Rotating Projection Algorithm for Computer Tomography of Discrete Structures

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Abstract: - Traditional computer tomography requires scanning the object to obtain a lot of projections. Then the image reconstruction is realized on the base of some mathematical model that corresponds to the concrete physical field producing this tomography. For example, in the X-rays tomography the inversion of the Radon transform is used. It seems necessary for difficult structures and can be realized in sufficiently fast manner. We consider in this paper the situation, when the investigating object has the “Discrete Structure”, so its reconstruction consists only in localization of some point-wise elements with different characteristic inside of the homogeneous (or quasi homogeneous) substance in the considered region. We propose for this case the Rotating Projection algorithm with a little number of scanning angles. This algorithm do not requires application of some inverse transforms. It simplifies the image reconstruction. Proposed approach is faster in its computer realization, gives possibility to reduce the time of the radiation treatment. The good properties of the developed algorithm are demonstrated on simulated numerical examples.

Key-Words: - Image reconstruction, rotating projection, computer tomography

1 Introduction

We consider the problem of the image reconstruction of the structure, consisting of the component with different characteristics. This problem can be resolved by tomography corresponding to some external physical field that we use to obtain indirect boundary observations as projections. There are some tomographies such as electrical, infrared, acoustic, Roentgen (X-rays radiography), etc. [1], [2]. The traditional approach requires scanning the object and calculating the inversion of the Radon transform. The detailed scanning of object seems necessary for difficult structures and can be realized in sufficiently fast manner [3] – [5]. But sometimes the investigating object has the simple discrete structure, so its reconstruction consists only in **localization** of some elements with different characteristic inside of the homogeneous (or quasi homogeneous) region. In [6] it was proposed the simplified variant of scanning algorithm without application of the inverse Radon transform. We call it as Rotating Projection algorithm. In [6] this algorithm was applied for electric tomography. Here we use this idea and develop the approach for abstract Discrete Computer Tomography.

2 Plane Rotating Projection Algorithm

We explain the idea of this algorithm in the simplest plane case for one element to be localized. Suppose that we know values of function $v(p,\varphi)$ (projections), which characterizes the intensiveness of passed through the object rays for some fixed angles φ and all linear coordinates p of scanning. By another words, we know for some fixed angles $\varphi_i, i = 1,\dots,n$ corresponding number of one-dimensional images (projections) as functions of one variable p.

We need the next steps.

1. Prolongation on a plane of calculated projections $v(p,\varphi_i)$ for every p along the direction, corresponding to angle φ_i, to obtain the extended two dimensional image.
2. Rotation, i.e., changing number i (scanning for different φ) of prolonged projections and localization of the areas of intersection of projections with the same values of density.

Let us suggest that we have homogeneous substance in the region and only one element with another density. If we do not interesting in the exact geometrical form of the element and want to localize it as a rectangle, it is sufficient [7] to use $n=2$. We suppose also that $\varphi_1 = 0, \varphi_2 = 90\ grad$, that means we have two orthogonal projections. The intensiveness $v_i = v(p, \varphi_i), i = 1, 2$ of rays, passed through the homogeneous substance, is equal to 0, and for rays, passed through the element, is equal to 1. So, our variable *p* is *x* for *i=1*, or *y* for *i=2*, and we can localize one dimensional images at exes X and Y. The graphical illustration of the above supposition is presented at Fig. 1: one-dimensional images as little squares at axes x and y, which correspond to the value 1 of the projection function. Illustration of the projection algorithm is presented at Fig. 2: two bidimensional extensions and final image of localized element.

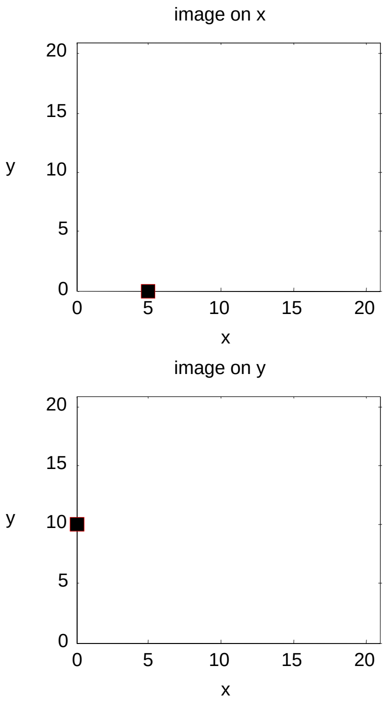

Fig. 1. One-dimensional images.

3 Space Case of Rotating Projection Algorithm

We developed the explained scheme for the space (three dimensional) case. Now we suppose that we have some elements with different characteristic inside of quasi homogeneous substance in the space region. Suppose that the density of these elements is such that their X-rays tomography images have a strictly greater intensiveness than intensiveness of the image of the quasi homogeneous fund of the region.

