT

The STFT is performed by dividing the signal into small sequential or overlapping data frames. Then, Fast Fourier Transform (FFT) has been applied to each data frame. The output of successive STFT can provide a time–frequency representation of the signal. In order to accomplish this, the signal is truncated into short data frames by multiplying it by a window so that the modified signal is zero outside the data frame. In order to analyse the whole signal, the window is then translated into a time and reapplied to the signal.

For the resolution, the length of the window used in this method is fixed on every time and frequency axis. Window size used will determine the obtained resolution, where small windows present good time resolution, and longer windows represent good frequency representations [10].

The STFT is composed by the local spectra of segments of the primary function, as viewed through a translating window of fixed shape. The local spectra at all points on the primary time axis constitute the STFT. Generally, the STFT is expressed as [11]:

$$STFT(t, f) = \int_{-\infty}^{\infty} h(t) w(t-\tau) \exp(-2\pi i f \tau) d\tau \qquad (1)$$

where h is the primary function, τ is the time, and f is the frequency. The position of the translating window w is determined by t, which has the same units as τ. If w is replaced with the value of 1, the STFT reduces to H, i.e. the Fourier transform of h. The modulus of the STFT is also known as the spectrogram.

2.2 The Morlet wavelet

This approach is probably the most recent solution to overcome the nonstationary signals. This time-frequency technique is applied by cutting time domain signal into various frequency components through the compromise between time and frequency-based views of the signal. It presents information in both time and frequency domain in a more useful form [12-14].

The WT analysis is started with a basic function (called the mother wavelet) scaled and translated to represent the signal being analysed [15]. The transform shifts a window along the signal and calculates the spectrum for every position. The process is repeated many times with a slightly shorter (or longer) window for every new cycle. The result will be a collection of time-frequency representations of the signal with different resolutions. The WT provides information on when and at what frequency the change in signal behaviour occurs [12].

Obviously, the WT represents a windowing technique with variable-sized regions. This technique allows the use of long time intervals (more precise low frequency information) and shorter regions (high frequency information). It means the wavelet method solves the resolution problem because the window length is long for low frequency and short for high frequency. Therefore, the frequency resolution is good for low frequency (at high scales) and the time resolution is good at high frequency (at low scales). The major advantage is the ability to analyse a localized area of larger signal, also known as local analysis [7].

The wavelet decomposition calculates a resemblance index between the signal being analysed and the wavelet, called the coefficient. It is the result of regression of an original signal produced at different scales and different sections on the wavelet. It represents the correlation between the wavelet and a section of the signal. If the index is large, the resemblance is strong, otherwise it is slight. Generally, the wavelet coefficient C is expressed with the following integral [7]:

$$C_{(scale,position)} = \int_{-\infty}^{\infty} f(t)\psi(scale, position, t)dt \tag{2}$$

The Morlet wavelet is one of functions that are generally used in the Continuous Wavelet Transform (CWT) analyses [16]. The WT of any time-varying signal $f(t)$ is defined as the sum of all of the signal time multiplied by a scaled and shifted version of the wavelet function $\psi(t)$ [10]. The CWT is expressed by the following integral:

$$CWT_{(a,b)} = \int_{-\infty}^{+\infty} f(t)\psi_{a,b}(t)dt \tag{3}$$

The parameter a represents the scale factor which is a reciprocal of frequency, the parameter b indicates the time shifting or translation factor, and t is time.

$\Psi_{a,b}(t)$ denotes the mother wavelet, i.e. [17]:

$$\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}}\psi\left(\frac{t-b}{a}\right) a,b \in R; a \neq 0 \tag{4}$$

$$CWT_{(a,b)} = \int_{-\infty}^{+\infty} f(t)\frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right)dt \tag{5}$$

In addition, the wavelet coefficient indicates how the energy in the signal is distributed in the time-frequency plane [8]. The energy spectrum (the energy density over frequency) is plotted in order to observe the signal behaviour and its content gives significant information about the random signal pattern.

