Modeling and Stability Analysis of AC-DC Power System with Controlled Rectifier and Constant Power Loads

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Abstract: - Power converters with their controls normally behave as a constant power load. This load can significantly degrade the system stability. Therefore, the dynamic model of the power system with CPLs is very important. It is well known that the power converter model is time-varying because of the switching behaviour. Several approaches are commonly used for eliminating the switching actions to achieve time-invariant model. The classical linear control theory can be easily applied to the model for a system analysis and design. The first method is the generalized state-space averaging (SSA) modeling method. This method has been used to analyze many power converters in DC distribution systems [5]-[7], as well as uncontrolled and controlled rectifiers in single-phase AC distribution systems [8],[9] and 6- and 12- pulse diode rectifiers in three phase systems [10]. The second is an average-value modeling method, which has been used for 6- and 12- pulse diode rectifiers in many publications [11]-[13], as well as generators with line-commutated rectifiers [14]-[18]. These rectifiers can be modeled with good accuracy as a constant DC voltage source. However, this method is not easily applicable to the analysis of the stability of the general AC power system with multi-converter power electronic systems.

Key-Words: - Constant power loads, DQ method, simulation, modeling, controlled rectifier, stability analysis

1 Introduction
Electrical loads based on power electronic converters are widely used in many applications. Unfortunately, power electronic driven loads often behave as a constant power load (CPL). The CPL can significantly degrade the system stability [1]-[4]. Therefore, the dynamic model of the power system with CPLs is very important.

It is well known that the power converter model is time-varying because of the switching behaviour. Several approaches are commonly used for eliminating the switching actions to achieve time-invariant model. Then, the classical linear control theory can be easily applied to the model for a system analysis and design. The first method is the generalized state-space averaging (SSA) modeling method. This method has been used to analyze many power converters in DC distribution systems [5]-[7], as well as uncontrolled and controlled rectifiers in single-phase AC distribution systems [8],[9] and 6- and 12- pulse diode rectifiers in three phase systems [10]. The second is an average-value modeling method, which has been used for 6- and 12- pulse diode rectifiers in many publications [11]-[13], as well as generators with line-commutated rectifiers [14]-[18]. These rectifiers can be modeled with good accuracy as a constant DC voltage source. However, this method is not easily applicable to the analysis of the stability of the general AC power system with multi-converter power electronic systems.

Another technique widely used for AC system analysis is that of DQ-transformation theory [19]-[21], in which power converters can be treated as transformers. The DQ modeling method can also be easily applied for modeling a power system comprising vector-controlled converters where the SSA model and the average-value model are not easily applicable. Moreover, the resulting converter models can be easily combined with models of other power elements expressed in terms of synchronously rotating frames such as generators, front-end converters, and vector-controlled drives. The DQ models of three-phase AC-DC power systems have been reported in the previous works [19]-[21]. But these do not include a constant power load (CPL). Applying the DQ modeling approach for stability studies of the power system including a CPL has been addressed in [22]-[24]. The DQ method for modeling the three-phase uncontrolled and controlled rectifier has been reported in [22] and [25], respectively. This paper extends the work in [25] with the eigenvalue theorem to analyze the stability of the system due to a CPL. The stability results from the theory will be supported by using the intensive time domain simulation. The paper is structured as follows. In Section 2, the power system definition and assumptions are explained.
The DQ dynamic model of the system from [25] is explained again in Section 3. The steady-state value calculation for the small-signal model derived from the proposed method is presented in Section 4. In Section 5, the model validation using a small-signal simulation and stability analysis for each firing angle of controlled rectifier are shown. Finally, Section 6 concludes and discusses the advantages of the DQ method to model the power converter for stability analysis.

2 Power System Definition and Assumptions

The power system studied in this paper is depicted in Fig. 1. It consists of a three-phase voltage source, transmission line, 6-pulse controlled rectifier, DC-link filters, and an ideal CPL connected to the DC bus. The ideal CPL is used to represent actuator drive systems by assuming an infinitely fast controller action of the drive system. Hence, the ideal CPL can be considered as a voltage-dependent current source given by:

$$I_{cpl} = \frac{P_{cpl}}{V_{out}} \quad (1)$$

where $V_{out}$ is the voltage across the CPL and $P_{cpl}$ is the power level of CPL.

