Shuffle from Sequential to Parallel in Production Planning

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Abstract: This paper presents an implementation of shuffle operation in production planning. We present a computational formula for shuffle and some optimizations to reduce the sets of shuffle strings. Our idea is to combine shuffle with parallelism for a planning of production phases.

Key-words: shuffle, production phases, production planning, linguistic model, execution time.

1 Linguistic Model of Production Process

By production process we understand the transformation action of resources (material, energy) in final products according of a fabrication recipe.

The model that we will show has the purpose to determine the set of actions strings that represents right evolutions of process with the help of linguistic mechanism.

A production system is defined as a 7-tuple:

$$\Sigma = (O, R, M, A, T, I, S)$$

where
a) O= a finite and non empty set of finite products.
b) R= a finite and non empty set of resources.
c) M= set of materials.
d) A= set of activities (phases) set 1,2...,qi that must be executed for the fabrication of each other of aiєO products.
e) T= deliver plan
f) I= economical indicators
g) S= set of fabrication processes

Between the phases from A exists a series of precedence established relations, that can be represented through many graphs:

$$\Gamma_i = (\{1, 2, ..., q_i\}, U_i)$$

These graphs don’t have circuits and always qi is the last activity from the respective graph. We note with Ui the arcs of graph \(\Gamma_i\). There are one graph \(\Gamma_i\) for each product \(a_i\єO\). By linear the graphs \(\Gamma_i\) results \(Z_i\) sets for all products. By successive applications of Shuffle operation results the plan set of fabrication:

$$W = Shuf^p \left( \bigcup_{i=1}^{p} Z_i \right)$$

where \(p\) = number of products or sets of products.

We note with \(Shuf^*\) the transitive closure for \(Shuf\) defined in 2.1.

2 Shuffle Operation

Definition

For \(V\) a vocabulary and two strings \(x, y \in V^*\) the shuffle operation is defined:

$$Shuf(x, y) = \{x_{i1} y_{i1} x_{i2} y_{i2} ... x_{ip} y_{ip} : x_i, y_i \in V^*, x=x_{i1}x_{i2}...x_{ip}, \ y=y_{i1}y_{i2}...y_{ip}, \ \ p\geq1 \}$$ (1)

The note \(x_{i}, y_{i} \in V^*\) means that \(x_{i}\) and \(y_{i}\) can be any symbols sequences (substrings) from the two strings \(x\) and \(y\).

2.2 Computational formula

The Shuffle operation for two characters set \(x\) and \(y\) of different lengths has two phases: the two strings division in substrings sequences \(x_i\) and \(y_i\) and then the mixing of all sequences \(x_i\) and \(y_i\) that result from the first phase.

2.2.1 The string division

Note a string with \(s=a_1a_2...a_n\) and \(|s|=n\). We note by \(f(i, s)\) the function for first \(i\) string symbols:
\[ f(1, s) = a_1 \]
\[ f(2, s) = a_1 a_2 \]

\[ \ldots \ldots \ldots \]
\[ f(n, s) = a_1 a_2 \ldots a_n \quad (2) \]

Note \( s^i \) the characters beginning from \( i \) position from \( s \) string. One results the next substrings:

\[ s^1 = s = a_1 a_2 \ldots a_n \]
\[ s^2 = a_2 \ldots a_n \]
\[ s^3 = a_3 \ldots a_n \]
\[ \ldots \ldots \ldots \]
\[ s^n = a_n. \]

For example if we have \( s = |abc| \) string it can be divided in substrings this way:

\[ |ab|c|, |a|bc\] , |ab|c|, |abc|.