2.3 Fatigue Life Assessment

There are three major approaches to predicting fatigue life namely stress-life, strain-life, and fracture mechanics. At below the transition point (approximately 1000 cycles), the ε-N-based approach is appropriate method and is commonly used to predict fatigue life for ductile materials at relatively short fatigue life. The crack initiation method relates the plastic deformation that occurs at a localized region where fatigue cracks begin to the durability of the structure under influence of mean stress [18].

Current industrial practice uses the Palmgren-Miner [19-20] linear cumulative damaging rule normally associated with the established strain-life fatigue damaging models. The total strain amplitude ε_a is produced by the combination of elastic and plastic amplitudes, i.e.:

$$\varepsilon_a = \varepsilon_{ea} + \varepsilon_{pa} \tag{6}$$

where ε_{ea} is the elastic strain amplitude and ε_{pa} is the plastic strain amplitude. The elastic strain amplitude is defined by:

$$\varepsilon_{ea} = \frac{\sigma'_f}{E}\left(2N_f\right)^b \tag{7}$$

while the plastic strain amplitude is given as:

$$\varepsilon_{pa} = \varepsilon'_f\left(2N_f\right)^c \tag{8}$$

where N_f is the numbers of cycle to failure for a particular stress range and mean, σ'_f is the fatigue strength coefficient, b is the fatigue strength exponent, ε'_f is the fatigue ductility coefficient, c is the fatigue ductility exponent and E is the material modulus of elasticity.

Combining Equations (7) and (8) gives the Coffin-Manson relationship [21-22], which is mathematically defined as:

$$\varepsilon_a = \frac{\sigma'_f}{E}\left(2N_f\right)^b + \varepsilon'_f\left(2N_f\right)^c \tag{9}$$

which is essentially Equation (6) above and is the foundation of the strain-life approach.

The fatigue damage caused by each cycle of repeated loading is calculated by reference to material life curves, such as S-N or ε-N curves. The fatigue damage D for one cycle and the total fatigue damage ΣD caused by cycles are expressed respectively as [19-20]:

$$D = \frac{1}{N_f} \tag{10}$$

$$\Sigma D = \Sigma\left(\frac{N_i}{N_f}\right) \tag{11}$$

where N_i is the numbers of cycle within a particular stress range and mean.

Fatigue damage has value in the range (0-1) where zero denotes no damage (extremely high or infinite number of cycles to failure) and 1 means total failure (one cycle to failure).

2.4 Signal Statistical Parameters

In the case of the fatigue research, a signal consists of a measurement of cyclic loads, i.e. force, strain, and stress against time. A time series typically consists of a set of observations of a variable being taken at equally spaced intervals of time. Global signal statistical parameters are frequently used to classify random signals and monitor the pattern of analysed signals. For a signal with a numbers of data point n in a sampled sequence, the mean $\bar{x}$ is given by:

$$\bar{x} = \frac{1}{n}\sum_{j=1}^{n} x_j \tag{12}$$

For a fatigue signal, the calculation of the root-mean-square (r.m.s.) and the kurtosis are important in order to retain a certain amount of the signal amplitude range characteristics. The r.m.s. value is the signal 2nd statistical moment used to quantify the overall energy content of the oscillatory signal. The r.m.s relationship is defined as:

$$r.m.s. = \left\{\frac{1}{n}\sum_{j=1}^{n} x_j{}^2\right\}^{1/2} \tag{13}$$

The kurtosis is the signal 4th statistical moment. In engineering field, it is used as a measure of nongaussianity for detection of fault symptoms since it is highly sensitive to spikiness or outlier signal among the instantaneous values. Mathematically, the kurtosis expression is defined as [23]:

$$K = \frac{1}{n(r.m.s.)^4}\sum_{j=1}^{n}\left(x_j - \bar{x}\right)^4 \tag{14}$$

where x_j is the amplitude of signal.

In some definitions of the kurtosis, a deduction of 3.0 is added to the definition in order to maintain the kurtosis of a Gaussian distribution to be equal to zero. For clarity and convenience, in this study the original definition of the kurtosis, where the Gaussian distribution has a kurtosis value is approximately 3.0, was used for the analysis. Therefore, a kurtosis value of higher than 3.0 indicates the presence of more extreme values than the one that should be found in a Gaussian distribution [3]. This situation indicated that the fatigue damage is higher than Gaussian stresses due to higher amplitude fatigue cycles [24].

