INHARMONIC DISPERSION TUNABLE COMB FILTER DESIGN USING MODIFIED IIR BAND PASS TRANSFER FUNCTION

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Abstract :- An excitation/filter system of Inharmonic sound synthesis signal is presented with an application to piano, a stiff string instrument. Specific features of the piano string important in wave propagation, dispersion due to stiffness, frequency dependent losses and presence of phantom partials are included in the proposed model. The modified narrow bandpass filter is used as a basic building block in modeling the vibrating structure. The parallel bank of narrow band pass filters is used to model the dispersion. The center frequencies of narrow bandpass filters can be tuned to follow the same relation as the partial frequencies of piano tone. Novel loss filter is designed to implement frequency dependent losses. The resulting model is called as Inharmonic Dispersion Tunable Comb filter.

Key-words:- Bandpass, Bandstop, Dispersion, Inharmonicity, Synthesis

1 INTRODUCTION

Digital representation of a sound is just a list of numbers; any list of numbers can theoretically be considered a digital representation of sound. In order for the list of numbers to be audible as sound, the numerical values must fluctuate up and down at an audio rate. Any arithmetic operation performed with those numbers becomes a form of audio processing. The effects that such operation can have on the shape of the signal are so many and varied that they comprise a subject called Digital Signal Processing. Various techniques of Digital Signal processing can be used to implement synthesis of sound signals. Such designed sounds can be used in:

*Complex Systems

*Multi-sensorial systems

*Animations and film

*Virtual reality systems

Basic wave shape such as sine, square and triangle play an important role in digital signal processing applications. If we play a sine wave it is a pure tone. A real music is much more complex. A tone on a real instrument is not a single frequency but the sum of many frequencies with different amplitudes.

A complex tone can be harmonic or inharmonic. A complex signal is called the harmonic signal if all the frequencies are integer multiples of fundamental frequency that is the lowest frequency of the signal. Various harmonics of this signal can be calculated using following equation:

$$f_n = n f_0 \tag{1}$$

Where n = number of harmonic

In case of inharmonic signal the frequencies of harmonics are not exact integral multiples of the fundamental frequency. The stiffness of the strings results in inharmonic sound. For e.g. the sound waves generated from the struck or plucked type of musical instruments. The sound of piano is inharmonic in nature [4]. Due to stiffness of the string the waves are dispersed .The partial frequencies of transverse vibration of piano string [5] are given by

$$f_n = n f_0 \sqrt{1 + Bn^2} \tag{2}$$

where B is known as inharmonicity coefficient given

by

$$B = (\pi^3 Q d^4 / 64 l^2 T)$$
(3)

Where n = partial number

Q = Young's modulus,

d = diameter of the string

l= length of the string

T = tension of the string

 $f_0 =$ fundamental frequency

Typical values of B for piano strings lie roughly between 0.00005 for low bass tones and 0.015 for the high treble tones.

The sound generated by the piano strings is very complicated. The sound builds up rapidly and decays slowly. Moreover, partials decay at different rates.

2 LITERATURE SURVEY

Jean Laroche and Jean Louis Meiller 1994 [8], have discussed source filter models of percussive instrument. The computationally efficient digital waveguide piano model known as commuted piano was proposed by Smith and Van Duyne, 1995 [14]. The hammer string force interactions were modeled as discrete events one or a few successive impulse responses of low-order digital filters. Balázs Bank and Vesa Välimäki 2003 [3], presented a robust loss filter design method for digital waveguide string models, which can be used with high filter orders. Julien Julien Bensa and Laurent Daudet,2004 [10] proposed a method of modeling phantom partials in piano tones. Lehtonen, H-M., Rauhala J and Valimki V.2005[9] presented a loss filter design method for

The spectrum varies over time and differs from key to key; at the bass end over 50 partials can be extracted while at the treble end the corresponding number is only about 3 or 4. Other important phenomenon in the piano sound is the beating, which results from unison groups of strings. If multiple strings are tuned exactly equal no two stage decay occurs. If these are tuned to pitches more than 1 or 2 Hz apart they sound simply out of tune. In order to obtain two stage decay rate in the piano model output, one must set the tuning of the strings to some small fraction of a Hz apart (Kirk, 1959)[11]. Jaffe and Smith, 1983 used all pass interpolation method to obtain fractional tuning. Piano spectra for lowpitched notes also exhibit a number of partials known as phantom partials. These are caused by nonlinear coupling between the longitudinal and transverse vibrations of the strings and are perceptually important. [2].

