Using Pisarenko Harmonic Decomposition for the design of 2-D IIR Notch filters

Nikos E. Mastorakis

***Abstract* — In this paper, the Pisarenko Harmonic Decomposition is used for the design of 2-D (Two-Dimensional) notch filters. An appropriate transformation, recently proposed by the author is used.**

***Keywords* — Pisarenko Harmonic Decomposition, Notch Filters, 2-D Notch Filters, Multidimensional Systems, Multidimensional Filters, Filter Design,**

I. Introduction

The Pisarenko Harmonic Decomposition, which is a well-known frequency estimation method, is widely used in many areas of Signal Estimation, Signal Reconstruction and Adaptive Filtering. The Pisarenko method uses the eigenvector associated with the smallest eigenvalue to estimate the frequencies of the input signal [11]. Adaptive Notch filters are used in live sound reproduction, in instrument amplifiers design, in electrocardiogram (ECG) signal processing etc. So, for every case in signal processing and communications that an elimination of an undesirable frequency is necessary, an appropriate Notch Filter is necessary in order to cut-off this unwanted frequency. For example in ECG signal processing there is a need to eliminate the power line noise as it is added in the bandwidth of the ECG signal. In the one-dimensional (1-D) case, several methods for the design and performance analysis of IIR and FIR notch filters have been developed [1]÷[3]. In this paper, we use the results of [3] and an attempt to extend them in 2-D case via appropriate transformations is presented. On the other hand, the adaptation here is achieved by using a 2-D adaptation law. This paper is organized as follows: Section II presents First-Order 2-D IIR Notch Filters design together with a numerical example. In Section III, the design of a family of Second-Order 2-D IIR Notch Filters is presented. Some remarks can be found in IV and finally there is a Conclusion. The adaptation law is based on Pisarenko Method and is given in Section IV.

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II. The Proposed Method for First-Order IIR 2-D Notch Filters (Without Adaptation – Without Pisarenko Method)

First we consider the transfer function

$$H\left(z^{-1}\right)=K\frac{1-z^{-1}}{1-rz^{-1}} \qquad (1)$$

with $z^{-1}=e^{j\omega T}$ $-\pi\le\omega\le\pi$, T is the Sampling Period, and $0<<r<1$. For $0<<r<1$ this 1-D transfer function is stable [3].

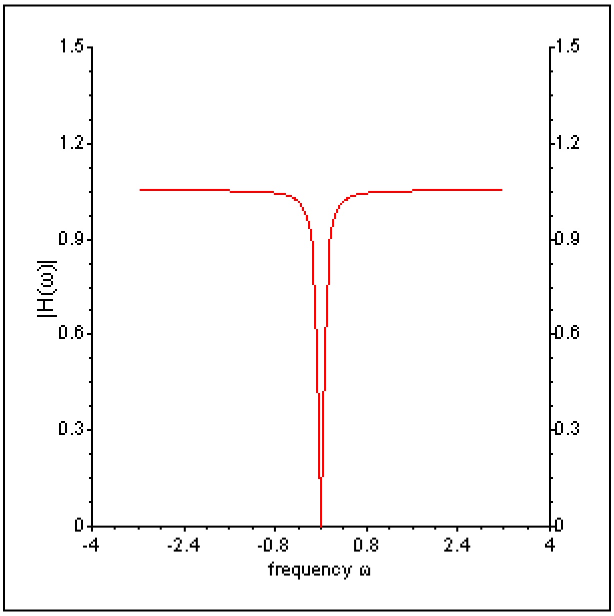

Fig. 1.a

K is a scaling factor such that the maximum gain of the filter to be equal to 1. With the pole radius almost equal to 1, the pole almost cancels the effect of the zero except in the case z=1. So, this filter is an all-pass filter that rejects the frequency of $\omega=0$ (e.g. DC frequency). The magnitude response is illustrated in Fig.1.a in the case of $r=0.9$, (T =1 without loss of generality).