2.5 Fatigue Feature Extraction

This fatigue signal summarising algorithm uses peak to peak amplitude range as a parameter to determine gate value for the eliminating process. The value obtained from the wavelet coefficient amplitude at a cut off point or fatigue limit of the particular material is used to slice the original signal. The extracted segment identification is performed by searching the events start and finish points which define the temporal extent of the extracted segment. The identification is based on energy loss concept, i.e. selected segments are at the start and finish points. The example of the segment identification is described in Fig. 1. In the figure, the selected segment is at gate value of 400 $\mu\varepsilon^2$/Hz. Start point is a valley point if the peak before is higher than the peak after the point. While the finish point is selected if peak after is higher than peak before the point. This concept is performed by [3] based on transient vibration where start and finish points are selected based on transient form. The points are determined based on the signal where the shortening in signal background occurs.

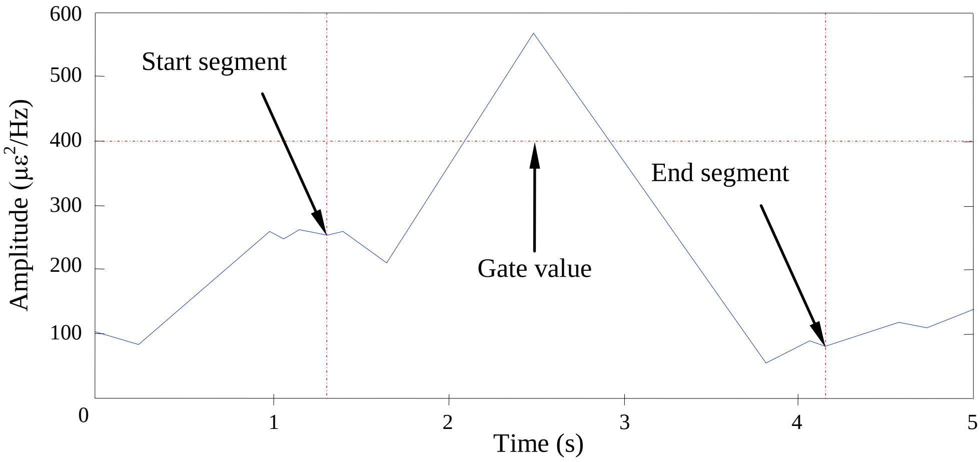
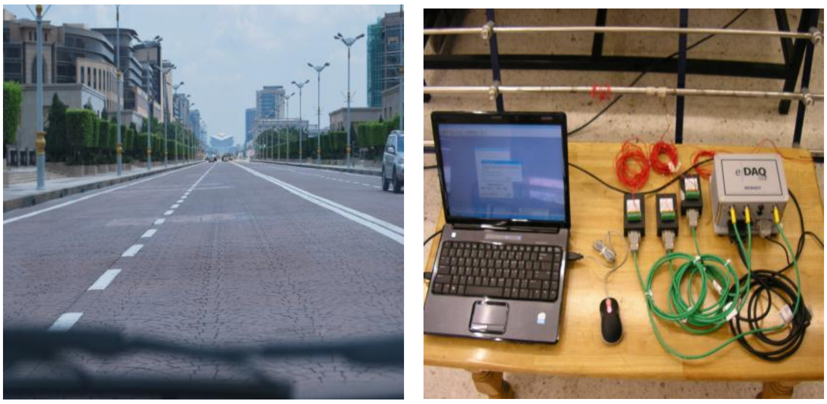


Fig. 1 The extracted segment identification

After all the segments are identified, the fatigue time history is then sliced in order to remove lower wavelet coefficient amplitudes (less than the gate value) contained in the original time history range. For this reason, the majority of the original fatigue damage is retained in the edited signal. All extracted segments (the complete section between the start and the end of the segments) selected based on time location of the wavelet coefficient amplitude are then combined together to produce a new mission time history. The mission signal replicates the signal statistical parameter and total fatigue damaging characteristics of the original time history. The optimum gate value is accordingly determined and it is based on the effectiveness of retaining the characteristics of the original signal in the mission signal. Ideally, the signal has shorter time length but is equivalent in the characteristic values.

