A Reliable Fuzzy Logic approach for Measurement Data Validation through line susceptance Estimation

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Abstract: - A novel approach is described to estimate the transmission line susceptances using the linear power flow model with linear relation between the measurement vector and the susceptance vector. The presently estimated susceptance is compared with the already stored correct value, to determine validity of the present measurement using the fuzzy inference

Key-Words: - Data Validation, lower and upper bounds of state estimation, Interval arithmetic, Fuzzy logic, Reliabality

1 Introduction

Branch impedances are important parameters in the State Estimation process. Series conductance values and shunt admittances do not play important roles in the state estimation. Therefore they are ignored and only the series line reactances are taken into consideration in this paper. The active power flow in a branch depends very much on the corresponding branch reactance. Errors in the accurate knowledge of line reactances will lead to permanent and serious errors in State Estimation [1]. Hence, correct and accurate line reactance values are very much essential in the state estimation process. The transmission line series reactances are essentially time invariant during short to moderate time spans and change slowly over a long time period due to aging and other reasons [2]. Therefore they are stored in the power system's parameter data-base and used during state estimation. The parameter data-base is regularly updated to reflect the correct present line reactance values. A sudden or a substantial change in the present line reactance value from the previous value indicates an error in the measurement system or an abnormal condition in the measurement setup. This fact is used, in the method we are going to present in this paper, to check the validity of the measurement results.

Several techniques are available for the estimation of line parameters [3]. The method of *Augmented State Vector* and similar methods involve non linear equations for the measurement models and the solution involves the iterative solution of the normal equation [3]. The method based on the sensitivity analysis of measurement residuals is another approach [4]-[5]. The method proposed in this paper uses the linear measurement model and hence the solving process is quick and efficient.

2 Problem Formulation

Data validation (Bad data detection) is done in two stages.

Stage 1: To estimate the line susceptances correctly.

Transmission line susceptances are estimated correctly and stored in the SCADA data base for the subsequent use.

Stage 2: To validate the Data.

The present power measurement data is validated by comparing the correspondingly estimated susceptance with the available correct susceptances.

Terminology and Basic Relations

Consider the basic DC power flow model shown in Fig.1, where the active power flow and the node phase angles are related as [1],

$$P_{km} = x_{km}^{-1}(\theta_k - \theta_m) = y_{km}(\theta_k - \theta_m) \qquad (1)$$

Here, P_{km} is the branch active power flow, θ_k and θ_m are the node voltage phase angles, x_{km} is the line reactance and $y_{km} = x_{km}^{-1}$ is the series line susceptance. For any k and m,

$y_{km} = y_{mk}$.

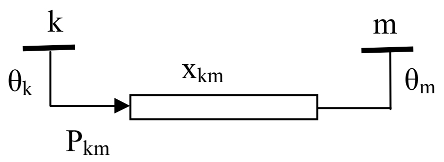

Fig.1. Transmission Line DC Power Flow Model

Eq.(1) is rewritten as,

$$P_{km} = (\theta_k - \theta_m) y_{km} \quad (2)$$

Now, the equation for power injection at node k is [1],

$$P_k = \sum_{m \in \Omega_k} x_{km}^{-1} (\theta_k - \theta_m) = \sum_{m \in \Omega_k} (\theta_k - \theta_m) y_{km} \quad (3)$$

Where: Ω_k is the set of buses adjacent to bus k. Here after, we use y_{km}'s in our equations and calculations, because the use of y_{km}'s makes the measurement equations linear. θ_j's, used in Eqs.(2) and (3), are accurately determined either by direct measurements or by separate estimation. Then they are used as the coefficients in Eqs.(2) and (3).

Stage 1: Estimation of line susceptances correctly

After correctly measuring the values of P_{km}'s, P_k's and knowing θ_j's, the set of equations corresponding to Eqs.(2) and (3) are used to estimate y_{km}'s by the standard linear method. During the estimation process, if any bad data is detected, they are eliminated and only the correct and validated measurements are used to estimate y_{km}'s. These correct values of y_{km}'s are stored in the parameter data-base for future comparison.

2.1 Algorithm for the Estimation of Line Susceptances

Mark the network nodes as 1,2,…3 and so on. Select the Line Susceptances y_{km}'s to be estimated by listing the values of k and m appropriately.

1. Measure the active power flows P_{km}'s and P_k's needed, based on the observability criterion.
2. Predetermine or pre-estimate the node voltage phase angles, θ_j's.
3. Write a set of equations linear in y_{km}'s with $(\theta_k - \theta_m)$ as the coefficients with suitable values for k's and m's.
4. Rewrite the above equations in the vector-matrix form as $z = Hx$ where the measurement vector z contains the branch flow and node injection power measurements, x is the vector of susceptances (y_{km}'s) to be estimated and H is the coefficient matrix involving θ's.
5. Solve The resulting linear over determined set of equations to get the estimate for y_{km}'s using the solution as given by [3] $\hat{x} = (H^T W H)^{-1} H^T W z$ With usual notations.
6. Store this for comparison to be used in stage 2.

Stage 2: Data Validation

Now the Present power measurements are checked for correctness as follows. From the measured data, the susceptances are re-estimated. If these values match with the corresponding correct values, which are already available in the data base, then there is no error. Otherwise error is present in the measured data.