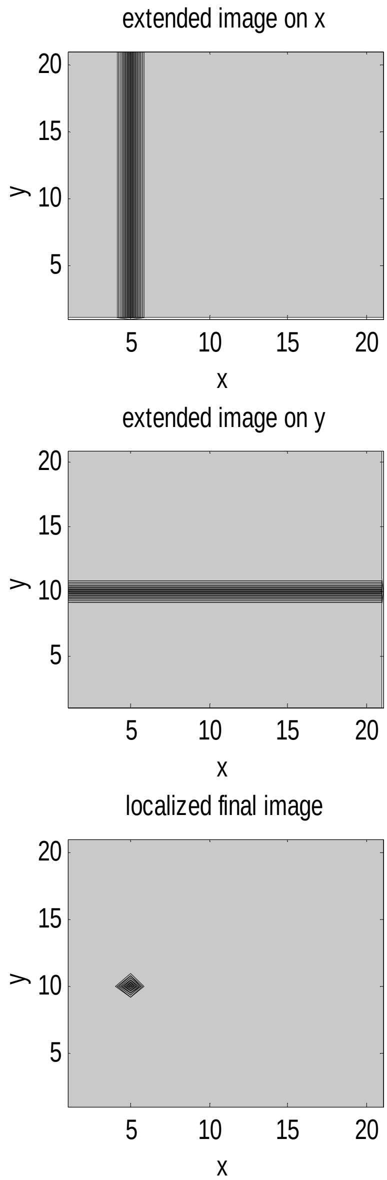

Fig. 2. Bidimensional extensions and final image of localized element

Hence, we include in the scheme of the algorithm the procedure of “clearing” of the images as the first step. It consists in normalization of the range of the image in a grey scale and changing intensiveness for 0 in the pixels with initial intensiveness less then 1. The description of the next steps is similar to description of the plane scheme. Constructed algorithm was realized as a package of computer programs in MATLAB system and justified by a lot of numerical model experiments.

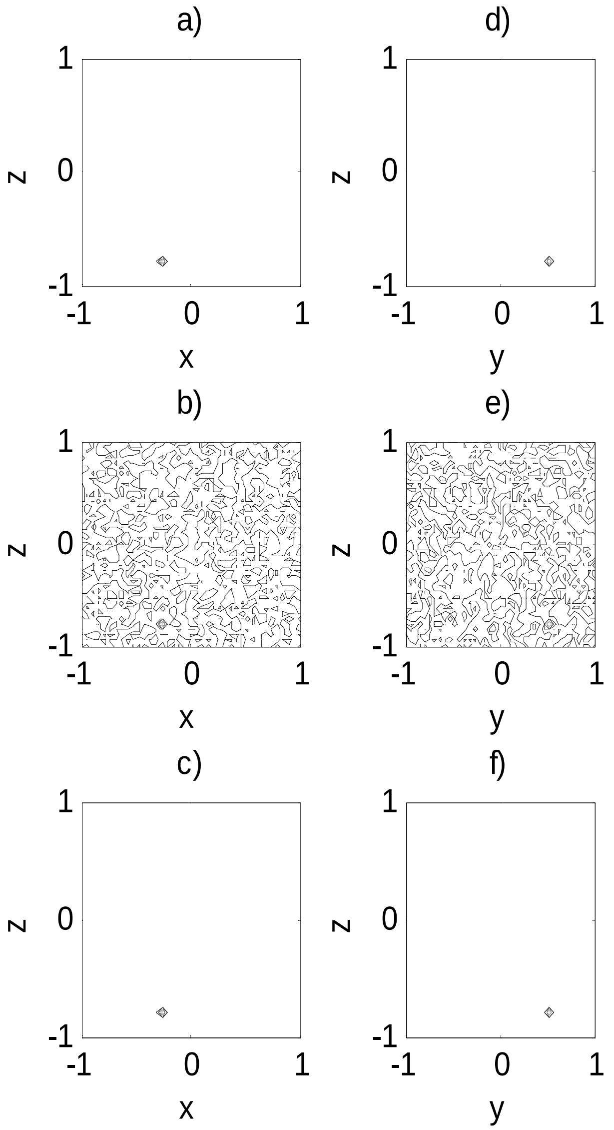

Fig. 3. Exact, noised and recuperated
XZ and YZ simulated projections.