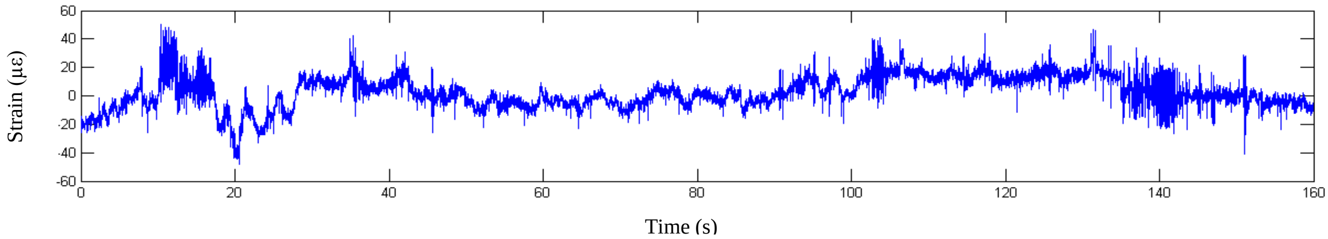
3 Methodology

In this study, the input signal was measured at a front lower suspension arm of a passenger car driven over a public road surface (Fig. 2). The signal was a variable amplitude loading sampled at 200 Hz for 32,000 data points. It gave the total signal record length of 160 seconds. The collected signal was recorded using a fatigue data acquisition system containing many small amplitude and high frequency in the signal background, as illustrated in Fig. 3.a for the time series and Fig. 3.b for its Power Spectral Density (PSD). For the fatigue damaging calculation, the selected material for the simulation purpose was the SAE1045 carbon steel shaft. It was chosen as a common material used in automotive industries for fabricating a vehicle lower suspension arm structure [25]. The material properties and their definitions are given in Table 1 [26].



a b

Fig. 2. Data collection: (a) a section of the test track, (b) the data acquisition set-up



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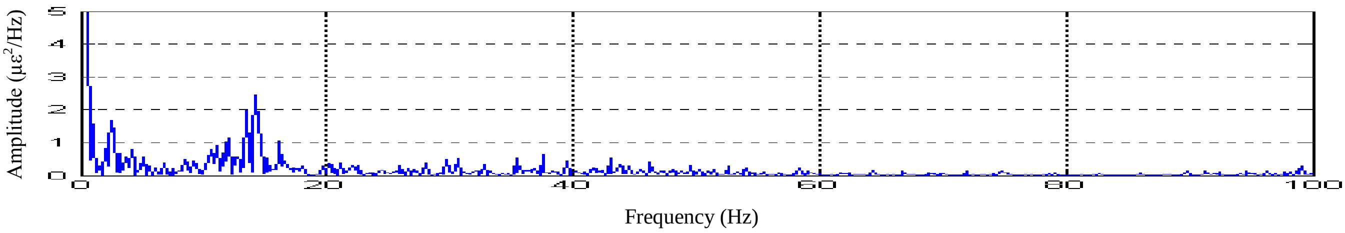

b

Fig. 3. (a) the time history plot of the original test signal, (b) the Power Spectral Density (PSD)

Table 1: The mechanical properties of the SAE1045 carbon steel shaft

Properties	***Values***
Ultimate tensile strength, S_u (MPa)	621
Modulus of elasticity, E (GPa)	204
Fatigue strength coefficient, σ'_f (MPa)	948
Fatigue strength exponent, b	-0.092
Fatigue ductility exponent, c	-0.445
Fatigue ductility coefficient, ε'_f	0.26

For solving the subject matters of this paper, the fatigue data editing using the STFT and Morlet wavelet methods was based on the following main stages: the time-frequency analysis, the generation of a new edited signal, and fatigue damaging analysis. Computational algorithms based on the STFT and the Morlet wavelet were developed in order to analyze the signal according to the fatigue damaging calculation and also to remove amplitude containing lower power. The locations of fatigue damaging event were identified according to the higher power level in time domain plot.