The first string \( |ab|c| \) is obtained by:

\[ f(1, s) \quad f(1, s^2) \quad f(1, s^3) \quad \text{that's equivalent with} \quad f(1, abc) \quad f(1, bc) \quad f(1, c) \]

The second string \( |a|bc| \) is obtained by:

\[ f(1, s) \quad f(2, s^2) \quad \text{that's equivalent with} \quad f(1, abc) \quad f(2, bc) \]

The third string \( |ab|c| \) is obtained by:

\[ f(2, s) \quad f(1, s^3) \quad \text{that's equivalent with} \quad f(2, abc) \quad f(1, c) \]

The last string \( |abc| \) is:

\[ f(3, s) \quad \text{that's equivalent with} \quad f(3, abc) \]

We can make the next notes for the recurrence formula:

\[ f^3(s) = \{ f^2(s), \ f(3, s) \ f^0(0) \} \quad (3) \]

With \( f^3(s) \) we have noted the third order of the function \( f \). With \( f^0(0) \) we have noted the order 0 of the \( f \) function. Function \( f^0(0) \) is applied to the empty string and returns the null string, and performs an write in the exit file of the function before it, namely \( f(3, s) \).

\[ f^2(s) = \{ f^1(s), \ f(2, s) \ f^1(s^2) \} \quad (4) \]
\[ f^1(s) = \{ f^0(s), \ f(1, s) \ f^2(s^2) \} \quad (5) \]

2.2.2 Return from the mixing of strings

The \( f^0(s) \) function returns null string, it is being used just for the returning of recursive function for \( f^1(s) \).

Next for \( f^1(s^2) \) results:

\[ f^1(s^2) = \{ f^0(s^2), \ f(1, s^3) \ f^0(0) \} \text{ for } |s^2| = 1 \quad (6) \]

For \( f^2(s^2) \):

\[ f^2(s^2) = \{ f^1(s^2), \ f(2, s^2) \ f^0(0) \} \text{ for } |s^2| = 2 \quad (7) \]

Making the replacements for \( s = |abc| \) results:

\[ f^3(abc) = \{ f^2(abc), \ f(3, abc) \ f^0(0) \} \]

exit for \( f(3, abc) \ f^0(0) = |abc| \)

\[ f^2(abc) = \{ f^1(abc), \ f(2, abc) \ f^1(c) \} \]
\[ f(2, abc) \ f^1(c) = |ab| \ f^1(c) = |ab| \ \{ f^0(c), \ f(1, c) \}
\[ f^0(0) \} = |ab| \ f(1, c) \ f^0(0) = |ab|c| \]

exit = |abc|

\[ f^1(abc) = \{ f^0(abc), f(1, abc) \ f^2(bc) \} =
\[ = |a| \ f^1(bc) \]
\[ f^2(bc) = \{ f^1(bc), f(2, bc) \ f^0(0) \}
\[ |a| \ f^1(bc) = |a| \ \{ f^1(bc), |bc|f^0(0) \}
\[ = |a| \ f^1(bc) \ |a| |bc| f^0(0) \}

exit = |abc|
2.2.3 General formula of division

In general for $|s| = n$ we have:

$$f^n(s) = \{ f^{n-1}(s), f(n, s) f^0(0) \}$$

$$f^{n-1}(s) = \{ f^{n-2}(s), f(n-1, s) f^1(s^n) \}$$

$$f^{n-2}(s) = \{ f^{n-3}(s), f(n-2, s) f^2(s^{n-1}) \}$$

$$\vdots$$

$$f^3(s) = \{ f^2(s), f(3, s) f^3(s^3) \}$$

$$f^2(s) = \{ f^1(s), f(2, s) f^2(s^2) \}$$

$$f^1(s) = \{ f^0(s), f(1, s) f^1(s^1) \}$$

For any $x$ between $n$ and 1 we have the general formula:

$$f^x(s) = \{ f^{x-1}(s), f(x, s) f^{x-1}(s^{x-1}) \}$$  \hspace{1cm} (10)

with $|s| = n$.

