Applying mathematical programming elements to answer market needs: case studies of optimization of electrical power flow

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Abstract: This paper seeks to investigate the application of mathematical programming, considering it as a tool for optimal electrical power generation and management. Nowadays, observing signals of crisis in various countries, electrical power emerges not only as a commodity but as a valuable, renewable and sometimes rare resource. Modeling and studying electrical power systems with application of mathematical programming can produce alternatives for optimal management of resources and allow better consumer satisfaction, impacting positively as a typical marketing indicator.

Key-words: Mathematical Programming; Linear Programming; Energy flow in electrical energy networks and markets; Optimum Power Flow DC; Marketing theory.

1. Introduction

Energy markets are facing tough challenges in actual globalized economic scenarios. Increase of competition, transnational competitive participation of enterprises, growth of technology-based industries such computing and communications and many other business solutions need excellence for infrastructure support. As an indispensable infrastructure component, energy turns out to be a major concern when its supplying restrictions are observed.

Some facts, frequently noticed and analyzed by independent organizations, reflect such problematic market situation:

Unbalanced availability of generation, transmission and distribution systems result in new market problems, evolving to discussions about regulatory and legal aspects. This can be noticed from negotiations – some of them long and complex – involving countries, agencies and firms which have the ownership or legal rights of at least one of these industries. [4], [12], [6].

Increase on environmental worries, remarkably pointed out by arguments about use of some energy sources. Nuclear, gas, coal based and many others sources have been questioned over its efficiency, polluting potential and costs, resulting in a challenging problem of energy provision and consumption.[12], [9].

Integration of electrical systems, as an answer to strategic government or firms' needs.[6]

Search of renewable sources, due also to environmental and costs problems, as a repercussion of the above, resulting in a demand for research funding and market needs. [4], [11].

New marketing approach for energy: new economic models, adopted by many countries, transferred energy supply and provision to private enterprises, changing from original state-controlled model, modifying expressively this service for customer's point of view. [4], [11], [12], [6].

Emergence of new markets, as national economics arise, such as BRIC – Brazil, Russia, India and China – and local markets, with additional energy demands as basic infrastructure demand for its affirmation and growth. Some of these economic systems shows unexpected performance rates, causing various disturbances as environmental and social issues (migration, racial disputes, etc.) [15]

Factors like these changed energy supply from a technical problem to a business oriented (marketing) engineering problem, as market aspects are increasingly being taken into account by economic players such as those which deal with supply, distribution, commerce and regulation. Energy markets change from a government owned, centered and geographically constrained models to transnational, flexible, fragmented and open types, showing high level of complexity, as long as is considered critical, for reasons such as affirmed above.

In a previous work [5], we analyzed mathematical alternatives to address mathematical programming algorithms applied in order to optimize power flow in regular systems, concluding with demonstration of results that confirm algorithms efficiency for energy supply and dispatch to consumers. In this article, we reintroduce this evaluation, aiming now to observe it from customer's point of view, analyzing such mathematical programming application as an answer to real energy supply problems, constrained by the economical scenario discussed above.

2. Operational Research

2.1. Initial Considerations

According to [8], during the Second World War, a group of scientists were united in England to study strategy problems and the tactics associated to the country's defence. The objective was to decide about the most efficient use of the limited military resources. The call of this group's meeting is identified as the first operational research formal activity.

The positive results that were obtained by the British operational research team motivated the Americans to initiate similar activities. Even though the origin of the Operational Research is accounted to England, its propagation is due mainly to a group of scientists leaded by George B. Dantzig from the United States of America, drafted during the Second World War. The result of the effort involved in this research, which was concluded in 1947, was named *Simplex Method.*

A very important characteristic of operational research and which made the process of analysis and of decision easier was the usage of models. They allow the experimentation of the proposed solution. This means that, before a decision is implemented, it can be better evaluated and tested. The obtained economy and the experience that is acquired by this experimentation, justifies it usage.

In the beginning of the 50s, several areas began to appear, which are today collectively known as *mathematical programming*. With the linear programming the mathematical programming sub-areas grew rapidly, having a fundamental performance in this development. Among these sub-areas are the non-linear programming, which started around 1951 with the famous Karush-Kuhn-Tucker condition, commercial utilization, network flows, linear programming, integer programming, dynamic programming and stocking programming.

The *linear programming* is used to analyze models where the restrictions and the objective function are linear; the *integer programming* is applied in models that have integer variables (or discreet); the *dynamic programming* is used in models where the entire problem can be decomposed into smaller sub-problems; the *stocking programming* is applied in a special class of models where the parameters are described by probability functions; finally, the *non-linear programming* is used in models containing non-linear functions.

