A New Method of DOA Estimation for Uniform Antenna Array

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Abstract: - A novel method of direction-of-arrival (DOA) estimation based on subarray beamforming for uniform circular arrays is proposed. In this method, the beamform manifold of uniform circular arrays is transformed via virtual structure, and then the virtual array is divided into two subarrrays. The target DOA is estimated from the phase shift between the reference signal and its phase-shifted version by subarray beamforming. Since the reference signal is obtained after interference rejection, the effect of interference The computation of the proposed method is simple, and the number of the signal sources of target is not bounded by the number of antenna elements. Simulation results demonstrate that proposed method has significantly improved estimation resolution, capacity, and accuracy relative to other method.

Key-Words: direction of arrival (DOA), estimation, virtual array, subarray beamforming

1. Introduction

In the last two decades, Smart antenna has been widely used in many applications such as radar, sonar and wireless communication systems. Considerable research efforts have been made to estimate the direction of arrival (DOA) and various array signal process techniques for DOA estimation have been proposed. In particular, the DOA estimation for uniform circular arrays (UCAs) has been developed in these scenarios, which desired all-azimuth angle coverage. By the virtue of their geometry, UCAs are able to provide 360°of coverage in azimuth plane. Moreover, they are known to be is isotropic. That is, they can estimate the DOA of incident signal with uniform resolution in the azimuth plane. In addition, direction patterns synthesized with UCAs can be electronically rotated in the plane of the array without significant change of beam shape [1-4].

The most commonly used DOA estimation techniques are (1) spectrum based method, such as Bartlett [5] and capon [6]; (2) subspace-based algorithm solutions, such as multiple signal classification (MUSIC) [7]; (3) parametric methods, such as estimation of signal parameters via rotational invariance technique (ESPRIT) [8]. The techniques mentioned were mainly developed for uniform linear arrays (ULAs). Some of these algorithm, such as UCA-MUSIC [9], UCA-ESPRIT [10], and their variations [2] [11], have been expended for UCAs. UCA-MUSIC techniques as the MUSIC for ULAs, the DOAs are determined by finding the directions for which their antenna response vectors lead to peaks in the MUSIC spectrum formed by the eigenvectors of the noise subspace. Thus, the capacity of DOA estimation using UCA-MUSIC is bounded by the number of antenna elements. UCA-ESPRIT techniques are transform the steering vectors of UCAs to

Vandermonde form via virtual arrays [2][3][4][12], and then use the techniques of ESPRIT for ULAs to analyze the transformed array structure. As a result, the capacity of DOA estimation using UCA-ESPRIT is bounded by the number of virtual arrays. The application of the aboved techniques is limited to cases where the number of signal sources is less than the number of antenna elements. These techniques require subspace estimation and eigendecomposition leading to high computational complexity, thereby limited their use to applications where fast DOA estimation is not required. Further, in the presence of interference, these techniques need to estimated the DOAs of all target signals and interference, which decreases the accuracy of the DOA estimation.

A novel subarray beamforming-based DOA (SBDOA) estimation technique [13] for UCAs (UCA-SBDOA) has been proposed in this paper. The method consists of two steps. The first step is the transformation of the signals received by the array to virtual arrays whose structure is amenable to the application of SBDOA. The second step is estimation of target DOA via SBDOA estimation techniques for virtual ULAs. The advantages of the UCA-SBDOA estimation are as follow:

(1) In UCA-SBDOA estimation techniques, the target DOA is estimated after interference rejection using beamforming. In this way, the estimation resolution and accuracy of UCA-SBDOA are significantly improved.

(2)Since the use of subarray beamforming in this technique makes subspace, eigendecomposition unnecessary and reduce the complexity of computation, the UCA-SBDOA can be easily applied in terms of hardware.

(3) The capacity of DOA estimation of proposed UCA-SBDOA technique can be far larger than the number of antenna elements.

The paper is organized as follows. In Section 2 we describe the basic structure of UCA-SBDOA system. In the Section 3, the signal formation, beamforming, and DOA computing of the proposed UCA-SBDOA are presented. Design examples and simulation results are given in Section 4, and conclusion are drawn in the Section 5.

