

# Optimal Power System Stabilizer Based Enhancement of Synchronizing And Damping Torque Coefficients

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**Abstract:-** Design of power system stabilizer for enhancement power system stability is proposed. The effect of the proposed PSS on the synchronizing and damping torque coefficients is proved. To study the effectiveness of the proposed linear quadratic regulator (LQR) power system stabilizer, a sample power system in a linearized model is simulated and subjected to different operating conditions. The Linear Quadratic Gaussian (LQG) power system stabilizer is proposed. The power system dynamic responses after applying a variety of operating points with the proposed LQG-PSS stabilizer are plotted. The output of such stabilizer is fed directly to the automatic voltage regulator (AVR) of the synchronous machine. The input to such stabilizer are four state variables, two are accessible which are the deviation of the speed ( $\Delta\omega$ ) and rotor angle ( $\Delta\delta$ ). The other two inaccessible states that are the deviation of the Voltage proportional to q-axis flux linkage ( $\Delta E'_q$ ) and the Generator field voltage ( $\Delta E_{fd}$ ). An observer has been designed to access the two inaccessible states. Further, the effect of connecting the proposed power system stabilizer on synchronizing and damping torque coefficients is tabulated. A comparison between the effect of the power system stabilizer based on either LQR approach, the proposed LQG stabilizer in terms of either power system responses or its eigenvalues due to different load condition is reported.

**Key-Words :** Power Systems, LQR Control, LQG Power System Stabilizer.

## List of symbols:

$\Delta\delta$	Is the rotor angle (Rad.)
$\Delta\omega$	Is the rotor speed (pu.)
$P_m$	Is the mechanical power (pu.)
$D$	Is the self damping coefficient (pu.)
$M$	Is the inertia constant in (s)
$\Delta e'_q$	Is the quadrature axis voltage(pu.)
$\Delta E_{fd}$	Is the excitation voltage (pu.)
$X_d$	Is the direct axis reactance (pu)
$X'_d$	Is the transient direct axis reactance (pu)
$T'_{do}$	Is the direct axis short circuit time constant (s)
$T_A$	Is the exciter time constant (s)
$K_A$	Is the exciter gain (pu.)
$K$	Is the optimal control gain
$A$	Is the system matrix

$X$  Is the state variables

$B$  Is the input matrix

## 1 Introduction

Improved dynamic stability of power system can be achieved through utilization of supplementary excitation control signal [1-4]. The controller which generates this supplementary signal is called power system stabilizer PSS. The conventional lead-lag power system stabilizer PSS is widely used. Other types such as proportional-integral and proportional-integral-derivative PSSs have been proposed [5-6]. However, the gain settings of these stabilizer are determined based on the linearized model of the power system around a nominal operating point to provide optimal performance at this point. While the power systems are highly non-linear and the operating conditions can vary over a wide range as a

result of load changes , line outages, and unpredictable major disturbances such as three-phase faults. Consequently , these stabilizers no longer ensure the optimal performance . Alternative adaptive control techniques have been proposed to overcome such problems [7-10] .

However, most adaptive controllers are designed on the basis of a linear model and parameter identification of the system model in real-time which is a time consuming task. The phenomenon of stability of synchronous machine has received a great deal of attention in the past and will receive increasing attention in the future Small signal stability analysis of power systems becomes more important nowadays. Under small perturbations, this analysis is to predict the low frequency electromechanical oscillations resulting from poorly damped rotor oscillations. These oscillations stability becomes a very important issue as reported in [11]. The operating conditions of the power system are change with time due to the dynamic nature, so it is need to track the system stability on-line. To track the system, some stability indicators will be estimated from given data and these indicators will be updated as new data received.

Synchronizing torque coefficients  $K_s$  and damping torque coefficients  $K_d$  are used as stability indicators. To achieve stable operation of the machine, both  $K_s$  and  $K_d$  must be positive [12,16]. Certain techniques have been proposed to estimate the value of  $K_s$  and  $K_d$  which involved optimization technique. Some techniques have been explored by means of frequency response analysis decomposes the change in electromagnetic torque into two orthogonal components in the frequency domain. The two equations are expressed in terms of the load angle deviation then solved directly.

The present paper uses LQR and LQG control approach to design a power system stabilizer[9]. An expression for synchronizing and damping torque coefficients with robust controller is established.