Algorithm 2

Data Validation using Estimated Susceptances

1. Using the presently measured P_{km}'s, P_k's and known θ_j's get the over determined state equation obtained from Eq.(2) and (3).
2. Solve it by WLS method to estimate the line susceptances. Call the resulting state vector as $\hat{x}$. These are estimated y_{km}'s.
3. Compare this $\hat{x}$ with x obtained in Algorithm 1. If $(x - \hat{x})$ is zero or very near to zero, there is no error. The measured data are good. Else, bad data is present.

2.2 Evaluating 3 Bus System

2.21 Case-1

Consider the 3-bus system shown in Fig.2. y_{12}, y_{23} and y_{13} are the line susceptance parameters to be estimated. The bus voltage phase angles are: $\theta_1 = 0$ is the reference, θ_2 and θ_3 are already determined by the previous estimate as,

$\theta_2 = -0.0400$ and $\theta_3 = -0.0201$.

The power flow measurement values are,

$P_1 = 3.90$ p.u. $P_2 = -4.07$ p.u.

$P_3 = -0.04$ p.u. $P_{13} = 2.04$ p.u.

The corresponding measurement variances given are,

$\sigma_1^2 = 0.004$ p.u. $\sigma_2^2 = 0.004$ p.u.

$\sigma_3^2 = 0.001$ p.u. $\sigma_{13}^2 = 0.002$ p.u.

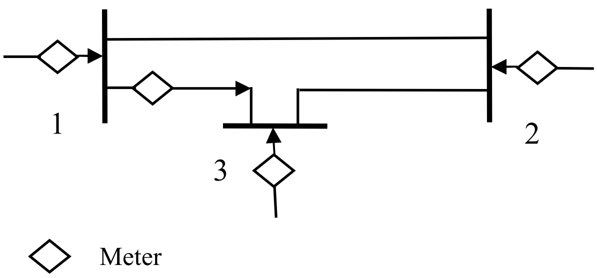

Fig.2. Three-Bus System

The basic equations for the measurement model are,

$$P_1 = (\theta_1 - \theta_2)y_{12} + (\theta_1 - \theta_3)y_{13}$$
$$P_2 = (\theta_2 - \theta_1)y_{12} + (\theta_2 - \theta_3)y_{23}$$
$$P_3 = (\theta_3 - \theta_1)y_{13} + (\theta_3 - \theta_2)y_{23}$$
$$P_{13} = (\theta_1 - \theta_3)y_{13}$$

Since $\theta_1 = 0$, the above equations become,

$$P_1 = (-\theta_2)y_{12} + (-\theta_3)y_{13}$$
$$P_2 = (\theta_2)y_{12} + (\theta_2 - \theta_3)y_{23}$$
$$P_3 = (\theta_3)y_{13} + (\theta_3 - \theta_2)y_{23}$$
$$P_{13} = (-\theta_3)y_{13}$$

These equations are rewritten in the vector-matrix form as,

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_{13} \end{bmatrix} = \begin{bmatrix} -\theta_2 & -\theta_3 & 0 \\ \theta_2 & 0 & \theta_2 - \theta_3 \\ 0 & \theta_3 & \theta_3 - \theta_2 \\ 0 & -\theta_3 & 0 \end{bmatrix} \begin{bmatrix} y_{12} \\ y_{13} \\ y_{23} \end{bmatrix} \qquad (4)$$

Eq.(4) is written as,

$$z = Hx \qquad (5)$$

Here:

$$z = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_{13} \end{bmatrix} = \begin{bmatrix} 3.9 \\ -4.07 \\ -0.04 \\ 2.04 \end{bmatrix}$$

$$H = \begin{bmatrix} -\theta_2 & -\theta_3 & 0 \\ \theta_2 & 0 & \theta_2 - \theta_3 \\ 0 & \theta_3 & \theta_3 - \theta_2 \\ 0 & -\theta_3 & 0 \end{bmatrix},$$

$$= \begin{bmatrix} 0.0400 & 0.0201 & 0 \\ -0.0400 & 0 & -0.0199 \\ 0 & -0.0201 & 0.0199 \\ 0 & 0.0201 & 0 \end{bmatrix}$$

$$x = \begin{bmatrix} y_{12} \\ y_{13} \\ y_{23} \end{bmatrix}$$

Here, vector x is a collection of y_{km}'s and km's take the values 12, 13 and 23 in that order.

In this case the measurement covariance matrix is given by,

$$R_z = \text{diag}\left(\sigma_1^2 \quad \sigma_2^2 \quad \ldots \quad \sigma_m^2\right)$$
$$= \text{diag}(0.004 \quad 0.004 \quad 0.001 \quad 0.002)$$

The weight matrix **W**, is given by,

$$W = R_z^{-1} = \text{diag}(250 \quad 250 \quad 1000 \quad 500)$$

With these values the estimate for **x** is given by [2]

$$\hat{x} = \begin{bmatrix} \hat{y}_{12} \\ \hat{y}_{13} \\ \hat{y}_{23} \end{bmatrix} = \left(H^T W H\right)^{-1} H^T W z = \begin{bmatrix} 48.8408 \\ 101.4307 \\ 101.7690 \end{bmatrix}$$

Thus the estimated line susceptances are (in p.u.),

$$\begin{bmatrix} \hat{y}_{12} \\ \hat{y}_{13} \\ \hat{y}_{23} \end{bmatrix} = \begin{bmatrix} 48.8408 \\ 101.4307 \\ 101.7690 \end{bmatrix}$$

Data Validation

If these values match with the correct available values of y_{km}'s, then there is no error. Else there is error in the measured data.