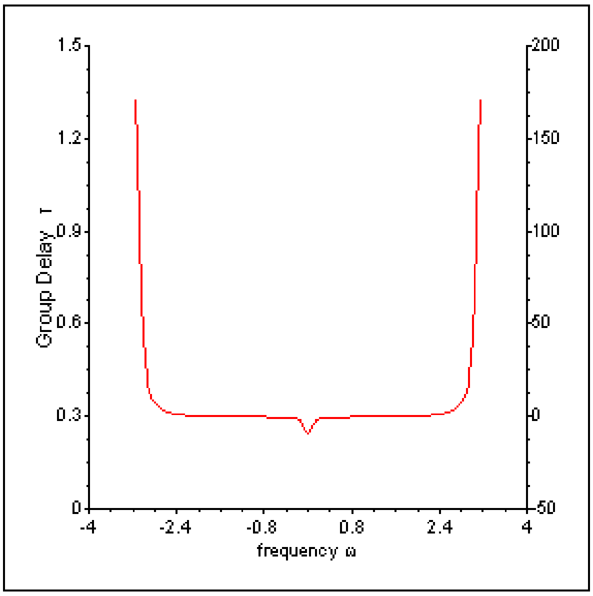

Fig. 1.b

The Group Delay $\tau = -\dfrac{\partial ArgH(j\omega)}{\partial \omega}$ is depicted in Fig. 1.b and shows almost linear behavior in a big part of the frequency domain. The Notch filter of (1) is presented in [3]. In this section, we extend it to 2-D case as follows:

In this paper, we propose 2-D filters, based on the results of [3] by applying appropriate transformations.
So, for the first-order notch filter of (1) considering the transformation

$$z^{-1} = \frac{z_1^{-1} + z_2^{-1}}{2} \qquad (2)$$

we take

$$H\left(z^{-1}\right) = K\frac{2-(z_1^{-1}+z_2^{-1})}{2-r(z_1^{-1}+z_2^{-1})}$$

with $z_1^{-1} = e^{j\omega_1 T_1}$ $-\pi \le \omega_1 \le \pi$, $z_2^{-1} = e^{j\omega_2 T_2}$, $-\pi \le \omega_2 \le \pi$
T_1, T_2 are the sampling periods to horizontal and vertical direction whereas: $0 << r < 1$

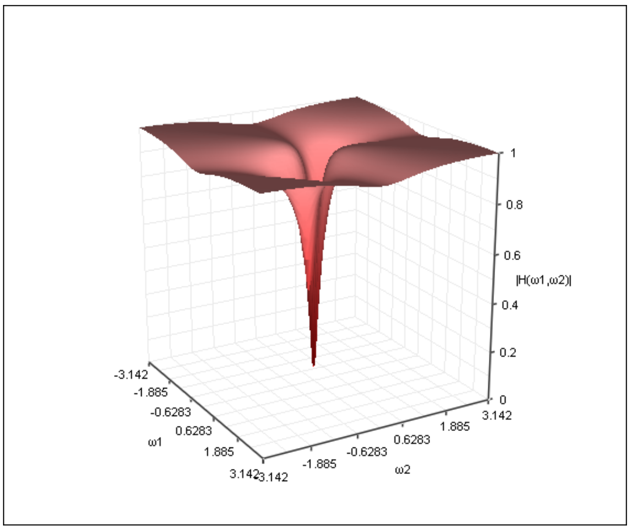

Fig. 2.a

Transformation (2) is remarkable because the Equation $2-(z_1^{-1}+z_2^{-1})=0$ has the unique solution $z_1^{-1}=1, z_2^{-1}=1$ since $z_1^{-1} = e^{j\omega_1 T_1}, z_2^{-1} = e^{j\omega_2 T_2}$ and can be easily extended to a family of transformations as follows. Introducing

$z^{-1} = \dfrac{\lambda_1 z_1^{-1} + \lambda_2 z_2^{-1}}{\lambda_1 + \lambda_2}$ with λ_1, λ_2 real numbers or simply

$z^{-1} = \lambda z_1^{-1} + (1-\lambda) z_2^{-1}$ with $0 < \lambda < 1$
one obtains

$$H\left(z^{-1}\right) = K\frac{1-(\lambda z_1^{-1}+(1-\lambda)z_2^{-1})}{1-r(\lambda z_1^{-1}+(1-\lambda)z_2^{-1})} \qquad (3)$$

with $z_1^{-1} = e^{j\omega_1 T_1}$ $-\pi \le \omega_1 \le \pi$, $z_2^{-1} = e^{j\omega_2 T_2}$, $-\pi \le \omega_2 \le \pi$
($0 << r < 1$)
The $r < 1$ condition guarantees the 1-D and 2-D filter stability in all the above cases.