2. SIGNAL MODEL

Suppose the array is composed by a uniform circular array of M elements which are distributed over a circular ring of radius r, all elements are identical and omnidirection. Assume that K narrowband signal sources and P unknown interference sources, planar waves of different direction incidence in the circular array, centered on a known wavelength (λ_0).

Using complex envelope representation, the output signals of can be expressed by

$$x(t) = \sum_{k=1}^{K+P} \mathbf{a(\theta_k)} s_k(t) + n(t) \tag{1}$$

where $s_k(t)$ denotes the kth signal component, $k = 1, 2, \cdots, K$ for target signals component and $k = K+1, K+2, \cdots, K+P$ for interference, The $a(\theta_k)$ in (1) denotes the steering vector of the array toward direction θ_k, given by

$$\mathbf{a}(\boldsymbol{\theta}_k) = \left[e^{j\beta r \cos\theta_k}, \cdots, e^{j\beta r \cos(\theta_k - 2\pi(m-1)/M)} \right]^T \tag{2}$$

Where $\beta = 2\pi / \lambda_0$, and $n(t)$ denotes the noise vector with zero mean and cross-covariance

$$E\left[n(t_1) n^H(t_2) \right] = \sigma^2 \delta(t_1 - t_2) I \tag{3}$$

Where I is the identity matrix.

Suppose that the received vector $x(t)$ is sampled at L time instants $t_1 \cdots, t_L$, the sampled data can be expressed by

$$\mathbf{X = A(\theta)S + N} \tag{4}$$

Where X and N are $M \times L$ matrixes,

$$\mathbf{X} = [x(t_1), \cdots, x(t_L)]$$

and

$$\mathbf{N} = [n(t_1), \cdots, n(t_L)]$$

and $A(\theta)$ is a $M \times K$ matrix,

$$\mathbf{A(\theta)} = [\mathbf{a(\theta_1)}, \cdots, \mathbf{a(\theta_K)}]$$

and S is a $K \times L$ matrix,

$$\mathbf{S} = [s(t_1), \cdots, s(t_L)]$$

3.UCA-SBDOA ESTIMATION

The proposed UCA-SBDOA uses the uniform circular antenna array geometry as the receiving element. Signals are transformed via the virtual ULA whose structure is amenable to application of SBDOA. In SBDAO techniques, two subarrays are used in conjunction with two subarray beamformers to obtain an optimum estimation of a phase-shift reference signal whose phase relative to that of the reference signal is a function of the target DOA. The target DOA is then computed from the estimated phase-shift between the phase shifted reference signal and the reference signal. The block diagram of the UCA-SBDOA system is illustrated in Fig.1.