A characteristic that is present in almost of all the mathematical programming is that the optimum solution of the problem can not be obtained with only one step, having to be obtained iteratively. An initial solution is chosen (which usually is not an optimum solution). One algorithm is specified to determine, starting from it, a new solution that normally is superior to the preceding one. This step is repeated until the optimum solution is achieved (supposing it does exist).

2.2. Marketing

It is possible to use marketing theory to analyze this new competitive scenario in which energy supplying is a complex infrastructure problem, using mathematical programming algorithms analysis, proposing it as a tool for market problems.

As defined in [13] and [10] marketing is a managerial discipline which aims to add perceived value to a product or service. Perceived value is detailed in those same works as the set of benefits which a costumer identify in a product or service minus its apparent cost, which comprise every element he faces in order to receive such benefits.

Energy, by its turn, is classically considered as a commodity, items where price and distribution are mandatory criteria for purchase [4], [13], [10]. Thinking by this way, and even disregarding some discussions as continuity and quality of service (parameters used for supply quality measurement), the complex scenario stated in the introduction of this article is enough to show a special marketing application opportunity, as one could perceive energy cost and distribution as a complex issues related to its services, which are offered to a costumer after elaborate negotiations involving questions over source types, transmission, distribution and many others questions. If those specific parameters, as quality of service, are also observed, such complexity would result in a genuine marketing problem. But for the purpose of this article, marketing is an opportune evaluation discipline, if energy is just considered as a commodity.

Such competitive situations conform opportunities to see mathematical programming techniques, specially optimization algorithms, as it was marketing tools, where application could be done to calculate optimal conditions for overall costs components, resulting in marketing value study, as stated before.

2.3. Linear Programming

The general problem of the linear programming, according to [3], is used to optimize (maximize or minimize) a linear function of variables, called "objective function", which is subject to a succession of linear equations or inequalities, called restrictions. The formulation of the problem to be solved by linear programming follows some basic steps, as described below:

1. The basic objective of the problem should be defined, in other words, the optimization to be reached. For example, the profit's maximization, or performance, or social welfare, cost, loss, time minimization. This objective will be represented by an objective function, to be maximized or minimized.
2. For this function to be mathematically specified, the variables of decision involved should be defined. For example, number of machines, the area to be explored, and the classes of investment that are available, etc. Normally, it is expected that all these variables can assume only positive values.
3. These variables normally are subjected to a series of restrictions, usually represented by equations. For example, quantity of equipment that is available, size of the area to be explored, the capacity of a reservoir, nutritional requirements of a determined diet, etc.

All these expressions, however, should be according to the main hypotheses of the linear programming, in other words, all the relations between the variables should be linear. This implies in the proportionality of the quantities involved. This linearity characteristic can be interesting as for simplifying the mathematical structure involved, but prejudicial when representing non-linear phenomenon (for example, cost functions that are typically quadratic).

The canonical form of a linear programming problem is presented as followed:

$$Max. \quad c_1x_1 + c_2x_2 + ... + c_nx_n$$

$$\text{subject to} \begin{cases} a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n \le b_1 \\ a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n \le b_2 \\ ..... \\ a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n \le b_m \end{cases} \quad (1)$$

$$x_1, x_2, ..., x_n \ge 0$$

where $c_1x_1 + c_2x_2 + ... + c_nx_n$ represents a linear objective function to be maximized and can be expressed or represented by *z*. The coefficients $c_1, c_2, ..., c_n$ represent costs (known values) and $x_1, x_2, ..., x_n$ represent the decision variables; their values should be obtained by the solution, if the solution of the problem exists. The inequality $\sum_{j=1}^{n} a_{ij}x_j \le b_i$ represents the set of linear restrictions with $i \in \{1, 2, ..., m\}$ and $j \in \{1, 2, ..., n\}$. The set of all the coefficients a_{ij} make up the technological coefficients' matrix. And $x_1, x_2, ..., x_n \ge 0$ restriction guarantees that decision variables are not negatives.

2.4. The Simplex Method

According to [8], a procedure is a finite sequence of instructions and algorithm is a procedure that ends in a finite number of operations.