2.3 Multiple scans of measurements

When the number of state equations m is less than the no of parameters n, the matrix $H^T WH$ is not a full rank one and has no inversion. This is an underdetermined system and we cannot apply the WLS method. To overcome this problem we can take multiple scan of the measurements with suitable intervals. The interval between successive measurements should be so selected that the values of measured quantities are sufficiently different from one scan to the next scan. Since the line susceptances are same over successive scans, their number n, remains same where as the number of state equations get increased by m for each additional scan of measurements. Thus an underdetermined system is converted into an over determined system. This is illustrated in the example given below.

2.4 Evaluating 5 bus 6 branch network
2.41 Case-2

Consider the 5-bus 6-branch network shown in Fig.3.

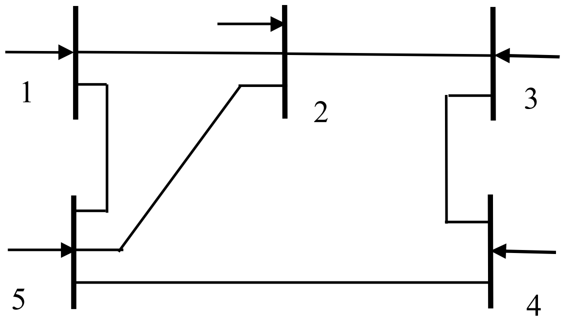

Fig.3. 5-Bus 6-branch sample System

Bus 5 is taken as the reference. Hence $\theta_5 = 0$. Four bus injection powers P_1, P_2, P_3 and P_4 are measured. Also the four bus voltage phase angles $\theta_1, \theta_2, \theta_3$ and θ_4 with respect to θ_5 are measured.

The linear (DC model) power flow equations are,

$$P_1 = (\theta_1 - \theta_2)y_{12} + (\theta_1 - \theta_5)y_{15}$$
$$P_2 = (\theta_2 - \theta_1)y_{12} + (\theta_2 - \theta_3)y_{23} + (\theta_2 - \theta_5)y_{25}$$
$$P_3 = (\theta_3 - \theta_2)y_{23} + (\theta_3 - \theta_4)y_{34}$$
$$P_4 = (\theta_4 - \theta_3)y_{34} + (\theta_4 - \theta_5)y_{45}$$

Taking $\theta_5 = 0$ and rearranging, we get

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix} = \begin{bmatrix} \theta_1-\theta_2 & \theta_1 & 0 & 0 & 0 & 0 \\ \theta_2-\theta_1 & 0 & \theta_2-\theta_3 & \theta_2 & 0 & 0 \\ 0 & 0 & \theta_3-\theta_2 & 0 & \theta_3-\theta_4 & 0 \\ 0 & 0 & 0 & 0 & \theta_4-\theta_3 & \theta_4 \end{bmatrix} \begin{bmatrix} y_{12} \\ y_{15} \\ y_{23} \\ y_{25} \\ y_{34} \\ y_{45} \end{bmatrix}$$

Here the number of equations are m=4 and the number of parameters to be estimated are n=6. Therefore an additional set of measurements are taken after sufficient interval to get a different set of values for P's and θ's. Using the additional sub-script 1 and 2 for the first set and second set respectively, the resulting state equation in vector matrix notation is written as,

$$H = \begin{bmatrix} \theta_{11}-\theta_{21} & \theta_{11} & 0 & 0 & 0 & 0 \\ \theta_{21}-\theta_{11} & 0 & \theta_{21}-\theta_{31} & \theta_{21} & 0 & 0 \\ 0 & 0 & \theta_{31}-\theta_{21} & 0 & \theta_{31}-\theta_{41} & 0 \\ 0 & 0 & 0 & 0 & \theta_{41}-\theta_{31} & \theta_{41} \\ \theta_{12}-\theta_{22} & \theta_{12} & 0 & 0 & 0 & 0 \\ \theta_{22}-\theta_{12} & 0 & \theta_{22}-\theta_{32} & \theta_{22} & 0 & 0 \\ 0 & 0 & \theta_{32}-\theta_{22} & 0 & \theta_{32}-\theta_{42} & 0 \\ 0 & 0 & 0 & 0 & \theta_{42}-\theta_{32} & \theta_{42} \end{bmatrix}$$

Now, the number of equations is 8 and the number of parameters to be estimated is 6. This is an over determined system and can be solved as usual. The weight matrix values $\mathbf{W_1}$ and $\mathbf{W_2}$ for the first and second set of measurements respectively are same, because the same meters are used in both the cases. Therefore, $\mathbf{W_2= W_1}$ and the overall 8x8 weight matrix is,

$$W = \begin{bmatrix} W_1 & 0 \\ 0 & W_2 \end{bmatrix} = \begin{bmatrix} W_1 & 0 \\ 0 & W_1 \end{bmatrix}$$

Numerical values and results.

