A hybrid model of Mathematical Programming and analytic hierarchy process for the GISMR: The Industrial localization

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Abstract: - One of the major questions which arise for the decision makers is the localization choice of places of their establishment related to the constraint of the space, social, economic and policy difference between the places of production and those of consumption. This question is related to the diversity of the criteria integrated in the decision-making, and to the very great number of possible space alternatives. We propose in this article an approach of aid to the industrial localization decision by profiting from the advantages offered by the geographical information systems to multiple representation compounds to the AHP method and from the advantages of the mathematical programming models. The use of a GISMR combined with AHP will help us to reduce the number of the space alternatives and to evaluate them according to real criteria of decision which are quantitative or qualitative. The evaluations obtained will be integrated in a mathematical model to make the final choice of the best alternatives. In fact this model is used to identify the best solution taking into account the criteria fixed by the decision-maker and the data provisions generated by the GISMR.

Key-Words: - Geographical Information Systems to Multiple Representation (GISMR), Multi-Criteria Decision Analysis (MCDA), Analytic Hierarchy Process (AHP), Mathematical Programming (MP), Industrial localization.

1 Introduction

The industrial localization is one of the most significant strategic decisions because it conditions the long-term operation of the firm, and it often depends on several contradictory factors. To choose an optimal location, the decision-makers must consider some rational criteria such as profit or efficiency and use rigorous and reliable methods.

To make this decision, we must raise the difficulties of the diversity of the criteria and factors of decision, the importance of the number of possible solutions, the heterogeneity of socio-economic space and the two dimensions of localization: inter-regional (level of the great economic space) and intra-regional (level of the regions and localities) [1].

Our work appears in the context of development of models of territorial decision-making aid. Our interest was focused on the power of space analysis of the GISMR, and the decisional capacities of the AHP [2] and the MP.

We will present in this paper, the GISMR and the MCDA (section 2 and 3). In section 4, we will describe our hybrid model of aid to the decision making. An illustration of our model is presented in section 5. The conclusion and the perspectives of our work are exposed in the last section.

2 Geographical information systems to multiple representation

At present, the GIS are not regarded any more as spatial data management systems. They are from now on able to represent the spatial data according to various possible points of view. Thus for each real data, the system will be able to maintain several different representations simultaneously. They are multiple representations GIS or multi-representation (GISMR). The fact that GISMR takes into account these various factors is a very significant base to meet the requirements which the new function of the GIS imposes; that is to say to be a tool of the multicriterion decision aid.

In fact, the GISMR have the possibility of aggregating all information necessary concerning a decisional project in a coherent and structured way. This aggregation of information is of primary importance for the problems of the industrial localization. Moreover, the integration of the powerful tools for analysis such as the MCDA methods will permit the GISMR to propose relevant solutions to help the decision makers to make the best choice.

3 Multicriterion Decision Aid

The MCDA, often called multicriterion analysis, is a domain that has known a lightning development these last years. The encouraging aspect of this development is that it does not exist in an isolated manner, but concerns all chapters of the operational research. In other words, practitioners are more and more aware of the presence of the multiple criteria in the concrete problems of management and decision whatever their nature is. These kinds of problems do not have a best decision that go simultaneously with all points of view. Therefore, the word «optimization " no longer has sense in such a context. This is the reason why the MCDA intends to look for a solution of "compromise". Its main goal is to help decision-makers to organize and to synthesize their information so that they feel at ease with their decision-making.

The MCDA analysis generally refers to a set of methods permitting to aggregate several criteria with the objective to select one or several actions, options or solutions. These methods are used a lot in case of the decision of the localization in general, and the industrial localization in particular.

Among these methods, we can mention, the utility multiattributs methods (MAUT, SMART, UTA, TOPSIS) [3] and [4], the methods of outranking ELECTRE [5] and [6], PROMETHEE [7], and the AHP method [2].