Numerical Example 1:
Consider without loss of generality T_1, T_2 equal to 1. Then, for $r = 0.9$ and $\lambda = \dfrac{1}{2}$ in (3), one finds $K = 1.05260$, the magnitude response is depicted in Fig.2.a,
while the Group Delays

$\tau_1 = -\dfrac{\partial ArgH(j\omega_1, j\omega_2)}{\partial \omega_1}$, $\tau_1 = -\dfrac{\partial ArgH(j\omega_1, j\omega_2)}{\partial \omega_2}$
are depicted in Fig.2.b and Fig.2.c.

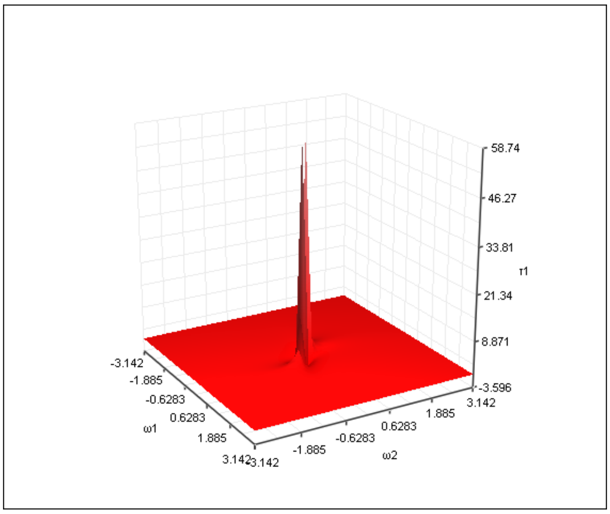

Fig.2.b

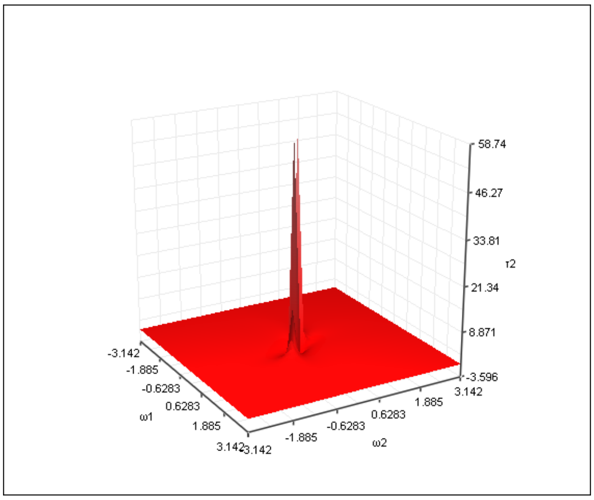

Fig.2.c

It is apparent that the family of the filters of (3) eliminates the 2-D frequency $(\omega_1,\omega_2)=(0,0)$. Using this First-Order 2-D Notch filter, the only frequency that can be eliminated is $(\omega_1,\omega_2)=(0,0)$. If elimination of another 2-D frequency $(\omega_1,\omega_2)=(\omega_{10},\omega_{20})\neq(0,0)$ is necessary, a second-order 2-D IIR notch filter must be used. As we prove in Section IV the 2-D first-order Notch filter is also Stable for $0<<r<1$.