For the eliminating process, the magnitude of time domain spectrum level was used as the parameter to set the gate value. Various gate values were used in order to exhibit the effectiveness of the edited signal with respect to the fatigue damaging retention. It means that, the segments with amplitude level below the gate value were eliminated from the time domain signal based on the location in the time history distribution. The segment with magnitude higher than the gate value was sliced as the retained segment. The sliced segment identification was performed by searching the two inversion points (one on either side of peak value). The retained segments, selected based on time location of the sliced segment, were joined to produce the new edited signal. For this reason, the majority of the original fatigue damage is related in the edited signal. The flowchart of both methods is illustrated in Fig. 4.

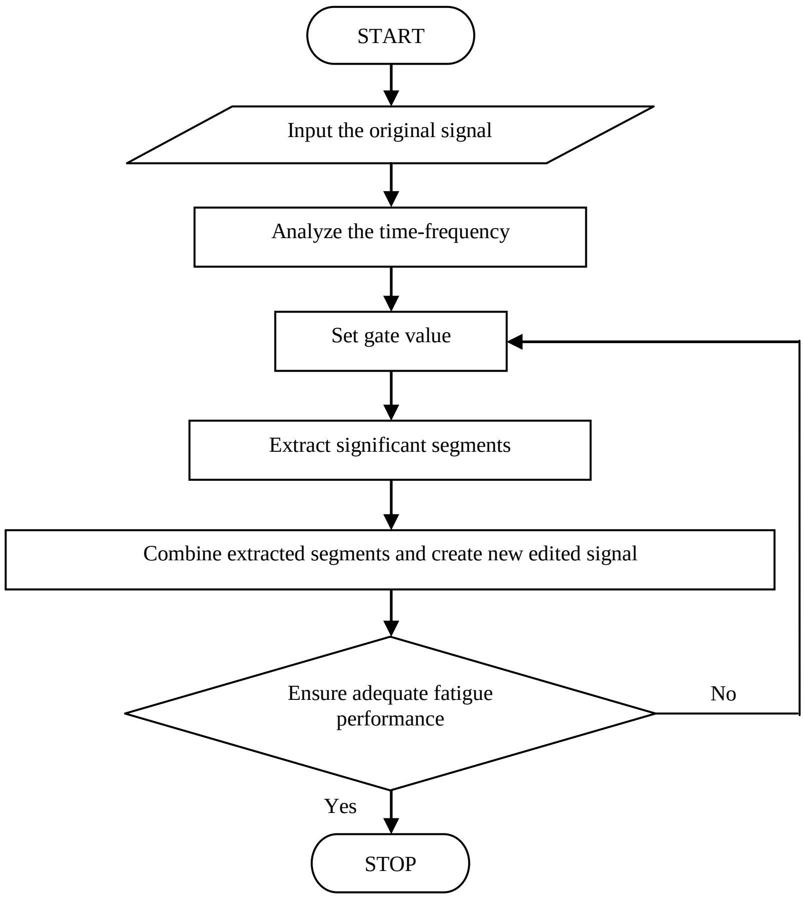

Fig. 4. Simplified flowchart of the computational fatigue data editing algorithm using the STFT and Morlet wavelet

4 Results and Discussions

In this analysis, the signal in time domain was converted into time-frequency domain using the STFT method. The time history signal was separated into a number of windows using the Gaussian window with window size of 128. The number of overlaps used in order to provide the high resolution in the time representation was 120. For each window, the Fourier transform was applied for the calculation of the signal energy contained in each window. The energy calculation was gained from the PSD that produced the spectrogram of the STFT.

Using a specific commercial software package, the STFT plot of the original fatigue signal showed a two dimensional view of the signal energy distribution, as observed in the time-frequency plane. This result is plotted in Fig.5.a. The level was presented by a colour contour, where the red colour showed the highest energy content and followed by yellow, green and blue. Based on the energy parameter, the spectrogram value was decomposed into a time domain display in order to represent the signal energy distribution in time history. The energy display provided the time location containing the lower energy cycle, as illustrated in Fig. 5.b.