## 2 Studied Power System Modeling

The linearized model of the studied power system consisted of synchronous machine connected to infinite bus bar through transmission line is represented in a block diagram as shown in Fig.1. Its state space formulation can be expressed as follows [10]:

$$\Delta \dot{\delta} = \omega_o \Delta \omega \tag{1}$$

$$\Delta \dot{\omega} = \frac{1}{M} (-K_1 \Delta \delta - D \Delta \omega - K_2 \Delta E' q) \tag{2}$$

$$\Delta \dot{E}'_q = \frac{1}{T'_{do}} (-K_4 \Delta \delta - \frac{\Delta E' q}{K_3} + E_{FD}) \tag{3}$$

$$\Delta \dot{E}_{FD} = \frac{1}{T_A} (-K_A K_5 \Delta \delta - K_A K_6 \Delta E' q - \Delta E_{FD} + K_a u) \tag{4}$$

In a matrix form as follows:

$$\dot{X}(t) = AX(t) + Bu(t) \tag{5}$$

where,

$$A = \begin{bmatrix} 0 & \omega_o & 0 & 0 \\ \frac{-k_1}{M} & \frac{-D}{M} & \frac{-k_2}{M} & 0 \\ \frac{-k_4}{T'_{do}} & 0 & \frac{-1}{k_3 T'_{do}} & \frac{1}{T'_{do}} \\ \frac{-k_A k_5}{T_A} & 0 & \frac{-k_A k_6}{T_A} & \frac{-1}{T_A} \end{bmatrix}$$

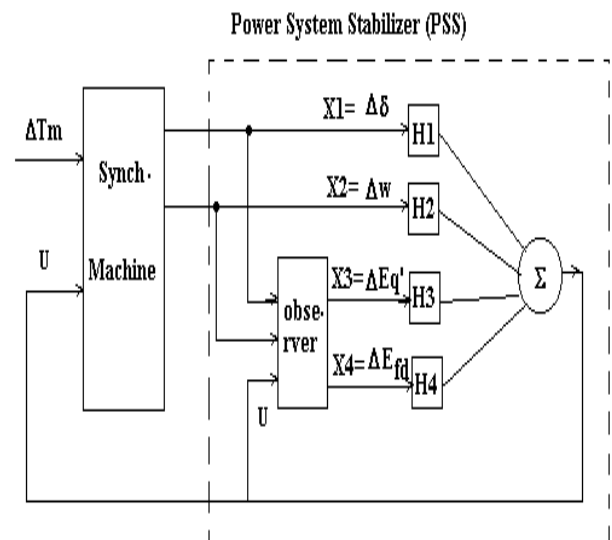


Fig. 1: Block diagram of power system under study

$$, B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K_A}{T_A} \end{bmatrix}, X = \begin{bmatrix} \Delta\delta \\ \Delta\omega \\ \Delta E'q \\ \Delta E_{FD} \end{bmatrix} \quad (6)$$

The parameters and coefficients of A matrix and B vector are defined in ref. [10]. The implementation of the state feedback control law for the infinite bus synchronous machine connected to power system stabilizer PSS and reduced-order observer is shown in Fig. 2

### 3 Reduced- Order Observer Design

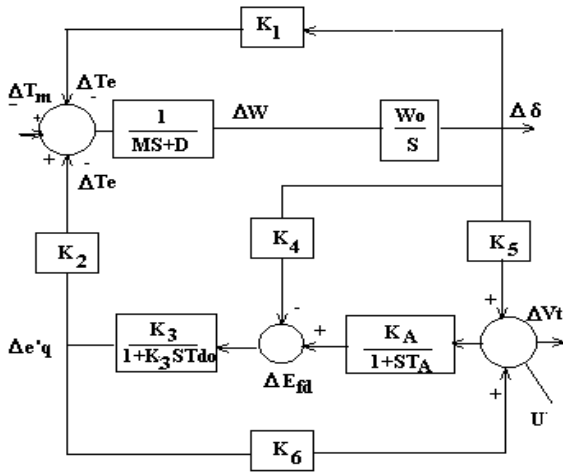


Fig.2: Block diagram of the synchronous machine connected to the power system stabilizer

The reduced-order observer feedback and gain parameters which are the F, G and H matrices are calculated for the given power system as follows [13]

$$\dot{X} = AX + BU$$

$$\begin{bmatrix} \dot{X} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} X \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} U \quad (7)$$

Where;

$$A_{11} = \begin{bmatrix} 0 & \omega_o \\ -\frac{K_1}{M} & -\frac{D}{M} \end{bmatrix}, A_{12} = \begin{bmatrix} 0 & 0 \\ -\frac{K_2}{M} & 0 \end{bmatrix},$$

$$A_{21} = \begin{bmatrix} -\frac{K_4}{T_{do}} & 0 \\ -\frac{K_A K_5}{T_A} & 0 \end{bmatrix}, A_{22} = \begin{bmatrix} -1 & 1 \\ -\frac{K_A K_6}{T_A} & -1 \end{bmatrix}$$

and

$$B_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{and} \quad B_2 = \begin{bmatrix} 0 \\ \frac{K_A}{T_A} \end{bmatrix}$$