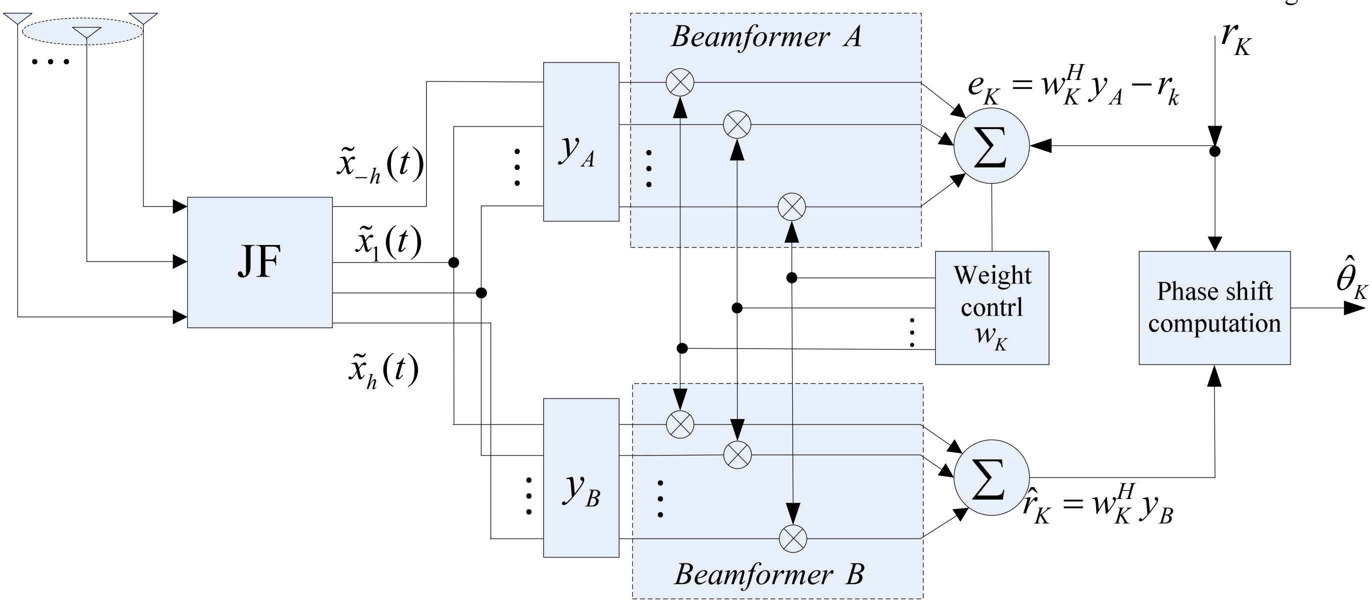

Fig .1. Block diagram of the UCA-SBDOA system

Suppose the array elements are isotropic. In [2].Wax showed that if the sensor outputs are transformed by the matrix JF as illustrated in Fig.1, where the matrices J and F are defined as follows:

$$\mathbf{F} = \frac{1}{\sqrt{M}} \begin{bmatrix} 1 & \omega^{-h} & \omega^{-2h} & \cdots & \omega^{-(M-1)h} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & \omega^{-1} & \omega^{-1} & \cdots & \omega^{-(M-1)} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & \omega^{h} & \omega^{h} & \cdots & \omega^{(M-1)h} \end{bmatrix} \quad (5)$$

$$\mathbf{J} = diag\{(j^m \sqrt{M} j_m(\beta r)^{-1})\}, m = -h, \cdots, 0, \cdots, h \quad (6)$$

Where $\omega = e^{j2\pi/M}$, the size of the virtual array is $N_V = 2h+1$, and $J_m(\bullet)$ is the Bessel function of the first kind of order m, and then the array response vector of the resultant virtual array will take approximately the Vandermonde form

$$\mathbf{a}_{\mathbf{v}}(\boldsymbol{\theta}_k) = \mathbf{JFa}(\boldsymbol{\theta}_k) \approx \left[e^{-jh\theta_k}, \cdots, 1, \cdots e^{jh\theta_k}\right]^T \quad (7)$$

The criterion for the choice of h is given in [12], i.e.

$$\max\left\{h \middle| h \leq \frac{M-1}{2} and \frac{J_{h-M}(\beta r)}{J_h(\beta r)} < \varepsilon\right\} \quad (8)$$

Consider the application of a transformation to the array vector $x(t)$.From (4) we can express the result as

$$\tilde{x}(t) = \tilde{\mathbf{A}}(\boldsymbol{\theta})s(t) + \tilde{n}(t) \quad (9)$$

where

$$\tilde{x}(t) = \mathbf{JF}x(t)$$

$$\tilde{\mathbf{A}}(\boldsymbol{\theta}) = \mathbf{JFA}(\boldsymbol{\theta}) = \left[a_v(\theta_1), \cdots, a_v(\theta_K)\right]$$

and

$$\tilde{n}(t) = \mathbf{JF}n(t)$$

Thus, in applying this preprocessing technique, the receive signals for UCAs transform the virtual ULAs which adapt to SBDOA estimation techniques.