III. The Proposed Method for Second -Order IIR 2-D Notch Filters (Without Adaptation – Without Pisarenko Method)

In this session, we extend (3) as follows in order to create a filter for rejection $(\omega_1,\omega_2)=(\omega_{10},\omega_{20})$

$$H\left(z_1^{-1},z_2^{-1}\right)=K\frac{\lambda e^{j\omega_{10}T_1}+(1-\lambda)e^{j\omega_{20}T_2}-(\lambda z_1^{-1}+(1-\lambda)z_2^{-1})}{\lambda e^{j\omega_{10}T_1}+(1-\lambda)e^{j\omega_{20}T_2}-r(\lambda z_1^{-1}+(1-\lambda)z_2^{-1})}\cdot$$
$$\cdot\frac{\lambda e^{-j\omega_{10}T_1}+(1-\lambda)e^{-j\omega_{20}T_2}-(\lambda z_1^{-1}+(1-\lambda)z_2^{-1})}{\lambda e^{-j\omega_{10}T_1}+(1-\lambda)e^{-j\omega_{20}T_2}-r(\lambda z_1^{-1}+(1-\lambda)z_2^{-1})} \quad (4)$$

with $0<\lambda<1$, $0<<r<1$ and K is a scaling factor such that the maximum gain of the filter to be equal to 1. For $0<<r<1$ this 1-D transfer function is stable [3]. Eq. (4) can be written also as

$$H\left(z_1^{-1},z_2^{-1}\right)=K\frac{A\left(z_1^{-1},z_2^{-1}\right)}{B\left(z_1^{-1},z_2^{-1}\right)}$$

where

$$A\left(z_1^{-1},z_2^{-1}\right)=\lambda^2+(1-\lambda)^2+2\lambda(1-\lambda)\cos(\omega_{10}T_1-\omega_{20}T_2)$$
$$-2(\lambda z_1^{-1}+(1-\lambda)z_2^{-1})(\lambda\cos\omega_{10}T_1+(1-\lambda)\cos\omega_{20}T_2)+(\lambda z_1^{-1}+(1-\lambda)z_2^{-1})^2$$

and

$$B\left(z_1^{-1},z_2^{-1}\right)=\lambda^2+(1-\lambda)^2+2\lambda(1-\lambda)\cos(\omega_{10}T_1-\omega_{20}T_2)$$
$$-2r(\lambda z_1^{-1}+(1-\lambda)z_2^{-1})(\lambda\cos\omega_{10}T_1+(1-\lambda)\cos\omega_{20}T_2)+r^2(\lambda z_1^{-1}+(1-\lambda)z_2^{-1})^2$$

Let's examine for what $\left(z_1^{-1},z_2^{-1}\right)$ we have $H\left(z_1^{-1},z_2^{-1}\right)=0$

Because for the frequency response $\left|z_1^{-1}\right|=1,\left|z_2^{-1}\right|=1$, we have to examine the frequencies (ω_1,ω_2) for which

$\lambda e^{j\omega_1T_1}+(1-\lambda)e^{j\omega_2T_2}-\lambda e^{j\omega_{10}T_1}-(1-\lambda)e^{j\omega_{20}T_2}=0$ or

$\lambda e^{j\omega_1T_1}+(1-\lambda)e^{j\omega_2T_2}-\lambda e^{-j\omega_{10}T_1}-(1-\lambda)e^{-j\omega_{20}T_2}=0$

Using now: $c=\dfrac{1-\lambda}{\lambda}$

$$(e^{j\omega_1T_1}+ce^{j\omega_2T_2})-e^{j\omega_{10}T_1}-ce^{j\omega_{20}T_2}=0 \quad (5.1)$$
$$(e^{j\omega_1T_1}+ce^{j\omega_2T_2})-e^{-j\omega_{10}T_1}-ce^{-j\omega_{20}T_2}=0 \quad (5.2)$$

We examine two cases

a) $c=1$, that means $\lambda=\dfrac{1}{2}$ and

b) $c\neq1$, that means $\lambda\neq\dfrac{1}{2}$

a) The first case yields the two equations:

$$e^{j\omega_1T_1}+e^{j\omega_2T_2}-e^{j\omega_{10}T_1}-e^{j\omega_{20}T_2}=0 \quad (6.1)$$
$$e^{j\omega_1T_1}+e^{j\omega_2T_2}-e^{-j\omega_{10}T_1}-e^{-j\omega_{20}T_2}=0 \quad (6.2)$$

