A Reliable Fuzzy Logic approach for Measurement Data Validation through line susceptance Estimation

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Abstract: - A novel approach is described to estimate the transmission line susceptances using the linear power flow model with linear relation between the measurement vector and the susceptance vector. The presently estimated susceptance is compared with the already stored correct value, to determine validity of the present measurement using the fuzzy inference

Key-Words: - Data Validation, lower and upper bounds of state estimation, Interval arithmetic, Fuzzy logic, Reliability

1 Introduction

Branch impedances are important parameters in the State Estimation process. Series conductance values and shunt admittances do not play important roles in the state estimation. Therefore they are ignored and only the series line reactances are taken into consideration in this paper. The active power flow in a branch depends very much on the corresponding branch reactance. Errors in the accurate knowledge of line reactances will lead to permanent and serious errors in State Estimation [1]. Hence, correct and accurate line reactance values are very much essential in the state estimation process. The transmission line series reactances are essentially time invariant during short to moderate time spans and change slowly over a long time period due to aging and other reasons [2]. Therefore they are stored in the power system’s parameter data-base and used during state estimation. The parameter data-base is regularly updated to reflect the correct present line reactance values. A sudden or a substantial change in the present line reactance value from the previous value indicates an error in the measurement system or an abnormal condition in the measurement setup. This fact is used, in the method we are going to present in this paper, to check the validity of the measurement results.

Several techniques are available for the estimation of line parameters [3]. The method of Augmented State Vector and similar methods involve non linear equations for the measurement models and the solution involves the iterative solution of the normal equation [3]. The method based on the sensitivity analysis of measurement residuals is another approach [4]-[5]. The method proposed in this paper uses the linear measurement model and hence the solving process is quick and efficient.

2 Problem Formulation

Data validation (Bad data detection) is done in two stages. Stage 1: To estimate the line susceptances correctly. Transmission line susceptances are estimated correctly and stored in the SCADA data base for the subsequent use. Stage 2: To validate the Data. The present power measurement data is validated by comparing the correspondingly estimated susceptance with the available correct susceptances.

Terminology and Basic Relations

Consider the basic DC power flow model shown in Fig.1, where the active power flow and the node phase angles are related as [1].

\[ P_{km} = x_{km}^{-1} (\theta_k - \theta_m) = y_{km}(\theta_k - \theta_m) \]  

(1)

Here, \( P_{km} \) is the branch active power flow, \( \theta_k \) and \( \theta_m \) are the node voltage phase angles, \( x_{km} \) is the line reactance and \( y_{km} = x_{km}^{-1} \) is the series line susceptance. For any \( k \) and \( m \),

\[ y_{km} = y_{mk} . \]
Fig. 1. Transmission Line DC Power Flow Model

Eq. (1) is rewritten as,

\[ P_{km} = (\theta_k - \theta_m)y_{km} \]  

(2)

Now, the equation for power injection at node k is [1],

\[ P_k = \sum_{m\in\Omega_k} x^{-1}_{km}(\theta_k - \theta_m) = \sum_{m\in\Omega_k} (\theta_k - \theta_m)y_{km} \]  

(3)

Where: \( \alpha_k \) is the set of buses adjacent to bus k. Hereafter, we use \( y_{km} \)'s in our equations and calculations, because the use of \( y_{km} \)'s makes the measurement equations linear. \( \theta_j \)'s, used in Eqs.(2) and (3), are accurately determined either by direct measurements or by separate estimation. Then they are used as the coefficients in Eqs.(2) and (3).

**Stage 1: Estimation of Line Susceptances correctly**

After correctly measuring the values of \( P_{km} \)'s, \( P_k \)'s and knowing \( \theta_j \)'s, the set of equations corresponding to Eqs.(2) and (3) are used to estimate \( y_{km} \)'s by the standard linear method. During the estimation process, if any bad data is detected, they are eliminated and only the correct and validated measurements are used to estimate \( y_{km} \)'s. These correct values of \( y_{km} \)'s are stored in the parameter data-base for future comparison.

**2.1 Algorithm for the Estimation of Line Susceptances**

Mark the network nodes as 1, 2, …, 3 and so on. Select the Line Susceptances \( y_{km} \)'s to be estimated by listing the values of k and m appropriately.

1. Measure the active power flows \( P_{km} \)'s and \( P_k \)'s needed, based on the observability criterion.
2. Predetermine or pre-estimate the node voltage phase angles, \( \theta_j \)'s.
3. Write a set of equations linear in \( y_{km} \)'s with \((\theta_k - \theta_m)\) as the coefficients with suitable values for k’s and m’s.
4. Rewrite the above equations in the vector-matrix form as \( z = Hx \) where the measurement vector \( z \) contains the branch flow and node injection power measurements, \( x \) is the vector of susceptances \( (y_{km}) \) to be estimated and \( H \) is the coefficient matrix involving \( \theta \)'s.
5. Solve The resulting linear over determined set of equations to get the estimate for \( y_{km} \)'s using the solution as given by [3]

\[ \hat{x} = (H^TWH)^{-1}H^TWz \]

With usual notations.
6. Store this for comparison to be used in stage 2.