In SBDOA estimation, consider the virtual ULAs is divided into two equal-sized subarrays such that for each element in one subarray, there is the corresponding element in the other subarray displace by fixed translational distance. Two virtual subarray signal vectors y_A and y_B are formed such that the phase shift between each signal component in y_A and its corresponding signal component form the same source in y_B is a function of the DOA. The weight vector w_k is obtained by minimizing the mean-square error e_k between the output signals of the beamformer A and the reference r_k .Using the weight vector w_k obtained from beamformer y_A , the subarray signal y_B is weighted and consist of beamformer y_B [13]. The output of beamformer y_B is an optimum estimation of the phase-shifted reference signal, i.e., $\hat{r}_k$, and further, the phase of $\hat{r}_k$ relative to that of the reference signal r_k is a function of target DOA. Finally, the estimation $\hat{\theta}_k$ of the target θ_k is obtained based on the computation of the phase shift between the phase-shifted reference signal $\hat{r}_k$ and the reference r_k.

The vectors of the A and B are given by

$$\mathbf{y}_A = \left[\tilde{x}_{-h}(t), \cdots 1, \cdots \tilde{x}_{h-1}(t)\right]^T \quad (10)$$

$$\mathbf{y}_B = \left[\tilde{x}_{-h+1}(t), \cdots 1, \cdots \tilde{x}_h(t)\right]^T \quad (11)$$

respectively. If we let

$$\mathbf{b}(\theta_k) = \left[e^{-jh\theta_K}, \cdots, 1, \cdots e^{j(h-1)\theta_K}\right]^T \quad (12)$$

subarray signals $\mathbf{y}_A$ and $\mathbf{y}_B$ can be written as

$$\mathbf{y}_{\mathbf{A}}(t) = \sum_{k=1}^{K+P} s_k(t)\mathbf{b}(\theta_k) + n_A(t) \quad (13)$$

$$\mathbf{y}_{\mathbf{B}}(t) = \sum_{k=1}^{K+P} e^{j\theta_k} s_k(t)\mathbf{b}(\theta_k) + n_B(t) \quad (14)$$

where vectors $n_A(t)$ and $n_B(t)$ are the background noise at the subarray respectively.

In [13], Wang show that way to obtain the weight, we can obtain at output of the beamformer B by using beanforming weights obtained from A in minimum mean-square error. The relationship between the optimal weight vector w_k^A and w_k^B can be readily obtained in closed form as

$$\mathbf{w}_k^A = \mathbf{R}_A^{-1}\mathbf{h}_K^A \tag{15}$$

$$\mathbf{w}_k^B = \mathbf{R}_B^{-1}\mathbf{h}_K^B \tag{16}$$

$$\mathbf{w}_k^A = \mathbf{w}_k^B \tag{17}$$

where

$$\mathbf{R}_A = E[y_A(t)y_A(t)^H]$$
$$\mathbf{h}_k^A = E\{y_A(t)[r_k(t)]^*\}$$

$$\mathbf{R}_B = E[y_B(t)y_B(t)^H]$$
$$\mathbf{h}_k^B = E\{e^{-j\theta_k}y_B(t)[r_k(t)]^*\}$$

are the autocorrelation matrix of the input signal y_A and y_B ,the cross-correlation vector between the input signal and the reference signal $r_k(t)$,respectively.

Let $\hat{r}_k(t) = (\mathbf{w}_k^B)^H \mathbf{y}_B(t)$ donate the output signal of beamformer B. Thus, $\hat{r}_k(t)$ is an optimum estimation of the phase-shifted reference signal $e^{j\theta_k}r_k(t)$ in the MMSE sense, it can be written as

$$\hat{r}_k(t) = e^{j\theta_k}r_k(t) + n_k(t) \tag{18}$$

Let $\hat{\mathbf{r}}_k = [\hat{r}_k(1),\cdots,\hat{r}_k(L)]^T, \mathbf{r}_k = [r_k(1),\cdots,r_k(L)]^T$.