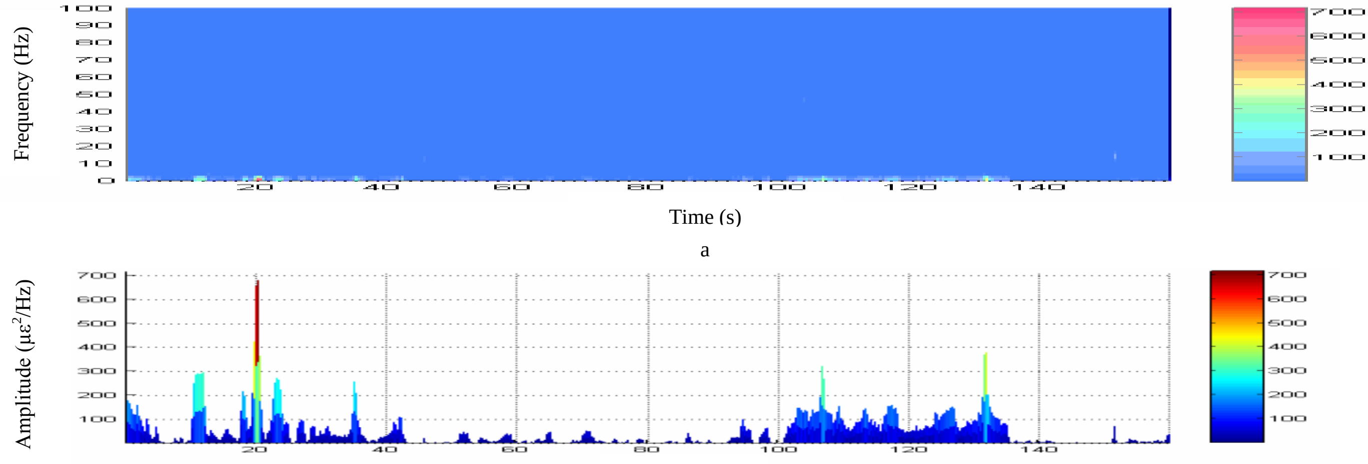

Fig. 5. (a) The STFT localization, (b) the STFT energy distribution

In this STFT study, nine signals (the original and 8 edited signals) were simulated for the purpose of the verification the efficiency of fatigue data editing using STFT method. The edited signals were produced at eight gate values, i.e. 5 $\mu\varepsilon^2$/Hz, 10 $\mu\varepsilon^2$/Hz, 15 $\mu\varepsilon^2$/Hz, 20 $\mu\varepsilon^2$/Hz, 25 $\mu\varepsilon^2$/Hz, 30 $\mu\varepsilon^2$/Hz, 35 $\mu\varepsilon^2$/Hz, and 40 $\mu\varepsilon^2$/Hz. From the fatigue damaging calculation results, as shown in Fig. 6, it was found that 5 $\mu\varepsilon^2$/Hz to be an optimum gate value since the total fatigue damaging value produced from this edited signal had only 6 % deviation when compared to the original signal. The new edited signal of 124 seconds was produced, which was 36 seconds shorter than the original signal length, as shown in Fig. 7. This value gave a reduction of 22 % the original time length.

For the Morlet wavelet-based edited signal, it started by analyzing the wavelet coefficients, as shown in Fig. 8.a using Equation (5). In the presented scalogram, the *x*-axis denoted the time parameter and the *y*-axis represented the scale that has an inversely related to the frequency value. The colour intensity at each *x*-*y* point was proportional to the absolute value of the wavelet coefficients as a function of the dilation and translation parameters. It provided the energy distribution display with respect to the particular time and frequency information. Accordingly, a lower scale indicated higher frequency and had small amplitude which means these cycles had lower energy, indicating minimal or no fatigue damaging potential. A large scale was indicative of lower frequency and higher amplitude that indicates these cycles had higher energy causing the fatigue damage.

