A Wideband digital Beamforming Method Based on Stretch Processing

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Abstract: - The calculation of wideband digital beamforming using traditonal methods is so large that it is hard to realize in project. This paper describes a wideband digital beamforming method on the base of stretch processing for linear frequency modulated (LFM) waveforms. This method offers advantages that are moderate data rate for wideband signal processing by reducing the signal bandwidth greatly. In addition, the method not only can get high range resolution of wideband array radar, but also can form good shape pattern with null at interference, as the simulation results show. To the most important, this method compared to traditional methods is easier for engineering implementation greatly.

Key-Words: Digital beamforming (DBF), linear frequency modulated (LFM), Stretch Processing , wideband, Digital array radar

1 Introduction

Digital array radar is new phased array radar, uses digital beamforming technology in transmitting and receiving. It has a very good application prospect, with digital, modular, scalable, and other excellent features. For the need of enhancing anti-interference ability, improving identification, recognition of the target, and solving the problem of radar imaging, radars transmit wideband signals for high-resolution.

Wideband digital array radar (WDAR) is digital array radar using wideband waveforms, combines the advantages of wideband signals and digital array radar. In the recent years, WDAR attracts more and more attention, becomes an important development direction of the radar. And wideband digital beamforming technology is one of the key technologies on wideband digital array radar [1].

The purpose of digital beamforming is to suppress interference and to retain desired signal, so that the signal, interference and noise ratio (SINR) of the array output reaches the maximum. Early digital beamforming is for narrowband signals, however, narrowband digital beamformer can't solve the problem of wideband digital beamforming. Wideband signals can be seen as some narrowband signals being superposed in frequency field, but it doesn't mean that wideband array model equal to the sum of these narrowband array models, and wide band array processing is more complex.

At present, wideband signal algorithms can be divided into two categories [1][2]. One is based on the adaptive array structure of wideband signal processing which is proposed by Frost [3], called space-time processing approach. This method is complex for hardware implementation, and computation is large. The other one is called frequency-domain approach. It transforms wideband signals into multi-way signals in different frequency bands by FFT, and then does digital beamforming in each frequency respectively. This method requires high-speed digital devices for FFT.

The wideband digital beamforming methods mentioned above can be applied to any forms of wideband signals, but are complex in computing and hard for project implementation. In some applications, the required processing rates will exceed the current state of digital devices greatly, especially when the waveform bandwidth is very large.

On the radar, the main purpose of using wideband signals is high-resolution, sometimes do not need deal in the whole course, and the signal form is known and identified usually, such as LFM signal[4]. Linear frequency modulated(LFM) signal has large time-bandwidth product ($TB \gg 1$), can solve the contradiction between the range and distance resolution better, applicable to long-range, ultra long-range surveillance radar.

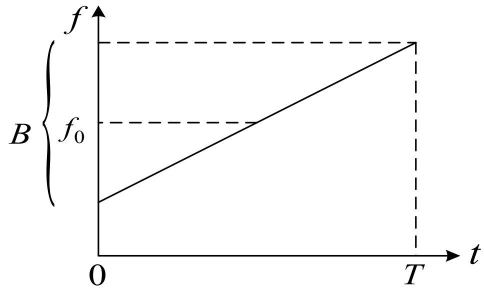


Figure.1 Linear frequency modulated signal

For a LFM signal, the relationship between instantaneous frequency and time can be described as figure 1. Its carrier frequency changes in linear from initial frequency to end frequency, and can be written as $f(t) = 2\pi f_0 + 2\pi\mu t(-T/2 < t < T/2)$. There, f_0 is the pulse center frequency, $\mu = B/T$ is frequency modulation slope, B is bandwidth, T is the pulse duration.

The paper describes a wideband digital beamforming method, which uses the special properties of LFM waveforms. The method does not require doing FFT to the received data, so that it does not need high speed A/D converters and digital processors. When radars transmit LFM signal as wideband pulse, the target signal bandwidth can be reduced greatly through stretch processing and narrow-band filtering. Then beamforming can be finished in a small bandwidth, the data sampling rate reduced greatly, in favor of the project to achieve.

In this paper, section 2 introduces the basic theory of stretch processing. Section 3 improves stretch processing to carry out beamforming flexibility, and researches interference nulling technology based on stretch processing at subarray level in section 4 followed. A summary is given in Section 5.