4 Proposed approach

This approach is based on the remarkable potentialities of the integration of AHP and the MP in GISMR. This makes it possible to enrich the decision-making process of industrial localization through the complementarity between these tools, as illustrate in what follows:

– The taking into account of the real criteria of decision which are quantitative or qualitative, the use of a very thorough analysis of all the elements necessary to a good evaluation of the various possible solutions, and the study of the sensitivity of these solutions;

– Great capacity to solve the problem by considering a great number of possible solutions subject to a set of constraints;

– GISMR offers an effective visual multi-representation of the possible solutions and is used for the management of a significant volume of the data;

– Finally, this integration has the merit to answer various recent requirements which the new function of the GIS imposes: to evolve to really computerized decision-making systems with spatial reference.

Indeed, GISMR has the possibility of incorporating all information necessary concerning the decisional problem and in a coherent and structured way. This aggregation of information is of primary importance in order to draw a classification or a choice by integrating AHP and the MP.

However, the various stages, processes, and the relations ships between these elements remain to be specified. This will allow the description of the advantages above to answer specificities of the industrial localization problems.

4.1 General approach

If the scale is adopted as the main factors, the definition of the problem will be categorically influenced. Otherwise, the criteria are more or less relevant according to the scale of work since any criterion cannot be considered on any scale [8]. With the scale of a country for example, it is not necessary to take into account the facility of connection to the sewerage system. This criterion is on the other hand of primary importance when it is about only one area.

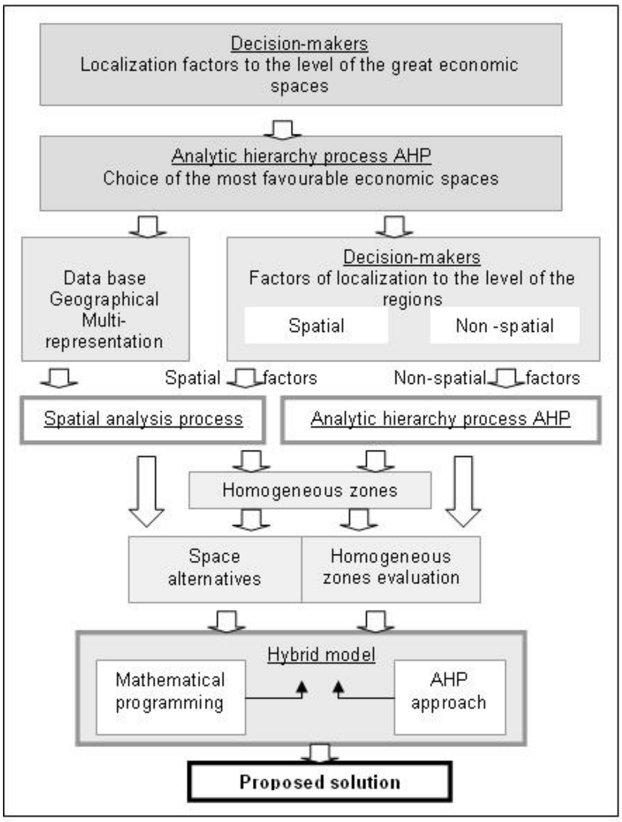

Fig. 1. Proposed approach

ISSN: 1991-8755 Issue 4, Volume 3, April 2008

We propose a hierarchical approach of localization (figure 1) by considering that this decision must be done in two levels: level of great space (for example a country) and level of localities (for example an area).

We use mainly four processes:

– The first process is charged to describe the perimeter of study using a geographical data base managed by a GISMR;

– The second process makes it possible to build the homogeneous zones, through factors related to the level of localities. It is also used for the multi-representation of these zones that makes it possible to present the spatial alternatives in the most reliable way;

– The third is the analytic hierarchy process AHP. It is charged to evaluate homogeneous zones resulting from the space analysis process;

The fourth is based on MP; its objective is to maximize the total utility of choice.