Let $\hat{\theta}_k$ donates an estimation of θ_k ,it can be computed using the least square method such that the square error between the two signal vectors $\hat{r}_k(t)$ and $r_k(t)$ is minimized, i.e.,

$$\text{minmize} \quad \hat{\theta}_k \quad \left\| \hat{r}_k - e^{j\hat{\theta}} r_k \right\|_2 \tag{19}$$

The optimization $\hat{\theta}_k$ can be obtained as

$$\hat{\theta}_k = \arg(\hat{r}_k r_k^H) \tag{20}$$

4. SIMULATION RESULATS

In this section, to demonstrate the performance of the proposed method, the resolution, capacity, and accuracy of the UCA-SBDOA techniques will be evaluated through simulations. In Simulation 1 and 2, the resolution and the capacity of the DOA estimation using the UCA-SBDOA techniques will be illustrated and compared with other techniques, such as UCA-MUSIC and UCA-ESPRIT. In Simulation 3, the effects of snapshot length on the estimation accuracy will be investigated.

Simulation 1: Resolution of DOA Estimation

A UCA of radius $r = \lambda$, with $h = 8$ being the maximum phase mode excited, the number of array elements $M = 21$, was employed for the simulations, to deal with a case where the DOAs of three signals are closely distributed. It was further assumed that the DOAs of the target incoherent signal components were at 8° ,10° and 12°. The DOAs of the interference components were at 6° and 14° . The information bit-to-background noise power spectral density ratio of the received signal was set to $20dB$ .Ten thousand simulation runs were performed.

The histograms of resolution of DOA estimation obtained for these three techniques are shown in Fig.2-4. The histogram depicts the number of occurrences estimated DOA as a function of DOA degrees. The actual DOAs of different signals are marked at the top of figure. In Fig.2and Fig.3, the histograms show two peak values, indicating that using the UCA-MUSIC and UCA-ESPRIT techniques can not get the desired results when the

DOAs of signal are very close. Corresponding, in Fig.4, the histogram shows three peak values, indicating that using the proposed UCA-SBDOA, all three DOAs are successfully estimated.