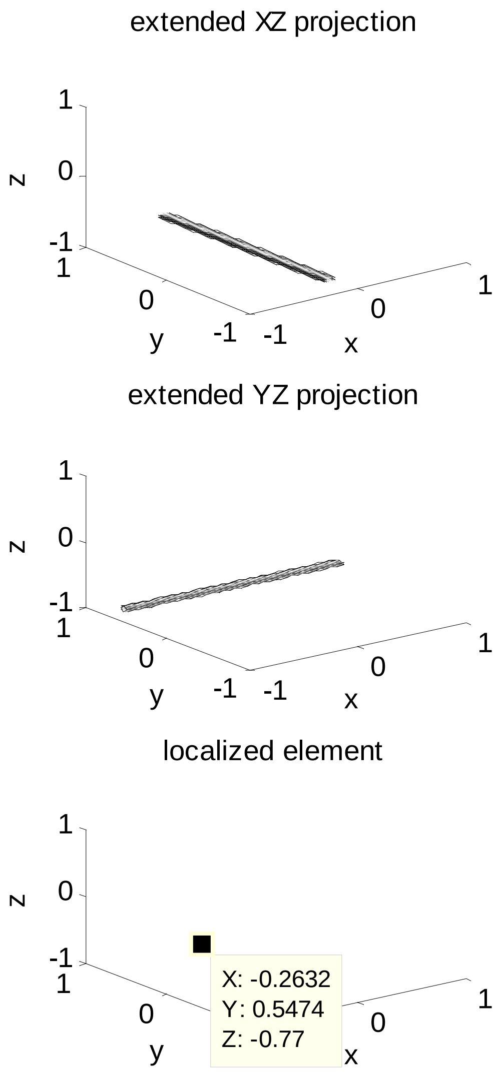

Fig. 4. Model example for space projection algorithm: three dimensional extensions and final image.

One illustration is presented at Fig. 3, 4. At Fig. 3 graphs a) and d) present exact XZ and YZ simulated projections correspondently; graphs b) and e) present noised XZ and YZ projections that simulate real Roentgen tomography images; graphs c) and f) present recuperated by “clearing” XZ and YZ simulated projections, which were used in Projection algorithm to construct images presented at Fig. 4. We can calculate and mark coordinates of the detected point as we made at the bottom graph of Fig. 4.

Sometimes two projections are not sufficient to reconstruct uniquely the structure. In the example presented bellow we see at Fig. 5 the original image with the homogeneous fund (density 0) and two elements with density 1. Using two projections corresponding to angles 0 *grad* and 90 *grad* presented at Fig. 6, 8 we obtain in reconstruction two real and two false elements, presented at Figure 9. If we use the third projection corresponding to angle 60 *grad* explained at Fig. 7, the reconstructed image at Fig. 10 has only real elements.

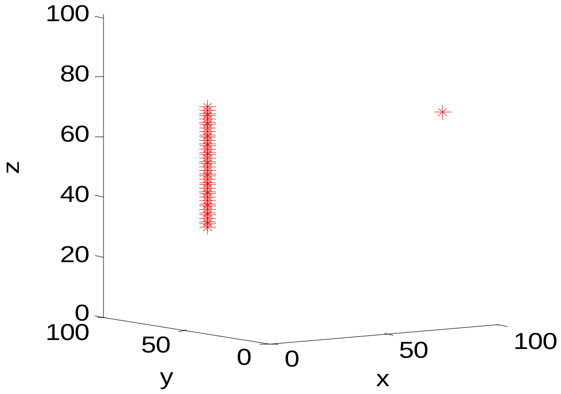

Fig. 5. Original image.

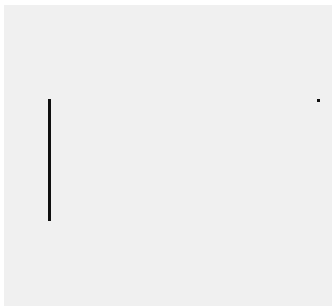

Fig. 6.

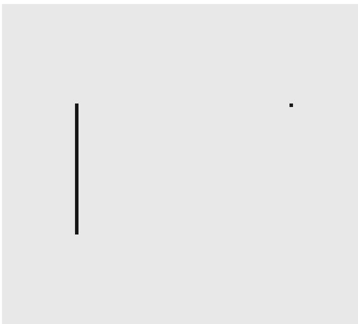

Fig. 7.

Fig. 8.