The simplex method through its iterative algorithm searches for the solution, if it exists, by the vertices of a viable region until it finds a solution which does not have better neighbors than itself. This is an optimum solution. The optimum solution may not exist in two cases: when a viable solution does not exist, due to incompatible restrictions; or when the maximum does not exist (or minimum), in other words, one or more variables can incline to the infinitive and the restrictions continue to be satisfied which gives a value without limits to an objective function.

The simplex algorithm stands out as one of the greatest contributions to the mathematical programming of the twentieth century. It is an extremely efficient general algorithm, as mentioned by [8], for the solution of linear systems and adaptable for computational calculus. Its functional comprehension will give a base for several other methods. Refuting this statement, Latoree quoting [1], declares that even though the simplex method is in practice very efficient, it presents exponential complexity, in other words, the number of iterations grows exponentially with the number of the problem's variables.

2.5. The Interior Points Methods (IPM)

The interior points methods had their recognition in 1984, when Karmarkar proposed an polynomial algorithm that requires ($n^{3,5}$L) arithmetic operations and (nL) iterations in the worse cases, assuring that the iterative process is of an order of 50 times more rapid than the simplex method [18]. Initially the performance of this method was very criticized by the scientific community, but the results present by (ADLER, 1989) quoted in [16], gave a new impulse to the development of this class of methods. The revolution of the interior points' methods, as many other revolutions, includes earlier ideas which are rediscovered or seen in a different way, together with genuine new ideas [18].

Given a solution's feasible region of a linear (or non-linear) programming problem, a interior point is that in which all the variables (coordinates) meet inside this region, named region of viable solutions.

The Karmarkar algorithm is significantly different from George Danzig's simplex method, which solves a Linear Programming Problem (LPP) starting from an extreme point along its limit to, finally, reach an optimum extreme point. The method that was projected by Karmarkar rarely visits the extreme points before the optimum point is reached, in other words, the algorithm finds viable solutions in the interior of the solution.

The IPM tries to find a solution in the center of the polygon, finding a better direction for the next move, in the direction to find an optimum solution for the problem. Choosing the correct steps, an optimum solution is reached after a few iterations.

Even though to find a direction of movement, the IPM approach requires a longer computing time than the traditional simplex method and a smaller number of iterations will be required by the IPM to reach an optimum solution. In this manner, the IPM approach has become a competitive tool with the simplex method and, for this reason, has attracted the attention of the optimization community.

Fig.1 illustrates how the two methods approach the optimum solution. In this example, the IPM algorithm requires approximately the same quantity of iterations than the simplex method. How ever, for a bigger problem, this method requires only a fraction of the numbers of repetitions demanded by the simplex method and, the IPM also works perfectly with non-linear problems.

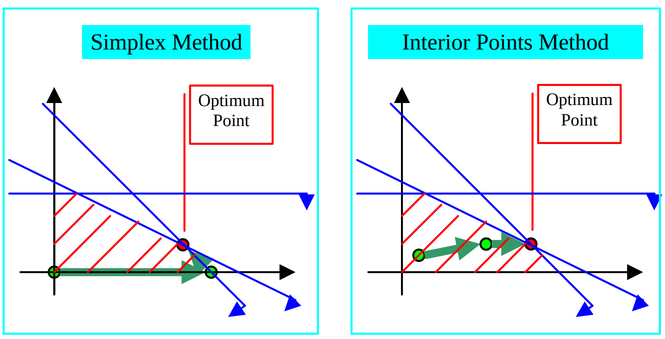

Fig.1 – Simplex Method X Interior Points Method

3. Optimum Power Flow

Electric energy has an important role in the society, from domestic, commercial usage to industrial usage. Knowing this, it is impossible conceive the lack of this important input in any kind of activity. That is the reason of the importance of these studies related to the improvement of the generation and transmission of this energy.

According to [14], up to the year of 1970, the final energy consumed from electrical energy in Brazil had less than 20% of participation of the final consumption. After the first petroleum crises in 1975, the percentage of the final consumption of electrical energy reached 22% and in 1999 it reached a percentage of 40%. It is important to remember that the hydroelectric plants are responsible for about 80% of the generation of electrical energy, as declared by Oliveira (1999), quoted in [14].

The Brazilian electrical energy generation system has characteristics that make it unique in the world:

1. Hydroelectric predominance;
2. Great geographic extensions and great distances between the generation sources and the main consumer centers.
3. Several potentials to be utilized in the same river;
4. Diversity of hydrologic and pluviometric regimes;
5. Relative high degree of interconnection between the systems (south/southeast/centre-west regions), in comparison with other countries;
6. Great unexplored hydroelectric potential.