4.2 Spatial analysis process

The spatial criteria specified by the decision-makers to establish favourable choices are taken into account by the spatial analysis process. This process uses the possibilities of spatial combinations offered by the GISMR [9]. And it builds the homogeneous zones and makes the choice of the candidate sites according to the aspects meeting the specific needs for the actors.

To evaluate the various criteria, the GISMR explores the same geographical area according to several representations where each one reflects an interpretation, a point of view or a quite precise scale. The result of this evaluation is the combination of answers for each criterion according to the corresponding representation. This result is generally translated into term of chart in a multi-scale chart and multi-topics.

4.3 Analytic hierarchy process AHP

The objective of this process is to overcome the complexity of the problem of industrial localization by the hierarchical decomposition, and the evaluation of the various actions considered during the decision-making process. This process offers a methodology to rank alternative courses of action based on the decision's judgments concerning the importance of the criteria and the extent to which they are met by each alternative. This makes it possible to provide tools to a decision-maker allowing him to take a decision when several, often contradictory, points of view must be taken into account [10].

The first step of the AHP is to form a hierarchy of objectives, criteria and all other elements involved in the problem. Once the hierarchical structure has been formed, comparison matrices are to be developed. These are evaluations made by the decision-makers on the intensity of difference in importance, expressed as a rank number on a given numerical scale, for each level in the hierarchy. From these weights, priorities are determined. An expert would be asked to make pairwise comparisons between two criterion at a time, decide which factor is more important, then specify the degree of importance on a scale between 1 and 9 in which 9 is most important. These evaluations would result in reciprocal matrices of the components of each level against the items in the level above.

Next, assign a relative weight to each one. Each criterion has a local (immediate) and global priority. The sum of all the criteria beneath a given parent criterion in each tier of the model must equal one. Its global priority shows its relative importance within the overall model.

Finally, after the criteria are weighted and the information is collected, put the information into the model. Scoring is on a relative basis, not an absolute basis, comparing one choice to another. Relative scores for each choice are computed within each leaf of the hierarchy. Scores are then synthesized through the model, yielding a composite score for each choice at every tier, as well as an overall score.

4.4 Mathematical programming process

The goal of this process is to select one or more space alternatives who maximize an objective function subject to a set of constraints related basically to the industrial localization. Our model takes into account the factors of localization and different criteria of decision.

Constants:

- u_i: the total mark of site given by using of AHP method and spatial analysis process;
- DR_i : order quantity assigned for retailer i;
- $umax_i$: the maximum order quantity for the site i;
- $umin_i$: the minimum order quantity for the site i;
- Mu_i : minimum activity of site i (minimum order quantity to select the site i);

Variable of decision:

- Su_i : binary variable, with 1 indicate that the site i is selected, and 0 otherwise;

- $expu_{ij}$: expedition quantity between the site i and the retailer j.

Proposed mathematical model:

The purpose of this model is to maximize a function U (Maximize U) which represents the total utility of the decision. We have to take tow decisions: to choice a set of sites and to determine the expedition quantity between the sites and the retailers. We assign to each one a weight of importance by using the pairwise comparisons of these decisions according to the global objective of the firm. We note p_1 and p_2 respectively these weights. Cu_{ij} the evaluation of expedition quantity decision between the sites i and the retailer j. The mathematical model retained is:

$$\text{Maximize } U = p_1 \sum_i Su_i \times u_i - p_2 \times \sum_i Su_i \left(\sum_k Cu_{ik} \times \frac{expu_{ik}}{\sum_j expu_{ij}} \right)$$