**Stage 2: Data Validation**

Now the Present power measurements are checked for correctness as follows. From the measured data, the susceptances are re-estimated. If these values match with the corresponding correct values, which are already available in the data base, then there is no error. Otherwise error is present in the measured data.

**Algorithm 2**

Data Validation using Estimated Susceptances

1. Using the presently measured \( P_{km} \)'s, \( P_k \)'s and known \( \theta_j \)'s get the over determined state equation obtained from Eq.(2) and (3).
2. Solve it by WLS method to estimate the line susceptances. Call the resulting state vector as \( \hat{x} \).

These are estimated \( y_{km} \)'s.
3. Compare this \( \hat{x} \) with \( x \) obtained in Algorithm 1. 

If \((x-\hat{x})\) is zero or very near to zero, there is no error. The measured data are good. Else, bad data is present.

**2.2 Evaluating 3 Bus System**

**2.21 Case-1**

Consider the 3-bus system shown in Fig.2. \( y_{12}, y_{23} \) and \( y_{13} \) are the line susceptance parameters to be estimated. The bus voltage phase angles are: \( \theta_1 = 0 \) is the reference, \( \theta_2 \) and \( \theta_3 \) are already determined by the previous estimate as,

\[ \theta_2 = -0.0400 \quad \text{and} \quad \theta_3 = -0.0201. \]

The power flow measurement values are,

\[ P_1 = 3.90 \quad \text{p.u.} \quad P_2 = -4.07 \quad \text{p.u.} \]
\[ P_3 = -0.04 \quad \text{p.u.} \quad P_{13} = 2.04 \quad \text{p.u.} \]

The corresponding measurement variances given are,

\[ \sigma_1^2 = 0.004 \quad \text{p.u.} \quad \sigma_2^2 = 0.004 \quad \text{p.u.} \]
\[ \sigma_3^2 = 0.001 \quad \text{p.u.} \quad \sigma_{13}^2 = 0.002 \quad \text{p.u.} \]
The basic equations for the measurement model are,

\[ P_1 = (\theta_1 - \theta_2)y_{12} + (\theta_1 - \theta_3)y_{13} \]
\[ P_2 = (\theta_2 - \theta_1)y_{12} + (\theta_2 - \theta_3)y_{23} \]
\[ P_3 = (\theta_3 - \theta_1)y_{13} + (\theta_3 - \theta_2)y_{23} \]
\[ P_{13} = (\theta_1 - \theta_3)y_{13} \]

Since \( \theta_1 = 0 \), the above equations become,

\[ P_1 = (-\theta_2)y_{12} + (-\theta_3)y_{13} \]
\[ P_2 = (\theta_2)y_{12} + (\theta_2 - \theta_3)y_{23} \]
\[ P_3 = (\theta_3)y_{13} + (\theta_3 - \theta_2)y_{23} \]
\[ P_{13} = (-\theta_3)y_{13} \]

These equations are rewritten in the vector-matrix form as,

\[
\begin{bmatrix}
P_1 \\
P_2 \\
P_3 \\
P_{13}
\end{bmatrix} =
\begin{bmatrix}
-\theta_2 & -\theta_3 & 0 \\
\theta_2 & 0 & \theta_2 - \theta_3 \\
0 & \theta_3 & \theta_3 - \theta_2 \\
0 & -\theta_3 & 0
\end{bmatrix}
\begin{bmatrix}
y_{12} \\
y_{13} \\
y_{23}
\end{bmatrix}
\] (4)

Eq.(4) is written as,

\[ z = Hx \] (5)

Here:

\[
\begin{bmatrix}
P_1 \\
P_2 \\
P_3 \\
P_{13}
\end{bmatrix} =
\begin{bmatrix}
3.9 \\
-4.07 \\
-0.04 \\
2.04
\end{bmatrix}
\]

\[
H =
\begin{bmatrix}
-\theta_2 & -\theta_3 & 0 \\
\theta_2 & 0 & \theta_2 - \theta_3 \\
0 & \theta_3 & \theta_3 - \theta_2 \\
0 & -\theta_3 & 0
\end{bmatrix}
\]

Here, vector \( x \) is a collection of \( y_{km} \)'s and \( km \)'s take the values 12, 13 and 23 in that order.