With these characteristics, it is possible to notice the importance of integrated expansion planning e usage of the generation and transmission system, so that it can work in an optimized manner.

3.1 The Optimum Power Flow Problem

There are, according to [1], several feasible points for the correct performance of the electric power system (EPS), but some of the operational points are more advantageous than others, depending in the aspects in which they are evaluated. For example, to diminish the system's losses, it is possible to distribute uniformly the generation by the generation systems; on the other hand, to minimize the generation costs, it is advantageous that this distribution stops being uniform and starts to being concentrated in generators with lower costs.

To solve this problem, it is common to use the optimum power flow (OPF) where, by means of an objective function, it is possible to find an optimum performance point to satisfy one or more objectives, being the system subjected to physical, performance, reliability restrictions, among others.

According to [2] maintains that OPF problem was proposed by Carpentier in the beginning of the 60s, starting from the economic dispatch (ED) problem. Historically, the ED problem, solved by equal incremental costs, was the predecessor of the optimum power flow problem, which marked the end of the ED classical period, which had been studied and developed during 30 years. Thus, the ED problem started to be approached as an OPF private case.

According to [17], the methods for the solution of the OPF can be united in four big families: Linear Programming (LP), Kuhn-Tucker (KT), Gradiente (GR) and metric variables (MV). During the last three decades, the problem's solutions used theses different mathematical programming techniques.

According to [16], the OPF can be applied in several analysis problems and power operational systems, such as generation and transmission reliability, security analysis, generation and transmission expansion planning and short term generation programming.

In most of these applications, the linearized representation (DC) of the power flow has been adopted, due to its bigger simplicity and the degree of satisfactory precision of its results. In the Fig.2 the functional structure of the EPSs are presented.

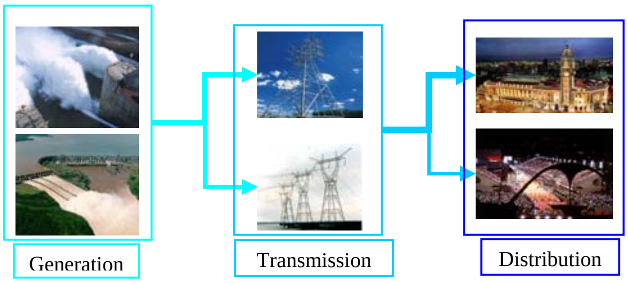

Fig. 2 – The EPSs' Functional Structure

According to [7], the functional structural components which are presented in the Fig.2 are:

- Generation: formed by generating plants or powerhouses. These powerhouses can be hydroelectric, thermal (coal, oil or natural gas) or nuclear. The hydroelectric powerhouses, generally, are located far from the consuming centers, making it necessary to have complex transmission systems and high tension.
- Transmission: constituted by the transmission auxiliary equipments which are needed to transmit the power produced in the generating powerhouses to the consumers' centers. The transmission systems can be of alternate current (AC) or continuous current (CC).
- Distribution: constituted by the substations and feeders which are responsible for the electric power distribution to the industrial, commercial and residence consumers.

The mathematical model of the OPF problem is represented by an optimizing problem formulated in the next section.

3.2 Formulation of the Optimum Power Flow Problem

The optimum power flow problem, as seen before, consists in determining the state of an electric network. It maximizes or minimizes an objective function while it satisfies a group of physical and operational restrictions.

The restrictions of equality correspond to the active and reactive power balance equations in each network's bus bar. The inequalities are functional restrictions, such as flow monitoring in lines and physical and operational limits of the system.

The Optimum Power Flow problem can be formulated mathematically and, generically, by:

$$\begin{aligned} \min.\ & f(x,u,p) \\ s.a.\ & g_i(x,u,p)=0, i=1,\dots,m \\ & h_j(x,u,p)\le 0,\ j=1,\dots,r \\ & x_{\min}\le x\le x_{\max}\ ,\ u_{\min}\le u\le u_{\max} \end{aligned} \tag{2}$$

where: $(x,u,p) \in R^n$ represents the state, control and disturbance variables respectively; *f(x,u,p)* represents the performance index of the system; $g(x,u,p)=0$ represents power flow equations; $h(x,u,p)\le 0$ represents functional restrictions, in other words, active and reactive power limits in the transmission lines and transformers, reactive power injection limits in the controlling tension bars and injection of active power in the reference bar; $x_{\min}\le x\le x_{\max}$ *e* $u_{\min}\le u\le u_{\max}$ represent limits on the state and controlling variables, respectively.