2.2.3 Strings mixture formula

For two strings:

$$s_1 = a_1a_2\ldots a_n$$

$$s_2 = b_1b_2\ldots b_m$$

and notes:

$$x^0(s_1, s_2) = s_1s_2 = a_1a_2\ldots a_nb_1b_2\ldots b_m$$

$$x^1(s_1, s_2) = a_1a_2\ldots a_{n-1}ba_nb_1b_2\ldots b_m$$

$$x^2(s_1, s_2) = a_1a_2\ldots a_{n-2}b_1a_nb_2a_1b_3\ldots b_m$$

$$\vdots$$

$$x^k(s_1, s_2) = a_1a_2\ldots a_{n-k}b_1a_{n-k-1}b_2\ldots a_nb_{k+1}b_{k+2}\ldots b_m$$

For $k = n-1$ we achieve the start of string $s_j$

$$n-k = (n-1)-1 = a_{n-k} = a_{l}$$

$$k+1 = n-l+1 = n$$

Later we have the cases:

$$x^{n-1}(s_1, s_2) = a_1a_2b_2\ldots aNb_{n+1}\ldots b_m$$

for $n < m$

$$x^n(s_1, s_2) = a_1a_2b_2\ldots a_nb_{n+1}\ldots b_m$$

for $n = m$

$$x^{n+1}(s_1, s_2) = b_1b_2a_1a_2b_2\ldots b_{m-1}a_nb_n\ldots b_m$$

for $n > m$

2.2.4 General formula for shuffle

By generalization of the two phases of shuffle formula (10) and (11) we obtain the final formula of shuffle.

$$n = m$$

Shuffle

For string $s_1 = a_1a_2\ldots a_n$ we have:

$$f^n(s_1) = S_I$$

Shuffle

$$s_2 = b_1b_2\ldots b_m$$

We note with

$$X^0(s_1, s_2) = \{ x_0(s_1, s_j) \mid s_j \in S_1, s_j \in S_2 \}$$  \hspace{1cm} (12)

the mixture of all substrings from $S_1$ and $S_2$ obtained by function $x_0$. Also we extend the notation to $X^1, X^2, \ldots, X^{n+m-1}$ and we obtain the final formula:

$$x^k(f^n(s_1), f^n(s_2)) X^k(f^n(s_1), f^n(s_2)) \mid k = 0, 1, 2, \ldots, n+m-1$$

or a shorter notation

$$\text{SHUFFLE}(s_1, s_2) = \{ x^k(f^n(s_1), f^n(s_2)) \mid k = 0, 1, 2, \ldots, m-1 \}$$

The last formula is the computational formula for Shuffle with two strings $s_1$ and $s_2$.

3 Shuffle Implementation

The source code in Visual FoxPro for implementation of the function $f^n(s)$ is shown below. First we suppose that an element of a string $s$ has one char length:
FUNCTION fn
PARAMETERS sir_f,x,p
* sir_f=string created before
* elements of string sir_f are separated by "|"
* x= function order
* p = position in initial string to be process
PRIVATE sir_ramas,n
sir_ramas=SUBSTR(sir_ini,p,x1dp+1)
n=LEN(sir_ramas) && length of remaining string
IF x=0.and.LEN(sir_ramas)=0 && f[0](0)
exit from recursive function
SELECT sir_gen && exit file
APPEND BLANK
replace string_gen WITH sir_f+"|
RETURN ""
ENDIF
IF x=0 && f[0](s) = ""
RETURN ""
ENDIF

* generation of expression: f[x-1](s)
=fn(sir_f,x-1,p)

*generation of expression: f(x,s) f[n-x](s[x+1])
sir_f2=sir_f+"|"+substr(sir_ini,p,x) &&=
=f(x,s) = string created before
=fn(sir_f2,n-x,p+x)

RETURN

For the mixture of two strings with one char per symbol we use the code below.