The observer gains depends upon the above system and can be written as

$$L_r = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix}$$

The observer feedback and gain are :

$$F = A_{22} - L_r A_{12} \quad (8)$$

$$G = FL_r - L_r A_{11} + A_{21} \quad (9)$$

$$H = B_2 - L_r B_1 \quad (10)$$

Substituted by the values of  $A_{11}$ ,  $A_{12}$ ,  $A_{21}$  and  $A_{22}$  in Eq.(8), then calculate the matrix F as follows:

$$F = \begin{bmatrix} -\left(\frac{1}{K_3 T_{do}} - \frac{K_2 L_{12}}{M}\right) & \frac{1}{T_{do}} \\ -\left(\frac{K_A K_6}{T_A} - \frac{K_2 L_{22}}{M}\right) & -\frac{1}{T_A} \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \quad (11)$$

Also, substitute by F,  $L_r$ , A in Eqn.(9) to find G as follows:

$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}, \text{ where;}$$

$$G_{11} = -L_{11} \left( \frac{1}{K_3 T_{do}} - \frac{K_2 L_{12}}{M} \right) + \frac{L_{21}}{T_{do}} + \frac{L_{12} K_1}{M} - \frac{K_4}{T_{do}}$$

$$G_{12} = -L_{12} \left( \frac{1}{K_3 T_{do}} - \frac{K_2 L_{12}}{M} \right) + \frac{L_{22}}{T_{do}} - L_{11} \omega_o$$

$$G_{21} = -L_{11} \left( \frac{K_A K_6}{T_A} - \frac{K_2 L_{22}}{M} \right) - \frac{L_{21}}{T_A} + \frac{L_{22} K_1}{M} - \frac{K_5 K_A}{T_A}$$

$$G_{22} = -L_{12} \left( \frac{K_A K_6}{T_A} - \frac{K_2 L_{22}}{M} \right) - \frac{L_{22}}{T_A} - L_{21} \omega_o$$

Similarly, substitute by B, L in Eq.(10) to find H as:

$$H = \begin{bmatrix} 0 \\ \frac{K_A}{T_A} \end{bmatrix} \quad (12)$$

The reduced-order observer differential equations are:

$$\begin{aligned}\dot{Z} &= FZ + GY + Hu \\ \hat{X} &= L_r Y + Z\end{aligned}\quad (13)$$

Equation (13) is rewritten as follows:

$$\begin{bmatrix} \dot{Z}_1 \\ \dot{Z}_2 \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} + \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{K_A}{T_A} \end{bmatrix} U$$

Also, the estimator of the observer equation can be written as:

$$\begin{bmatrix} \hat{X}_3 \\ \hat{X}_4 \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} + \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix}$$

### The Observer Gain Feedback

$A_{obs} = A_{22}^T$ ,  $B_{obs} = A_{12}^T$   
and using Matlab function [ *PLACE* ] to calculate observer gain feedback [  $L_r$  ], where,

$$L_r = place(A_{obs}, B_{obs}, P_{obs}) \quad (14)$$

## 4 LQR Power System Stabilizer

The feedback gain of the closed loop system design by The design of the power system stabilizer based on linear-quadratic regulator LQR control for continuous-time systems as follow:

$$[K, S, E] = lqr(A, B, Q, R, N) \quad (15)$$

calculates the optimal gain matrix K such that the state-feedback law  $u = -Kx$  minimizes the cost function

$$J = \text{Integral} (X' Q X + u' R u + 2X' N u) dt$$

Subject to the state dynamics

$$\dot{X} = AX + Bu.$$

The matrix N is set to zero when omitted. Also returned are the Riccati equation solution S and the closed-loop eigenvalues E:

$$SA + A'S - (SB + N) R^{-1} (B'S + N') + Q = 0, \quad (16)$$

$$E = \text{EIG} (A - B * K)$$

$$K = [H_1 \ H_2 \ H_3 \ H_4].$$

## 5 LQG Power System Stabilizer

The optimal control technique was presented for designing linear regulator that minimized a quadratic objective function. Using a separation principle, we can combine the optimal regulator with the optimal observer (the kalman filter), result the LQG compensator. In other words, if LQR control and the observer using Kalman filter are designed, the resulting system is referred to as Linear Quadratic Gaussian (LQG) Control or LQG-compensator. In short, the optimal compensator design process is the following [14,15]:

- Design an optimal regulator for a linear plant using full-state feedback. The regulator is designed to generate a control input,  $u(t)$ , based upon the measured state-vector,  $X$ .
- Design a Kalman filter for the plant assuming a known control input,  $u(t)$ , a measured output,  $y(t)$ , and white noises,  $V$  &  $Z$ . The Kalman filter is designed to provide an optimal estimate of the state vector,  $X$ .
- Combine the separately designed optimal regulator and Kalman filter into an optimal compensator (LQG), which generates the input vector,  $u(t)$ , based upon the estimated state-vector,  $X_o$ , rather than the actual state-vector,  $X$ , and the measured output,  $y(t)$ .