Subject to:

$$Su_i \in \{0,1\} \text{ and } expu_{ik} \text{ as integer ;} \quad (1)$$

$$0 \le expu_{ik} \le Su_i \times umax_i ; \quad (2)$$

$$DR_k = \sum_i expu_{ik} ; \quad (3)$$

$$\sum_i expu_{ki} \ge Su_k \times Mu_k ; \quad (4)$$

$$umin_i \le \sum_k expu_{ik} \le umax_i . \quad (5)$$

The global objective of this linear integer programming model is to privilege the sites and the retailers having the best marks. Constraint 1 represents the nature of the decision variables. Constraints 2 introduce the conjunctive rules into the model. Constraint 3 ensures that retailer k is satisfied. Constraint 4 represents the expedition quantity to satisfy the minimum activity of site. Constraints 5 make it possible to satisfy, for each site, the maximum ($umax_i$) and minimum ($umin_i$) order quantity.

5 Illustration of the proposed approach

To illustrate the proposed approach, we present, in this section, an example of industrial localization.

5.1 Stage I

In the first, we choose the most favourable great spaces, by questioning the decision-makers and by considering the factors able to influence the localization. This choice is based on consultations of several concerned actors. Thus we retained three Factors which appear to intervene in a consequent way in the decision-making process: the **market**, the **comparative advantages** and the **governmental settlement**. On this level we chose only one country "**Morocco**".

5.2 Stage II

In the continuation, the question is the choice of the localization factors related to the second level. These factors are used to build the homogeneous zones. The choice must be precise for discriminating between the zones, and not to be redundant to avoid raising the importance allotted to an unspecified dimension. The selected factors are: market, human resources, geographical situation and transport.

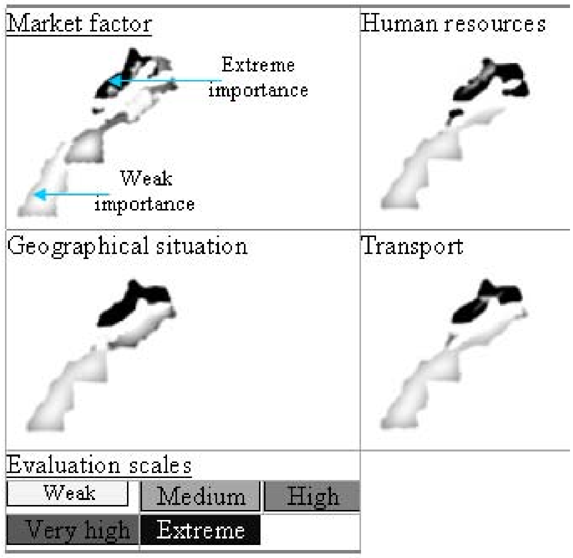

Fig. 2. Layer evaluation compared to the localization factors

To build homogeneous zones, the process of space analysis starts with the determination of the charts of the homogeneous zones reflecting each one a space factor. The space alternatives of the same zone are similar compared to the maximal and minimal values of indifference fixed by the decision-makers.

ISSN: 1991-8755 Issue 4, Volume 3, April 2008

Thereafter, through the technique of layers superposition, the process of space analysis determines the intersection zones. These zones are constituted of the regroupings of the space alternatives belonging to homogeneous zones on the level of all the layers. These zones are homogeneous compared to all the space factors. For the non-space factors we don't need a great territorial data analysis to determine the homogeneous zones.

The space analysis process explores the same chart in several representations (figure 2). Each representation reflects a qualification of the importance of a factor. These qualifications are as: **weak**, **medium**, **high**, **very high** and **extreme**.

Following this stage we determined 12 homogeneous zones. To evaluate them we use the AHP method.

5.3 Stage III

In this stage we use the AHP method to determine the weight of each criterion by a binary comparison method. The mark of each zone is calculated by comparing the zones with respect to each criterion. This method allows us to structure the problem in the form of a hierarchy.