In this case the measurement covariance matrix is given by,

\[ R_z = \text{diag}(\sigma_1^2, \sigma_2^2, \ldots, \sigma_m^2) \]

\[ = \text{diag}(0.004, 0.004, 0.001, 0.002) \]

The weight matrix \( W \) is given by,

\[ W = R_z^{-1} = \text{diag}(250, 250, 1000, 500) \]

With these values the estimate for \( x \) is given by [2]

\[
\hat{x} = \left[H^TH\right]^{-1}H^TWz
\]

Thus the estimated line susceptances are (in p.u.),

\[
\begin{bmatrix}
\hat{y}_{12} \\
\hat{y}_{13} \\
\hat{y}_{23}
\end{bmatrix} =
\begin{bmatrix}
48.8408 \\
101.4307 \\
101.7690
\end{bmatrix}
\]

**Data Validation**

If these values match with the correct available values of \( y_{km} \)'s, then there is no error. Else there is error in the measured data.

### 2.3 Multiple scans of measurements

When the number of state equations \( m \) is less than the no of parameters \( n \), the matrix \( H^TH \) is not a full rank one and has no inversion. This is an underdetermined system and we cannot apply the WLS method. To overcome this problem we can take multiple scan of the measurements with suitable intervals. The interval between successive measurements should be so selected that the values of measured quantities are sufficiently different from one scan to the next scan. Since the line susceptances are same over successive scans, their number \( n \), remains same where as the number of state equations get increased by \( m \) for each additional scan of measurements. Thus an underdetermined
system is converted into an over determined system. This is illustrated in the example given below.

2.4 Evaluating 5 bus 6 branch network

2.41 Case-2

Consider the 5-bus 6-branch network shown in Fig.3.

![Fig.3. 5-Bus 6-branch sample System](image)

Bus 5 is taken as the reference. Hence \( \theta_5 = 0 \). Four bus injection powers \( P_1, P_2, P_3 \) and \( P_4 \) are measured. Also the four bus voltage phase angles \( \theta_1, \theta_2, \theta_3 \) and \( \theta_4 \) with respect to \( \theta_5 \) are measured.

The linear (DC model) power flow equations are,

\[
\begin{align*}
P_1 &= (\theta_1 - \theta_2) y_{12} + (\theta_3 - \theta_5) y_{15} \\
P_2 &= (\theta_2 - \theta_1) y_{12} + (\theta_2 - \theta_3) y_{23} + (\theta_2 - \theta_3) y_{25} \\
P_3 &= (\theta_3 - \theta_2) y_{23} + (\theta_3 - \theta_4) y_{34} \\
P_4 &= (\theta_4 - \theta_1) y_{34} + (\theta_4 - \theta_4) y_{45}
\end{align*}
\]

Taking \( \theta_5 = 0 \) and rearranging, we get

\[
\begin{bmatrix}
P_1 \\ P_2 \\ P_3 \\ P_4
\end{bmatrix} =
\begin{bmatrix}
\theta_1 - \theta_2 & \theta_1 & 0 & 0 & 0 & 0 \\
\theta_2 - \theta_1 & 0 & \theta_2 - \theta_3 & \theta_2 & 0 & 0 \\
0 & 0 & \theta_3 - \theta_2 & 0 & \theta_3 - \theta_4 & 0 \\
0 & 0 & 0 & 0 & \theta_4 - \theta_3 & \theta_4
\end{bmatrix}
\begin{bmatrix}
y_{12} \\ y_{15} \\ y_{23} \\ y_{25} \\ y_{34} \\ y_{45}
\end{bmatrix}
\]

Here the number of equations are \( m=4 \) and the number of parameters to be estimated are \( n=6 \). Therefore an additional set of measurements are taken after sufficient interval to get a different set of values for \( P \)'s and \( \theta \)'s. Using the additional sub-script 1 and 2 for the first set and second set respectively, the resulting state equation in vector matrix notation is written as,

\[
\begin{bmatrix}
\theta_{11} - \theta_{21} \\ \theta_{11} - \theta_{11} \\ \theta_{21} - \theta_{31} \\ \theta_{21} - \theta_{21} \\ \theta_{31} - \theta_{41} \\ \theta_{31} - \theta_{31} \\ \theta_{31} - \theta_{41} \\ \theta_{31} - \theta_{41}
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\theta_{11} \\ \theta_{11} \\ \theta_{21} \\ \theta_{21} \\ \theta_{31} \\ \theta_{31} \\ \theta_{41} \\ \theta_{41}
\end{bmatrix}
\]

\[
H =
\begin{bmatrix}
\theta_{12} - \theta_{22} \\ \theta_{12} - \theta_{12} \\ \theta_{22} - \theta_{32} \\ \theta_{22} - \theta_{22} \\ \theta_{32} - \theta_{42} \\ \theta_{32} - \theta_{42} \\ \theta_{42} - \theta_{32} \\ \theta_{42} - \theta_{42}
\end{bmatrix}
\begin{bmatrix}
P_{11} \\ P_{21} \\ P_{31} \\ P_{41} \\ P_{12} \\ P_{22} \\ P_{32} \\ P_{42}
\end{bmatrix}
\]