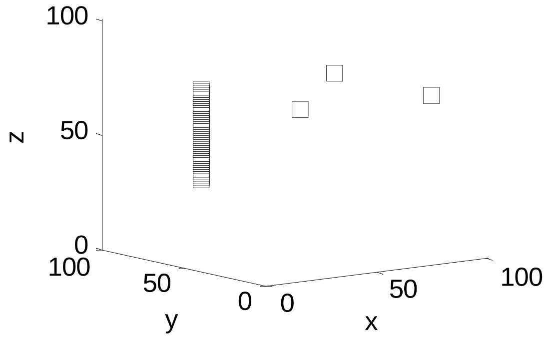

Fig. 9. Reconstruction by 2 projections

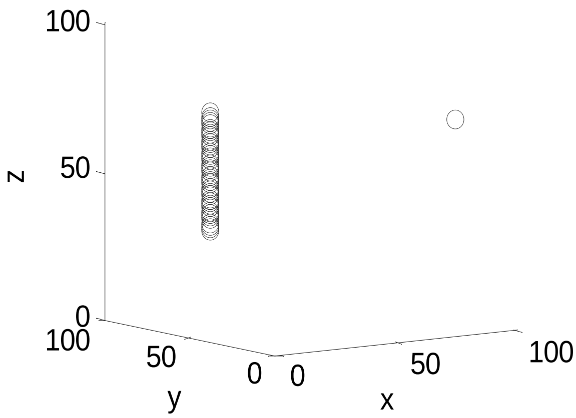

Fig. 10. Reconstruction by 3 projections

4 Discrete Computer Tomography

The Discrete Tomography consists in using of the developed algorithm for objects with the discrete structure. Here we demonstrate this concept using experimental optic images. At Fig. 11 - 14 there are presented photography of the glass filed by gelatin with 8 elements, which have the density satisfying the mentioned above quality. We consider these images as projections corresponding to the angles of 00 *grad* (XZ projection), 60 *grad*, 90 *grad* (YZ projection), 120 *grad*. We can see that our images have quasi homogeneous fund in the main part of the region, but there are also dark part near the border. So, we need preprocessing the images to eliminate its “no interesting” parts.

Fig. 11. Image of XZ projection.

Fig. 12. I mage of 60 *grad*

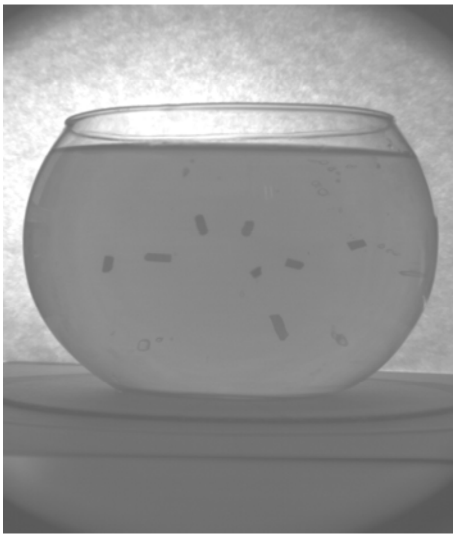

Fig. 13. Image of 90 *grad*

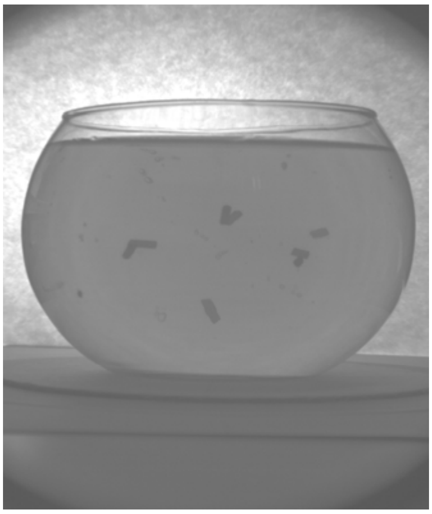

Fig. 14. Image of 120 *grad*

The special algorithms of data pre-processing were used also for reducing real data for the simplified form considered above. We make the reduction of the region using the program Adobe Image Ready to select the parts of images to be processed. These selected parts corresponding to angle 0 *grad*. and 90 *grad* are presented at Fig. 15, 16.

Fig. 15. Part of the image corresponding to XZ projection.

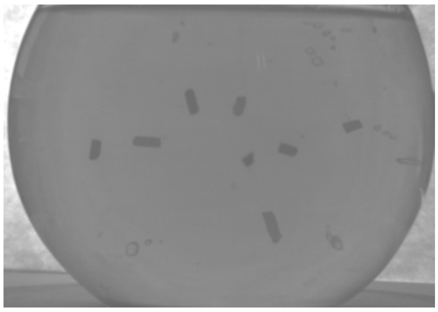

Fig. 16. Part of the image corresponding to YZ projection.

We used also the system FOTOSHOP and Picasa2 program to make better the illumination of images, eliminate borders, reduce the region to unify coordinates of the "interesting" elements and make them more contrast. Examples of this part of the preprocessing are presented at Fig. 17, 18.

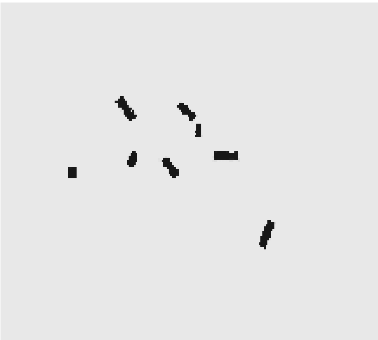

Fig. 17. Preprocessed image of XZ projection.