PROCEDURE mixt_2strings
SELECT 1
USE sir_shuffle excl
ZAP
sir_1="abcd" && test string 1
sir_2="wuxyz" && test string 2
n=LEN(sir_1)
m=LEN(sir_2)
FOR i=0 TO n+m-1
sir_f=""
n1=n-i
m1=ABS(n1)+1
IF n1>0 then
** l - n1 = sir_1
sir_f=sir_f+left(sir_1,n1)
c=MIN(n,m+n1)
FOR j=n1+1 TO c
ENDFOR
sir_f=sir_f+substr(sir_2,c-n1+1,m-(c-n1))
ELSE && remaining sir_2
sir_f=sir_f+STR(sir_1,c+1,m-c)
ENDIF
ELSE
** l - m1 = sir_2
sir_f=sir_f+left(sir_2,m1)
c=MIN(m,n+m1)
FOR j=m1+1 TO c && n1+1dc=sir_2+ sir_1
sir_f=sir_f+STR(sir_1,j-m1,1)+STR(sir_2,j,1)
ENDFOR
IF c>=m && remaining sir_2
sir_f=sir_f+substr(sir_1,c-m1+1,n-(c-1))
ELSE && remaining sir_2
sir_f=sir_f+substr(sir_2,c+1,m-c)
ENDIF
ENDIF
SELECT sir_shuffle
APPEND BLANK
replace string_shuffle WITH sir_f
ENDFOR
RETURN

In production planning each symbol of strings is an execution phase of manufacturing process. Each symbol contain information about product (product code), operation to be performed in this phase (operation code) and the manufacturing series. In our implementation each symbol is a 32 characters length.
A product may have many execution phases. The execution phases will be done in a sequential manner. Execution phases of one product make a string for shuffle operation.
We have a number of strings equally with the number of products (for different products) or equally with the numbers of manufacturing sets (for one product manufactured in many sets). Each set have a unique number, a series number.
Each execution phase have a unique execution time and will be done on one type of manufacturing post. Each post type may have...
one or many work places. Moreover each work place has its own start time for effective execution.

By shuffle operation on all the strings results a set of string of possible configurations of execution phases. For planning we scan the configurations set and append the start time, final time and execution time for each phase. At last we detect the minimum execution time for each configuration.

4 Optimization

We implemented our model first for one product manufactured in many sets. Each string of execution phases has completed with the series number of execution set.

4.1 Eliminating duplicate configurations

After shuffle operation we obtained a large number of configurations set.

For a product with only 2 execution phases we obtained the below number of configurations:

<table>
<thead>
<tr>
<th>Sets number</th>
<th>Configurations set</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>768</td>
</tr>
<tr>
<td>4</td>
<td>245760</td>
</tr>
</tbody>
</table>

On a better look of configurations we observed duplicate configuration.

The first step in our optimization was to eliminate all duplicate configurations by a new scanning in configuration set. After we obtained:

<table>
<thead>
<tr>
<th>Sets number</th>
<th>Configurations set</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>90</td>
</tr>
<tr>
<td>4</td>
<td>2520</td>
</tr>
</tbody>
</table>

4.2 Reducing the configurations number

Even after elimination of duplicate configuration the number of configurations is high.

After the planning step we make a summary of configurations that have the same execution time. For a set of 90 configurations we obtained only 7 different execution times.

4.3 Fluent allocation of manufacturing posts

The next steps in planning optimization are related to use more parallelism and fluent distribution of phases to posts.

To increase the use time of work posts is useful to have more sequences with different series interlaced. For two phases and three series 001,002,003 the following sequences have long execution times:

003 002 001 001 002 003
002 001 000 003 001 002

Fig.1 Summary configuration 3 sets with 2 phases

In fig.1 the two execution phases are named TAI_BAGH and INDOI_BAGH. There a 3 sets (series) of the same product manufacturing.

By analyzing the configurations that have the same execution time we extract the following rule for reduce the configurations set.

In a real production planning one execution phase may be done on many work places. If exits many work place for the same type of execution phase, these phases for different series will be executed in parallel. So the order of these phases in configurations set may be anyone. That’s the rule we have extracted.