4. Mathematical Programming Applied in the Optimum Power Flow Problem DC

4.1 Initial Considerations

An electric power system has a series of controlling devices which have a direct influence on the operational conditions and, therefore, should be included in the modeling of the system so that it can correctly simulate its performance.

The table 1 lists a synthesis of the case that were investigated, the network that was used, the problem in question, the objective function, the problem's restrictions and the optimizing method that was used.

Table 1 – Synthesis of the case to be investigated

Case	**Network**	**Investigated Problem**	**Objective Function**	**Restrictions**	**Optimizing Method**
I	6 bars 7 lines	Congestion Management	Minimum Load Cut	Line Flowes	MatLab (LinProg)

Source: [5]

4.2. Congestion Management – Case I

This case involves congestion management with the minimum of load cut. The limits of the controlling variables are described in Table 4. The limits of the power flowes were reduced in 50%, aiming in creating situations of multiple congestions. To solve the problem, the MatLab toolbox optimization from LINPROG routine will be maintained.

The Fig.3 shows a unifilar diagram of the 6 bars system, being 2 generation bars, 3 of charge and one of 3 type, that is, one of reference and 7 lines, used to illustrate Case 1.

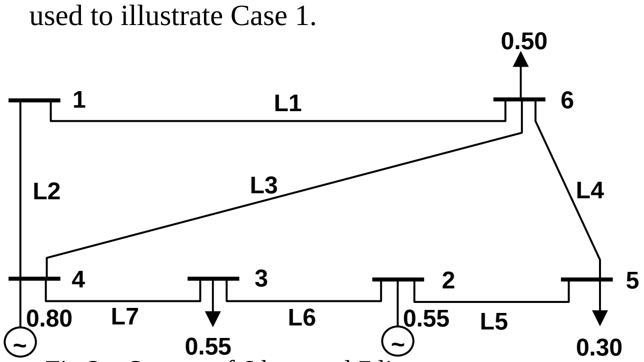

Fig.3 – System of 6 bars and 7 lines.

The data for the modeling, resolution and analysis of the problem, are found in the Tables 2 and 3. In the Table 2 the data about the lines are shown.

Table 2 – Data of the lines /Transformers for the 6 bars and 7 lines system

Lines	Initial Bar	Final Bar	X (pu)	Flows limits (pu)
L1	1	6	0.518	0.80
L2	1	4	0.370	0.80
L3	4	6	0.407	0.30
L4	5	6	0.300	0.18
L5	2	5	0.640	0.80
L6	2	3	1.050	0.90
L7	3	4	0.133	0.80

Source: [5]

In Table 3, the bar data are shown.

Table 3 – Bar Data for the 6 bars and 7 lines system

Bar	Bar Type		(+)P_G (pu) (-)P_D (pu)	$P_{G\,min}$ (pu)	$P_{G\,max}$ (pu)	Unitary Cost ($ / pu)
1	3	*Ref.*	-	-	-	-
2	1	PV	0.55	0	1	0.4798
3	2	PQ	-0.55	-	-	-
4	1	PV	0.80	0	1	0.6535
5	2	PQ	-0.30	-	-	-
6	2	PQ	-0.50	-	-	-

Source: [5]

Table 4 – Limits in the Controlling Variables

Controlling Variable	Inferior Limit (pu)	Superior Limit (pu)
P_{G2}	0.40	0.80
P_{D3}	-0.55	-0.35
P_{G4}	0.10	0.55
P_{D5}	-0.30	-0.25
P_{D6}	-0.50	-0.40

Source: [5]

The problem can be formuladed in an incremetal form:

$$\min \; f = [0.001 \;\; 0.5 \;\; 0.001 \;\; 0.5 \;\; 0.5] \,.\, [\Delta P_{G2} \;\; -\Delta P_{D3} \;\; \Delta P_{G4} \;\; -\Delta P_{D5} \;\; -\Delta P_{D6}]^T$$

s.a.