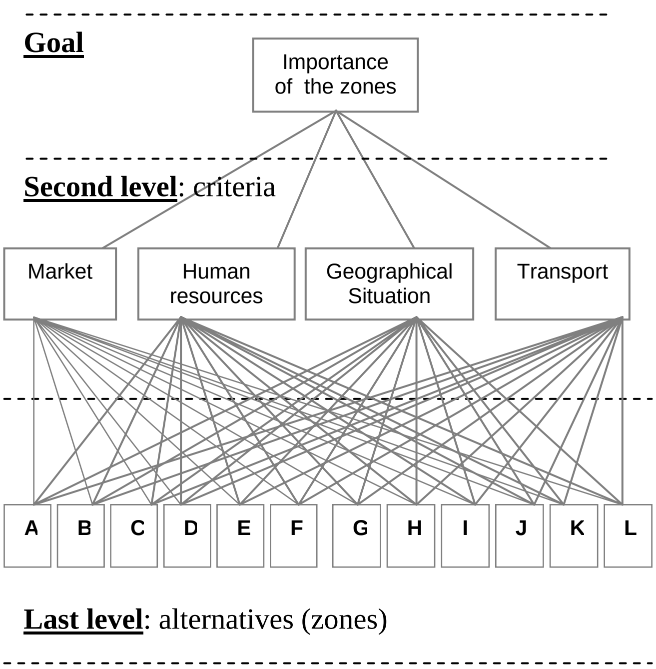

Fig. 3. The hierarchical decomposition

Generally, the hierarchy has at last three levels: the goal, the criteria, and the alternatives. For this example (figure3), the goal is to determine the global mark of each zone. The second level represents the criteria: market, human resources, geographical situation and transport. The last level of hierarchy represents the alternatives: zones A, B, C, D, E, F, G, H, I, J, K and L. Construction of hierarchy is the first step in the problem solving process of the AHP method.

To make pairwise comparisons and establish priorities among the elements in the hierarchy, the managerial judgements are expressed in terms of pairwise comparisons of items on a given level of the hierarchy with respect to their impact on the next higher level. Pairwise comparisons express the relative importance of one item versus another in meeting a goal or a criterion.

There are many scales that could be used for quantifying managerial judgments; the scale given in table 1 is the standard usage of AHP analysis.

Verbal judgment or preference	Numerical rating
Extremely preferred	9
Very strongly preferred	7
Strongly preferred	5
Moderately preferred	3
Equally preferred	1
Intermediate values between two adjacent judgments	2, 4, 6, and 8

Tab. 1. Measurement scales

The pairwise comparisons for the above evaluation of the importance of the zones are shown below.

For choosing between different zones, the decision-maker must weight and prioritize different criterions. That means he has not only to assign weights to each zones, but also to assign weights to criterions related to the goal.

Following an initial step of AHP that consists of making a hierarchy, the decision-maker has to compare in pairs all criterions with respect to goal by using Table 1, and to put results into the matrix of criterions. If the decision-maker, for example, believes that transport is moderately more important than human resources, he will put number 3 in the position where row transport crosses column human resources, and number 1/3 in the position symmetrical with respect to main diagonal. Proceeding in the same manner he should make (4x3)/2=6 pairwise comparisons. Resulting matrix could look like this one:

	Market	Human resources	Geographical situation	transport	Total	Weights
Market	1	6	2	2	11.00	0.46
Human resources	1/6	1	1/3	1/3	1.82	0.07
Geographical situation	1/2	3	1	1	5.50	0.23
Transport	1/2	3	1	1	5.50	0.23
				Total	23.82	1.00

Tab. 2. Pairwise comparisons of evaluation criteria

The vector of weights is obtained by the normalization of the vector of the sums of the comparisons values corresponding to each criterion.

In the same way, we obtain the vectors of weights (table 3) of zones A, B, C, D, E, F, G, H, I, J, K and L (Elements of the last level in the hierarchy) according to each criterion.