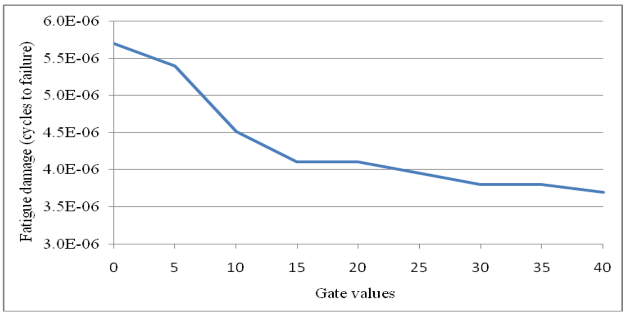

Fig. 6. Graph of parameter changes over gate values of the STFT

Obviously, the lower frequency indicated higher magnitude distribution, and the lower magnitude distribution was presented at higher frequency event. With the newly Morlet wavelet-based developed algorithm, the wavelet coefficients were transposed into time domain signal, as shown in Fig. 8.b.

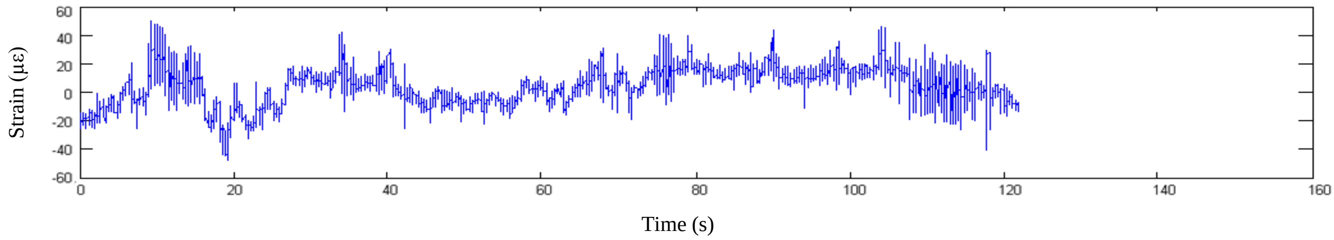

Fig. 7. The 124 seconds of the STFT-based edited signal

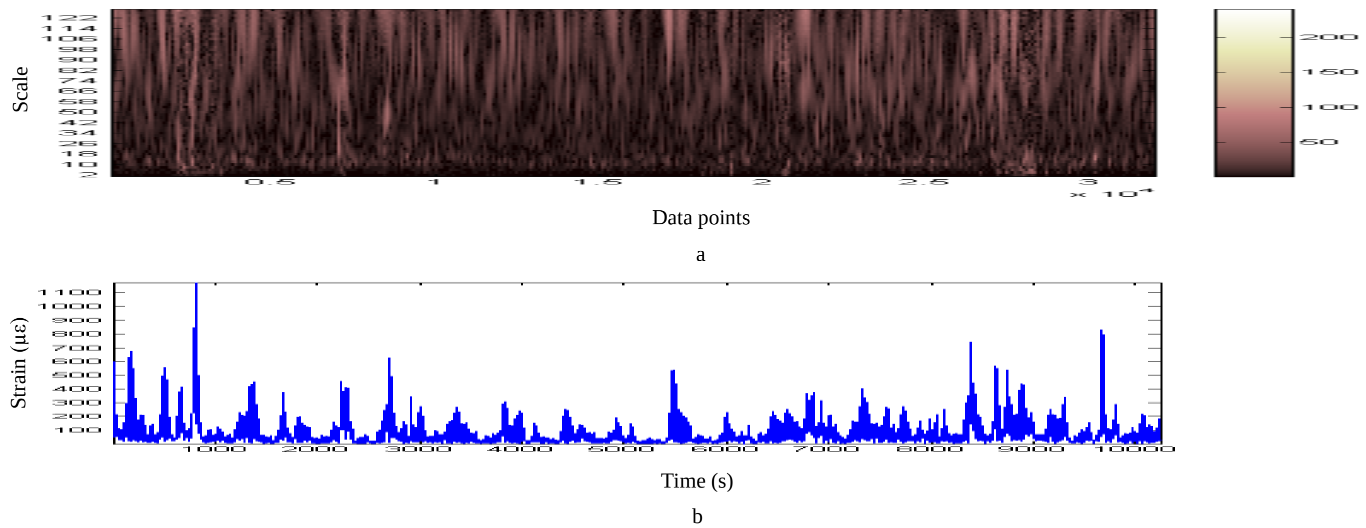

Fig. 8. The distribution of the Morlet wavelet coefficients: (a) time-frequency representation, (b) time representation