It is assumed that the three-phase voltage source is balanced. The equivalent parameters of a transmission line are represented by $R_{eq}$, $L_{eq}$, and $C_{eq}$. The DC-link filters are shown by elements $r_F$, $L_F$, and $C_F$. $E_{dc}$ and $V_{out}$ are the output terminal voltage of a controlled rectifier and the voltage across the DC-link capacitor $C_F$, respectively. A phase shift between the source bus and the AC bus is $\lambda$ as shown in Fig.1. The effect of $L_{eq}$ on the AC side causes an overlap angle $\mu$ in the output waveforms that causes a commutation voltage drop. This drop can be represented as a variable resistance $r_\mu$ that is located on the DC side [22],[26] as shown in Fig.2. The $r_\mu$ can be calculated by:

$$r_\mu = \frac{3\omega L_{eq}}{\pi} \quad (2)$$

where $\omega$ is the source frequency.

It can be seen from Fig.2 that $E_{dc1}$ represents the output voltage from the switching signal without an overlap angle effect, while $E_{dc}$ represents the voltage at the rectifier output terminal taking onto account the voltage drop effect.

Since the commutation effect has been moved on to the DC side, the switching signals for 3-phase controlled rectifier can be applied without considering the effect of overlap angle. This is show in Fig. 3 in which $\alpha$ is the firing angle of thyristors. The switching function of $S_\alpha$ in Fig.3 can be expressed by a Fourier series.
In this paper, neglecting the harmonics of the power system, the switching functions can be written for three phases as:

\[
S_{abc} = \frac{2\sqrt{3}}{\pi} \begin{bmatrix}
\sin(\alpha + \phi - \alpha) \\
\sin(\alpha - \frac{2\pi}{3} + \phi - \alpha) \\
\sin(\alpha + \frac{2\pi}{3} + \phi - \alpha)
\end{bmatrix}
\]

where \(\phi\) is a phase angle of the AC bus voltage and \(\alpha\) is the firing angle.

The relationship between input and output terminal of controlled rectifier is given by:

\[
I_{in,abc} = S_{abc} I_{dc}
\]

\[
E_{dc,1} = S_{abc}^T V_{bus,abc}
\]

It can be seen from (4) that the fundamental input current is in phase with the switching signals. In addition, for a controlled rectifier, the fundamental input current lags the fundamental input voltage by \(\alpha\) [26].

Equations (3)-(5) will be used to derive the model of controlled rectifier by using DQ modeling method in Section 3. The model assumptions in this paper are as follows:

- The rectifier is operated under a continuous conduction mode (CCM).
- The output DC current of the rectifier is constant.
- The amplitude of the three-phase source is constant and balanced.
- Only one commutation occurs at a time.
- All harmonics in the system are neglected.

3. DQ Dynamic Model of the System

In this section, the DQ modeling method is applied to derive a mathematical model of the system as depicted in Fig.1.

Firstly, the controlled rectifier is transformed into a two axis frame (DQ frame) rotating at the system frequency \(\omega\) by means of:

\[
T[\theta] = \begin{bmatrix}
\cos(\theta) & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\
\sin(\theta) & \sin(\theta - \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3})
\end{bmatrix}
\]

Combining equations (4)-(6) results in:

\[
I_{in,dq} = S_{dq} I_{dc}
\]

\[
E_{dc,1} = S_{dq}^T V_{bus,dq}
\]

Secondly, the switching functions in (3) can be transformed into a DQ frame by means of (6) to give:

\[
S_{dq} = \frac{3}{2} \frac{2\sqrt{3}}{\pi} \begin{bmatrix}
\cos(\phi_1 - \phi + \alpha) \\
-\sin(\phi_1 - \phi + \alpha)
\end{bmatrix}
\]

The vector diagram for the DQ transformation is as shown in Fig. 4 where \(V_s\) is the peak amplitude phase voltage, \(I_m\) is the peak amplitude current, \(V_{bus}\) is the peak amplitude AC bus voltage, and \(S\) is peak amplitude of the switching signal, here equal to \(2\sqrt{3}/\pi\) as shown in (3).