Now, the number of equations is 8 and the number of parameters to be estimated is 6. This is an over determined system and can be solved as usual. The weight matrix values \( W_1 \) and \( W_2 \) for the first and second set of measurements respectively are same, because the same meters are used in both the cases. Therefore, \( W_2 = W_1 \) and the overall 8x8 weight matrix is,

\[
W =
\begin{bmatrix}
W_1 & 0 \\
0 & W_2
\end{bmatrix}
\]

**Numerical values and results.**

In the above example, let the first and second scan measurements be as given below

\[
\begin{bmatrix}
P_{11} \\ P_{21} \\ P_{31} \\ P_{41} \\ P_{12} \\ P_{22} \\ P_{32} \\ P_{42}
\end{bmatrix} =
\begin{bmatrix}
+1.3021 \\ -0.1064 \\ -0.6092 \\ -0.1250 \\ +1.2970 \\ -0.0917 \\ -0.6021 \\ -0.1229
\end{bmatrix}
\]

The Actual susceptance parameter vector denoted by \( x \) is given by

\[
\begin{bmatrix}
y_{12} \\ y_{15} \\ y_{23} \\ y_{25} \\ y_{34} \\ y_{45}
\end{bmatrix} =
\begin{bmatrix}
16.6667 \\ 4.1667 \\ 3.3333 \\ 5.5556 \\ 4.1667 \\ 33.3333
\end{bmatrix}
\]

Now the 8x1 measurement vector \( z \) is obtained by combining \( P_1 \) and \( P_2 \) as,
Determined by,

\[ z = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} +1.3021 \\ -0.1064 \\ -0.6092 \\ -0.1250 \\ +1.2970 \\ -0.0917 \\ -0.6021 \\ -0.1229 \end{bmatrix} \]

The corresponding H matrix is,

\[ H = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} = \begin{bmatrix} 0.0537 & 0.0977 & 0 & 0 & 0 & 0 \\ -0.0537 & 0 & 0.0653 & 0.044 & 0 & 0 \\ 0 & 0 & -0.0653 & 0 & -0.0156 \\ 0 & 0 & 0 & 0 & 0.0156 & -0.0057 \\ 0.0530 & 0.0980 & 0 & 0 & 0 \\ -0.0530 & 0 & 0.650 & 0.0450 & 0 \\ 0 & 0 & -0.650 & 0 & -0.0145 \\ 0 & 0 & 0 & 0 & 0.0145 & -0.0055 \end{bmatrix} \]

When there is no error in the measurement vector z, the given x and the estimated \( \hat{x} \) determined by,

\[ \hat{x} = (H^T WH)^{-1} H^T Wz \]

are same. When there is some error in z, x and \( \hat{x} \) differ. Let an 2% error be introduced in the third element of z as, \( z(3) = 1.02z(3) \). Then the estimated \( \hat{x} \) is found to be markedly different from x, as shown in the Table-1 below.

<table>
<thead>
<tr>
<th>Sequence Number</th>
<th>x</th>
<th>( \hat{x} )</th>
<th>(x-( \hat{x} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16.6667</td>
<td>15.1540</td>
<td>1.5127</td>
</tr>
<tr>
<td>2</td>
<td>4.1667</td>
<td>4.9914</td>
<td>-0.8247</td>
</tr>
<tr>
<td>3</td>
<td>8.3333</td>
<td>6.3683</td>
<td>1.9651</td>
</tr>
<tr>
<td>4</td>
<td>5.5556</td>
<td>6.6190</td>
<td>-1.0634</td>
</tr>
<tr>
<td>5</td>
<td>4.1667</td>
<td>13.0785</td>
<td>-8.9119</td>
</tr>
<tr>
<td>6</td>
<td>33.3333</td>
<td>57.2920</td>
<td>-23.9587</td>
</tr>
</tbody>
</table>

### 3.3 Basis of determination of \( y_{km} \)

The value of \( y_{km} \) has to be determined carefully with as much accuracy and reliability as possible. The true value of \( y_{km} \), represented by \( y_{km}^M \), is generally time invariant and changes slowly over a long time period due to aging and other reasons. This property is used to determine \( y_{km}^M \) by taking the healthy snapshots of the required measurements, while discarding suspicious measurement sets \( [2] \).

Calculation of \( y_{km}^M \) based on the inductance and length of the line can also be used to arrive at the initial estimate and then further refined by successive sequential valid estimations. Thus using the historical data sequence is used to fix \( y_{km}^M \).