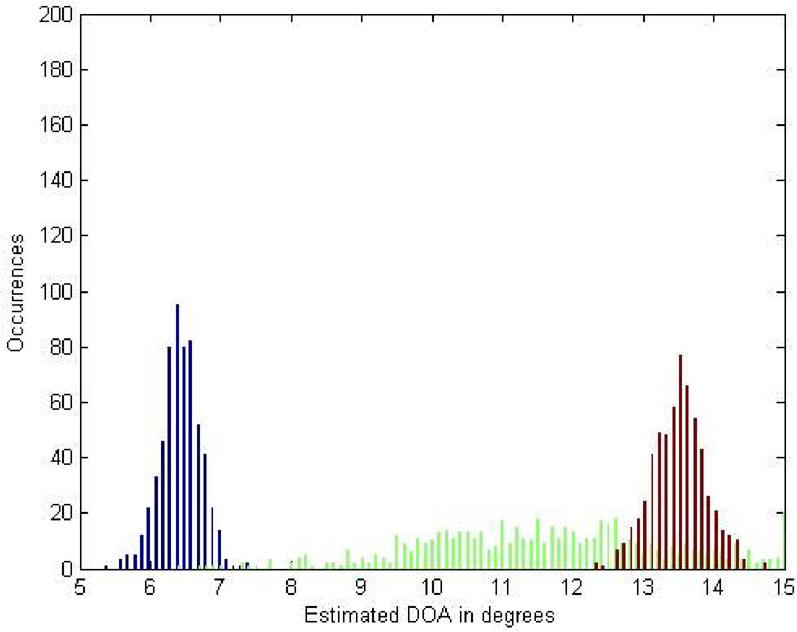

Fig.2 Resolution of UCA-MUSIC estimation

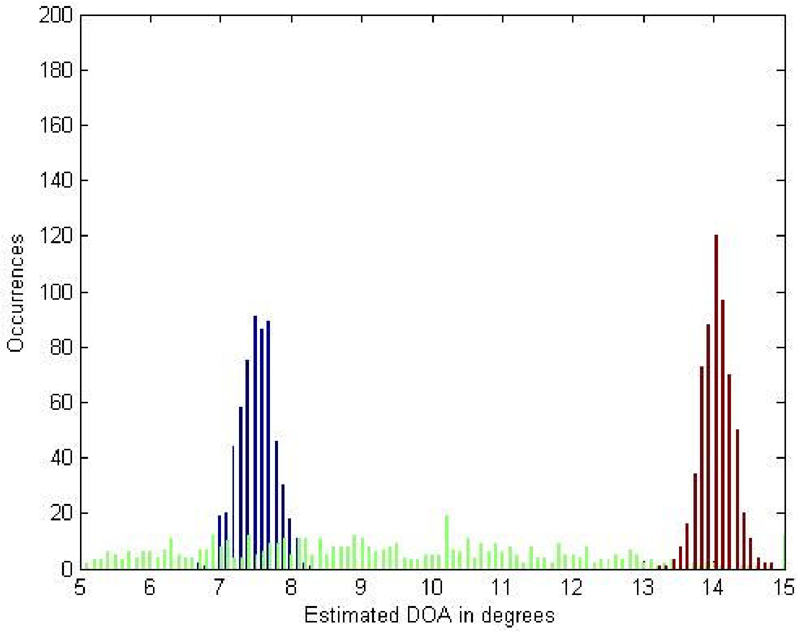

Fig.3 Resolution of UCA-ESPRIT estimation

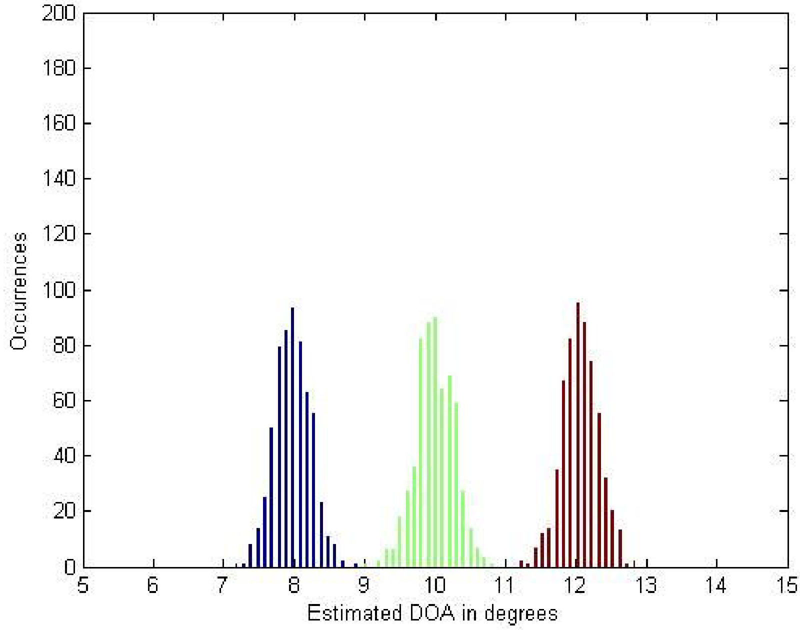

Fig.4 Resolution of UCA-SBDOA estimation

Simulation 2: Capacity and Accuracy of DOA Estimation

This simulation deals with a case where the number of signal is larger than the number of antenna elements. All simulation conditions were kept the same as in Simualtion 1 except the number of signal sources considered. The DOAs of 21 target signal components were set from -150° to $150°$ with interval $15°$. The DOAs of the 16 interference components were set from -110° to 100° with interval $15°$.