From (7)-(9), the controlled rectifier can be easily represented as a transformer having \(d\) and \(q\)-axis transformer ratio \(S_d, S_q\) that depend on the
phase of the DQ frame ($\phi_1$), the phase of $V_{bus}$ ($\phi$), and the firing angle of thyristors ($\alpha$). As a result, the equivalent circuit of the controlled rectifier in the DQ frame derived by using DQ modeling method is shown in Fig.5.

![Fig.5 The controlled rectifier equivalent circuit in the DQ frame](image)

Finally, using (6), the cable section can be transformed into DQ frame [27]. The DQ representation of the cable is then combined with the controlled rectifier as shown in Fig.5. As a result, the equivalent circuit of the power system in Fig.1 can be represented in the DQ frame as depicted in Fig.6.

The equivalent circuit in Fig.6 can be simplified by fixing the rotating frame on the phase of the switching function ($\phi = \phi - \alpha$). This results in the circuit as shown in Fig.7.

Applying the Kirchhoff’s voltage law (KVL) and the Kirchhoff’s current law (KCL) to the circuit in Fig.7 obtains the set of nonlinear differential equations. We define:

State variables:
$$\mathbf{x} = [I_{ds}, I_{qs}, V_{bus,d}, V_{bus,q}, I_{dc}, V_{out}]$$

Input:
$$\mathbf{u} = [V_m, P_{CPL}]$$

Output:
$$\mathbf{y} = [V_{out}]$$

The set of nonlinear differential equations is given as follows:

- $$I_{ds} = \frac{R_{eq}}{L_{eq}} I_{ds} + \omega l_{qs} - \frac{1}{L_{eq} V_{bus,d}} + \frac{1}{L_{eq}} V_{sd}$$
- $$I_{qs} = -\omega l_{ds} - \frac{R_{eq}}{L_{eq}} I_{qs} - \frac{1}{L_{eq} V_{bus,q}} + \frac{1}{L_{eq}} V_{sq}$$
- $$V_{bus,d} = \frac{1}{c_{eq}} I_{ds} + \omega V_{bus,q} - \sqrt{\frac{3}{2}} \sqrt{3} l_{dc}$$
- $$V_{bus,q} = -\omega V_{bus,d} + \frac{1}{c_{eq}} I_{qs}$$
- $$I_{dc} = \sqrt{\frac{3}{2}} \sqrt{3} V_{bus,d} - \left( \frac{r_{F}}{L_{F}} + \frac{r_{\mu}}{L_{F}} \right) I_{dc} - \frac{1}{L_{F}} V_{out}$$
- $$V_{out} = \frac{1}{C_{F}} I_{dc} - \frac{1}{C_{F}} \frac{P_{CPL}}{V_{out}}$$

The equation (10) is a nonlinear equation. It is well known that the linearized model can be used for a controller system design via a linear control theory. In addition, the linearized model can be also used to analyze the small-signal stability of the power system including a CPL [22]-[24]. Therefore, (10) is linearized using the first order terms of the Taylor expansion so as to achieve a set of linear differential equations around an equilibrium point. The DQ linearized model of (10) is then of the following form:

$$\begin{align*}
\dot{\mathbf{x}} &= \mathbf{A}(\mathbf{x}, \mathbf{u}_o) \dot{\mathbf{x}} + \mathbf{B}(\mathbf{x}, \mathbf{u}_o) \dot{\mathbf{u}} \\
\dot{\mathbf{y}} &= \mathbf{C}(\mathbf{x}, \mathbf{u}_o) \dot{\mathbf{x}} + \mathbf{D}(\mathbf{x}, \mathbf{u}_o) \dot{\mathbf{u}}
\end{align*}$$

(11)

where

- $$\dot{\mathbf{x}} = [\delta I_{ds}, \delta I_{qs}, \delta V_{bus,d}, \delta V_{bus,q}, \delta I_{dc}, \delta V_{out}]$$
- $$\dot{\mathbf{u}} = [\delta V_m, \delta P_{CPL}]$$
- $$\dot{\mathbf{y}} = [\delta V_{out}]$$

$$\mathbf{A}(\mathbf{x}, \mathbf{u}_o) = \begin{bmatrix}
\cos(\lambda_o + \alpha) & 0 \\
\frac{3}{2} \sin(\lambda_o + \alpha) & \frac{3}{2} \sin(\lambda_o + \alpha)
\end{bmatrix}
\begin{bmatrix}
\frac{1}{L_{eq}} & 0 \\
0 & \frac{1}{C_{F} V_{out}}
\end{bmatrix}
$$
Fig. 6 The equivalent circuit of the system in Fig. 1 on DQ frame.

Fig. 7 The simplified equivalent circuit of the power system.

4. Calculation the Steady-State Value

The DQ linearized model in (11) needs to define $V_{\text{out, } o}$ and $\hat{\lambda}_o$. The power flow equation can be applied to determine the steady state value at the AC side of the power system in Fig. 1. This leads to a system of nonlinear equations:

$$
\begin{align*}
V_{\text{bus, } o}^2 \cos(\gamma - \hat{\lambda}) - \frac{V_{\text{bus}}^2}{Z} \cos(\gamma) &= P_{\text{bus}} \\
V_{\text{bus, } o}^2 \sin(\gamma - \hat{\lambda}) - \frac{V_{\text{bus}}^2}{Z} \sin(\gamma) &= Q_{\text{bus}}
\end{align*}
$$

(12)

where the following steady-state values are: $V_{\text{bus, } o}$ – voltage at AC bus (rms), $\hat{\lambda}_o$ - phase shift between $V_s$ and $V_{\text{bus}}$ as mentioned above. Note that $Z \angle \gamma$ is the transmission line impedance, while the active and
reactive power (per phase) at the AC bus is given by:

\[
P_{\text{bus}} = V_{\text{bus}} I_{\text{bus}} \cos \alpha = (P_{\text{CPL}} / 3) \\
Q_{\text{bus}} = V_{\text{bus}} I_{\text{bus}} \sin \alpha
\]  

(13)

It can be seen from (13) that the \( P_{\text{bus}} \) and \( Q_{\text{bus}} \) depend on the firing angle of thyristors (\( \alpha \)). Equation (12) can be solved by using a numerical method such as Newton Raphson to achieve \( V_{\text{bus},o} \) and \( \lambda_{o} \) at the steady-state conditions. Consequently, \( V_{\text{out},o} \) for DQ linearized model in (11) can then be calculated by:

\[
V_{\text{out},o} = \frac{3\sqrt{3}}{\pi} \sqrt{2} V_{\text{bus},o} \cos(\alpha) - \frac{3 L_{\text{eq}} \omega}{\pi} I_{\text{dc},o} - r_{F} I_{\text{dc},o}
\]  

(14)

where

\[
I_{\text{dc},o} = -\frac{\sqrt{3} V_{e} e^{-j0} - V_{\text{bus},o} e^{-j\lambda_{o}}}{Z_{o}^{*} / \gamma}
\]

\[
Z = \sqrt{R_{\text{eq}}^2 + (\omega L_{\text{eq}})^2}, \quad \gamma = -\tan^{-1}\left(\frac{\omega L_{\text{eq}}}{R_{\text{eq}}} \right)
\]

5. Small-signal Simulation and Stability Analysis

The DQ linearized model in (11) is simulated for small-signal transients against a corresponding three-phase benchmark circuit model in SimPowerSystems™ of SIMULINK. The set of parameters for the example system according to Fig.1 is given as follows: \( V_{s}=230 \) V rms/phase, \( f=50 \) Hz, \( R_{\text{eq}}=0.15 \Omega \), \( L_{\text{eq}}=30 \mu \text{H} \), \( C_{\text{eq}}=2 \text{nF} \), \( C_{F}=1000 \mu \text{F} \), \( r_{F}=0.3 \Omega \), and \( L_{F}=6.5 \text{mH} \). Fig.8 shows the \( V_{\text{out}} \) response of the system in Fig.1 to a step change of \( P_{\text{CPL}} \) from 7 to 9kW that occurs at \( t = 0.4 \)s. (\( \alpha = 0 \) degrees). Similarly, Fig.9-Fig.13 are the responses to a step change of \( P_{\text{CPL}} \) from 7 to 9kW for \( \alpha \) equal to 10, 20, 30, 40, and 50 degrees, respectively. Note that the firing angle cannot be allowed to exceed 60 degrees to obtain the positive output voltage of the rectifier on the DC side [26].

From the results in Fig.8-Fig.13, an excellent agreement between both models is achieved under small-signal simulation. It confirms that the mathematical model of the power system with a controlled rectifier derived from the DQ method provide a high accuracy. Therefore, this model can then be used for stability analysis.
In addition, it can be seen from Fig.8-Fig.13 that the harmonic components due to the switching actions are included in the responses that are simulated by using the three-phase benchmark model, while the responses from the DQ linearized model provide only the fundamental component. However, it will be shown in the stability results that the mathematical model derived by considering only the fundamental component can correctly predict the unstable point of the system.

For stability analysis, the DQ linearized model in (11) is used with the eigenvalue theorem. The eigenvalue can be calculated from the Jacobian matrix $A(x, u)$ in (11) by:

$$\det[A - I\lambda] = 0$$

and the system is stable if

$$\text{Re}[\lambda] < 0$$

where $i = 1, 2, 3, \ldots, n$ ($n$ = the number of state variables).

To investigate the instability condition of the power system in Fig.1 due to a CPL, the eigenvalues of the system with the given parameters are calculated from the Jacobian matrix when the $P_{CPL}$ varies from 0 kW to 50 kW. The dominant root locus for $\alpha$ equal to 10 degrees is shown in Fig.14. According to (16), it can be seen that the system becomes unstable when the $P_{CPL}$ exceeds ~22 kW. Note that the results depend on the system parameters such as system frequency, DC-link filters etc. If the parameters are changed, the stability results will be changed. Fig.15 shows the time-domain simulations that support the theoretical results with instability occurring at $P_{CPL}$ equal to 24 kW. This is greater than 22 kW for the unstable condition predicted from the theory.
Similarly, Fig.16-Fig.19 show the eigenvalue plot for \( \alpha \) equal to 20, 30, 40, and 50 degrees, respectively. Fig.20-Fig.23 show the time-domain simulations that confirm the theoretical results of Fig.16-Fig.19, respectively. It can be seen that the mathematical model derived from the DQ method can be used to predict the unstable condition of the power system.

In the future, the dynamic model will be used to predict the instability point for variations in system parameters such as system frequency, DC-link parameters etc. Moreover, the dynamic CPL explained in [24] can be also used with the DQ model of three-phase controlled rectifier from this paper instead of the ideal CPL. This is to investigate the effect of the dynamic CPL to the stability margin.
Recently, the artificial intelligence (AI) techniques are widely used in the power system application [28], [29]. Therefore, the mathematical model derived by using the DQ method from this paper can be also used as the objective function of the AI algorithm for the system design.

6. Conclusion

In this paper, the DQ modeling method is presented for modeling a three-phase AC distribution system with a three-phase controlled rectifier, DC-link filters, and an ideal CPL connected to the DC bus. The proposed approach is very useful for modeling the AC distribution system and also concerning a phase shift between source bus and AC bus. Moreover, the resulting converter models can be easily combined with models of other power elements expressed in terms of synchronously rotating frames such as generators, front-end converters, and vector-controlled drives. This paper also present the DQ linearized model that is used to analyse the system stability due to the CPL. The three-phase benchmark model is used to verify the stability results in the paper. The results show that the mathematical model derived from the DQ method can predict the instability point with a high accuracy. Therefore, electrical engineers can use the mathematical model to study the power system behaviour and to avoid the unstable condition. In the future, the dynamic model will be used to predict the instability point for variations in system parameters. Moreover, the mathematical model from this paper can be also used for the AI application to the power system design to achieve the best performance.

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