Since the optimal regulator and Kalman filter are designed separately, they can be selected to have desirable properties that are independent of one another. The overall closed-loop eigenvalues consist of the regulator eigenvalues and the kalman filter eigenvalues, (i.e. for the 4<sup>th</sup> order system, there is 8 closed-loop eigenvalues, and for the 6<sup>th</sup> order system, there is 12 closed-loop eigenvalues.). The closed-loop system's performance can be obtained as desired by suitably selecting the optimal regulator's weighting matrices Q & R, and the kalman filter's spectral noise densities, V & Z. Hence, the matrices Q, R, V, and Z are the design parameters for the closed-loop system with an optimal compensator. For the noisy plant with the following state-space representation :

$$\dot{X} = AX + BU + V$$

$$Y = CX + DU + Z$$

Where; A is a system matrix, B is the input vector, C is the output vector, V is the process noise

spectral density matrix, and  $Z$  is the measurement noise spectral density matrix, the state-space realization of the optimal compensator is given by the following state and output equations as in [11] :

$$\begin{aligned} \dot{X}_o &= (A - BK - LC + LDK)X_o + LY \\ U &= KX_o \end{aligned}$$

Where:

$K$  and  $L$  are the optimal regulator and Kalman filter gain matrices, respectively, and  $X_o$  is the estimated state vector. Figure 3 shows the block diagram of the optimal LQG-compensator

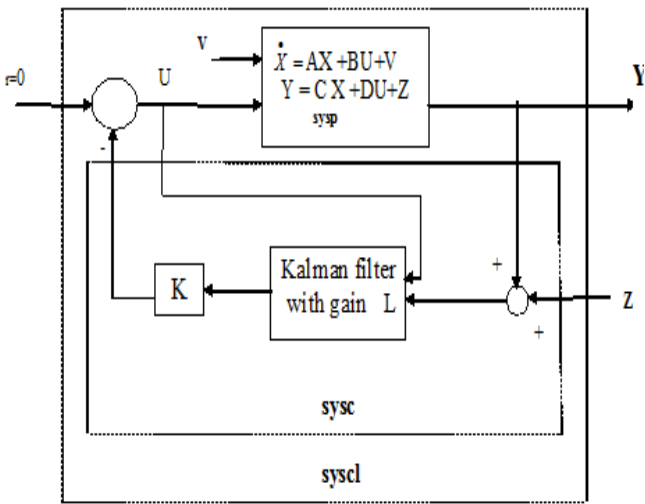


Fig. 3: Block diagram of the optimal LQG-compensator.

Using MATLAB's Control System Toolbox, a state-space model of the regulating closed-loop system, *syscl*, can be constructed as follows :

$$\begin{aligned} [L, P, E] &= lqe(A, F, C, V, Z) \\ sysp &= ss(A, B, C, D) \\ sysc &= ss(A - BK - LC + LDK, \\ &L, K, zeros(size(D'))) \\ syscl &= feedback(sysp, sysc) \end{aligned} \quad (17)$$

Where :

*sysp* : is the state-space model of the plant,

*sysc* : is the state-space model of the LQG compensator, and *syscl* : is the state-space model of the closed loop system

## 6 Effect of LQR-PSS On Synchronizing and Damping Torque Coefficients

Fig. (1) shows the block diagram of sample power system under study. From this figure, the electric torque relation can be expressed as follows [12]:

$$\Delta T_e = K_1 \Delta \delta + K_2 \Delta E'_q \quad (18)$$

where;

$$\begin{aligned} \Delta E'_q &= \frac{K_3}{1 + ST} \{-K_4 \Delta \delta - \Delta E_{fd}\} \\ T &= K_3 T'_{do} \end{aligned} \quad (19)$$

By adding the LQR control signal defined in equation (15), the feedback gain  $K$  defined as:

$$K = [H_1 \quad H_2 \quad H_3 \quad H_4]$$

Using thyristor type excitation, where, its transfer function can be expressed by

$$G_e = \frac{K_A}{1 + ST_A}$$

where;  $K_A$  is the excitation gain ,  $T_A$  is time constant of excitation system .

The following equation is obtained

$$\Delta E_{fd} = G_e (K_5 \Delta \delta + K_6 \Delta E'_q + u)$$

Substituting  $u$  by equation (9) yields the following

$$\Delta E_{fd} = \frac{K_A}{1 + ST_A} \left\{ \begin{aligned} &K_5 \Delta \delta + K_6 \Delta E'_q + H_1 \Delta \delta + \\ &H_2 \Delta \omega + H_3 \Delta E'_q + H_4 \Delta E_{fd} \end{aligned} \right\} \quad (20)$$