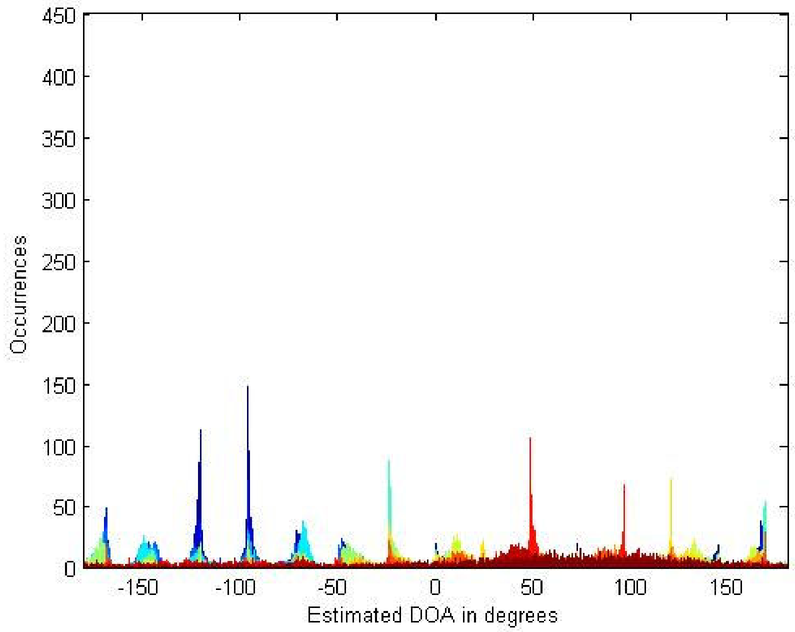

Fig.5 Capacity of the UCA-MUSIC estimation

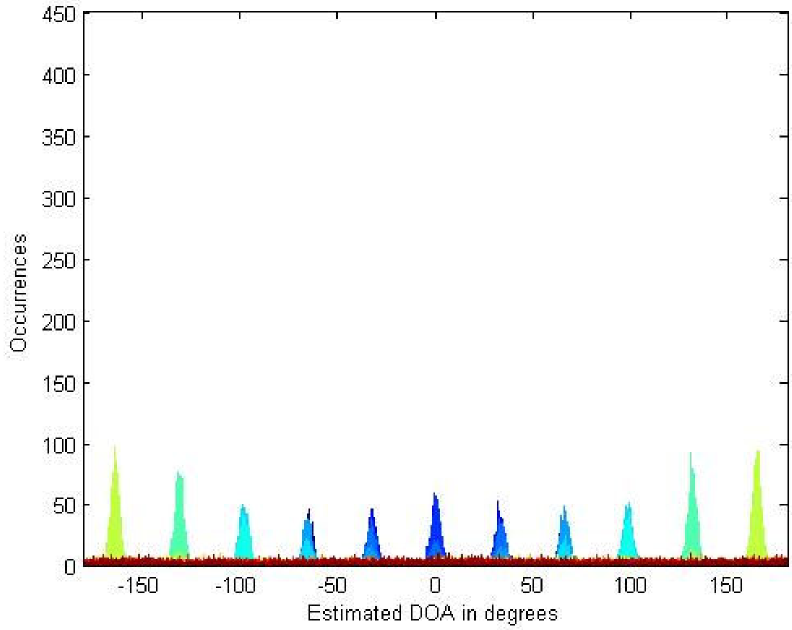

Fig.6 Capacity of the UCA-ESPRIT estimation

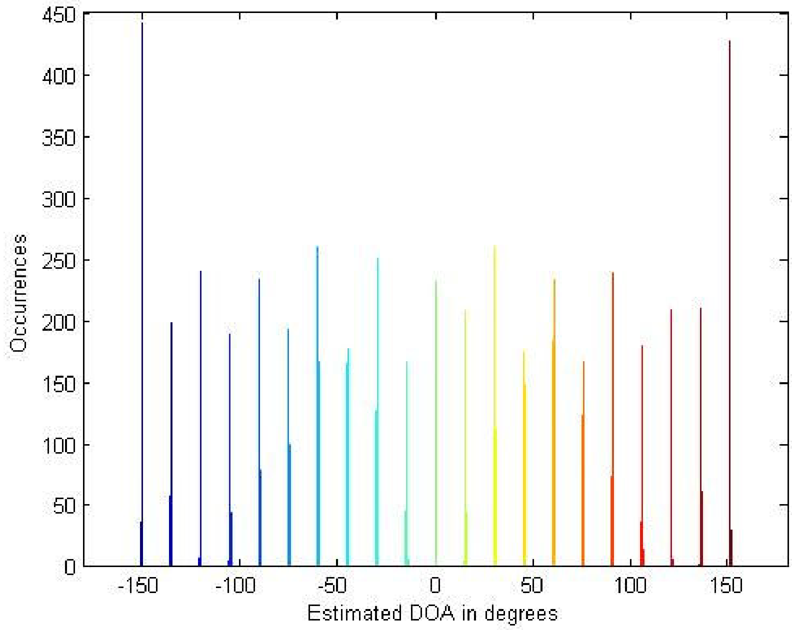

Fig.7 Capacity of the UCA-SBDOA estimation

Histograms of the estimated DOAs obtained are shown in Fig.5-7. In Fig.5 and Fig.6, the histograms demonstrate that these two techniques can not provide acceptable DOA estimation, when the number of antenna elements less than the number of signal. In contrast, in Fig.7 the histogram shows that all 19 target DOAs were successfully estimated when the proposed UCA-SBDOA techniques was used.

Simulation 3: Effect of Snapshot Length on Estimation Accuracy

In this simulation, the snapshot length for subarray beamforming and DOA computation was set to different values 20, 50, 100 and 1000. The DOA of the target signal was fixed at 0° and the DOAs of unknown signals from other sources were randomly distributed from -90° to 90° .The root mean square error (RSME) of the estimated target DOA averaged over ten thousand simulation runs versus the SNR of the target DOAs and the snapshot length are illustrated in Fig.8.