Criteria \ ZONE	A	B	C	D	E	F	G	H	I	J	K	L
Market	0.0728	0.1460	0.0344	0.1093	0.0364	0.0364	0.0727	0.1093	0.1443	0.0727	0.0727	0.0910
Workforce	0.1095	0.1097	0.0821	0.0546	0.0993	0.0993	0.0993	0.0546	0.0840	0.0993	0.0546	0.0546
Geographical situation	0.1818	0.1363	0.0911	0.0454	0.1818	0.0911	0.0454	0.0454	0.0454	0.0454	0.0454	0.0454
Transportation	0.1000	0.1000	0.0670	0.0828	0.0828	0.0828	0.0670	0.0670	0.1025	0.0828	0.0828	0.0828

Tab. 3. Results of pairwise comparisons of Zones

The weights of the zones and the weights of the criteria are combined by moving through the hierarchy, starting at the last level. The global marks of zones are calculated as follow:

$$U_X = \sum_{i=1}^{4} C_i W_{XC_i}$$

With:

- U_X: Global mark of zone X;
- C_i: weight of criterion i;
- W_{XCi}: Weights of zone X according to criterion i;

Zones	Global utility
A	u_1 = 0.46 x 0.0728 +0.07 x 0.1095 +0.23 x 0.1818 +0.23 x 0.1000= 0.1059
B	u_2 = 0.46 x 0.1460 +0.07 x 0.1097 +0.23 x 0.1363 +0.23 x 0.1000= 0.1291
C	u_3 = 0.46 x 0.0344 +0.07 x 0.0821 +0.23 x 0.0911 +0.23 x 0.0670= 0.0579
D	u_4 = 0.46 x 0.1093 +0.07 x 0.0546 +0.23 x 0.0454 +0.23 x 0.0828= 0.0835
E	u_5 = 0.46 x 0.0364 +0.07 x 0.0993 +0.23 x 0.1818 +0.23 x 0.0828= 0.0845
F	u_6 = 0.46 x 0.0364 +0.07 x 0.0993 +0.23 x 0.0911 +0.23 x 0.0828= 0.0636
G	u_7 = 0.46 x 0.0727 +0.07 x 0.0993 +0.23 x 0.0454 +0.23 x 0.0670= 0.0662
H	u_8 = 0.46 x 0.1093 +0.07 x 0.0546 +0.23 x 0.0454 +0.23 x 0.0670= 0.0799
I	u_9 = 0.46 x 0.1443 +0.07 x 0.0840 +0.23 x 0.0454 +0.23 x 0.1025= 0.1062
J	u_{10} = 0.46 x 0.0727 +0.07 x 0.0993 +0.23 x 0.0454 +0.23 x 0.0828= 0.0698
K	u_{11} = 0.46 x 0.0727 +0.07 x 0.0546 +0.23 x 0.0454 +0.23 x 0.0828= 0.0667
L	u_{12} = 0.46 x 0.0910 +0.07 x 0.0546 +0.23 x 0.0454 +0.23 x 0.0828= 0.0751

Tab. 4. Global utilities of Zones

5.4 Stage IV (Choice of the candidates places)

This choice is based on the data generated by the process of space analysis. By using a multi-criterion methodology the decision-makers considered a set of space alternatives. During this analysis the decision-makers use the multiple representation of the chart to have all information necessary to make the most relevant possible decision.

To make the choice in the most interesting way the GISMR offers to the decision-makers, through the possibility of incorporating the most useful and relevant information and data, to express the preferences in the most objective and most convincing way.

Thus the decision makers determined 16 sites candidates in various cities (table 5); ***Casa1***, ***Casa2***, ***Rabat***, ***Sale***, ***Marrakech***, ***Beni Mellal***, ***Fes***, ***Meknes***, ***Agadir***, ***Essaouira***, ***Nador***, ***Elhouceima***, ***Eljadida***, ***Settat***, ***Safi*** and ***Tanger***.