In the above example, let the first and second scan measurements be as given below

$$P_1 = \begin{bmatrix} P_{11} \\ P_{21} \\ P_{31} \\ P_{41} \end{bmatrix} = \begin{bmatrix} +1.3021 \\ -0.1064 \\ -0.6092 \\ -0.1250 \end{bmatrix} \text{ and } \begin{bmatrix} \boldsymbol{\theta}_{11} \\ \boldsymbol{\theta}_{21} \\ \boldsymbol{\theta}_{31} \\ \boldsymbol{\theta}_{41} \end{bmatrix} = \begin{bmatrix} +0.0977 \\ +0.0440 \\ -0.0213 \\ -0.0057 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} P_{12} \\ P_{22} \\ P_{32} \\ P_{42} \end{bmatrix} = \begin{bmatrix} +1.2970 \\ -0.0917 \\ -0.6021 \\ -0.1229 \end{bmatrix} \text{ and } \begin{bmatrix} \boldsymbol{\theta}_{12} \\ \boldsymbol{\theta}_{22} \\ \boldsymbol{\theta}_{32} \\ \boldsymbol{\theta}_{42} \end{bmatrix} = \begin{bmatrix} +0.0980 \\ +0.0450 \\ -0.0200 \\ -0.0055 \end{bmatrix}$$

The Actual susceptance parameter vector denoted by x is given by

$$x = \begin{bmatrix} y_{12} \\ y_{15} \\ y_{23} \\ y_{25} \\ y_{34} \\ y_{45} \end{bmatrix} = \begin{bmatrix} 16.6667 \\ 4.1667 \\ 8.3333 \\ 5.5556 \\ 4.1667 \\ 33.333 \end{bmatrix}$$

Now the 8x1 measurement vector z is obtained by combining P_1 and P_2 as,

$$z = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} +1.3021 \\ -0.1064 \\ -0.6092 \\ -0.1250 \\ +1.2970 \\ -0.0917 \\ -0.6021 \\ -0.1229 \end{bmatrix}$$

The corresponding H matrix is,

$$H = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} =$$

$$\begin{bmatrix} 0.0537 & 0.0977 & 0 & 0 & 0 & 0 \\ -0.0537 & 0 & 0.0653 & 0.044 & 0 & 0 \\ 0 & 0 & -0.0653 & 0 & -0.0156 & \\ 0 & 0 & 0 & 0 & 0.0156 & -0.0057 \\ 0.0530 & 0.0980 & 0 & 0 & 0 & \\ -0.0530 & 0 & 0.650 & 0.0450 & 0 & \\ 0 & 0 & -0.650 & 0 & -0.0145 & \\ 0 & 0 & 0 & 0 & 0.0145 & -0.0055 \end{bmatrix}$$

When there is no error in the measurement vector z, the given x and the estimated $\hat{x}$ determined by, $\hat{x} = \left(H^T W H\right)^{-1} H^T W z$ are same. When there is some error in z, x and $\hat{x}$ differ. Let an 2% error be introduced in the third element of z as, z(3)=1.02*z(3). Then the estimated $\hat{x}$ is found to be markedly different from x, as shown in the Table-1 below.

Table 1 Indicating the difference of (x- $\hat{x}$)

Sequence Number	x	$\hat{x}$	(x- $\hat{x}$)
1	16.6667	15.1540	1.5127
2	4.1667	4.9914	-0.8247
3	8.3333	6.3683	1.9651
4	5.5556	6.6190	-1.0634
5	4.1667	13.0785	-8.9119
6	33.3333	57.2920	-23.9587

3 Measurement data validation by Fuzzy logic

Fuzzy Logic is useful when the values of the variables under consideration are imprecise and uncertain. In the case of the line susceptance y_{km} is calculated from Eq.(2), as

$$y_{km} = \frac{P_{km}}{(\theta_k - \theta_m)}$$

Here, P_{km}, θ_k and θ_m are all obtained from the noisy measurement data from measuring equipments which may not be 100 percent accurate. Therefore the line susceptance y_{km} is imprecise and uncertain to some degree. Hence Fuzzy Logic reasoning is used dealing with y_{km}. For a specified appropriate k and m, y_{km} is represented by the fuzzy set *validity* as shown in Fig.4. For the sake of simplicity, we assume the trapezoidal membership function [6]. The normal value or the middle value y_{km}^M has the membership value of 1. The lower and the upper bounds with membership value of 1 are denoted by y_{km}^L and y_{km}^U respectively. The base points are a and b (see Fig.4).

3.1 The Criteria of determination of y_{km}^M

The value of y_{km}^M has to be determined carefully with as much accuracy and reliability as possible. The true value of y_{km}, represented by y_{km}^M, is generally time invariant and changes slowly over a long time period due to aging and other reasons. This property is used to determine y_{km}^M by taking the healthy snapshots of the required measurements, while discarding suspicious measurement sets [2]. Calculation of y_{km}^M based on the inductance and length of the line can also be used to arrive at the initial estimate and then further refined by successive sequential valid estimations. Thus using the historical data sequence is used to fix y_{km}^M

3.2 Basis of Fixing of y_{km}^L and y_{km}^U.

y_{km}^L and y_{km}^U of y_{km} values are fixed based on the existing measurement tolerance and standard deviations of measurements used in the calculations. Since the meters used and their configurations do not change in short time spans, the successive valid estimations of permissible deviations are used to determine y_{km}^L and y_{km}^U. Thus, we build up the best values y_{km}^L and y_{km}^U from the successive valid estimations.

3.3 Basis of determination of corner points a and b.

The corner points a and b (see Fig.4) of the membership function are selected suitably, say, 0.9* y_{km}^L and 1.1* y_{km}^U respectively. These values are flexible and can be selected suitably according to the requirements of the Fuzzy Logic design.