From (6.1) one obtains the notch frequencies

$\omega_1=\omega_{10},\omega_2=\omega_{20}$

and the symmetric solution

$\omega_1=\dfrac{T_2}{T_1}\omega_{20},\omega_2=\dfrac{T_1}{T_2}\omega_{10}$

while from (6.2) two other couple of notch frequencies, i.e. $\omega_1=-\omega_{10},\omega_2=-\omega_{20}$

$\omega_1=-\dfrac{T_2}{T_1}\omega_{20},\omega_2=-\dfrac{T_1}{T_2}\omega_{10}$

are obtained.

b) The second case yields also two equations:

$$e^{j\omega_1T_1}+ce^{j\omega_2T_2}-e^{j\omega_{10}T_1}-ce^{j\omega_{20}T_2}=0 \quad (7.1)$$
$$e^{j\omega_1T_1}+ce^{j\omega_2T_2}-e^{j\omega_{10}T_1}-ce^{j\omega_{20}T_2}=0 \quad (7.2)$$

with $c\neq1$.

So, from (7.1) one obtains the notch frequencies $\omega_1=\omega_{10},\omega_2=\omega_{20}$, and from (7.2) the notch frequencies $\omega_1=-\omega_{10},\omega_2=-\omega_{20}$

Evidently, as 2-D IIR filter we can use only the case b) since the elimination of the "symmetric frequencies" ($\omega_1 = \frac{T_2}{T_1}\omega_{20}, \omega_2 = \frac{T_1}{T_2}\omega_{10}$) is not required.

Therefore our 2-D IIR Notch Filter is given by (4) that can be also written as

$$H\left(z_1^{-1}, z_2^{-1}\right) = K\frac{A\left(z_1^{-1}, z_2^{-1}\right)}{B\left(z_1^{-1}, z_2^{-1}\right)}$$

where

$$A\left(z_1^{-1}, z_2^{-1}\right) = \lambda^2 + (1-\lambda)^2 + 2\lambda(1-\lambda)\cos(\omega_{10}T_1 - \omega_{20}T_2)$$
$$-2(\lambda z_1^{-1} + (1-\lambda)z_2^{-1})(\lambda\cos(\omega_{10}T_1) + (1-\lambda)\cos(\omega_{20}T_2)) + (\lambda z_1^{-1} + (1-\lambda)z_2^{-1})^2$$

and

$$B\left(z_1^{-1}, z_2^{-1}\right) = \lambda^2 + (1-\lambda)^2 + 2\lambda(1-\lambda)\cos(\omega_{10}T_1 - \omega_{20}T_2)$$
$$-2r(\lambda z_1^{-1} + (1-\lambda)z_2^{-1})(\lambda\cos(\omega_{10}T_1) + (1-\lambda)\cos(\omega_{20}T_2)) + r^2(\lambda z_1^{-1} + (1-\lambda)z_2^{-1})^2$$

with $0 << r < 1$ $0 < \lambda < 1$, $\lambda \neq 0.5$ and K is a scaling factor such that the maximum gain of the filter to be equal to 1.

Using $c = \frac{1-\lambda}{\lambda}$ now, a further simplification of the second-order 2-D IIR Notch Filter transfer function is

$$H\left(z_1^{-1}, z_2^{-1}\right) =$$
$$K\frac{1^2 + c^2 + 2c\cos(\omega_{10}T_1 - \omega_{20}T_2) - 2(z_1^{-1} + cz_2^{-1})(\cos(\omega_{10}T_1) + c\cos(\omega_{20}T_2)) + (z_1^{-1} + cz_2^{-1})^2}{1^2 + c^2 + 2c\cos(\omega_{10}T_1 - \omega_{20}T_2) - 2r(z_1^{-1} + cz_2^{-1})(\cos(\omega_{10}T_1) + c\cos(\omega_{20}T_2)) + r^2(z_1^{-1} + cz_2^{-1})^2} \quad (8)$$