2 Stretch Processing

The basic idea of stretch processing is that two LFM signals having the same frequency modulation slope are mixed together and filtered, the output will be a narrowband sinusoidal signal. The frequency of the sinusoidal signal is proportional to the time-offset between the two LFM signals. Therefore, a high range resolution can be achieved by analyzing the frequency component. By the way, the time-offset is determined by the difference between actual distance and estimated distance of the target. If the target is multi-point, the output of mixer is a group of narrow-band sinusoidal signals.

Figure.2 shows stretch processing block diagram for a single channel. After transmitting a wideband LFM pulse $x(t)$, radar will receive a delayed version of the transmitted pulse $x(t-t_0)$ as the target echo, and t_0 is unknown two-way propagation delay determined by the target's range . At the same time, the transmitted signal was delayed to the estimated target range as the deramping oscillator $x(t-t_0^{'})$. The echo is demodulated in the mixer using the local oscillation signal. The following lowpass filter converts the target range difference into frequency difference, and the bandwidth of the filter limits the range region. Then, the information of the target can be obtained by taking FFT.

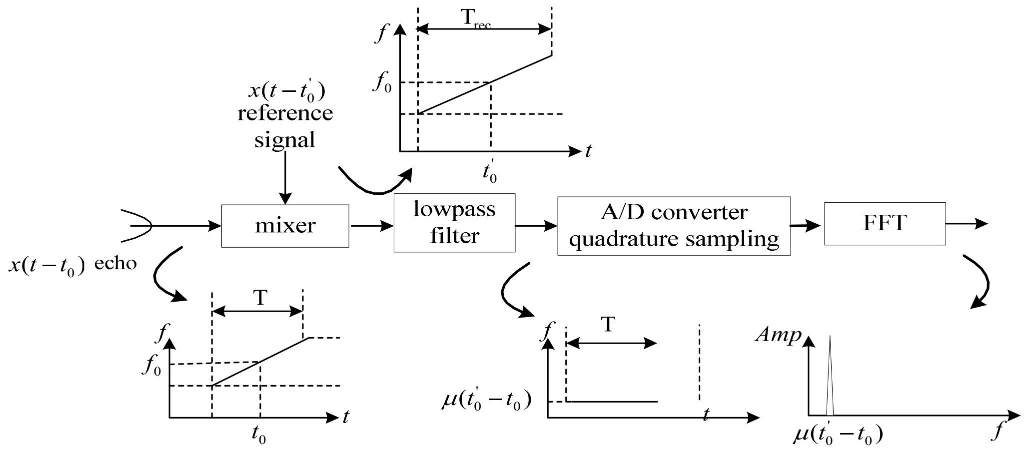


Figure.2 stretch processing block diagram

Single-channel stretch processor is described above. For a array radar, a channel or a subarray can be seen as a channel, and each receiver is a stretch processor for multi-channel stretch processing, shown in figure.3.

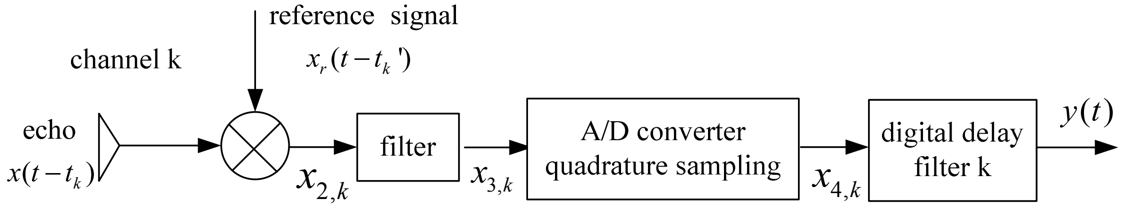


Figure.3 stretch processor for the kth channel

If every channel transmits the same LFM signal which is expressed as

$$\begin{aligned} x(t) &= \cos(2\pi(f_0 t + 0.5\mu t^2)) \\ -T/2 &< t < T/2 \end{aligned} \tag{1}$$

The echo of the kth channel $x(t-t_k)$ is

$$x_{1,k} = x\left(t\text{-}t_k\right) = a\times \cos\left(2\pi\left(f_0\left(t\text{-}t_k\right)+0.5\mu\left(t\text{-}t_k\right)^2\right)\right)rect\left(\frac{t\text{-}t_k}{T}\right) \tag{2}$$

There, $t_k = \tau_r + \tau_k$ is the delay of the echo. $\tau_r = 2R/c$ is the two-way propagation delay time, and R is the fact range of the target. τ_k is determined by channel space d and direction of the target θ, can be obtained by $\tau_k = (k-1)d\sin\theta$ previously. Besides, a is the attenuation factor dependent upon transmission distance, the target RCS, antenna gain, etc. Assume $a=1$.