This extraction process only involved 150 $\mu\varepsilon^2$/Hz and 200 $\mu\varepsilon^2$/Hz gate values. From the total fatigue damaging calculation results, it was found that 200 $\mu\varepsilon^2$/Hz was selected to be the optimum gate value giving lower than 10 % difference of the fatigue damage, as can be observed in Fig. 9. The total fatigue damage produced from this edited signal had only 6 % deviation compared to the original signal. Furthermore, the signal contained more than 90 % of the original signal statistical parameter values. It means that the algorithm preserved the originality of the fatigue damage and the signal behaviour. At this fatigue damaging ratio, the new edited signal of 59 seconds was produced, which was 101 seconds shorter than the original signal length, as shown in Fig. 10. This value gave 63 % of the original time length reduction.

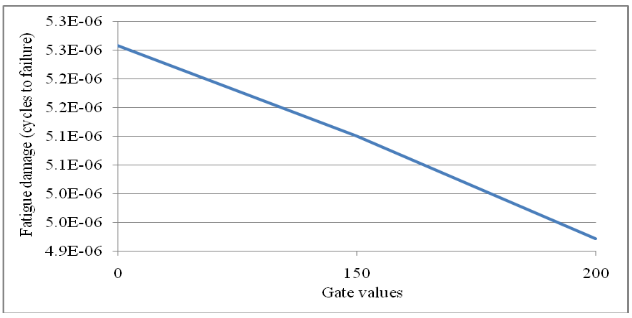

Fig. 9. Graph of parameter changes over gate values of the wavelet transform

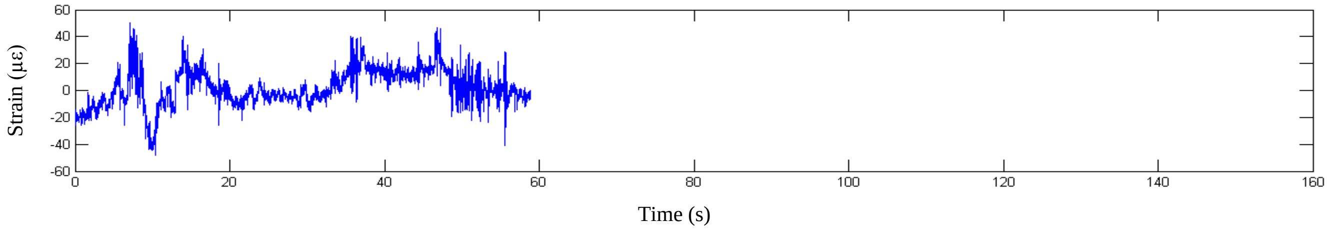

Fig. 10. The 59 seconds of the Morlet-based edited signal

Based on these two approaches, finally, the applicability of fatigue data editing with the adaptation of the Morlet wavelet method was proven for the situation to shorten the signal length with the retention of the majority of the original fatigue damage. The energy spectrum showed relatively adequate with damaging event in the fatigue signal and was a very useful tool for damaging detection in the fatigue signal. The extraction of fatigue damaging events successfully removed the lower energy cycles in the time history and created a new edited signal which retains higher fatigue damaging segments containing the majority of the fatigue damage.

5 Conclusion

This paper discusses the study of a fatigue data editing technique in time-frequency domain by using the STFT and Morlet wavelet methods. Overall, based on the simulation analysis, the findings of this paper suggested that the Morlet wavelet was more suitable for the fatigue data editing. The Morlet wavelet-based edited signal contained at least 94 % of the original fatigue damage in the 59 second edited signal, i.e. only 37 % of the original signal time length. Whereas the STFT-based edited signal contained 94 % of the original fatigue damage in the 124 second edited signal, i.e. 78 % of the original signal time length. In terms of the applicability of the shortened signal, this kind of signal was normally used in the laboratory fatigue testing for the purpose of accelerated fatigue testing.

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