With this rule we obtained the reduction below:

<table>
<thead>
<tr>
<th>Phases</th>
<th>Sets</th>
<th>Configurations</th>
<th>Reduced configurations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>4</td>
<td>9</td>
</tr>
</tbody>
</table>
Sequences have the same characteristic, the series are embedded: 001 in 002 in 003 or 003 in 001 in 002.

Sequences like

001 002 003 001 002 003 or
002 001 003 002 001 003

have a minimum execution time. The last series are interlaced and increase the parallelism in distribution of phase to work posts.

We changed the code for the mixture of two strings:

\[ s_1 = a_1a_2...a_n \text{ and } s_2 = b_1b_2...b_n . \]

The new code returns first

\[ x^{(s_1, s_2)} = a_1b_1a_2b_2...a_nb_n b_{n+1}...b_m \]

and not:

\[ x^0 (s_1, s_2) = s_1s_2 = a_1a_2...a_nb_1b_2...b_m \]

In this manner we obtain first configurations with a minimum execution time.

After the last optimization we observed that the same configuration may have different execution times. The difference was in the distribution step. The high execution times are obtained by a distribution of a sequence of two phases of the same series with the same type of manufacturing post to different work places.

The last step in our optimization was to keep the same post assigned to a series how long it is possible. If the next phase of the same series may be executed on the same type of manufacturing post, this phase will be executed on the same post. This fact increases the fluent allocation of manufacturing posts and also the use time of work posts.

5 Modeling by matrix grammars

Modeling production planning by shuffle operations generates a high number of configurations.

In our point of view the shuffle can’t replace the parallelism. The linguistic model with shuffle may be replaced by a model with more parallelism. A model with matrix grammar is a better solution.

5.1 Classical Grammars

Grammars generate strings over an alphabet symbols. The meaning of symbols involves the meaning of strings. If symbols are program instructions the strings are lines of program and the grammar is used in compilers. If symbols are state of cells the strings are the evolutions of cells and the grammar is useful in bioinformatics. If symbols are actions in a fabrication process the strings are the fabrication process planning.

One use the definition of Chomsky grammars: a grammar is a construct

\[ G = (N, T, S, P), \]

where:

\( N \) is the nonterminal alphabet,
\( T \) is the terminal alphabet,
\( S \) the initial letter or axiom and
\( P \) the set of rewriting rules or productions.

The rewriting rules are on the form:

\[ A \rightarrow w, A \in N, w \in (N \cup T)^* \]

for free-context grammars.

Given \( w, v \in (N \cup T)^* \), an immediate or direct derivation (in 1 step) denoted:

\[ w \Rightarrow_G v \]

holds if and only if there exist \( u_1, u_2 \in (N \cup T)^* \) such that \( w = u_1\alpha u_2 \) and \( v = u_1\beta u_2 \) and there exist \( \alpha \rightarrow \beta \in P. \)

\[ \Rightarrow_G^* \] denotes the reflexive transitive closure and \[ \Rightarrow_G^+ \] the transitive closure, respectively of \[ \Rightarrow_G \].

The language generated by a grammar is defined by:

\[ L(G) = \{ w : S \Rightarrow_G^* w \text{ and } w \in T^* \} \]

In other words \( L(G) \) is the set of terminal strings generated by a process of sequential derivations starting from \( S \).