This paper is primarily concerned with synthesis of inharmonic tones of piano. In this paper, a musical instrument will be said to be percussion instrument whenever the sound it produces results from the free vibration of a structure or medium that has been set into motion by a short excitation. Inharmonic dispersion tunable comb filter is presented to model stiff string of piano. Source signal for this model is assumed to be an impulse. Our contribution focuses mainly on dispersion due to stiffness, frequency dependent losses and presence of phantom partials.

digital waveguide piano synthesis. The structure of Loss filter consists of FIR filters Heidi-Maria Lehtonen, Jukka Rauhala, Vesa Välimäki 2005 [9], proposed new filter structure and a design method for the loss filter that is used in digital waveguide synthesis. The filter structure is based on a cascade of sparse FIR filters. Jukka Rauhala, Vesa Välimäki 2006 [15], proposed the concept of tunable dispersion filter design, which provides a closedform formula to design a dispersion filter which is applied to a cascade of first-order allpass filters. Balazs Bank 2006, presented a novel model for two stage decay. The resonator bank is implemented using multi-rate approach.

The following sections present the design and implementation of inharmonic dispersion tunable comb filter, a stiff string model with application to synthesis of piano tones. The model can be implemented using software to create a virtual instrument.

3 PROPOSED DESIGN

3.1 Theory

Band pass filter can be implemented using FIR as well as IIR filters. Several simple IIR filters can be designed with first order and second order function. Systems like Digital Oscillator or Digital Resonator can be used to generate a sinusoidal wave which is a basic building block of any sound wave. To achieve the band pass filter with extremely narrow band you need to have very high order of FIR filter. Using IIR filter the band pass filter can be designed with comparatively very low order but there are limitations on the narrow width of the pass band that can be achieved.

It is necessary to find the method to get the narrower pass band generating a sine wave as an impulse response which can be used as basic building block in synthesis process of musical sound. Transfer function of bandpass filter is modified in the proposed method. Transfer function of bandstop filter is given by equation (4) [13].

$$H_{BS}(z) = \frac{(1+\alpha)(1-2\beta z^{-1}+z^{-2})}{2(1-\beta(1+\alpha)z^{-1}+\alpha z^{-2})}$$
(4)

$$\omega_0 = \cos^{-1} \beta \tag{5}$$

The difference between the frequencies $\omega c1$, $\omega c2$ is

given by

$$\Delta \omega = \cos^{-1} \left(\frac{2\alpha}{\left(1 + \alpha \right)^2} \right) \tag{6}$$

Conventionally bandpass filter response is equivalent to the inverse response of bandstop filter. Using the fact that inverse system corresponding to a transfer function H (z) is denoted by H-1(z) and is defined as

$$H^{-1}(z) = \frac{1}{H(z)}$$
(7)

The modified transfer function of the second order

Bandpass filter can be written as

$$H_{\text{mod ified.BP}} = \frac{2(1 - \beta(1 + \alpha)z^{-1} + \alpha z^{-2})}{(1 + \alpha)(1 - 2\beta z^{-1} + z^{-2})}$$
(8)



Figure1 : Structure of Narrow bandpass filter

3.2 Design of Inharmonic Dispersion Tunable

Comb Filter and Loss filter

The modified narrow bandpass filter is used in modeling vibrating string of piano. The model consists of Parallel bank of these filters. The center frequencies of Inharmonic Dispersion Tunable Comb Filter are tuned to follow the same relation as the partial frequencies of equation 2.

The partials of a piano tone decay at different rates

and the higher partials die down at a faster rate.

These Frequency dependent losses are further

incorporated using modified loss filter design.

The single pole loss filter is designed to generate a decaying envelope. The rate of decay depends on the relative partial number. The transfer function of novel single pole loss filter which is designed to have impulse response of decaying amplitude is

$$H_{loss} = \frac{1}{1 - e^{-a} z^{-1}} \tag{9}$$

where

 $a \leq 1$

Impulse response of every narrow band pass filter is multiplied with an impulse response of the modified single pole filter which is given in equation 9. The value of 'a' is calibrated as

$$a = 0.5^{(p-nd)}$$
(10)

p= Total number of partials

n=number of partial of which envelope is to be obtained

d= parameter which decides the decay rate.

The synthesized tone using above method is further processed to incorporate the effect of phantom partials. Phantom partials are obtained by multiplying the synthesized signal S(n) with one of its components Sc(n).