where $c \neq 1$

Numerical Example 2:
Consider the 2-D IIR Notch Filter of (8). Suppose that we want the cancellation of $\omega_{10} = \frac{\pi}{2}, \omega_{20} = \frac{\pi}{4}$ (and of course the symmetric $\omega_{10} = -\frac{\pi}{2}, \omega_{20} = -\frac{\pi}{4}$). One can choose for example c = 2, $r = 0.9$. Consider also without loss of generality T_1, T_2 = 1. Then

$$H\left(z_1^{-1}, z_2^{-1}\right) = K\frac{(z_1^{-1} + 2z_2^{-1} + 3j)(z_1^{-1} + 2z_2^{-1} - 3j)}{(rz_1^{-1} + r2z_2^{-1} + 3j)(rz_1^{-1} + r2z_2^{-1} - 3j)}$$

with $z_1^{-1} = e^{j\omega_1}$ $-\pi \leq \omega_1 \leq \pi$, $z_2^{-1} = e^{j\omega_2}$, $-\pi \leq \omega_2 \leq \pi$
Hence, the magnitude response is depicted in Fig.3.a, while the Group Delays

$$\tau_1 = -\frac{\partial ArgH(j\omega_1, j\omega_2)}{\partial \omega_1}, \quad \tau_1 = -\frac{\partial ArgH(j\omega_1, j\omega_2)}{\partial \omega_2}$$