Fig. 18. Preprocessed image of YZ projection.

Then we applied developed "Rotating Projection" algorithm to preprocessed images of projections corresponding to the angles of 00 *grad* (XZ projection), 60 *grad*, 90 *grad* (YZ projection), 120 *grad*. Result of reconstruction is presented at Figure 19 and completely coincides with the original image of distribution of no homogeneous elements in the area.

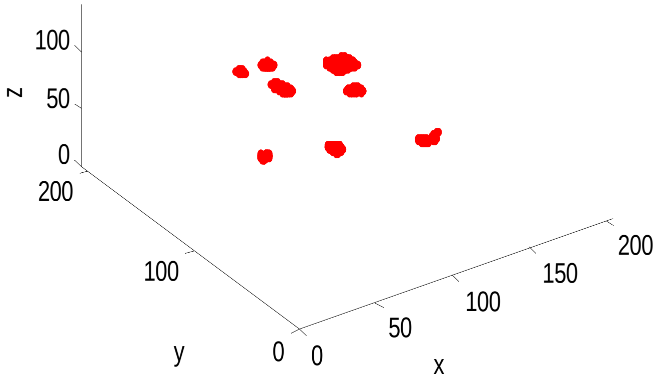

Fig. 19. Image of localized elements.

5 Affine Rotating Projection algorithm

We propose for considered type of structures to use three plane projections, two of which correspond to the XZ and YZ planes in rectangular coordinates; the third projection corresponds to some affine to XY plane [9]. Proposed scheme is three dimensional generalization of the Rotating Projection algorithm.

We explain this variant on the simulated example of the image recuperation of the rectangle, displaced at the plane that is parallel to XY plane, as we can see at Fig. 20.

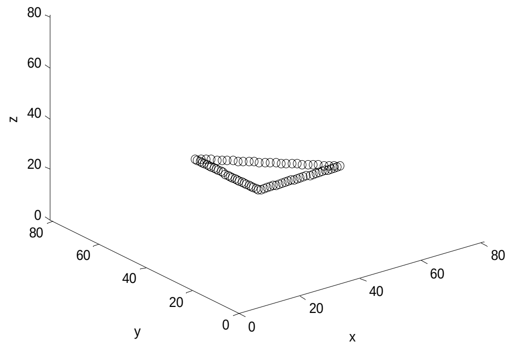

Fig. 20. Original image.