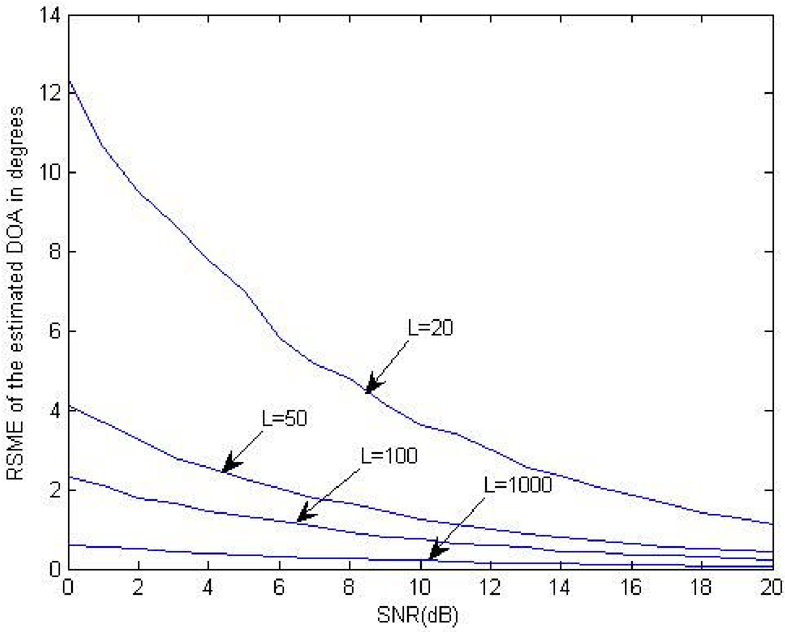

Fig.8.Root mean square error of the estimated DOA for different snapshot length L and the SNR of signal

The proposed UCA-SBDOA technique leads to an RSME of less than 4° in presence of signal, as can be seen, when using a small snapshot length such as 50. This demonstrates the fast DOA tracking capacity of the UCA-SBDOA technique.

5. CONCLUSION

A novel DOA estimation method based on for uniform circular arrays has been proposed. In this technique, beamform manifold is transformed via virtual ULAs, and the target DOA is estimated from the phase shift between the reference signal and its phase-shifted version. In this way, the number of sources detectable can exceed the number of antenna elements and reduce the complexity of computation. Further more, this technique are able to provide the 360°of coverage in azimuth plane. And computer simulation that demonstrates the efficacy and advantage of this technique are presented.

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