After some algebraic manipulation the following equation is obtained

$$\therefore \Delta E_{fd} = \frac{K_A}{1 + ST_A} \left\{ \begin{aligned} &(K_5 + H_1) \Delta \delta + (K_6 + H_3) \Delta E'_q + H_2 \Delta \omega \\ &1 - \frac{K_A H_4}{1 + ST_A} \end{aligned} \right\} \quad (21)$$

and by substituting in equation (19), the following yields :

$$\Delta E_q' = \frac{K_3}{1+ST} * \left[ -K_4 \Delta\delta - \frac{K_A}{1+ST_A} \left\{ \frac{(K_5 + H_1)\Delta\delta + (K_6 + H_3)\Delta E_q' + H_2\Delta\omega}{\left(1 - \frac{K_A H_4}{1+ST_A}\right)} \right\} \right]$$

$$\Delta E_q' \{1 + A\} = -\frac{K_3}{1+ST} K_4 \Delta\delta - \frac{K_3}{1+ST} * \left( \frac{K_A}{1+ST_A} \right) \left\{ (K_5 + H_1)\Delta\delta + H_2\Delta\omega \right\} \left( 1 - \frac{K_A H_4}{1+ST_A} \right)$$

where;  $A = \frac{K_3}{1+ST} * \left( \frac{K_A}{1+ST_A} \right) * \left( 1 - \frac{K_A H_4}{1+ST_A} \right) (K_6 + H_3)$

then,

$$\therefore \Delta E_q' = \left( -\frac{K_3}{1+ST} K_4 \Delta\delta - \frac{K_3}{1+ST} * \left( \frac{K_A}{1+ST_A} \right) \left\{ (K_5 + H_1)\Delta\delta + H_2\Delta\omega \right\} \right) / (1 + A) \quad (22)$$

Substitute  $\Delta\omega = S\Delta\delta$ . Where, S is the Laplace transform factor . The relation between  $\Delta E_q'$  and  $\Delta\delta$  can be estimated

Substitute the value of  $\Delta E_q'$  in equation (18), the deviation of the electrical torque as a function of delta deviation can be obtained as follows:

$$\Delta T_e = \left\{ K_1 - \frac{a_o + S a_1}{(b_o + b_2 S^2) + S b_1} \right\} \Delta\delta$$

where;

$$a_o = K_2 K_3 (K_4 + K_5 K_A - K_4 K_A H_4 + K_A H_1)$$

$$a_1 = K_2 K_3 (K_4 T_A + K_A H_2)$$

$$b_o = (1 + K_3 K_6 K_A - K_A H_4 + K_3 K_A H_3)$$

$$b_1 = T + T_A - T K_A H_4$$

$$b_2 = T T_A$$

To determine the damping and synchronizing torque coefficient , the s factor is substituted by jw

$$\Delta T_e = \left\{ K_1 - \frac{a_o + jw a_1}{(b_o - b_2 w^2) + jw b_1} \right\} \Delta\delta$$

$$\Delta T_e = \left\{ K_1 - \frac{a_o (b_o - b_2 w^2) + w^2 a_1 b_1}{(b_o - b_2 w^2)^2 + w^2 b_1^2} \right\} \Delta\delta + \left\{ \frac{a_o b_1 - a_1 (b_o - b_2 w^2)}{(b_o - b_2 w^2)^2 + w^2 b_1^2} \right\} jw \Delta\delta$$

$$\Delta T_e = K_s \Delta\delta + K_d \Delta\omega$$

where;

$K_s$  is a synchronizing torque coefficient.  
 $K_d$  is a damping torque coefficient.

To evaluate the  $K_s$  and  $K_d$  over a wide range of input frequency oscillation, the following analysis is obtained.

### 6.1 $K_s$ and $K_d$ at lower frequency $\omega t \ll 1$ :

The synchronizing torque coefficient with LQR control can be expressed by the following equation:

$$K_s = K_1 - \frac{a_o}{b_o}$$

$$= K_1 - \frac{K_2 K_3 (K_4 + K_5 K_A - K_4 K_A H_4 + K_A H_1)}{(1 + K_3 K_6 K_A - K_A H_4 + K_3 K_A H_3)} \quad (23)$$

While the synchronizing torque coefficient without proposed LQR controller can be found by putting the H's equal zero .The following equation gives the synchronizing torque coefficient .

$$K_s = K_1 - \frac{K_2 K_3 (K_4 + K_5 K_A)}{1 + K_3 K_6 K_A} \quad (24)$$

Moreover, the damping torque coefficient can be calculated with proposed LQR controller as follows:

$$K_d = \frac{a_o b_1 - a_1 b_o}{b_o^2}$$

or;

$$K_d = \frac{K_2 K_3}{1 + K_3 K_6 K_A - K_A H_4 + K_3 K_A H_3}$$

$$* \left\{ \begin{array}{l} \frac{(K_4 + K_A K_5 - K_4 K_A H_4 + K_A H_1)(T + T_A - T K_A H_4)}{1 + K_3 K_6 K_A - K_A H_4 + K_3 K_A H_3} \\ -(K_4 T_A + K_A H_2) \end{array} \right\} \quad (25)$$

Without LQR controller, the damping torque coefficient can be written as follows:

$$K_d = \frac{K_2 K_3}{1 + K_3 K_6 K_A} \left\{ \frac{(K_4 + K_A K_5)(T + T_A)}{1 + K_3 K_6 K_A} - K_4 T_A \right\} \quad (26)$$