Zones	A	B	C	D	E	F
Sites	Casa 2	Casa 1, Rabat	Salé	Elhouceima, Tanger	Nador	Eljadida, Settat
Utility	0.1059	0.1291	0.0579	0.0835	0.0845	0.0636
Zones	G	H	I	J	K	L
Sites	Essaouira, Safi	Agadir	Marrakech	Beni Mellal	Fès	Meknès
Utility	0.0662	0.0799	0.1062	0.0698	0.0667	0.0751

Tab.5. Global utilities of Sites

5.5 Stage V

Mathematical model: The distribution strategy of the industrialist in question neglects the costs of distribution and subdivides the territory in eight areas of distribution in which we must not install more than one site base for each one.

Areas	Large Casablanca	Center - North	Center - South	East - South
Candidates sites	Su - Casa1 Su - Casa2	Su - Rabat Su - Salé	Su - Marrakech Su - Béni-Méllal	Su - Fès Su - Meknès
Areas	South	East	Center	North
Candidates sites	Su - Agadir Su - Essaouira	Su - Nador Su - El Houceima	Su - El Jadidda Su - Settat Su - Safi	Su - Tanger

Tab. 6. Areas of sites

The choice of only one site per area satisfies the constraints related to the minimal activity. Thus the mathematical model relating to this case and the final solution are presented as follows:

Mathematical model	Final solution
$\text{Max } U = \sum_{i=1}^{16} Su_i \times u_i$	$U = \mathbf{0.7536}$ $\mathbf{Su_1} = 1, Su2 = 0,$
subject to: $\begin{cases} Su_1 + Su_2 = 1, \\ Su_3 + Su_4 = 1, \\ Su_5 + Su_6 = 1, \\ Su_7 + Su_8 = 1, \\ Su_9 + Su_{10} = 1, \\ Su_{11} + Su_{12} = 1, \\ Su_{13} + Su_{14} + Su_{15} = 1, \\ Su_{16} = 1, Su_i \in \{0,1\}, \forall i \in \{1,\ldots,16\}. \end{cases}$	**Su** = 1, Su[illegible] = 0, **Su** = 1, Su[illegible] = 0, Su[illegible] = 0, **Su** = 1 **Su** = 1, Su[illegible] = 0 **Su** = 1, Su[illegible] = 0, Su[illegible] = 0, Su[illegible] = 0, **Su** = 1, **Su** = 1.

Tab. 7. Final result

By the application of our model for this case we obtained a very convincing total utility (U=0.7536) by choosing only 8 sites among 16. These sites are in the following cities: Casa1, Rabat, Marrakech, Meknès, Agadir, Nador, Safi and Tanger.

6 Conclusion

In this paper, we have explored the idea of the use of the new generation of geographical information systems (GIS), called multiple representations GIS (GISMR), capable of maintaining several representations simultaneously. Therefore, the user has a global and complete vision of all the working space according to the different possible representations. The research and the analysis according to the spatial criteria are becoming more efficient and particularly more pertinent.

The approach suggested in this report requires that the industrial localization decision be made into two principal stages. In the first stage, we determine the homogenous zones by using analytic hierarchy process method (AHP) and geographical information systems to multiple representation GISMR. The others parameters to be considered in this stage are the industrial localization factors. In the second stage of this approach, we proposed a linear integer model to select the best sites by considering the constraints of the industrialist.

To illustrate this approach of aid to the industrial localization, an example was presented. This example showed us the interest of the use of GISMR, AHP method and mathematical programming.

The integration of the mathematical programming and the AHP analysis method within GISMR have created a new way of research that we have started and that seems very promising. We think that the use of the mathematical programming tools coupled with the multi-criteria decision analysis methods (MCDA) can be a pertinent decision aid tool for most choice and decision aid situations. It is thus about proposing thought processes used in a simple way and conceived in a structured and coherent manner to help the decision makers to make the best choice.

Our work is directed on the one hand towards the implementation of a GIS to multiple representation dedicated to the industrial localization and on the other hand toward the test and the assessment of the different MCDA methods.

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