The piano tone with phantom partials is synthesized

using equation (11)

$$S(n) = S(n) + gS(n)S_c(n)$$
⁽¹¹⁾

where g=gain

Complete block diagram of the model of stiff string is as shown in figure (2).



Figure2: Block Diagram of the model of Single Stiff

String

The designed Inharmonic Dispersion Tunable Comb filter with loss filter is driven by delta function which is used as a common excitation signal for generation of any piano tone.

Impulse response and power spectrum of the filter are plotted to verify the inharmonic nature of the signal. Audibility test is conducted to have the opinions of the subjects having knowledge of the timber and quality of the sounds of musical instruments.

4 ALGORITHM

To design the complete model of strings of Piano an algorithm is implemented in MATLAB, which includes following steps:

- 1. Input the fundamental frequency or the key number of the piano tone to be generated.
- 2. Calculate the number of audible partials for the selected tone
- 3. Set the value of inharmonicity coefficient
- 4. Calculate Coefficients of Inharmonic dispersion tunable comb filter using following iterative loop to generate parallel filter bank structure:
 - a) Current partial number is set to n=1.
 - b) Center frequency of nth narrow band pass filter is tuned to the frequency of partial calculated using equation
 - c) Implement Narrow band pass filter system's transfer function.
 - d) Compute the coefficients of this filter
 - e) Multiply the impulse response of narrow band pass filter with an impulse response of the modified single pole filter.
 - f) Set n = n+1; If n less than or equal p, Go to step 4
- Use delta function as an excitation signal to drive the model to generate piano tone.

5 RESULTS

Time domain representation and spectrum analysis of the synthesized tones is shown in figures 3to 7. Fundamental frequency of the synthesized tone is set to 146.8 Hz. The effect of dispersion and frequency dependent losses present in the synthesized tones can be observed from the power spectrum and the time domain representation respectively. Figure 3 shows the decay rates of fifteen different partials of synthesized tone. Figure 4 and 5 show time domain and power spectrum of the synthesized tone without phantom partials respectively. Figure 6 and 7 show time domain and power spectrum of the synthesized tone with phantom partials respectively. Audibility tests were also conducted using synthesized inharmonic tones.



Figure 3 Different decay rates of partials of the synthesized piano tone



Figure 4 Synthesized piano tone without phantom partials with fun.freq.=146.8Hz



Figure5 Power spectrum of synthesized piano tone without phantom partials with fundamental frequency =146.8Hz



Figure 6 Synthesized piano tone with beating effect and phantom partials with fundamental frequency=146.8Hz



Table1: Theoretical and obtained results of IDTC filter for fun.freq.=146.8Hz; Inharmonicity coeff.=50.10⁻⁶

Frequency of	Frequency of	Difference
Partials	Partials obtained	
calculated in Hz	in Hz	
(Theoretical)	(Prectical)	
146.8	147.0	-0.1904
293.6	293.5	0.1485
440.5	440.5	0.0278
587.4	587.5	-0.0269
734.5	734.5	0.0064
881.6	881.4	0.1497
1028.9	1028.9	-0.0750
1176.3	1176.4	-0.1459
1323.9	1323.9	-0.0411
1471.7	1471.9	-0.2388
1619.7	1619.9	-0.2171
1767.9	1767.9	0.0455
1916.4	1916.4	0.0708
2065.2	2065.4	-0.1196
2214.4	2214.4	-0.0042

Figure7 Power spectrum of synthesized piano tone without phantom partials with fundamental frequency=146.8Hz

Figure 5 shows the spectrum of synthesized signal which is inharmonic and the frequency components of the spectrum are not equidistant. Figure 7 when compared with figure 5 shows the introduction of phantom partials in the synthesized tone.

Subjects who participated in the audibility tests also confirmed the effect of inharmonicity as well as phantom partials.

6 CONCLUSION AND FUTURE SCOPE

Inharmonic Dispersion Tunable Comb filter along with loss filter can be used to model stiff string of any musical instrument which generates inharmonic sound. The model can be used in general to study the behavior of the instrument for different parameters like inharmonicity coefficient, number of partials, decaying rate of partials, frequencies phantom components etc. The model can be used to make a virtual instrument.

The inharmonic dispersion tunable comb filter can also be used to find the accurate onsets of the piano notes played during the song. The other application of the method is in development of automatic transcription system which extracts pitch, amplitude, duration of the note from the audio data.

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