are depicted in Fig.3.b and Fig.3.c

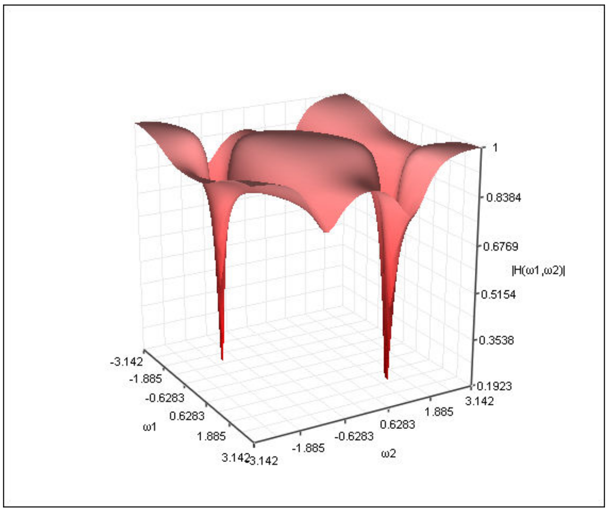

Fig.3.a

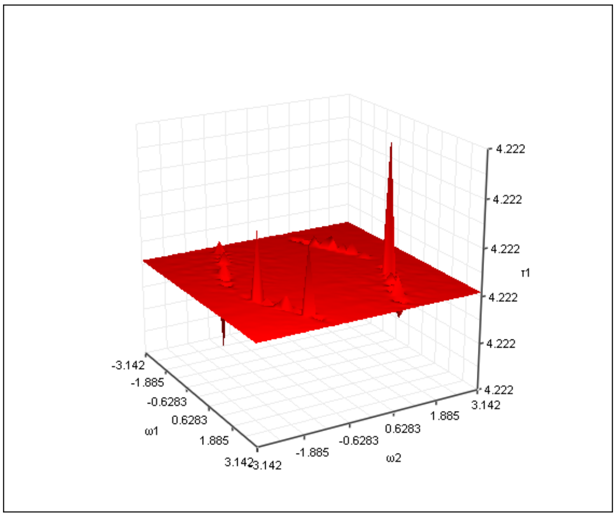

Fig.3.b

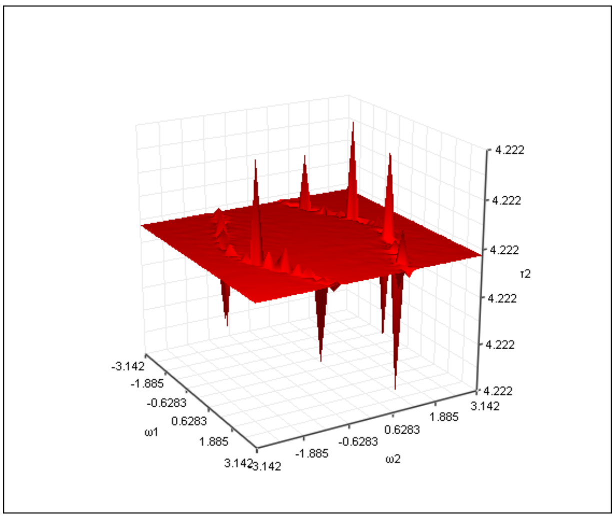

Fig.3.c

As we prove in the next Section, our 2-D second-order Notch filter is also Stable for $0 << r < 1$.

REMARKS

A first remark is that 2-D filters with several notch frequencies can be easily implemented by cascade design, while by using the new transformations

$z_1^{-1} = z_1^{-P_1}$ and $z_2^{-1} = z_2^{-P_2}$ where P_1, P_2 are positive integers, except the notch frequencies

$\omega_1 = \pm\omega_{10}, \omega_2 = \pm\omega_{20}$

the following notch frequencies are obtained

$$\omega_1 = \pm\frac{k_1}{P_1}\omega_{10}, \omega_2 = \pm\frac{k_2}{P_2}\omega_{20}$$

$k_1 = 1, 2, ..., P_1$ and $k_2 = 1, 2, ..., P_2$

Therefore periodic 2-D notch filters can easily implemented.

IV. PISARENKO HARMONIC DECOMPOSITION FOR THE DESIGN OF 2-D NOTCH FILTERS

The question is how to find the Notch frequencies $\omega_1 = \omega_{10}, \omega_2 = \omega_{20}$. To this end, we apply the usual 1-D Pisarenko Method separately to each direction. More detailes about Pisarenko Method are given in [11] and [12]. First we start from the horizontal direction (i.e. to determine ω_{10}

We apply the following algorithm for the horizontal dimension

(1) Estimate the covariance matrix R_a of size $N_1 x N_1$ from the N_1 measured samples of our 2-D signal in the horizontal dimension. The exact covariance matrix R_a is given as follows

$$R_a = \begin{pmatrix} R(0,0) & R(1,0) & \cdots & R(N_1-1,0) \\ R(1,0) & R(0,0) & \cdots & R(N_1-2,0) \\ \vdots & \vdots & \ddots & \vdots \\ R(N_1-1,0) & R(N_1-2,0) & \cdots & R(0,0) \end{pmatrix}$$

(2) Compute the eigenvector corresponding to the smallest eigenvalue of the estimated covariance matrix R_a.

(3) Compute the roots of the polynomial formed by the elements of the above eigenvector. This polynomial will have roots located at $\exp(\pm j\omega_{10})$

and the following algorithm for the vertical dimension

(1) Estimate the covariance matrix R_b of size $N_2 x N_2$ from the N_2 measured samples of our 2-D signal in the vertical dimension.. The exact covariance matrix R_b is given as follows

$$R_b = \begin{pmatrix} R(0,0) & R(0,1) & \cdots & R(0,N_2-1) \\ R(0,1) & R(0,0) & \cdots & R(0,N_2-2) \\ \vdots & \vdots & \ddots & \vdots \\ R(0,N_2-1) & R(0,N_2-2) & \cdots & R(0,0) \end{pmatrix}$$

(2) Compute the eigenvector corresponding to the smallest eigenvalue of the estimated covariance matrix R_b.

(3) Compute the roots of the polynomial formed by the elements of the above eigenvector. This polynomial will have roots located at $\exp(\pm j\omega_{20})$

Except the method of Pisarenko, we can use alternatively in each dimension (See [12])

- MUSIC Algorithm
- Minimum-Norm Method
- ESPRIT Algorithm

V. CONCLUSION

A new efficient and elegant technique for adaptive 2-D Notch Filter Design is investigated in this paper. Some other studies of the author for the stability of *m*-D systems can be found in [4] ÷[10]. Work is in progress by the author towards of statement new *m*-D design techniques better and more effective than the McClellan Transformations.

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