## 6.2 $K_S$ and $K_d$ at higher frequency $\omega t \gg 1$ :

The synchronizing torque coefficient at higher frequency with LQR controller can be calculated as :

$$K_S = K_1 + \frac{1}{\omega^2 b_2} (a_o - \frac{a_1 b_1}{b_2})$$

Without LQR controller, the  $K_S$  is determined by putting H's equal zero and the results can be obtained by

$$K_S = K_1 - \frac{K_2 K_3 K_4}{\omega^2 T^2}$$

Moreover, the damping torque coefficient  $K_d$  with LQR controller is obtained as

$$K_d = -\frac{a_1}{\omega^2 b_2} = -\frac{K_2 K_3 (K_4 T_A + K_A H_2)}{\omega^2 T T_A}$$

Without LQR controller, the  $K_d$  is expressed by

$$K_d = -\frac{K_2 K_3 K_4}{\omega^2 T}$$

## 7 Digital Simulation Results

Choosing the machine parameters at nominal operating point as

$$\begin{aligned} X_d &= 1.6; X_q = 1.55; X_d' = 0.32; X_e = 0.4 p.u. \\ \therefore M &= 10; w_o = 377; T_{do}' = 6; D = 0; \\ P &= 1 p.u.; Q = 0.25 p.u.; K_A = 25; T_A = 0.06s \end{aligned}$$

The design of the proposed LQR PSS is established as follows,

The nominal A matrix given in eqn.(6) is evaluated as :

$$A_n = \begin{bmatrix} 0 & 377 & 0 & 0 \\ -.1317 & 0 & -.1104 & 0 \\ -.2356 & 0 & -.463 & .1667 \\ 15.47 & 0 & -194.81 & -16.667 \end{bmatrix}$$

and also vector B is obtained as follows

$$B = [0 \ 0 \ 0 \ 25/0.06]^T$$

Referred to Eq.(14), the observer gains is calculated as follows after choosing the eigenvalues of reduced-order observer  $P_{obs} = [-8+j6, -8-j6]$ , then calculate the observer gains  $L_r$  as follows:

$$L_r = \begin{bmatrix} 0 & 0 \\ 10.2 & -4272.7 \end{bmatrix}$$

After calculates observer gain the  $L_r$ , then substituted in Eq.(8), Eq.(9) and Eq.(10)

$$F = \begin{bmatrix} 0.6667 & 0.1667 \\ -666.56 & -16.667 \end{bmatrix},$$

$$G = \begin{bmatrix} 1 & -705 \\ -547 & 64392 \end{bmatrix}, \text{ and}$$

$$H = \begin{bmatrix} 0 \\ 416.667 \end{bmatrix}$$

The assumption in Eqn.(15) are matrix  $N=0$  and matrix  $R=50$ , and Q as follow:

$$\text{matrix } Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & .001 \end{bmatrix},$$

Solving Eqn. (15), the Riccati equation S is:

$$S = \begin{bmatrix} 1.8 & 0 & 1.5 & 0 \\ 0 & 5329 & -34.4 & 0 \\ 1.5 & -34.4 & 1.9 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The matrix S is a positive definite  
Also, the optimal gain matrix K is calculated as:

$$K = [H_1 \ H_2 \ H_3 \ H_4].$$

$$K = [0.1217 \ -0.1090 \ 0.1292 \ 0.0015] \quad (27)$$

From LQG design assume the  $V=1.73 \times 10^7$  and  $Z=0.01$ . The output LQG Kalman gain vector  $L=10^7[0 \ 0 \ -0.0065 \ -1.05]^T$ . Fig.4 depicts the power system model in a block diagram with different power system stabilizers for comparison. Using the value of feedback gain vector  $K$  obtained in equation (27), the dynamic stability of the linearized studied power system subjected to load disturbances is simulated on the computer using Matlab program Package.

Figs. 5,6,7 and 8 show the speed deviation responses due to load disturbances of 0.1 p.u. without and with PSSs (either based on conventional LQR approach and proposed LQG - PSS at different operating points. To validate the effectiveness of the proposed LQR- PSS. The digital simulation results prove the robustness of the proposed PSS in terms of fast damping with system uncertainties defined in parameter and operating point changes. Table 1 displays the synchronizing and damping torque with and without LQR Controller at low frequency  $\omega t \ll 1$ . Also, Table 2 depicts the synchronizing and damping torque with and without LQR Controller at high frequency  $\omega t \gg 1$ . Table 3 display the eigenvalues analysis at different controller and different operating conditions.

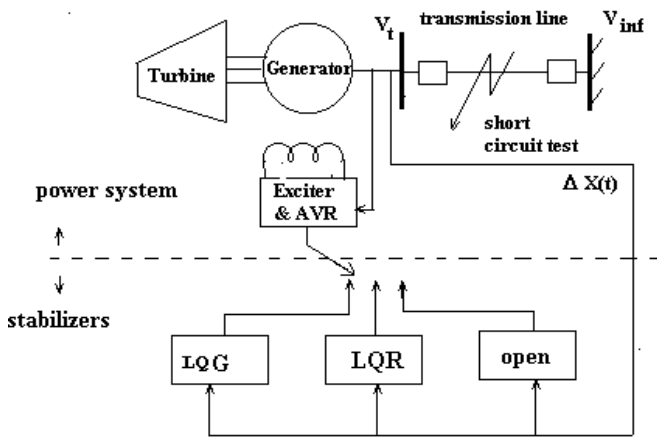


Fig. 4: Schematic diagram of power system model with different power system stabilizers

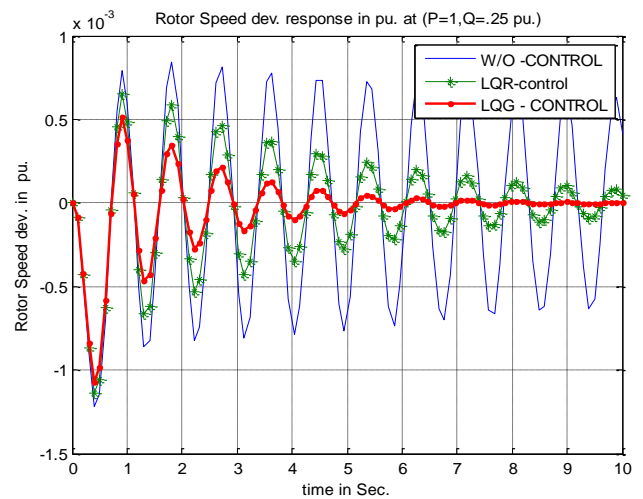


Fig. 5: speed deviation responses due to 0.1 p.u load disturbances with and without PSSs. At normal

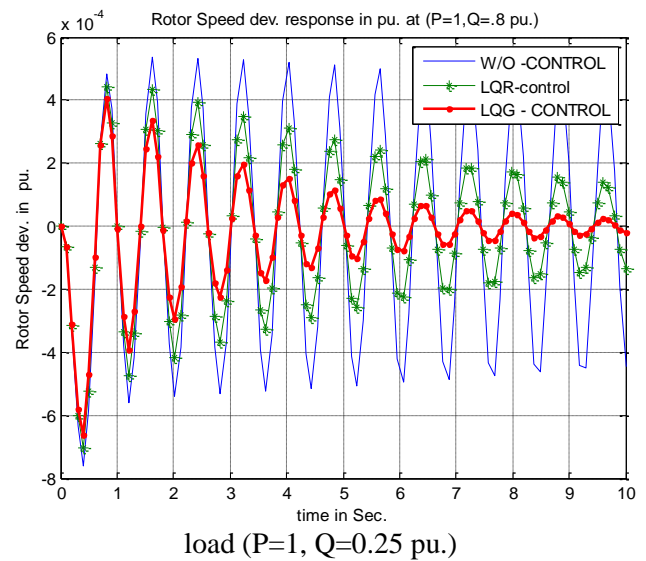


Fig. 6: speed deviation responses due to 0.1 p.u load disturbances with and without PSSs. At heavy load (P=1, Q=0.8 pu.)



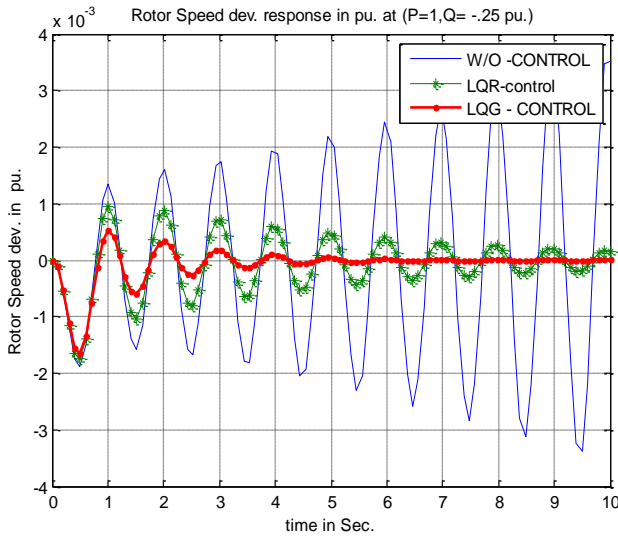


Fig. 7: speed deviation responses due to 0.1 p.u load disturbances with and without PSSs. At lead power factor load (P=1, Q= - 0.25 pu.)

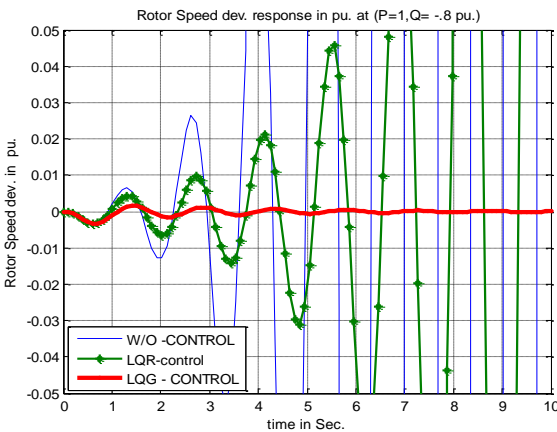


Fig. 8: speed deviation responses due to 0.1 p.u load disturbances with and without PSSs. At heavy lead power factor load (P=1, Q= - 0.8 pu.)

Table 1: Synchronizing And Damping Torque With And Without LQR Controller at  $wt \ll 1$

load	KS		Kd	
	Without control	With LQR	Without control	With LQR
P=1, Q=0.25 pu	1.27	1.09	0.0093	0.23
P=1, Q=0.8 pu	1.619	1.48	0.0026	0.16
P=1, Q=-.25 pu	1.206	0.905	-0.07 (un-stable)	0.32
P=1, Q=-.8 pu	1.86	0.99	-1.346 (un-stable)	0.07

Table 2: Synchronizing And Damping Torque With And Without LQR Controller at  $wt \gg 1$

load	KS		Kd	
	Without control	With LQR	Without control	With LQR
P=1, Q=0.25 pu	1.316	1.96	0.0012	0.036
P=1, Q=0.8 pu	1.62	2.123	-0.0067	0.027
P=1, Q=-.25 pu	1.03	1.832	-0.0017 (un-stable)	0.045
P=1, Q=-.8 pu	0.38	1.18	-0.0017 (un-stable)	-0.040 (un-stable)

Table 3  
The Eigenvalues Analysis At Different Controller With The Different Operating Point.

	Without control	With LQR control	With LQG control
P=1, Q=0.25 pu Normal load	-14.2951 -0.0363 + 7.074i -0.0363 - 7.0074i -2.7619	-14.3968 -0.2429 + 7.0208i -0.2429 - 7.0208i -2.8719	-14.5723 -0.5581 + 7.0172i -0.5581 - 7.0172i -3.0487 -34.2895 + 58.433i -34.2895 - 58.433i -67.7134 -0.0058
P=1, Q=0.8 pu Heavy load	-0.0146 + 7.8306i -0.0146 - 7.8306i -13.9757 -3.1246	-14.0302 -0.1384 + 7.8436i -0.1384 - 7.8436i -3.4475	-14.1276 -0.3253 + 7.8429i -0.3253 - 7.8429i -3.9630 -31.2950 + 53.363i -31.2950 - 53.363i -61.7440 -0.0098
P=1, Q= -0.25 pu Lead power factor load	-14.9002 +0.1021 + 6.32i +0.1021 - 6.327i -2.4336 (un-stable)	-15.0445 -0.2068 + 6.3348i -0.2068 - 6.3348i -2.2966	-15.2866 -0.6861 + 6.3549i -0.6861 - 6.3549i -2.0754 -36.6306 + 62.376i -36.6306 - 62.376i -72.3842 -0.0041
P=1, Q= -0.8 pu Lead power factor and heavy load	-15.7698 + 1.0279 + 4.64i +1.0279 - 4.64i -3.4157 (un-stable)	-15.9354 + 0.5611 + 4.50i +0.5611 - 4.50i -2.9415 (un-stable)	-16.2105 -0.3181 + 4.3091i -0.3181 - 4.3091i -1.8868 -36.6386 + 62.10i -36.6386 - 62.10i -72.3925 -0.0055

## 8 Discussions

From table 3 it is clear that system after adding stabilizers model that taken under study be more stable especially at normal, heavy and lead power factor load. These eigenvalues results are confirmed after obtaining figures which express the effect of stabilizers on synchronizing and damping torques, and this effect clear by obtaining positive damping torques after adding stabilizer. Tables 1,2 describe the synchronizing and damping torque coefficient with and without optimal LQR control.

At the load ( $P=1$ ,  $Q= -0.8$  pu.) , the damping torque is negative with the mean that the system with and without optimal control is unstable. Also, figure 8 shows this unstable system in both with and without LQR control but the system is stable with robust LQG controller.

## 9 Conclusions

Design of power system stabilizer PSS based on LQR control is introduced. The proposed LQG-PSS has robustness control property with power system operating points change, parameters variation and uncertainty. Moreover, an expression for both the synchronizing and damping torque coefficients of the studied power system with and without proposed LQR- PSS are derived and obtained. To validate the effectiveness of the proposed LQG-PSS, a simple power system consisting of synchronous generator connected to infinite bus through transmission line subjected to different input disturbances is simulated. A comparison between the dynamic power system responses using conventional LQR control and proposed LQG- PSS is reported. The digital results show the power of the proposed LQG-PSS in terms of improvement of damping of the power system oscillations. Also, improvement of eigenvalues of the system. The synchronizing and damping torque coefficients is improved with LQR control than the system without controller.

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