The reference signal in the kth channel is

$$x_{0,k} = x\left(t-t_k^{'}\right) = \cos\left(2\pi\left(f_0\left(t-t_k^{'}\right)+0.5\mu\left(t-t_k^{'}\right)^2\right)\right)\cdot rect\left(\frac{t-t_k^{'}}{T_r}\right) \tag{3}$$

$t_k^{'}$ is the delay of the reference signal, and $T_r > T$ generally.

The output of the lowpass filter is the signal has been stretch processed, and can be described as follow after digitalization.

$$x_{4,k}(t)=\frac{1}{2}e^{j2\pi\mu\left(t_k^{'}-t_k\right)t}e^{j2\pi\left[f_0\left(t_k^{'}-t_k\right)+0.5\mu\left(t_k^2-t_k^{'2}\right)\right]}\cdot rect\left(\frac{t-t_k}{T}\right)\cdot rect\left(\frac{t-t_k^{'}}{T_r}\right) \tag{4}$$

At this point, the frequency and phase of the kth channel output respectively are $f_k=\mu\left(t_k^{'}-t_k\right)$ and $\phi_k=2\pi[f_0\left(t_k^{'}-t_k\right)+0.5\mu\left(t_k^2-t_k^{'2}\right)]$, and both of them are related to k. The frequency and phase of the signal in different channel are inconsistent; this is what we do not want.

In order to avoid the above-mentioned inconsistencies of frequency and phase, two groups delay are introduced. One is delay for the reference signal, the other one is digital delay filter after digitalization.

Firstly, we rewrite the kth channel delay of the reference as $t_k{'}=\tau_r{'}+\tau_k$, $\tau_r{'}$ is inexact target range, can be measured previously. Then, $-\tau_k$ can be delayed using the digital FIR filter whose impulse response is $h_k\left(t\right)=\delta\left(t+\tau_k\right)$. The output of the kth channel after the two groups of delay is

$$y(t)=\frac{1}{2}e^{j2\pi\mu\left(\tau_r^{'}-\tau_r\right)t}e^{j2\pi\left[f_0\left(\tau_r^{'}-\tau_r\right)+0.5\mu\left(\tau_r^2-\tau_r^{'2}\right)\right]}\cdot rect\left(\frac{t-\tau_r}{T}\right)\cdot rect\left(\frac{t-\tau_r^{'}}{T_r}\right) \tag{5}$$

Now, frequency and phase are the same between different channels, and range information of the target can be obtained by

$$R=\frac{c}{2}\left(\tau_r^{'}-\frac{f}{\mu}\right) \tag{6}$$

where f denotes to the frequency of the output signal.

Here we illustrate the high range resolution of using stretch processing for wideband signals. In this example, a array of 16 channels is considered. The channel space d is half of the wavelength corresponding with the highest frequency of the transmitted signal. The echo is a 200 MHz LFM pulse of L-band, with the pulse duration 20 μs, and SNR is 0 dB. If the direction of the single-point target is 20^0 ,and the range is 1101 km. The estimated range by a narrowband pulse is 1100 km, and set range region 5km. Beside, the noise of the receiver is Gaussian white noise with zero mean and variance of 1. A peak is formed at the 1101 km in the range region in figure.4, it means that the range estimated by this array radar is 1101 km, with the error of 0 km. Compared with the narrowband radar; this radar can extract the distance information to the target accurately. Because the bandwidth of this radar is 200 MHz, its range resolution can be calculated by $\Delta R=c/2B=0.75m$, then the largest range estimation error is $0.75m$, it is so little.

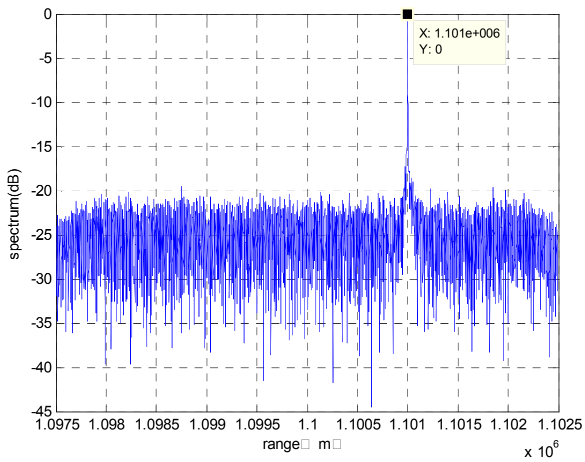


Figure.4 estimated range of single-point target

Assume the range resolution of the narrowband radar used for estimating range is 150m, its time resolution is $1/B=1\mu s$. For the wideband radar whose time-bandwidth product is 4000 in this example, the frequency of the signal after stretch processing is 10 MHz, the sampling rate is required just for 20 MHz, reduced greatly.