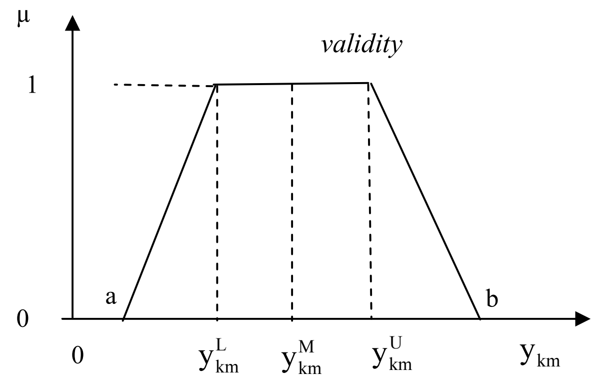

Fig.4. Trapezoidal Fuzzy membership function

3.4 Determination of y_{km}^M

Here, we assume that all the measurements are valid. If there are any bad data, they are discarded and only good data are retained. First, we determine P_{km} and then y_{km}^M.

3.5 Calculation of P_{km}'s.

In a given measurement system either all the P_{km}'s are measured or if not, they can be estimated from the measured values of node injection powers and the few measured branch power flows. (Measured values represent their Middle values.) This is demonstrated using the case-1 described earlier.

In case-1,(See Fig.2.) the measured values are, Three node injections and one branch flow. The relation among the node injection power values and all the branch power flows can be expressed, using the power balance property at the nodes, as follows,

$$P_1 = P_{12} + P_{13}$$

$$P_2 = P_{21} + P_{23} = -P_{12} + P_{23}$$

$$P_3 = P_{31} + P_{32} = -P_{13} - P_{23}$$

To this we also add one more equation involving P_{13}, which is a measured data, as,

$$P_{13} = P_{13}$$

This equation is an identity and is used to provide redundancy for the purpose of estimating branch flows. The above equations can be written in the vector-matrix notation as,

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_{13} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} P_{12} \\ P_{13} \\ P_{23} \end{bmatrix}$$

The above equation can be rewritten as,

$$z = H_P x_P \qquad (6)$$

Where,

$$z = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_{13} \end{bmatrix}, \quad H_P = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & -1 \\ 0 & 1 & 0 \end{bmatrix} \text{ and } x_P = \begin{bmatrix} P_{12} \\ P_{13} \\ P_{23} \end{bmatrix}$$

The above equation is an over determined set and can be solved for x_P using the state estimation technique as [2],

$$\hat{x}_P = \left(H_P^T W H_P\right)^{-1} H_P^T W z$$

Let $A_P = \left(H_P^T W H_P\right)^{-1} H_P^T W \qquad (7)$

Then, $\hat{x}_P = A_P z \qquad (8)$

$\hat{x}_P$ is the collection of P_{km}'s. Once $\hat{x}_P$ is known, P_{km}, the power flow in branch(k,m) can be easily picked up from $\hat{x}_P$.

3.6 Calculation of Y_{km}'s.

Consider the basic branch power flow equation given by Eq.(2), reproduced here for convenience,

$$P_{km} = (\theta_k - \theta_m) y_{km} \qquad (2)$$

Here we assume that, all the variables in Eq.(2) represent their middle values.

This equation can be solved for y_{km} as,

$$y_{km} = \frac{P_{km}}{(\theta_k - \theta_m)} = \frac{P_{km}}{\theta_{km}} \qquad (9)$$

Here, $\theta_{km} = (\theta_k - \theta_m) \qquad (10)$

3.7 Determination of y_{km}^L and y_{km}^U.

Once the middle value is found, the lower bound y_{km}^L and the upper bound y_{km}^U can be obtained by previous experience, by the specified tolerance limits from the engineering specification or from the knowledge of the estimated standard deviation and so on. In this paper we determine y_{km}^L and y_{km}^U from the knowledge of the lower and the upper bounds of the measurement data. Here we use interval arithmetic.

3.8 Interval arithmetic.

We use Interval arithmetic [7] to calculate the Fuzzy membership functions of the estimated parameters.

Let the intervals associated with P_{km}, θ_k, θ_m and y_{km} be written as,

$$\text{Interval}(P_{km}) = \left[P_{km}^L, P_{km}^U\right] \qquad (11)$$

$$\text{Interval}(\theta_k) = \left[\theta_k^L, \theta_k^U\right] \qquad (12)$$

$$\text{Interval}(\theta_m) = \left[\theta_m^L, \theta_m^U\right] \qquad (13)$$

$$\text{Interval}(y_{km}) = \left[y^L_{km}, y^U_{km}\right] \qquad (14)$$

$$\text{Interval}(z) = \left[z^L, z^U\right] \qquad (15)$$

$$\text{Interval}(x_P) = \left[x^L_P, x^U_p\right] \qquad (16)$$

3.9 The Calculation of P^L_{km} and P^U_{km}.

P_{km}'s and the measurement vector z are related by the linear relation, Eq.(8). In Eq.(8), vector $\hat{x}_P$ is the collection of P_{km}'s. By knowing $\hat{x}^L_p$ and $\hat{x}^U_p$ we can find P^L_{km} and P^U_{km} for different k's and m's. $\hat{x}^L_p$ and $\hat{x}^U_p$ are determined using Theorem 1, which is given below.