### 3.2 Basis of Fixing of \( y_{km}^L \) and \( y_{km}^U \)

\( y_{km}^L \) and \( y_{km}^U \) of \( y_{km} \) values are fixed based on the existing measurement tolerance and standard deviations of measurements used in the calculations. Since the meters used and their configurations do not change in short time spans, the successive valid estimations of permissible deviations are used to determine \( y_{km}^L \) and \( y_{km}^U \). Thus, we build up the best values \( y_{km}^L \) and \( y_{km}^U \) from the successive valid estimations.

### 3.3 Basis of determination of corner points \( a \) and \( b \).

The corner points \( a \) and \( b \) (see Fig.4) of the membership function are selected suitably, say, \( 0.9 \cdot y_{km}^L \) and \( 1.1 \cdot y_{km}^U \) respectively. These values are flexible and can be selected suitably according to the requirements of the Fuzzy Logic design.
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3.4 Determination of $Y_{km}^M$

Here, we assume that all the measurements are valid. If there are any bad data, they are discarded and only good data are retained. First, we determine $P_{km}$ and then $Y_{km}^M$.

3.5 Calculation of $P_{km}$'s.

In a given measurement system either all the $P_{km}$'s are measured or if not, they can be estimated from the measured values of node injection powers and the few measured branch power flows. (Measured values represent their Middle values.) This is demonstrated using the case-1 described earlier.

In case-1, (See Fig.2. ) the measured values are, Three node injections and one branch flow. The relation among the node injection power values and all the branch power flows can be expressed, using the power balance property at the nodes, as follows,

$$P_1 = P_{12} + P_{13}$$
$$P_2 = P_{21} + P_{23} = -P_{12} + P_{23}$$
$$P_3 = P_{31} + P_{32} = -P_{13} - P_{23}$$

To this we also add one more equation involving $P_{13}$, which is a measured data, as,

$$P_{13} = P_{13}$$

This equation is an identity and is used to provide redundancy for the purpose of estimating branch flows. The above equations can be written in the vector-matrix notation as,

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_{13} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} P_{12} \\ P_{13} \end{bmatrix}$$

The above equation can be rewritten as,

$$Z = H_p x_p$$

Where,

$$Z = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_{13} \end{bmatrix}, \quad H_p = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & -1 \\ 0 & 1 & 0 \end{bmatrix} \quad \text{and} \quad x_p = \begin{bmatrix} P_{12} \\ P_{13} \end{bmatrix}$$

The above equation is an over determined set and can be solved for $x_p$ using the state estimation technique as [2],

$$\hat{x}_p = \left(H_p^T W H_p\right)^{-1} H_p^T W z$$

Let $A_p = \left(H_p^T W H_p\right)^{-1} H_p^T W$ (7)

Then, $\hat{x}_p = A_p z$ (8)

$\hat{x}_p$ is the collection of $P_{km}$'s. Once $\hat{x}_p$ is known, $P_{km}$, the power flow in branch(k,m) can be easily picked up from $\hat{x}_p$.

3.6 Calculation of $Y_{km}$’s.

Consider the basic branch power flow equation given by Eq.(2), reproduced here for convenience,

$$P_{km} = (\theta_k - \theta_m) Y_{km}$$

(2)

Here we assume that, all the variables in Eq.(2) represent their middle values.

This equation can be solved for $Y_{km}$ as,

$$Y_{km} = \frac{P_{km}}{(\theta_k - \theta_m)} = \frac{P_{km}}{\theta_{km}}$$

(9)

Here, $\theta_{km} = (\theta_k - \theta_m)$

(10)

3.7 Determination of $Y_{km}^L$ and $Y_{km}^U$.

Once the middle value is found, the lower bound $Y_{km}^L$ and the upper bound $Y_{km}^U$ can be obtained by previous experience, by the specified tolerance limits from the engineering specification or from the knowledge of the estimated standard deviation and so on. In this paper we determine $Y_{km}^L$ and $Y_{km}^U$ from the knowledge of the lower and the upper bounds of the measurement data. Here we use interval arithmetic.

3.8 Interval arithmetic.

We use Interval arithmetic [7] to calculate the Fuzzy membership functions of the estimated parameters.

Let the intervals associated with $P_{km}$, $\theta_k$ and $\theta_m$ be written as,

$$\text{Interval}(P_{km}) = \left[ P_{km}^L, P_{km}^U \right]$$

(11)

$$\text{Interval}(\theta_k) = \left[ \theta_k^L, \theta_k^U \right]$$

(12)

$$\text{Interval}(\theta_m) = \left[ \theta_m^L, \theta_m^U \right]$$

(13)
3.9 The Calculation of $p_{km}^{L}$ and $p_{km}^{U}$

$P_{km}$’s and the measurement vector $z$ are related by the linear relation, Eq.(8). In Eq.(8), vector $\hat{x}_{p}$ is the collection of $P_{km}$’s. By knowing $\hat{x}_{p}^{L}$ and $\hat{x}_{p}^{U}$ we can find $p_{km}^{L}$ and $p_{km}^{U}$ for different $k$’s and $m$’s. $\hat{x}_{p}^{L}$ and $\hat{x}_{p}^{U}$ are determined using Theorem 1, which is given below.