$$[1 \;\; -1 \;\; 1 \;\; -1 \;\; -1] . [0.8+\Delta P_{G2} \;\; (0.55+\Delta P_{D3}) \;\; 0.55+\Delta P_{G4} \;\; (0.3+\Delta P_{D5}) \;\; (0.5+\Delta P_{D6})]^T = 0$$

$$\begin{bmatrix} -0.80 \\ -0.80 \\ -0.30 \\ -0.18 \\ -0.80 \\ -0.90 \\ -0.80 \end{bmatrix} \le \begin{bmatrix} -0.4557 & -0.3183 & -0.3009 & -0.5394 & -0.5787 \\ -0.5443 & -0.6817 & -0.6991 & -0.4606 & -0.4213 \\ -0.0852 & 0.2146 & 0.2525 & -0.2679 & -0.3535 \\ 0.5409 & 0.1038 & 0.0484 & 0.8073 & -0.0678 \\ 0.5409 & 0.1038 & 0.0484 & -0.1927 & -0.0678 \\ 0.4591 & -0.1038 & -0.0484 & 0.1927 & 0.0678 \\ 0.4591 & 0.8962 & -0.0484 & 0.1927 & 0.0678 \end{bmatrix} \begin{bmatrix} P_{G2} \\ P_{D3} \\ P_{G4} \\ P_{D5} \\ P_{D6} \end{bmatrix} \le \begin{bmatrix} 0.80 \\ 0.80 \\ 0.30 \\ 0.18 \\ 0.80 \\ 0.90 \\ 0.80 \end{bmatrix}$$

$$\begin{bmatrix} 0.40 \\ -0.55 \\ 0.10 \\ -0.30 \\ -0.50 \end{bmatrix} \le \begin{bmatrix} 0.80+\Delta P_{G2} \\ -(0.55+\Delta P_{D3}) \\ 0.55+\Delta P_{G4} \\ -(0.30+\Delta P_{D5}) \\ -(0.50+\Delta P_{D6}) \end{bmatrix} \le \begin{bmatrix} 0.80 \\ -0.35 \\ 0.55 \\ -0.25 \\ -0.40 \end{bmatrix}$$

Fig.4 shows the controlling variables before and after the solution of the load shedding problem.

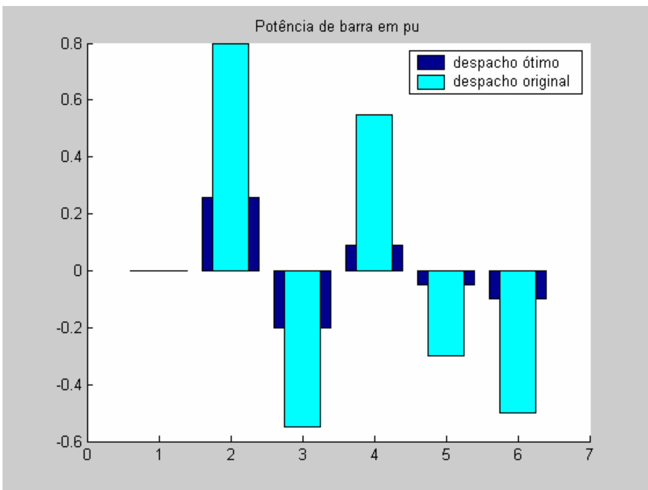

Fig.4 – Bar power

Source: [5]

The initial and final flowes are represented in Fig.5.

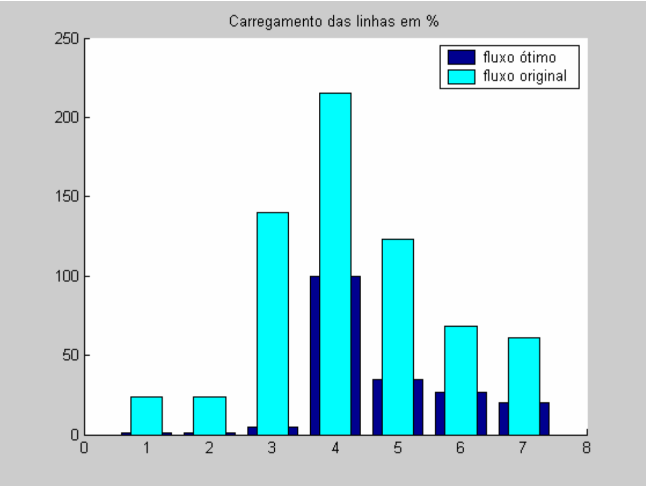

Fig.5 – Flows in the lines
Source: [5]

5. Final Considerations

This paper investigated the mathematic programming applications, more precisely, the linear programming to the linearlized optimum power flow problem.

Different methods of problem solutions in linear programming were presented and discussed. The interior points method was reported as being indicated for lager problems. The Simplex algorithm showed itself to be adequate for smaller and medium size networks.

The optimum power flow problem, in its several formulations, was examined and its objective functions and typical problems' restrictions were described.

Formulations of Network Congestion problems, in its linear version, were presented and numerical demonstrative examples were also presented.

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