3.10 The Description of the Theorem

Let vectors $\mathbf{x}$ and $\mathbf{z}$ be related as $\mathbf{x} = A\mathbf{z}$ where A is a constant matrix. Then, the upper and lower bounds of $\mathbf{x}$ and $\mathbf{z}$ are related as,

$$x^U = (A_+)z^U + (A_-)z^L \quad \text{and}$$

$$x^L = (A_-)z^U + (A_+)z^L$$

where: $A_+ = 0.5(A + \text{abs}(A))$ and

$$A_- = 0.5(A - \text{abs}(A))$$

From Theorem 1 and Eq.(8),

$$x^L_P = (A_{P-})z^U + (A_{P+})z^L \qquad (17)$$

$$x^U_P = (A_{P+})z^U + (A_{P-})z^L \qquad (18)$$

Here, A_{P+} is the matrix obtained from A_P by replacing all its negative elements by zeros and A_{P-} is the matrix obtained from A_P by replacing all its positive elements by zeros. Thus, P^L_{km}'s and P^U_{km}'s are determined.

3.11 Calculation of y^L_{km} and y^U_{km}

Assuming that Interval(θ_k) and Interval(θ_m) are known, we can calculate, from Eq.(8) and Interval Arithmetic, Interval(θ_{km}) as,

$$\text{Interval}(\theta_{km}) = \left[\theta^L_k - \theta^U_m, \theta^U_k - \theta^L_m\right] \qquad (19)$$

Therefore,

$$\theta^L_{km} = \theta^L_k - \theta^U_m \qquad (20)$$

$$\theta^U_{km} = \theta^U_k - \theta^L_m \qquad (21)$$

Now, coming to the division operation of Eq.(9), assuming that Interval(θ_{km}) does not contain 0, (If it contains 0, then the corresponding power flow will also be zero.) From Eq.(9), we see that the ratio P_{km}/θ_{km} has to be positive. Other wise susceptance y_{km} would be negative. Therefore, P_{km} and θ_{km} both have to be positive or both negative.
When both are positive, the interval division rule gives,

$$\text{Interval}(y_{km}) = \left[\frac{P^L_{km}}{\theta^U_{km}}, \frac{P^U_{km}}{\theta^L_{km}}\right] \qquad (22)$$

Therefore,

$$y^L_{km} = \frac{P^L_{km}}{\theta^U_{km}} \qquad (23)$$

$$y^U_{km} = \frac{P^U_{km}}{\theta^L_{km}} \qquad (24)$$

When both are negative, the interval division rule gives,

$$\text{Interval}(y_{km}) = \left[\frac{P^U_{km}}{\theta^L_{km}}, \frac{P^L_{km}}{\theta^U_{km}}\right] \qquad (25)$$

Therefore, $$y^L_{km} = \frac{P^U_{km}}{\theta^L_{km}} \qquad (26)$$

$$y^U_{km} = \frac{P^L_{km}}{\theta^U_{km}} \qquad (27)$$

Eqs.(23) and (26) can be combined into a single equation as,

$$y^L_{km} = \frac{P^L_{km+}}{\theta^U_{km}} + \frac{P^U_{km-}}{\theta^L_{km}} \qquad (28)$$

where: $P^L_{km+} = P^L_{km}$ if $P^L_{km} > 0$ else $P^L_{km+} = 0$,

and $P^U_{km-} = P^U_{km}$ if $P^L_{km} < 0$ else $P^U_{km-} = 0$.

Similarly, Eqs.(24) and (27) can be combined into a single equation as,

$$y^U_{km} = \frac{P^U_{km+}}{\theta^L_{km}} + \frac{P^L_{km-}}{\theta^U_{km}} \qquad (29)$$

Eqs. (28) and (29) can be expressed in the matrix form as,

$$y^L = \frac{x^L_{P+}}{\Theta^U} + \frac{x^U_{P-}}{\Theta^L} \qquad (30)$$

$$y^U = \frac{x^U_{P+}}{\Theta^L} + \frac{x^L_{P-}}{\Theta^U} \qquad (31)$$

Here, the matrix division operation is the element by element division (Array right division) operation.
Matrices x^L_{P+}, x^L_{P-} etc. have the same definition as in Theorem 1.

Initially, we make sure that there are no abnormal measurement situations or bad data. This is ascertained by state estimation followed by residual analysis techniques and hypothesis testing methods [2],[3]. Once there is no error in the measurement system, y_{km}^L, y_{km}^U and y_{km}^M are used to build the Fuzzy member ship function for the Fuzzy set *validity* as in Fig.4. Now, for any set of present measurements, the validity of the measurement system is determined as follows.

3.12 Proof of Theorem 1

Consider the vector-matrix equation $\mathbf{x} = A\mathbf{z}$ where A is a constant matrix.. This can be expanded as,

$$x_j = \sum_{k=1}^{m} a_{jk} z_k \quad \text{for j=1,2,…n} \qquad (32)$$

Since x_j is the summation of individual product terms, maximum of x_j occurs when all the individual product terms are maximum. Therefore, from Eq.(22), the upper bound x_j^U is given by,

$$x_j^U = \sum_{k=1}^{m} \left(a_{jk} z_k\right)^U \quad \text{for j=1,2,…n} \qquad (33)$$

Now, consider the upper bound of the product term $\left(a_{jk} z_k\right)^U$.