3.10 The Description of the Theorem

Let vectors $x$ and $z$ be related as $x = Az$ where $A$ is a constant matrix. Then, the upper and lower bounds of $x$ and $z$ are related as,

\[
x^{U} = (A_{+})z^{U} + (A_{-})z^{L} \quad \text{and} \\
x^{L} = (A_{-})z^{U} + (A_{+})z^{L}
\]

where: $A_{+} = 0.5(A + \text{abs}(A))$ and $A_{-} = 0.5(A - \text{abs}(A))$

From Theorem 1 and Eq.(8),

\[
x_{p}^{L} = (A_{p_{+}})z_{p}^{U} + (A_{p_{-}})z_{p}^{L} \quad (17) \\
x_{p}^{U} = (A_{p_{-}})z_{p}^{U} + (A_{p_{+}})z_{p}^{L} \quad (18)
\]

Here, $A_{p_{+}}$ is the matrix obtained from $A_{p}$ by replacing all its negative elements by zeros and $A_{p_{-}}$ is the matrix obtained from $A_{p}$ by replacing all its positive elements by zeros. Thus, $P_{km}^{L}$’s and $P_{km}^{U}$’s are determined.

3.11 Calculation of $y_{km}^{L}$ and $y_{km}^{U}$

Assuming that Interval($\theta_{k}$) and Interval($\theta_{m}$) are known, we can calculate, from Eq.(8) and Interval Arithmetic, Interval($\theta_{km}$) as,

\[
\text{Interval}(\theta_{km}) = [\theta_{km}^{L}, \theta_{km}^{U}] \quad (19)
\]

Therefore,

\[
\theta_{km}^{L} = \theta_{k}^{L} - \theta_{m}^{L} \quad (20)
\]

\[
\theta_{km}^{U} = \theta_{k}^{U} - \theta_{m}^{U} \quad (21)
\]

Now, coming to the division operation of Eq.(9), assuming that Interval($\theta_{km}$) does not contain 0, (If it contains 0, then the corresponding power flow will also be zero.) From Eq.(9), we see that the ratio $P_{km}/\theta_{km}$ has to be positive.

Other wise susceptance $y_{km}$ would be negative. Therefore, $P_{km}$ and $\theta_{km}$ both have to be positive or both negative.

When both are positive, the interval division rule gives,

\[
\text{Interval}(y_{km}) = \left[ \frac{p_{km}^{L}}{\theta_{km}^{U}}, \frac{p_{km}^{U}}{\theta_{km}^{L}} \right] \quad (22)
\]

Therefore,

\[
y_{km}^{L} = \frac{p_{km}^{L}}{\theta_{km}^{U}} \quad (23)
\]

\[
y_{km}^{U} = \frac{p_{km}^{U}}{\theta_{km}^{L}} \quad (24)
\]

When both are negative, the interval division rule gives,

\[
\text{Interval}(y_{km}) = \left[ \frac{p_{km}^{L}}{\theta_{km}^{L}}, \frac{p_{km}^{U}}{\theta_{km}^{U}} \right] \quad (25)
\]

Therefore,

\[
y_{km}^{L} = \frac{p_{km}^{L}}{\theta_{km}^{L}} \quad (26)
\]

\[
y_{km}^{U} = \frac{p_{km}^{U}}{\theta_{km}^{U}} \quad (27)
\]

Eqs.(23) and (26) can be combined into a single equation as,

\[
y_{km}^{L} = \frac{P_{km}^{L} + P_{km}^{U}}{\theta_{km}^{U}} \quad (28)
\]

where: $p_{km}^{L} = P_{km}^{L}$ if $P_{km}^{L} > 0$ else $P_{km}^{L} = 0$, and $p_{km}^{U} = P_{km}^{U}$ if $P_{km}^{U} < 0$ else $P_{km}^{U} = 0$.

Similarly, Eqs.(24) and (27) can be combined into a single equation as,

\[
y_{km}^{U} = \frac{P_{km}^{L} + P_{km}^{U}}{\theta_{km}^{L}} \quad (29)
\]

Eqs. (28) and (29) can be expressed in the matrix form as,

\[
y^{L} = \frac{x_{p}^{L}}{\theta^{U}} + \frac{x_{p}^{U}}{\theta^{L}} \quad (30)
\]

\[
y^{U} = \frac{x_{p}^{L}}{\theta^{U}} + \frac{x_{p}^{U}}{\theta^{U}} \quad (31)
\]

Here, the matrix division operation is the element by element division (Array right division) operation.