3 Proposed Flexible beamforming method

From the analysis above, we can see that the signals after stretch processing can not taking flexible beamforming, because of the array delay time for reference signal. It is the shortcomings of stretch processing. In the figure.3, we reconstruct reference signal as $x(t-\tau_r')$ without τ_k, and the echo is still shown as the formula of (2). After mixing, lowpass filtering and digitalization, the output can be written as

$$x_{4,k}(t)=\frac{1}{2}e^{j2\pi\mu\left(\tau_r'-\tau_r\right)t}e^{j2\pi f_0\left(\tau_r'-\tau_r\right)}e^{j\pi\mu\left(\tau_r^2-\tau_r'^2\right)} \cdot e^{-j2\pi\mu\tau_k t}e^{-j2\pi f_0\tau_k}e^{j\pi\mu\tau_k^2}e^{j2\pi\mu\tau_r\tau_k} \cdot rect\left(\frac{t-\tau_r-\tau_k}{T}\right)\cdot rect\left(\frac{t-\tau_r'}{T_r}\right) \tag{7}$$

The last four terms of the formula are related to τ_k ,that is, the direction of the target.

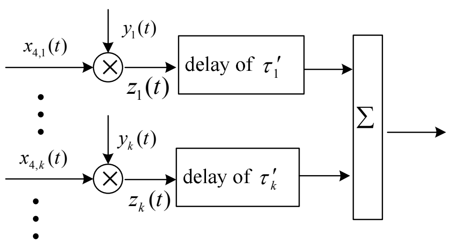


Figure.5 beamforming module

As shown in figure.5, While taking beam-forming, if the scanning angular is ϕ , let

$\tau_k' = (k-1)d\sin\phi / c$

$y_k(t)=e^{j2\pi\mu\tau_k' t}e^{j2\pi f_0\tau_k'}e^{-j\pi\mu\tau_k'^2}e^{-j2\pi\mu\tau'\tau_k'}$, we can describe $z_k(t)$ as

$$\begin{aligned} z_k(t) &= x_{4,k}(t)y_k(t) \\ &= \frac{1}{2}e^{j2\pi\mu\left(\tau_r'-\tau_r\right)t}e^{j[2\pi f_0\left(\tau_r'-\tau_r\right)-\pi\mu\left(\tau_r^2-\tau_r'^2\right)]} \\ &\cdot e^{j2\pi\mu(\tau_k'-\tau_k)t}e^{j2\pi f_0(\tau_k'-\tau_k)}e^{j\pi\mu\left(\tau_k^2-\tau_k'^2\right)} \\ &\cdot e^{j2\pi\mu\left(\tau_r\tau_k-\tau_r'\tau_k'\right)}rect\left(\frac{t-\tau_r-\tau_k}{T}\right)rect\left(\frac{t-\tau_r'}{T_r}\right) \end{aligned} \tag{8}$$

$y_k(t)$ here can be understood as scanning vector. It is different from that of Single-frequency signal because of the difference of the signal form.

Now, the realization of the array delay requires delaying $z_k(t)$ by τ_k', that is

$$\begin{aligned} z_k(t+\tau_k') &= \frac{1}{2}e^{j2\pi\mu\left(\tau_r'-\tau_r\right)t}e^{j2\pi f_0\left(\tau_r'-\tau_r\right)}e^{j\pi\mu\left(\tau_r^2-\tau_r'^2\right)} \\ &\cdot e^{j2\pi\mu(\tau_k'-\tau_k)t}e^{j2\pi f_0(\tau_k'-\tau_k)}e^{j\pi\mu\left(\tau_k^2-\tau_k'^2\right)} \\ &\cdot e^{j2\pi\mu\left(\tau_r\tau_k-\tau_r'\tau_k'\right)}e^{j2\pi\mu\left(\tau_r'-\tau_r\right)\tau_k'}e^{j2\pi\mu(\tau_k'-\tau_k)\tau_k'} \\ &\cdot rect\left(\frac{t-\tau_r-\tau_k+\tau_k'}{T}\right)rect\left(\frac{t-\tau_r'+\tau_k'}{T_r}\right) \end{aligned} \tag{9}$$

If $\phi=\theta$, that is $\tau_k'=\tau_k$

$$\begin{aligned} z_k(t+\tau_k') &= \frac{1}{2}e^{j2\pi\mu\left(\tau_r'-\tau_r\right)t}e^{j2\pi f_0\left(\tau_r'-\tau_r\right)}e^{j\pi\mu\left(\tau_r^2-\tau_r'^2\right)} \\ &\cdot rect\left(\frac{t-\tau_r}{T}\right)\cdot rect\left(\frac{t-\tau_r'+\tau_k'}{T_r}\right) \end{aligned} \tag{10}$$

$z_k(t+\tau_k')$ is unrelated to k, so that the receiving signal of every channel can sum up in the same phase, and get the top power. In addition, the frequency of (10) is $f_k=\mu\left(\tau_r'-\tau_r\right)$, are the same between any two different channels, so the range can be obtained by analyzing the frequency.

Using the proposed method to the example, the result is shown in figure 6. Figure 6 has a peak at 1101 km. We can get the conclusion that the range of the target is 1101 km.

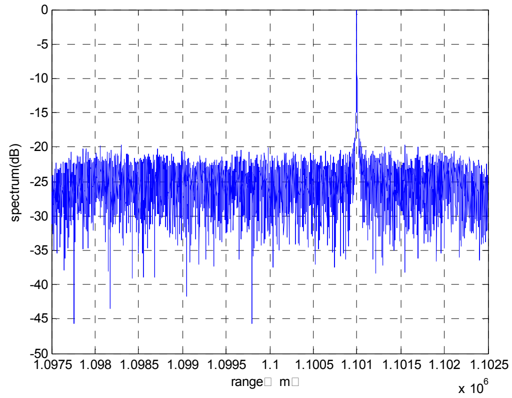


Figure.6 range estimation

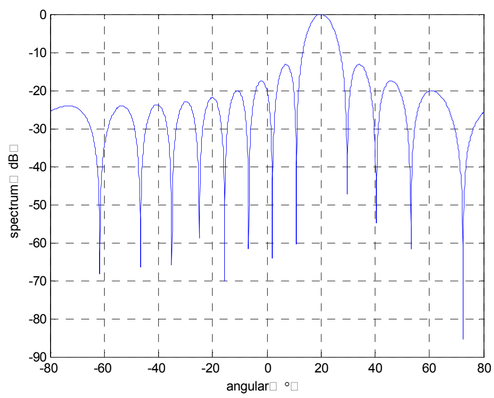
If the first channel is seen as the reference channel, and its corresponding target range is seen as the reference range, the delay in the formula derived in this section has nothing to do with the target range. That is,

$$z_k(t+\tau_k') = \frac{1}{2} e^{j2\pi\mu(\tau_k'-\tau_k)t} e^{j2\pi f_0(\tau_k'-\tau_k)} e^{j\pi\mu\left(\tau_k^2-\tau_k'^2\right)} \cdot e^{j2\pi\mu(\tau_k'-\tau_k)\tau_k'} rect\left(\frac{t-\tau_k+\tau_k'}{T}\right) \cdot rect\left(\frac{t+\tau_k'}{T_r}\right) \tag{11}$$

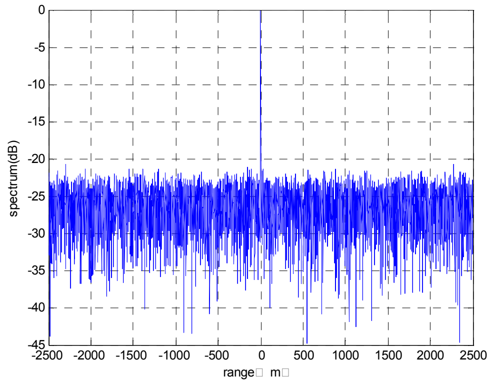
If $\phi = \theta$, that is $\tau_k' = \tau_k$

$$z_k(t+\tau_k') = \frac{1}{2} rect\left(\frac{t}{T}\right) \cdot rect\left(\frac{t+\tau_k'}{T_r}\right) \tag{12}$$

The frequency of each channel is zero; the range obtained by analyzing frequency is the reference range corresponding to the reference channel.



(a) Pattern



(b) Range information

Figure.7 Wideband digital beamforming

The pattern in Figure.7(a) has a good shape and forms a mainlobe at 20^0 which is the direction of the required signal, so that this radar can receive LFM signals from 20^0 better. It is just beamforming, the range information is 0.

In the discussion followed, the first channel is always seen as the reference channel, and its corresponding target range is seen as the reference range.

4 Wideband beamforming method based on stretch processing

When there is wideband interference, stretch processing and filter can reduce interference power to some extent. Even if interference power is not too high, one-dimensional range image can be obtained. However, it doesn't work when interference power is very strong, and the echo is still submerged in interference. In order to obtain greater signal interference and noise rate of output, and improve the detection and identification of the weak echo, null is needed at the interference.

Speaking on theory, beamforming method based on stretch processing can form mainlobe at the required direction, and the weight in the single-frequency can form null at the interference, then the combination of the two can nulling the interference.

Assumed that the direction of target is θ, and one interference comes from α, the following nulling method is after stretch processing and narrowband filtering. Consider a M-channel uniform linear array(ULA), the delay of the signal of the kth channel relative to that of the reference channel is

$$\tau_k = (k-1)d\sin\theta / c \qquad (13)$$

where c denotes the velocity of light.

To make the phase of every channel signals after stretch processing and narrowband filtering consistent, the weight of kth channel for phase compensation can be described as

$$w_{c,k}(t) = e^{j2\pi\mu\tau_k t} e^{j2\pi(f_0\tau_k - 0.5\mu\tau_k^2)} \qquad (14)$$

The delay of the interference which comes from α of the kth channel relative to that of the reference channel is

$$\tau_{\alpha,k} = (k-1)d\sin\alpha / c \qquad (15)$$

When the transmitting signal's frequency is f_0, The single-frequency weight w_n can form mainlobe at the target direction, and null at the interference direction. It can be generated by conventional narrowband method, such as MVDR. w_n satisfies $\sum_k w_k a_k^* = 1$, and $a_k^* = e^{-j2\pi f_0 \tau_k}$.

To enhance the algorithm robustness, wide notch formation technology is introduced, which is implemented by a simple modification of the covariance matrix.

$$\tilde{\mathbf{R}} = \mathbf{A} \circ \mathbf{R} \qquad (16)$$

In the formula (16), the modification is achieved by a matrix $\mathbf{A}$, the covariance matrix modified can be calculated as the Hadamard product of $\mathbf{A}$ and $\mathbf{R}$. The element of $\mathbf{A}$ is a sinc function[5][10], with the notch width factor. Here we write the new single-frequency weight as w_k.

If the weight for beamforming is

$$w_{o,k}(t) = w_{c,k}(t) w_k a_k^* \qquad (17)$$

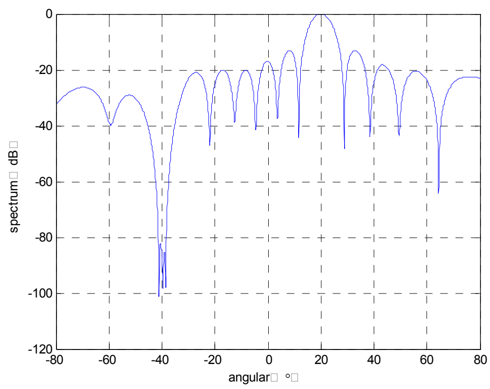
That mainlobe can be formed at θ, but can not form nullat α. Thus, compensation factor is needed, and can be described as

$$q_k(t) = e^{j2\pi(\tau_{\alpha,k} - \tau_k)t} e^{j\pi\mu(\tau_k^2 - \tau_{\alpha,k}^2)} \qquad (18)$$

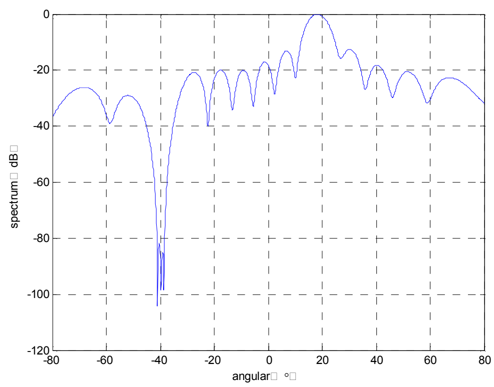
As a result, the weight after compensation is

$$W_k(t) = q_k(t) w_{o,k}(t) = q_k(t) w_{c,k}(t) w_k a_k^* \qquad (19)$$

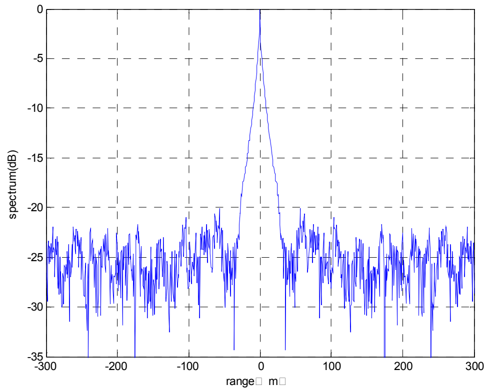
If interference is a LFM signal in the direction of -40^0, and has the same center frequency and bandwidth, taking wideband beamforming in (19). Figure.8(b) shows that notch is formed in the pattern by using the weight in (19), the same as that using single-frequency weight w_k in figure.8(a), so that interference can be suppressed well. The range information illustrates the reference range corresponding to the reference channel is 0.



(a) Pattern using narrowband weight



(b) Pattern using this section method



(c) Range information

Figure.8 Wideband digital beamforming with null

If the array is divided into L adjacent uniform subarrays, N is the number of elements in each subarray, that is, $M = L \times N$. Figure 9 is the block diagram for beamforming based on stretch processing at subarray level. Phase compensation is carried out to the element in the subarray, and weight for nulling is for the subarray.

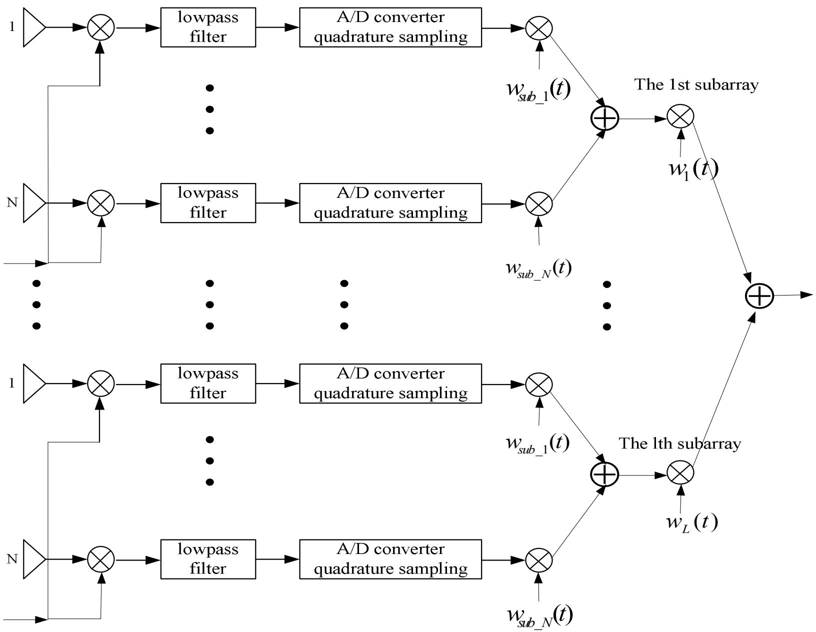


Figure.9 Wideband digital beamforming based on stretch processing at subarray level

Assumed that the first element in every subarray is the reference element, and the first subarray is the reference subarray for the array. The delay of the signal of the nth element relative to that of the reference element is

$$\tau_n = (n-1)d\sin\theta / c \quad n = 1,\cdots,N \tag{20}$$

And the delay of the signal of the lth subarray relative to that of the reference subarray is

$$\begin{aligned} &\tau_l = (l-1)Nd\sin\theta / c \\ &l = 1,\cdots,L \end{aligned} \tag{21}$$

Then, the receiving signal of the nth element in the lth subarray is

$$\begin{aligned} x_{l,n}(t) &= x(t-(\tau_n+\tau_l)) \\ &= \mathrm{e}^{j2\pi[f_0(t-\tau_n-\tau_l)+0.5\mu(t-\tau_n-\tau_l)^2]} \\ &= e^{j2\pi(f_0 t+0.5\mu t^2)} e^{j2\pi(-\mu\tau_n t-f_0\tau_n+0.5\mu\tau_n^2)} \\ &\quad \cdot e^{j2\pi(-\mu\tau_l t-f_0\tau_l+0.5\mu\tau_l^2)} e^{j2\pi\mu\tau_n\tau_l} \end{aligned} \tag{22}$$

Taking stretch processing to the signal of (22), obtains

$$x_n(t) = e^{j2\pi(-\mu\tau_n t-f_0\tau_n+0.5\mu\tau_n^2)} e^{j2\pi(-\mu\tau_l t-f_0\tau_l+0.5\mu\tau_l^2)} e^{j2\pi\mu\tau_n\tau_l} \tag{23}$$

If the cross-term $e^{j2\pi\mu\tau_n\tau_l}$ is neglected, the phase compensation weight of the nth in each subarray $w_{sub\_n}(t)$ can be described as

$$w_{sub\_n}(t) = w_{c,n}(t) = e^{j2\pi\mu\tau_n t} e^{j2\pi(f_0\tau_n - 0.5\mu\tau_n^2)} \quad (24)$$

As shown in section 3, the scanning vector of nth element in every subarray can be written as

$y_n(t,\varphi) = e^{-j2\pi\mu\tau_n(\varphi)t} e^{-j2\pi[f_0\tau_n(\varphi) - 0.5\mu\tau_n(\varphi)^2]}$, and $\tau_n(\varphi) = (n-1)d\sin\varphi / c$,denotes to the delay of the nth element related to that of the reference element when the scanning angular is ϕ .

Because the array is divided uniformly and adjacently, subarrays have the same output

$$z\_sub(t,\phi) = \sum_{n=1}^{N} w_{c,n}(t) y_n(t,\phi) \quad (25)$$

Interference nulling realized by digital weight at the subarray level. For the interference coming from α , the delay of the lth subarray related to that of the reference subarray is

$$\tau_{l,\alpha} = lNd\sin\alpha / c \quad (26)$$

Then the compensation factor in (18) can be rewritten as

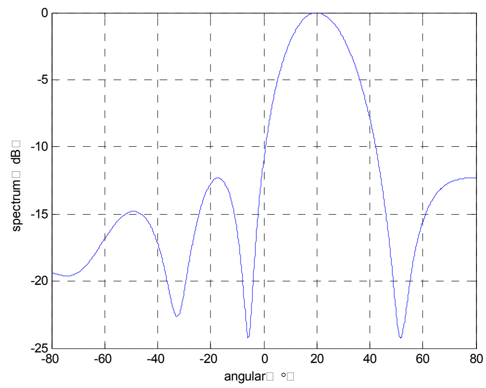
$$q_l(t) = e^{j2\pi(\tau_{l,\alpha} - \tau_l)t} e^{j\pi\mu(\tau_l^2 - \tau_{l,\alpha}^2)} \quad (27)$$

The weight at subarray can be described as

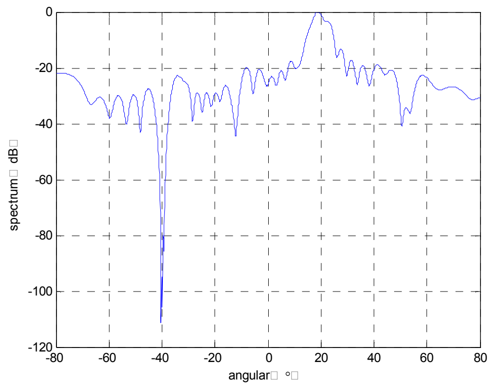
$$w_l(t) = q_l(t) w_l(t) w_l a_l^* \quad (28)$$

Where, $a_l^* = e^{-j2\pi f_0 \tau_l}$, $\tau_l = (l-1)Nd\sin\theta / c$.

Divided 40 channel ULA array into 8 subarrays uniformly, the pattern of subarrays are the same, shown in figure 10(a).It is statistic pattern because the weight in subarray is only for phase compensation. Figure 10(b) is the array pattern with nulling, notch is formed in the direction of -40^0, however, the mainlobe of the pattern has been offset, which is due to the unwanted null.



(a) Pattern of subarrays



(b) Pattern with null

Figure.10 wideband digital beamforming at subarray level

5 Conclusion

In order to reduce the computation of wideband signal digital processing, a wideband digital beamforming method based on stretch processing is present in this paper. The bandwidth of target signal after stretch processing and narrowband filtering, the data sample ratio is reduced greatly, so that beamforming can be carried out in a small bandwidth and the method is easy for engineering complementation. Simulation results demonstrated that the method proposed on the base of stretch processing can realize wideband interference nulling, achieve beamforming performance.

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