When a_{jk} is positive, $\left(a_{jk} z_k\right)^U = a_{jk} z_k^U$ (34)

When a_{jk} is negative, $\left(a_{jk} z_k\right)^U = a_{jk} z_k^L$ (35)

where: z_k^L is the lower bound of z_k. Eqs.(34) and (35) can be combined into a single equation as,

$$\left(a_{jk} z_k\right)^U = a_{jk+} * z_k^U + a_{jk-} * z_k^L \qquad (36)$$

Where: $a_{jk+} = a_{jk} * u\left(a_{jk}\right)$ (37)

and, $a_{jk-} = a_{jk} * u\left(-a_{jk}\right)$ (38)

a_{jk+} and a_{jk-} can be expressed as,

$$a_{jk+} = a_{jk} * u\left(a_{jk}\right) = 0.5 * \left(a_{jk} + \text{abs}(a_{jk})\right) \qquad (39)$$

$$a_{jk-} = a_{jk} * u\left(-a_{jk}\right) = 0.5 * \left(a_{jk} - \text{abs}(a_{jk})\right) \qquad (40)$$

Here $u(\bullet)$ is the standard unit step function. Substituting for $\left(a_{jk} z_k\right)^U$ from Eq. (35) in (33), we get

$$x_j^U = \sum_{k=1}^{m} \left(a_{jk+} * z_k^U + a_{jk-} * z_k^L\right) \quad \text{for j=1,2, …,n} \qquad (41)$$

In terms of vector-matrix notation, Eq.(41) becomes.

$$x^U = (A_+) z^U + (A_-) z^L \qquad (42)$$

Similarly it can be shown that,

$$x^L = (A_-) z^U + (A_+) z^L \qquad (43)$$

4 Validation of Measurement Data Using The Fuzzy Inference

We assume that the member ship functions for all Y_{km}'s are readily available. We also assume that the Observability criterion is satisfied for the measurement set.

1. Get all Y_{km}'s as described in section 2.1 from the set of measurements.
2. For each Y_{km}, Check whether the *validity* is 1 or less using the corresponding member ship function as in Fig. 4. If $y_{km}^L \le y_{km} \le y_{km}^U$, then, that y_{km} is valid. Other wise, *validity* of that y_{km} less than 1.

When all y_{km}'s are valid, then the measurement set is valid. Now, the *validity* member ship function of y_{km}'s can be updated if the situation needs, because of new meters, new configuration etc.

When even a single y_{km} is invalid (validity less than 1) the measurement set is invalid and should be discarded and suitable diagnostic and corrective action should be taken to remove the anomaly.

The over all measurement set is valid when all y_{km}'s are valid. This is a logical *and* operation. Therefore, From the Fuzzy Logic, Measurement validity = min{validity of y_{km}'s } over all km's under consideration.

4.1 Case 3

This is a continuation of Example 1, with the same circuit and the same numerical values.

4.2 Calculation of P_{km}'s

We start with Eq.(6) which gives the relation between the measurement vector z and x_P which is a collection of P_{km}'s. Eq.(6) is reproduced here.

$$z = H_P x_P$$

where:

$$z = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_{13} \end{bmatrix} = \begin{bmatrix} 3.9 \\ -4.07 \\ -0.04 \\ 2.04 \end{bmatrix}, \quad H_P = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

and

$$x_P = \begin{bmatrix} P_{12} \\ P_{13} \\ P_{23} \end{bmatrix}$$

Using the same W as in Example 1 and from Eq.(7),

$$A_P = \left(H_P^T W H_P\right)^{-1} H_P^T W = \begin{bmatrix} 0.5556 & -0.4444 & -0.4444 & -1 \\ 0 & 0 & 0 & 1 \\ 0.1111 & 0.1111 & -0.8889 & -1 \end{bmatrix}$$

Substituting this in Eq.(8),

$$\hat{x}_P = A_P z = \begin{bmatrix} P_{12} \\ P_{13} \\ P_{23} \end{bmatrix} = \begin{bmatrix} 1.9533 \\ 2.0400 \\ -2.0233 \end{bmatrix}$$

Thus P_{km}'s are obtained.

4.3 Calculation of θ_{km}'s.

In the example,

$\theta_1 = 0$, $\theta_2 = -0.0400$ and $\theta_3 = -0.0201$.

Therefore,

$$\Theta = \begin{bmatrix} \theta_{12} \\ \theta_{13} \\ \theta_{23} \end{bmatrix} = \begin{bmatrix} \theta_1 - \theta_2 \\ \theta_1 - \theta_3 \\ \theta_2 - \theta_3 \end{bmatrix} = \begin{bmatrix} 0.0400 \\ 0.0201 \\ -0.0199 \end{bmatrix}$$

From Eq.(2), $y_{km} = \frac{P_{km}}{\theta_{km}}$ and in the matrix notation, this can be written as,

$$Y = \begin{bmatrix} y_{12} \\ y_{13} \\ y_{23} \end{bmatrix} = \begin{bmatrix} \frac{P_{12}}{\theta_{12}} \\ \frac{P_{13}}{\theta_{13}} \\ \frac{P_{23}}{\theta_{13}} \end{bmatrix} = \begin{bmatrix} 48.8408 \\ 101.4307 \\ 101.7690 \end{bmatrix}$$

This is same as obtained in case 1.

To validate these y_{km} y_{km} values, we have to use the corresponding Fuzzy membership functions. In this example we consider the validation of y_{12}. (Other y_{km}'s can be tested similarly.) The Fuzzy membership function for y_{12} is shown in Fig.5. Here the following values are taken from an assumed database.

$y_{12}^M = 50$, $y_{12}^L = 45$ and $y_{12}^U = 55$

Call the present value of y_{12} as y_{12pr}. This is equal to 48.8408. Since $y_{12}^L \le y_{12pr} \le y_{12}^U$, y_{12pr} is valid. From Fig.5, we see that $\mu(y_{12pr})=\mu(48.8408)=1$.

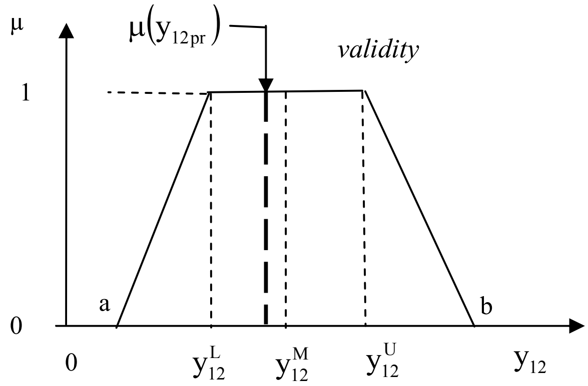

Fig.5. Trapezoidal Fuzzy membership Function for y_{12}

4.4 Calculation of P_{km}^L and P_{km}^U.

Assuming a ±2% change in z, z^L and z^U are obtained as,

$$z^L = \begin{bmatrix} 3.8220 \\ -4.1514 \\ -0.0408 \\ 1.9992 \end{bmatrix} \text{ and } z^U = \begin{bmatrix} 3.9780 \\ -3.9886 \\ -0.0392 \\ 2.0808 \end{bmatrix}$$

Now,

$$A_{P+} = 0.5(A_P + \text{abs}(A_P)) = \begin{bmatrix} 0.5556 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0.1111 & 0.1111 & 0 & 0 \end{bmatrix}$$

$$A_{P-} = 0.5(A_P - \text{abs}(A_P)) = \begin{bmatrix} 0 & -0.4444 & -0.4444 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -0.8889 & -1 \end{bmatrix}$$

Substituting the above values in Eqs.(17) and (18),

$$x_P^L = (A_{P-})z^U + (A_{P+})z^L = \begin{bmatrix} 1.8327 \\ 1.9992 \\ -2.0826 \end{bmatrix}$$

$$x_P^U = (A_{P+})z^U + (A_{P-})z^L = \begin{bmatrix} 2.0740 \\ 2.0808 \\ -1.9641 \end{bmatrix}$$

Assuming a 2% change in Θ, (collection of θ_{km}'s), Θ^L and Θ^U are obtained as,

$$\Theta^L = \begin{bmatrix} 0.0392 \\ 0.0197 \\ -0.0203 \end{bmatrix} \text{ and } \Theta^U = \begin{bmatrix} 0.04080 \\ 0.0205 \\ -0.0195 \end{bmatrix}$$

Now,

$$x_{P+}^L = \begin{bmatrix} 1.8327 \\ 1.9992 \\ 0 \end{bmatrix} \text{ and } x_{P-}^L = \begin{bmatrix} 0 \\ 0 \\ -2.0826 \end{bmatrix}$$

$$x_{P+}^U = \begin{bmatrix} 2.0740 \\ 2.0808 \\ 0 \end{bmatrix} \text{ and } x_{P-}^U = \begin{bmatrix} 0 \\ 0 \\ -1.9641 \end{bmatrix}$$

Substituting these values in Eqs.(30) and (31) gives,

$$y^L = \begin{bmatrix} 44.9252 \\ 97.4531 \\ 96.8532 \end{bmatrix} \text{ and } y^U = \begin{bmatrix} 52.9163 \\ 105.5708 \\ 106.8854 \end{bmatrix}$$

Here, $\mu(y_{12pr}^L) = \mu(44.9252) = 1$

and also, $\mu(y_{12pr}^U) = \mu(52.9163) = 1$.

Values of y^L, y and y^U are shown in the bar graph in fig6

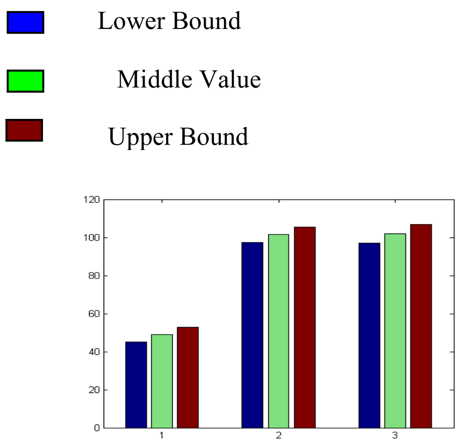

Fig. 6 Lower and Upper bounds of Y (size of Y is 3)

This means y_{12pr} along with its deviations is well within the valid region.

On the other hand if $y_{12pr} < y_{12}^L$ or $y_{12pr} > y_{12}^U$ then, the present y_{12pr} is inconsistent which indicates the presence of bad measurements or some other erroneous situation.

5 Conclusion

An entirely different but a simple method is described. This method provides an easy solution for line susceptance estimation. The Fuzzy Logic approach provides a good decision support tool when the values are uncertain. The above technique can be applied to other line parameters also.

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