Matrices $x_{p}^{L}$, $x_{p}^{U}$ etc. have the same definition as in Theorem 1.
Initially, we make sure that there are no abnormal measurement situations or bad data. This is ascertained by state estimation followed by residual analysis techniques and hypothesis testing methods [2],[3]. Once there is no error in the measurement system, $Y_{km}^L$, $Y_{km}^U$ and $Y_{km}^M$ are used to build the Fuzzy member ship function for the Fuzzy set validity as in Fig.4. Now, for any set of present measurements, the validity of the measurement system is determined as follows.

### 3.12 Proof of Theorem 1

Consider the vector-matrix equation $x = Az$ where $A$ is a constant matrix. This can be expanded as,

$$ x_j = \sum_{k=1}^{m} a_{jk} z_k \quad \text{for } j=1,2,\ldots,n $$  \hspace{0.5cm} (32)

Since $x_j$ is the summation of individual product terms, maximum of $x_j$ occurs when all the individual product terms are maximum. Therefore, from Eq.(22), the upper bound $x_j^U$ is given by,

$$ x_j^U = \sum_{k=1}^{m} (a_{jk} z_k)^U \quad \text{for } j=1,2,\ldots,n $$  \hspace{0.5cm} (33)

Now, consider the upper bound of the product term $(a_{jk} z_k)^U$.

When $a_{jk}$ is positive, $(a_{jk} z_k)^U = a_{jk} z_k^U$  \hspace{0.5cm} (34)

When $a_{jk}$ is negative, $(a_{jk} z_k)^U = a_{jk} z_k^L$  \hspace{0.5cm} (35)

where: $z_k^L$ is the lower bound of $z_k$. Eqs.(34) and (35) can be combined into a single equation as,

$$ (a_{jk} z_k)^U = a_{jk}+ * z_k^U + a_{jk}^- * z_k^L $$  \hspace{0.5cm} (36)

Where:

$$ a_{jk}+ = a_{jk} * u(a_{jk}) $$  \hspace{0.5cm} (37)

and,  \hspace{0.5cm} $a_{jk}^- = a_{jk} * u(-a_{jk})$  \hspace{0.5cm} (38)

$a_{jk}^+$ and $a_{jk}^-$ can be expressed as,

$$ a_{jk}+ = a_{jk} * u(a_{jk}) = 0.5 * (a_{jk} + \text{abs}(a_{jk})) $$  \hspace{0.5cm} (39)

$$ a_{jk}^- = a_{jk} * u(-a_{jk}) = 0.5 * (a_{jk} - \text{abs}(a_{jk})) $$  \hspace{0.5cm} (40)

Here $u(\cdot)$ is the standard unit step function. Substituting for $(a_{jk} z_k)^U$ from Eq. (35) in (33), we get

$$ x_j^U = \sum_{k=1}^{m} \left( a_{jk}+ * z_k^U + a_{jk}^- * z_k^L \right) \quad \text{for } j=1,2,\ldots,n $$  \hspace{0.5cm} (41)

In terms of vector-matrix notation, Eq.(41) becomes.

$$ x^U = (A_+) z^U + (A_-) z^L $$  \hspace{0.5cm} (42)

Similarly it can be shown that,

$$ x^L = (A_-) z^U + (A_+) z^L $$  \hspace{0.5cm} (43)

### 4 Validation of Measurement Data Using The Fuzzy Inference

We assume that the member ship functions for all $Y_{km}$’s are readily available. We also assume that the Observability criterion is satisfied for the measurement set.

1. Get all $Y_{km}$’s as described in section 2.1 from the set of measurements.

2. For each $Y_{km}$, Check whether the validity is 1 or less using the corresponding member ship function as in Fig.4. If $Y_{km}^L \leq Y_{km} \leq Y_{km}^U$, then, that $Y_{km}$ is valid. Other wise, validity of that $Y_{km}$ less than 1.

When all $Y_{km}$’s are valid, then the measurement set is valid. Now, the validity member ship function of $Y_{km}$’s can be updated if the situation needs, because of new meters, new configuration etc.

When even a single $Y_{km}$ is invalid (validity less than 1) the measurement set is invalid and should be discarded and suitable diagnostic and corrective action should be taken to remove the anomaly.

The over all measurement set is valid when all $Y_{km}$’s are valid. This is a logical and operation. Therefore, From the Fuzzy Logic, Measurement validity = min{validity of $Y_{km}$’s } over all $km$’s under consideration.

#### 4.1 Case 3

This is a continuation of Example 1, with the same circuit and the same numerical values.