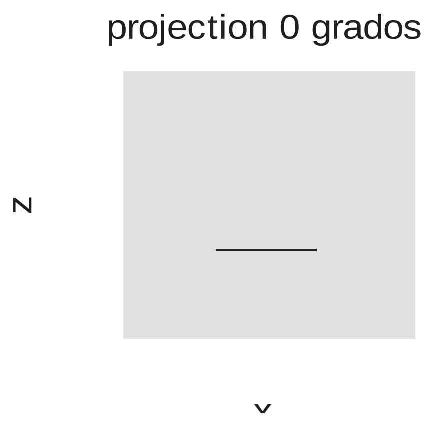

Fig. 21

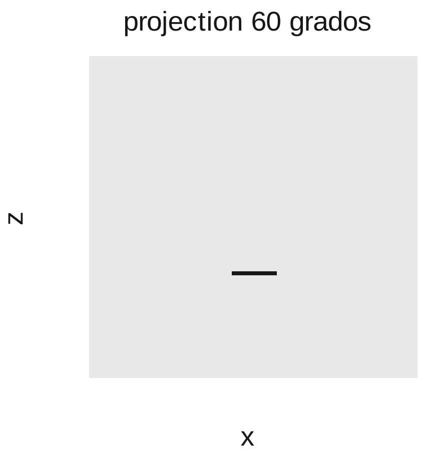

Fig. 22

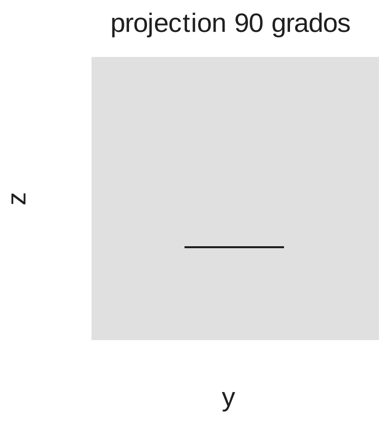

Fig. 23

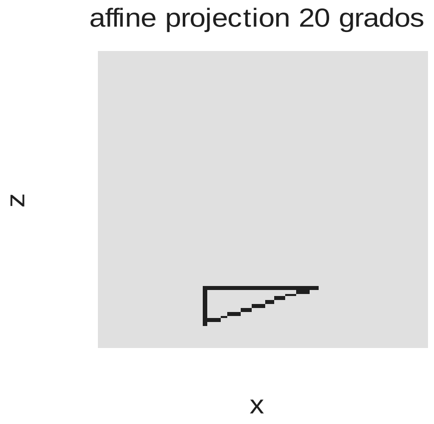

Fig. 24

Graphics at Fig. 21-23 correspond to projections 00, 60 and 90 *degree* calculated with parallel to XY plane beam. The graphic at Fig. 24 corresponds to projection onto the plane XZ with beam, which is inclined at angle of 20 *degree* to the XY plane. At Fig. 25 we can see results of reconstruction of the image by 3 parallel to XY plane projections, at Fig. 26 – reconstruction by 2 parallel to XY plane projections and 1 affine projection.

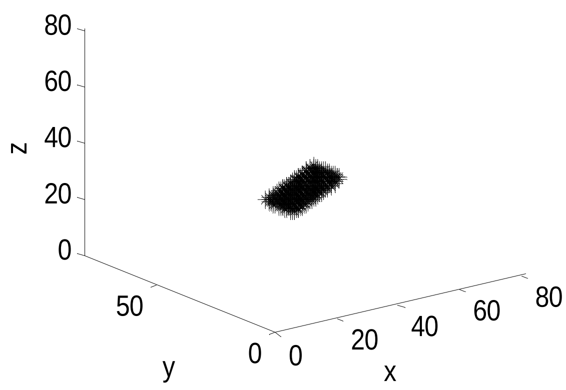

Fig. 25. Reconstruction by 3 parallel to XY plane projections

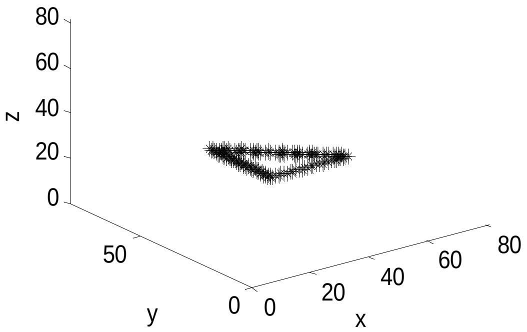

Figure 26. Reconstruction by 2 parallel to XY plane projections and 1 affine projection

From our point of view, such approach can be realized technically at most of consisting equipment for X-rays, electrical, infrared and others tomography [1], [2].

6 Numerical experiments for comparison of Discrete and model X-rays tomographies

We present some numerical experiments to illustrate comparison of the quality and rapidness of proposed discrete tomography and simulated X-rays tomography. The exact image to be recuperated is presented at Fig. 27. We calculated Radon transforms of this image for NA=2 angles of 0 degrees and 90 degrees that are explained at Fig. 28, 29 as normalized density in dependence of the variable p. Then we used corresponding one-dimensional images as input data for the Rotating Projection algorithm (RPA). Result of the reconstruction is presented at Fig. 30, parameter TIMEAlgRP means time of reconstruction by RPA.

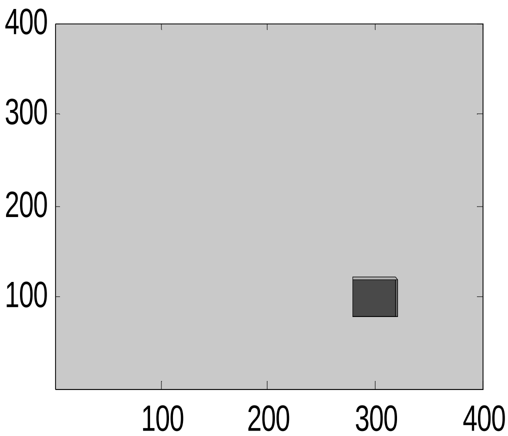

Fig. 27

Simulation of the X-rays tomography is based on construction the input data as the direct Radon transforms for test image for different number of angels *NA* and then application of the inverse Radon transform to reconstruct image. The MATLAB functions *radon* and *iradon* were used for this aim. Results are at Fig. 31 – 35, which explained that recuperation of image by simulated X-rays method is not so perfect as at Fig. 30 and require more time of calculation TIMEXR. At Fig. 36 the relation TIMEXR/TIMEAlgRP is presented in dependence of the parameter NA that confirms advantages of Rotating Projection algorithm in these series of numerical experiments.

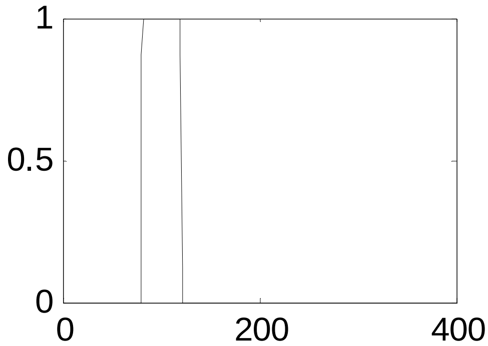

Fig. 28

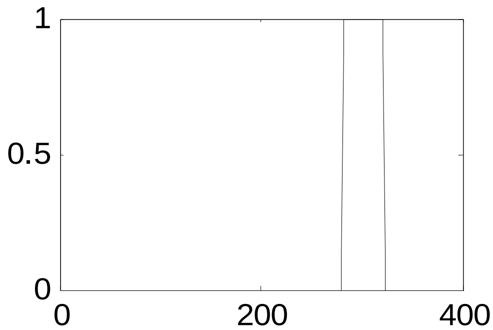

Fig. 29

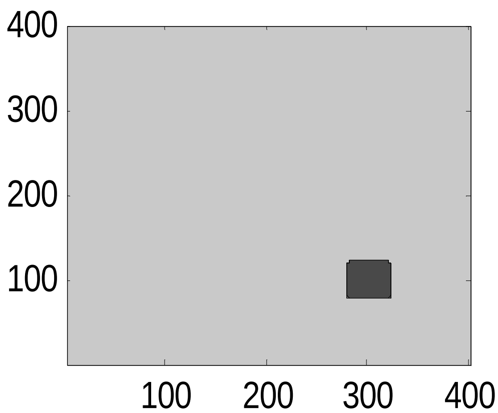

Fig. 30
NA=2; TIMEAlgRP = 1.0630 sec

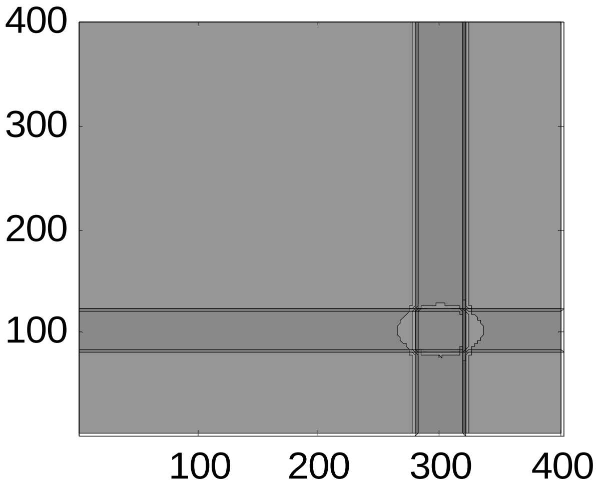

Fig. 31
NA=2; TIMEXR = 2.0470 sec

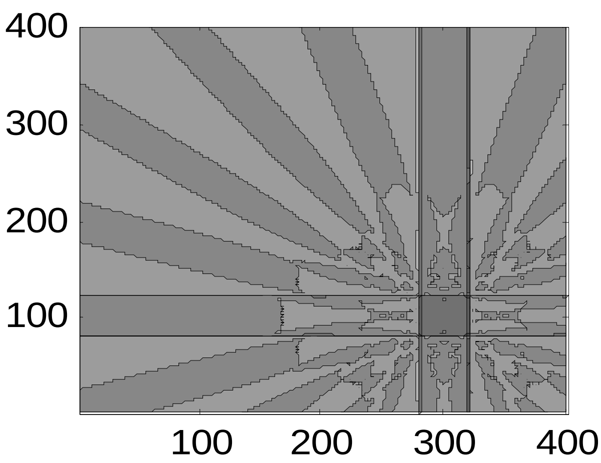

Fig. 32
NA=10; TIMEXR =3.6410 sec

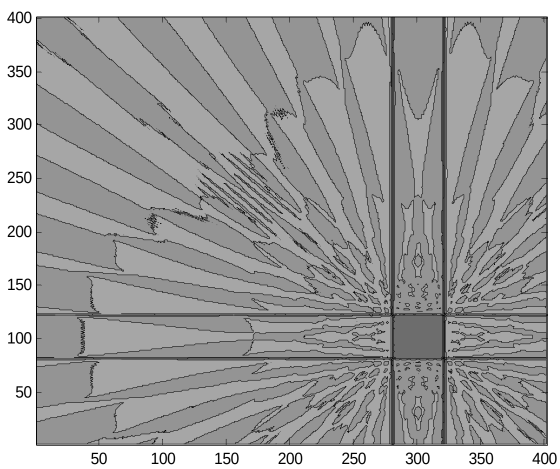

Fig. 33
NA= 20; TIMEXR =22.3910 sec

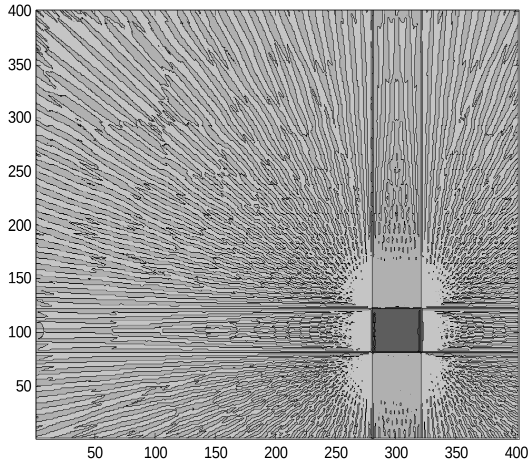

Fig. 34
NA=90; TIMEXR = 51.2190 sec

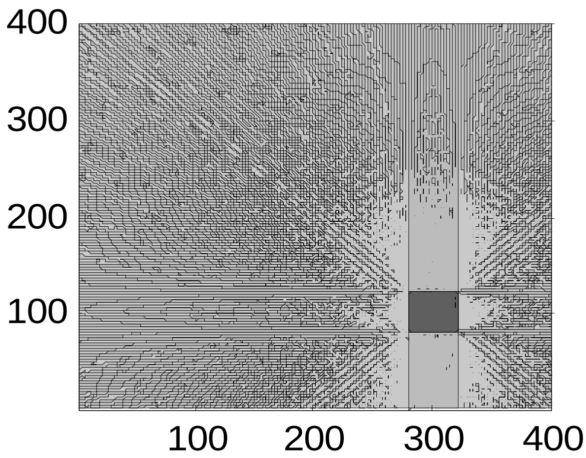

Fig. 35
NA=180; TIMEXR = 163.3440 sec

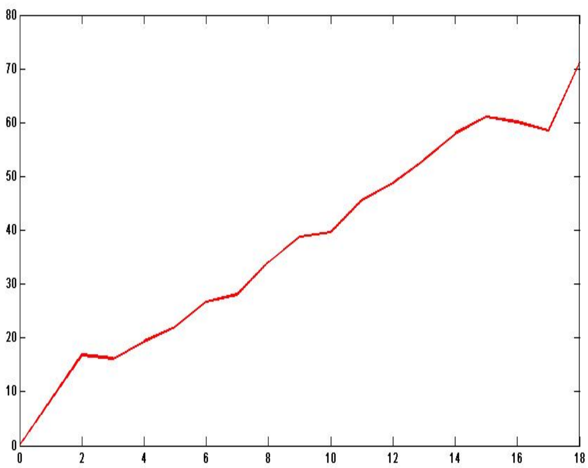

Fig. 36

7 Applications

One of the important applications of constructed algorithm and programs is related with the medicine diagnostics of cancer of the female bosom using X-rays tomography. This application is under the investigations of some groups of specialists [8]. In this case it is important to reduce the time of the radiation treatment in the tomography process, by other words – to reduce the number of angles of scanning. It seems possible to use the proposed approach for this applied problem.

A seismic tomography for recuperation of the underground structure by seismic methods can be formulated mathematically as a problem, which is analogue of X-rays tomography [9]. For detailed recognition of investigating structures it is necessary to use a lot of measured data and to reconstruct images by the inversion of the Radon transform. In some simplified cases, for example at preliminary recognition, it is possible to use supposition made above about the discrete structure of the investigating object, i. e. the structure consists of some separate elements with different characteristic inside of the homogeneous (or quasi homogeneous) substance in the considered domain. Under these suppositions it is possible to use the plane version of Rotating Projection algorithm [3]. It can simplify the image reconstruction for the considering case, because it does not require complicated calculation of the regularized inverse Radon transform or solving mal conditioned systems of linear algebraic equations [9]. This algorithm has also some perspectives in electromagnetic geophysics methods [10].

8 Conclusion

"Rotating Projection" algorithm is developed for the case of reconstructing images obtained in computer tomography of objects that have the simple structure, when its image reconstruction consists only in localization of some elements with different characteristic inside of the homogeneous (or quasi homogeneous) substance in the region. Good properties of the developed algorithm are demonstrated on numerical examples with simulated and experimental data.

Next steps at this line of investigations will be dedicated to mathematical description of discrete structures and theoretical analysis of Rotating Projection algorithm, so as practical and numerical experiments on image reconstruction. We will give also more attention to comparison with existent technologies and discussion of applicability of proposed approach.

9 Acknowledgements

Authors acknowledge for support of some part of investigations realized in the frame of Project No CB-2006-1-57479 SEP y CONACYT Mexico and the Project No 04/EXES/07 VIEP BUAP.

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