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Example 1.1
Let \( G = (N, T, S, P) \) be a grammar such that:
\[ N = \{ S, A, C \} \]
\[ T = \{ a, b, c \} \]
\[ P = \{ S \rightarrow abc, S \rightarrow aAbC, A \rightarrow aAb, A \rightarrow ab, C \rightarrow cC, C \rightarrow c \} \]
The language generated by \( G \) is the language:
\[ L = \{ a^n b^n c^k : n \geq 1 \text{ and } k \geq 1 \} \] (1)

1.4 Matrix grammars
In the previous grammars types (Chomsky grammars) derivation steps are made sequential (one occurrence of a nonterminal is rewritten) and in leftmost /rightmost fashion (extreme left/right derivation rule: the leftmost/rightmost nonterminal of string is rewritten first). In matrix grammars the derivations are made sequential or parallel, in one step many occurrences of the nonterminals are rewritten. A (simple) matrix grammar is a construct \( G_M = (N, T, S, M) \) where \( N, T, S \) are the same as in Chomsky grammars and \( M \) is a finite set of nonempty sequences(matrices)
\[ m_i : [r_1, r_2, \ldots , r_{ni}], ni \geq 1 \]
with context free rewriting rules:
\[ A_k \rightarrow w_k , A_k \in N, w_k \in (N \cup T)^* \]

A derivation in a matrix grammar is as follows: for every \( x, y \in (N \cup T)^* \),
\[ x \Rightarrow_{GM} y \]
if and only if there exist strings \( x_0, x_1, \ldots , x_{ni} \in (N \cup T)^* \) (intermediate sentential forms) such that \( x_0 = x, x_{ni} = y \), and for \( 1 \leq i \leq ni : \)
\[ x_{i-1} = u_{i-1} A u_{i-1}, x_i = u_{i-1} w_i u_{i+1}, u_{i-1}, u_{i+1} \in (N \cup T)^* \]
and there exist \( m_i : [A_1 \rightarrow w_{k_1}, A_2 \rightarrow w_{k_2}, \ldots , A_{ni} \rightarrow w_{k_{ni}}] \)

In a derivation step all the rules from matrix \( m \) are done and in a sequential manner, each rule has to be performed in leftmost fashion. The language generated by the matrix grammar \( G_M \) is defined usual as:
\[ L(G_M) = \{ w : S \Rightarrow^*_{GM} w \text{ and } w \in T^* \} \]

Despite of context-free rewriting rules the matrix grammars may generate context-sensitive languages
One notes with \( L_M \) the languages generated by matrix grammars with context-free rules without \( \lambda \) productions. \( L_M \) family includes context-free (CF) languages (every CF grammar is a matrix grammar with one single rule in every matrix \( m \)) and it is included in context-sensitive languages:
\[ L_2 \subseteq L_M \subseteq L_1 \]

Example 1.2
\( G_M = (N, T, S, M) \) is a matrix grammar such that:
\[ N = \{ S, A, B, C \} \]
\[ T = \{ a, b, c \} \]
\[ S = \{ S \} \]
\[ M = \{ m_1, m_2, m_3 \} \]
\[ m_1 : [S \rightarrow ABC] \]
\[ m_2 : [A \rightarrow aA, B \rightarrow bB, C \rightarrow cC] \]
\[ m_3 : [A \rightarrow a, B \rightarrow b, C \rightarrow c] \]

\( G_M \) generates the language:
\[ L(G_M) = \{ a^n b^n c^n : n \geq 1 \} \]

5.2 Vertical parallelism
For the grammar \( G_M \) the derivation tree for the string \( a^2 b^2 c^2 \) is shown in figure 2. The vertical dash lines show the vertical parallelism from the matrix grammar \( G_M \)

Fig 2. Derivation tree for \( a^2 b^2 c^2 \) in \( G_M \) of example 1.2
This derivation tree is like a tree structure of a product $P$ with three subassembly $A$, $B$ and $C$. The vertical parallelism between $A$, $B$ and $C$ is useful in modeling the parallel structure of the product $P$.

5 Conclusion
Modeling production planning by shuffle operations generates a high number of configurations.

Introduction of parallelism in production planning minimizes the execution times.

Our final conclusion is that the shuffle can’t replace the parallelism. The linguistic model with shuffle may be replaced by a model with more parallelism.

One must reconsider parallelism between the product parts and between the different series of the same product. In our point of view a model with matrix grammar is a better solution.

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