#### 4.2 Calculation of $P_{km}$’s

We start with Eq.(6) which gives the relation between the measurement vector $z$ and $X_p$ which is a collection of $P_{km}$’s. Eq.(6) is reproduced here.

$$ z = H_p x_p $$

where:

$$ x_j^U = \sum_{k=1}^{m} \left( a_{jk}+ * z_k^U + a_{jk}^- * z_k^L \right) \quad \text{for } j=1,2,\ldots,n $$  \hspace{0.5cm} (41)
can be tested similarly.) The Fuzzy membership function for $y_{12}$ is shown in Fig.5. Here the following values are taken from an assumed database.

$y_{12}^M = 50$, $y_{12}^L = 45$ and $y_{12}^U = 55$

Call the present value of $y_{12}$ as $y_{12pr}$. This is equal to 48.8408. Since $y_{12}^L \leq y_{12pr} \leq y_{12}^U$, $y_{12}$ is valid. From Fig.5, we see that $\mu(y_{12pr}) = \mu(48.8408) = 1$.

![Fig.5. Trapezoidal Fuzzy membership Function for $y_{12}$]

### 4.4 Calculation of $p_{km}^L$ and $p_{km}^U$.

Assuming a ±2% change in $z$, $z^L$ and $z^U$ are obtained as,

\[
\begin{bmatrix}
3.8220 \\
-4.1514 \\
-0.0408 \\
1.9992
\end{bmatrix}
= \begin{bmatrix}
0.5556 & 0 & 0 & 0 \\
0.1111 & 0.1111 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
1.8889 \\
0.0 \\
0 \\
2.0808
\end{bmatrix}
\]

Now,

\[
\begin{bmatrix}
0.5556 & 0 & 0 & 0 \\
0.1111 & 0.1111 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
0 & -0.4444 & -0.4444 & -1 \\
0 & 0 & 0 & 0 \\
0 & 0 & -0.8889 & -1
\end{bmatrix}
\]

Substituting the above values in Eqs.(17) and (18),

\[
x_p^L = (A_{p_m}^L)z^L + (A_{p_m}^U)z^L = \begin{bmatrix}
1.8327 \\
1.9992 \\
-2.0826
\end{bmatrix}
\]
\[ x_P^L = (A_{p_+})z^L + (A_{p_-})z^L = \begin{bmatrix} 2.0740 \\ 2.0808 \\ -1.9641 \end{bmatrix} \]

Assuming a 2% change in \( \Theta \), (collection of \( \theta_{km} \)’s), \( \Theta^L \) and \( \Theta^U \) are obtained as,

\[ \Theta^L = \begin{bmatrix} 0.0392 \\ 0.0197 \\ -0.0203 \end{bmatrix} \quad \text{and} \quad \Theta^U = \begin{bmatrix} 0.04080 \\ 0.0205 \\ -0.0195 \end{bmatrix} \]

Now,

\[ x_{p+}^L = \begin{bmatrix} 1.8327 \\ 1.9992 \\ 0 \end{bmatrix} \quad \text{and} \quad x_{p-}^L = \begin{bmatrix} 0 \\ 0 \\ -2.0826 \end{bmatrix} \]

\[ x_{p+}^U = \begin{bmatrix} 2.0740 \\ 2.0808 \\ 0 \end{bmatrix} \quad \text{and} \quad x_{p-}^U = \begin{bmatrix} 0 \\ 0 \\ -1.9641 \end{bmatrix} \]

Substituting these values in Eqs.(30) and (31) gives,

\[ y^L = \begin{bmatrix} 44.9252 \\ 97.4531 \\ 96.8532 \end{bmatrix} \quad \text{and} \quad y^U = \begin{bmatrix} 52.9163 \\ 105.5708 \\ 106.8854 \end{bmatrix} \]

Here, \( \mu(y_{12pr}^L) = \mu(44.9252) = 1 \)

and also, \( \mu(y_{12pr}^U) = \mu(52.9163) = 1 \).

Values of \( y^L \), \( y^M \) and \( y^U \) are shown in the bar graph in fig6

- **Lower Bound**
- **Middle Value**
- **Upper Bound**

![Bar graph showing lower and upper bounds of Y](image)

This means \( y_{12pr} \) along with its deviations is well within the valid region.

On the other hand if \( y_{12pr} \leq y_{12}^L \) or \( y_{12pr} \geq y_{12}^U \) then, the present \( y_{12pr} \) is inconsistent which indicates the presence of bad measurements or some other erroneous situation.

## 5 Conclusion

An entirely different but a simple method is described. This method provides an easy solution for line susceptance estimation. The Fuzzy Logic approach provides a good decision support tool when the values are uncertain. The above technique can be applied to other line parameters also.

### 7 References:


