A NEW MONITORING SPECTRAL TECHNIQUE FOR SUB-SYNCHRONOUS ELECTRICAL TORQUE FREQUENCIES

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Abstract: - The modern protection schemes are designed to protect against the negative effects that has the Subsynchronous Resonance (SSR) in the turbo-generators mechanical system damping. The studies which have been done with torsional monitoring data allows to propose the maintenance and inspection planning to mitigate the damages and overcome with this expensive repairs, this is because very few monitoring equipment is designed to capture transitory sporadic oscillations with torsional phenomenon characteristic. In this work, it is proposed a new monitoring technique through the torsional behavior modes forms based on a spectral analysis of the electrical subsynchronous torque component to detect its associated frequencies and to establish the coincidence with the mechanical turbo-generator torsional oscillation subsynchronous frequencies, transient load model is included indicating results of utmost importance in their modes damping

Key-Words: - Subsynchronous Resonance, Electrical Torque, Subsynchronous component, Prony, Torsional Modes Forms.

1 Introduction

The series compensation has the potential to produce the modes associated with the turbo-generators electrical torque. The influence of the load added to the problem of SSR in the transmission system, can cause problems of instability subsynchronous torsional interactions (SSTI).Most of the time these transient events do not excite the turbo-generator shaft torsional resonances, but occasionally there is a coincidence of the transient's wave form characteristics and torsional resonances resulting in several cycles of high stress. The accumulation of these cycles may lead to crack initiation and fatigue failure. The protection schemes that are implemented to protect the generators, must detect all the faults.

Nevertheless, before the presence of SSR, these schemes they are not adapted to identify the turbo-generator negative effects; this is because it is required a special logic that allows to detect the subsynchronous frequencies and to describe the impact in the generator mechanical turbines system, with the possibility to appear the phenomena associated with the SSR (induction generator effect, torque amplification and SSTI).

Although it is not a new phenomenon, many specialist engineers in electric power systems (E.P.S.) analysis and protections they do not know the effects and possible damages that can cause the appearance of this type of subsynchronous oscillating phenomena. Normally these schemes must take into account the results reported by the tests to the protection equipment against the presence of SSR made in the Navajo, Mohave generating plant during the winter of 1975-1976 [1, 2]. Basically the results describe that the components of subsynchronous frequency of the armature current are related directly to the electrical frequencies torque magnitudes and to the resulting mechanical fatigue in the shaft. The static filters combination, for the excitation control of the damping's and specialized relays in subsynchronous currents identification or torsional fatigue represents a coordination scheme protections to anticipate and to protect the turbo-generator in the presence of SSR and SSTI. The resonant frequencies of a generator system are different for each individual system. Typical frequencies usually lie within a frequency range of 10 to 50 Hz [1, 2, 3]. The sources of torsional current can be any number of mechanisms founded in a typical load profile of an electricity customer:

- Interaction with series capacitor compensated transmission.
- Motor starts.
- High current switching.
- Interaction with HVDC.
- Interaction with are furnaces.

These are just a few examples of torsional current sources. These mechanisms can cause phase imbalance, negative sequence currents and power swings [3].

2 Torsional Monitoring

Very few machinery oscillations monitors are designed for capturing transient, sporadic vibration as it is characteristic of the torsional phenomenon.

The development effort for the life assessment code, several power plants have had fatigue cracks on the rotor shaft. These failures can cause severe damage to the turbo-generator and are potential human safety problem.

The expense of downtime and repair may be in millions of dollars. The causes of these failures are oscillation fatigue initiated and driven by the rotor torsional oscillation. The used techniques to make a monitoring of the turbo-generators torsional behavior can include several stages [3].

A fatigue analysis must include the shaft determination loss life level by each event and with this to determine the fatigues that experience the elements of cumulative way by each generation unit.

The interpretation of these analyses is shown in special graphics that shows the damping which undergoes the turbo-generator mechanical elements [5,6]. In figure 1 the monitoring is helped by Prony analysis [13,14].

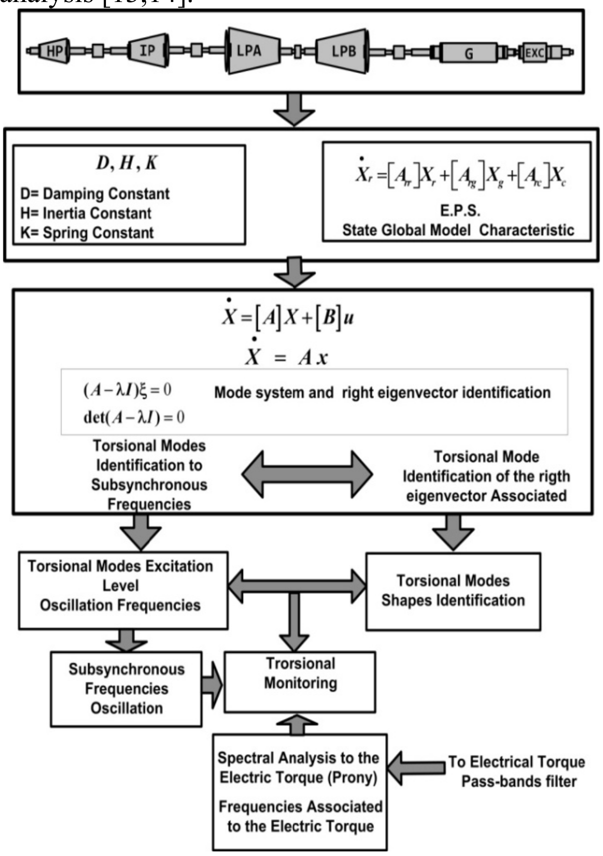


Fig.1 Torsional Monitoring Scheme

The torsional monitoring provides detail analysis of each torsional event. Using the data collected by the monitoring it is possible to assess the mechanical efforts in each shaft section and to determine the life of each. This device makes a continuous monitoring of the turbo-generator mechanical system due to the presence of torsional oscillations. During events with high level of mechanical stress the element must capture and send the data to an analyzer of torsional efforts; this device evaluates the answer of the system and analyzes the behavior due to the shaft mechanical effort

3 IEEE Test System Torsional Monitoring for SSR Study.

The system describes the model of a turbo-generator of 892, 4 MVA to 23 kV, connected to an infinite bus through a transmission system of 500 kV, compensated by capacitors bank at different compensation levels (XC = % XL). Figure 2 shows the test system.

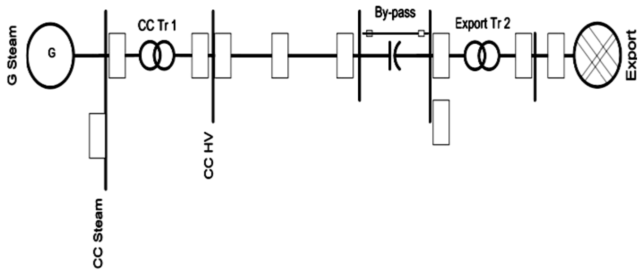


Fig. 2. IEEE Test system for SSR study [4].

All the specified data are in p.u. to the system base [7]. Figure 3 shows, the six masses shaft turbo-generator model, used to analyze the effect of the subsynchronous oscillating phenomenon.

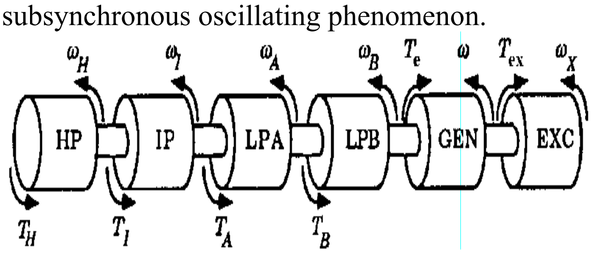


Fig 3. Six masses shaft model [7].

In order to make the monitoring the following considerations are made:

1. The EMTDC / PSCAD v.4.2.1 Program is used [12] to simulate the time domain analysis.
2. The time study settles down in 3 s to 16 samples by cycle.
3. The system values in per unit were determined.

4 Torsional Monitoring Without Fixed Series Compensation.

In the first study case it is considered that the transmission line does not include series fix compensation. Figure 5 shows the monitoring that is made with the PSCAD package [12].

Evidently the torsional monitoring allows developing diverse analyses with the purpose of determining the simply oscillation level or the each fatigue accumulation masses train section (see figure 3).

The torsional Monitoring through the torsional modes behavior identification allows to observe how the torsional mode is excited as the series fixed compensation level increases. In figure 4 shows this case. In this case the system does not perceives any disturbance of subsynchronous currents that can be associated with the torsional oscillation frequencies.

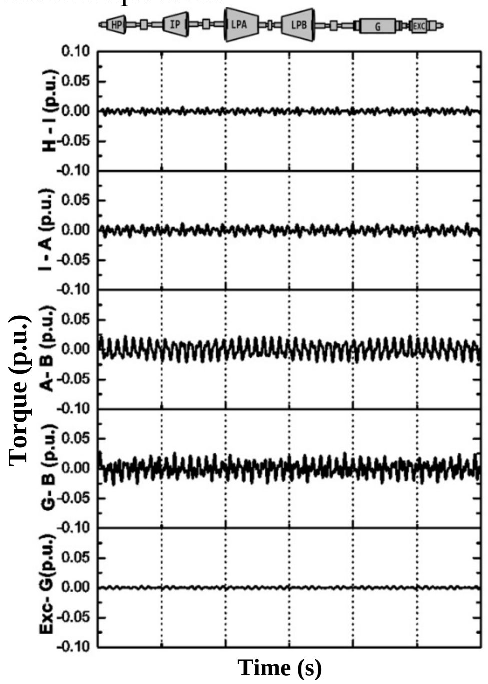


Fig. 4. Torsional Monitoring Xc=0%.

Nevertheless, an important condition of this torsional monitoring is to identify the subsynchronous electrical content torque and determine the associate frequencies to the armature current.

4.1 Electrical torque subsynchronous component determination without fixed series compensation

The excitation rotor winding creates the current flow through the three phase stator windings generator, which are is connected to the grid through transformers. The generator mass and the grid load coupled through the stator create a torsional resistance to the turbine rotational motion. Ideally under steady state conditions (constant rotational speed and constant electrical load) the torque remains essentially constant.

The technique to determine the electrical torque considers the voltages and currents measurements in the turbo-generator terminals. As it is, shown in the Eq. (1):

$$v_{abc}(t) = V\cos(2\pi f\ t + \phi)$$
$$i_{abc}(t) = I\cos(2\pi f\ t + \phi) \qquad (1)$$
$$Te(t) = \frac{v_a i_a + v_b i_b + v_c i_c}{\omega}$$

where:

va, vb, vc - are the phase voltage in the turbo-generator terminals, in [kV]

ia, ib, Ic - are the line currents in turbo-generator terminals, in [kA]

ω - angular velocity, [rad/s].

The justification of this proposal is based on that the electrical power in the terminals is an image of the generator air-gap torque, which reflects the turbine-generator mechanical system behavior. Figure 5 displays, the test system electrical torque measurement signal.

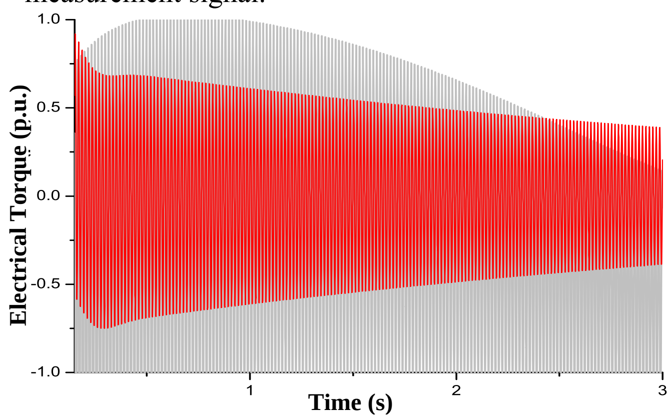


Fig. 5. Electric torque signal (Xc= 0 %).

First, the effect in the mechanical system, in this case is not evident, since series compensation does not exist and the only condition of a single oscillating phenomenon depends on the generator controls tunings. Next stage, considers the inclusion of a pass-band filter that uses the algorithm of Butterworth polynomial 4° filter [8], this works in band-wide of 3 to 50 Hz., detecting the electrical torque associated frequencies in this rank and determine the possible frequencies coincidences associated to the electrical torque. Consequently the dynamic analysis system characteristics will be described more exactly by the nonlinear mathematical models. In this work it was used transformed discrete Fourier series like in signal processing technique and the Prony analysis [8, 9] to sample the subsynchronous frequencies content that has the electrical torque, besides to analyze the phasor behavior using a cosine filter to obtain the electrical torque, which is directly associated with the torsional behavior through the subsynchronous positive sequence current flow in the generator armature.

Applying the Prony [13] analysis to the electrical torque, it was to identify its subsynchronous component; being observed that coincident frequencies within the torsional monitoring do not exist. The answer in the time domain was simulated by making a sampling of 208 μs and 16 samples by cycle, with a total time study of 3 s. In figure 6 show of the spectral analysis electrical torque.

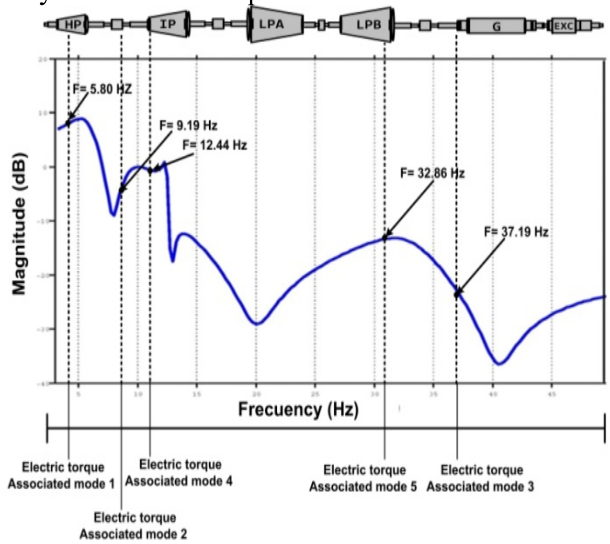


Fig. 6. Spectral analysis (Xc=0 %).

In table 1, appears the associated electrical torque, obtaining the of oscillation mode by the Prony analysis.

Table 1.
Oscillation modes associated to electric torque.

Xc=0%			
Oscillation mode	Frecuency (Hz)	Damping (rad/s)	Phase (^{o})
1	5.8077	0.2401	-140.09
2	9.1965	0.2244	-45.67
3	37.1964	0.9463	-87.08
4	12.4489	0.0174	122.33
5	32.8603	0.1127	-124.25

The monitoring frequency associated to the electrical torque that agrees with at least one of the torsional oscillation frequencies using the torsional behavior identification, in the previous section is not detected. The torsional monitoring is a very fast form to evaluate the fatigue level accumulated in the turbine due to the presence of SSR in the transmission system and the presence of SSTI phenomenon in the turbo-generator. This level of fatigue can be described by means of a loss of life curve due to the fatigue accumulation [6].

This curve normally is provided by the turbo-generators manufacturers where the construction data of each turbine and the fatigue prediction based on the power of the generator are included. The monitoring proposed does not need to create this loss of life curve, because with the frequencies associated identification to the electrical torque is possible to implement preventive actions that allow reducing to the torsional excitation effect and the damping level mitigation. In the following sections the monitoring results done to the test system appear when modifying the compensation level, and later analyzing the effect of the static transient loads models [10].

5 Torsional monitoring for the SSR study considering Xc =20%, 50% and 70% series compensation.

In this section, it is simulated the inclusion of the capacitor series bank in the test system, at different compensation levels (20, 50 and 70 %). In figure 7 it is show, the torsional monitoring in this graphic it is observed a certain increase of the mechanical torque oscillation.

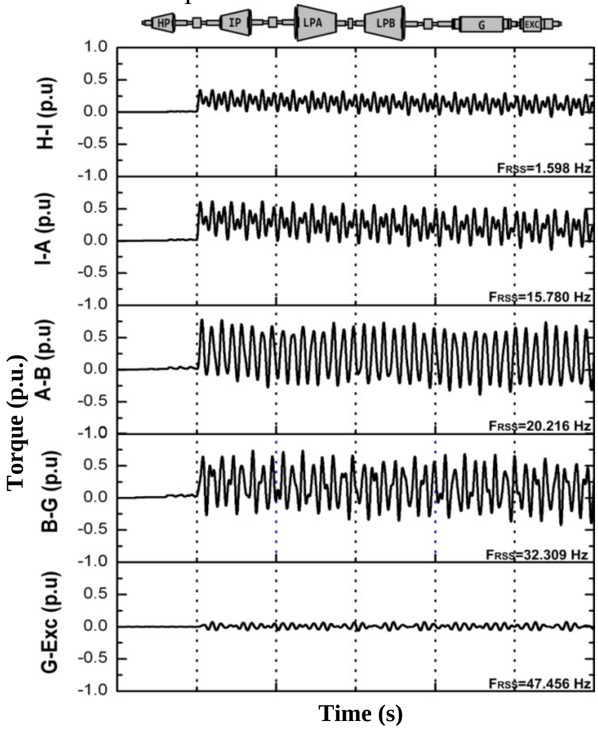
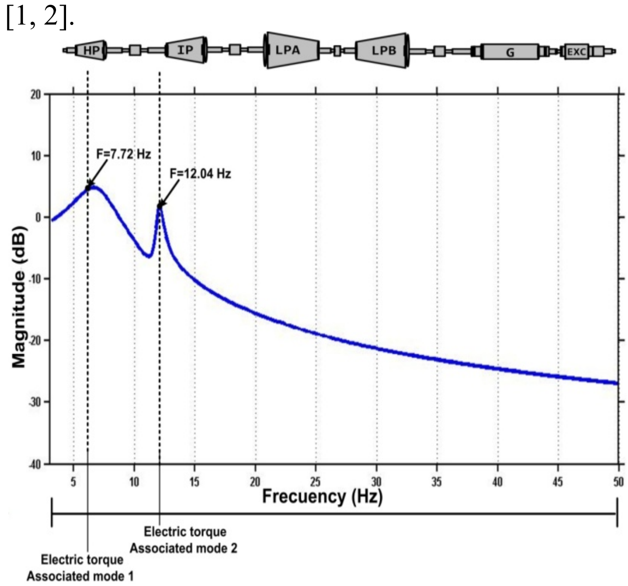


Fig.7. Torsional Monitoring.

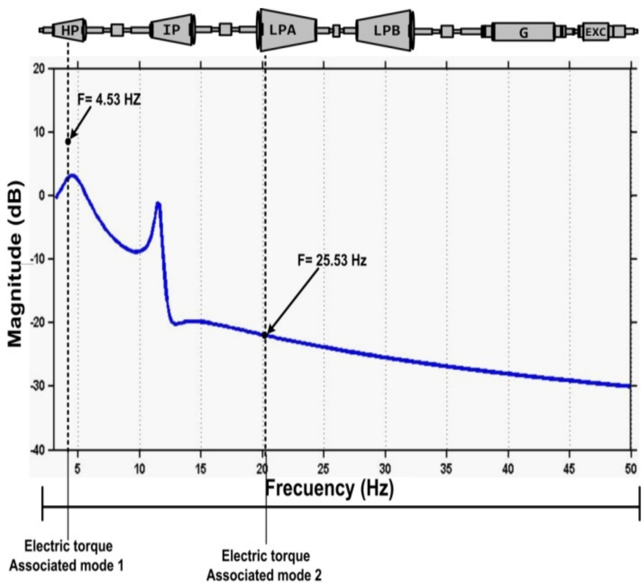
In order to determine the electrical torque, the methodology described previously, single section is used considering that exist different levels from fixed series compensation in the transmission system (20, 50 and 70 %). The fatigue level at these compensation levels is more critical.

The instabilities that appear in the turbo-generator (shaft) mechanical system due to the torsional efforts accumulation produce a drastic increase in the oscillation level which can cause severe damages to the turbines couples.

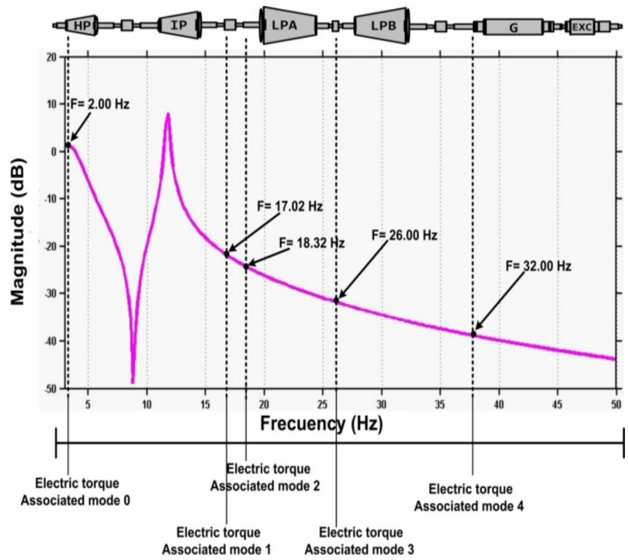
In figures 8 a), b) and c) appear the answers of these subsynchronous currents before a frequency sweeping. Assume that $Te = I_{armature}$ [1, 2].



a) Frequency analysis to Xc= 20%.



b) Frequency analysis to Xc= 50%.



c) Frequency analysis to Xc= 70%.

Fig. 8. Electric torque, frequencies analysis.

This can arrive causing an increase in the fatigue level, inclusively the shaft fracture. Immediately after entering the capacitors bank, a deformation of the waveform appears that evidently describes the electrical torque behavior that can indicate the presence of the subsynchronous content with different oscillation frequencies.

In table 2, the associated modes to the electrical torque oscillations appear at different compensation levels.

Table 2
Oscillation modes associated to electric torque.

Oscillation mode	Frecuency (Hz)	Damping (rad/s)	Phase (°)
Xc=20%			
1	7.7289	-0.2343	112.75
2	12.0423	0.0192	94.11
Xc=50%			
1	4.5378	0.2326	-118.965
2	**25.5373**	**-0.0165**	**67.61**
Xc=70%			
0	**2.0010**	**-0.2735**	**-79.192**
1	**17.0259**	**-0.1546**	**-118.965**
2	18.325	0.0368	135.658
3	**26.002**	**-0.0008**	**-115.26**
4	**32.005**	**-0.0084**	**12.356**
5	52.103	0.2022	100.282

Table 2, shows the oscillation modes, found with the spectral analysis, associated to the electrical torque; in this table the frequencies are indicated which are associated with the electrical torque and in addition it agrees with some of the torsional oscillation frequencies [7, 10, 11].

6.0 Torsional Monitoring for the SSR Study including Static Transient Loads Model.

The analysis for a complex network requires the establishment of a basic criterion for the identification of the small disturbances stability characteristics. Although it is possible in detail to analyze each oscillation modes, it is advisable to distinguish the typical characteristics of the interest modes.

Since it has been mentioned previously, the oscillation modes identification is based on criteria's that depend directly of the model. By means of the changes observation in the eigenvalues for each new power system configuration it is possible to establish relations cause-effect that indicates the main variables that they act in each mode. Other approaches have been based on the analysis of the first sensitivity or second order of each oscillation modes with respect to the state system variables [11]. In this study the dependencies of the variables are used that relate each system modes to the state variables to distinguish the main origin of each oscillation mode. The analysis of the torsional monitoring that the associated rotational displacements with the static transitory loads model characteristics produce an effect that influences in the synchronous speed of the turbo-generator system [10].

An influenced oscillating mode of subsynchronous nature must be located as rapidly as possible to mitigate this negative impact due to the series compensation variation in the system that includes static transient loads models implying a torsional modes modification.

It is important to observe that the modulation of the synchronous frequency of the system is function of the compensation level and by consequence of the reconfiguration by the inclusion of the static transient loads model [10].

The objective in this sections, is to show the results obtained in the study of the subsynchronous torsional interactions due to the connection of the static transient loads models, in the transmission system, in addition an analysis of the torsional monitoring form of the multi-masses system is done and identification of the coincident frequencies of the subsynchronous content in the electrical torque are shown. The same test system was used to analyze the monitoring; cases at three compensation levels and different characteristics were simulated for the static transient loads model [10] (constant power, current constant, constant impedance).

6.1 Torsional monitoring for the SSR considering compensation and loads model with transient impedance, power and constant current characteristic.

In [10], appears and describes to detail loads model used in this work, the model, is a voltage model dependent, which can be in three forms characterized:

1. Constant power (kp=kq=0).
2. Constant current (kp=kq=1).
3. Constant impedance (kp=kq=2).

In a first monitoring condition of the test system including static transient loads model characteristics are simulated. The operation points are: P=0.8 (p.u.), f.p.=0.9, Q=0.34874 (p.u.) and with compensation levels of 20%, 50% and 70%, including the power, current and constant impedance characteristic. Figure 9 show the monitoring of the turbo-generator mechanical system, where the load of the system was located between the nodes of high voltage (bus 2) and (bus 3) of the IEEE test system for the SSR study.

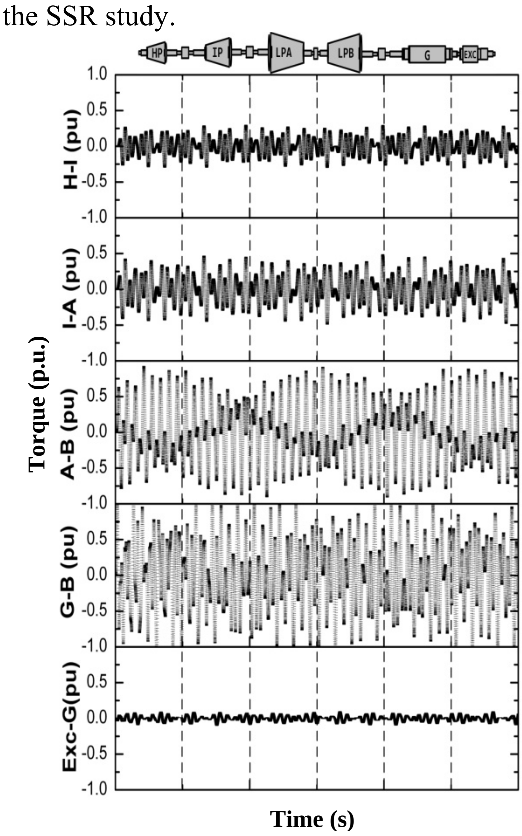


Fig. 9 Torsional Monitoring Including the Static transient Loads Model Effect with Constant Current Characteristics

6.2 Torsional monitoring for the SSR study considering compensation and static loads model with transient and constant power characteristic.

In figures 10 display the monitoring of the turbo-generator mechanical system, considering the static transient loads model of constant power.

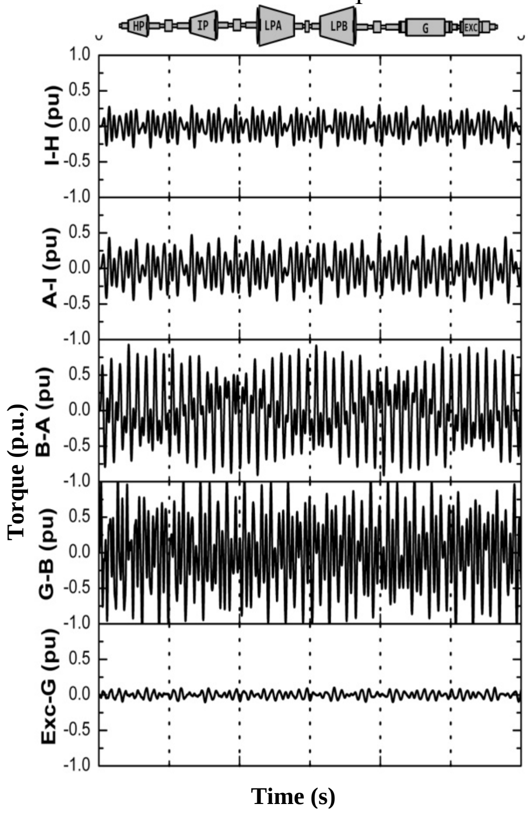


Fig. 10 Torsional Monitoring, considering Including the Static transient Loads Model Effect with Constant Power Characteristics.

6.3 Torsional for the SSR study considering compensation and static loads model with transient and constant impedance characteristic.

Finally in 11 figures show the monitoring, considering the static transient loads model characteristic of constant impedance.

When the compensation level is increased, as it is in figures 9 and 11, the influence of the static transient loads model modifies the damping levels and increases the rotational displacement that the turbines undergo before this the new system configuration.

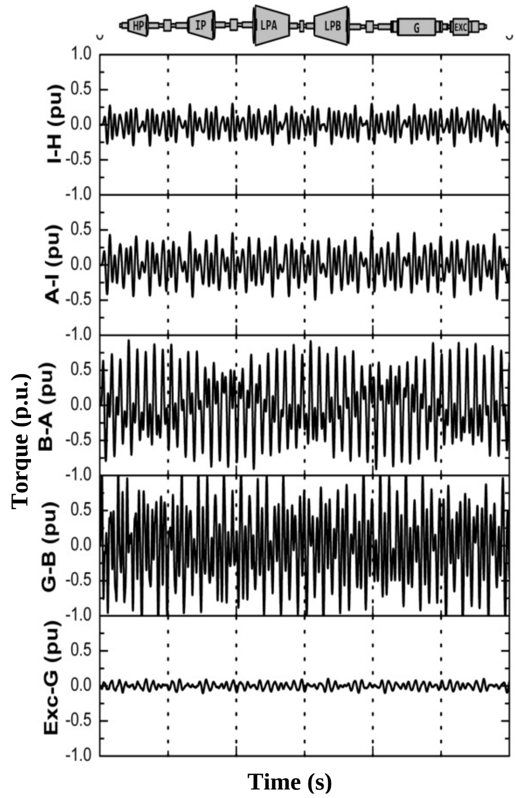
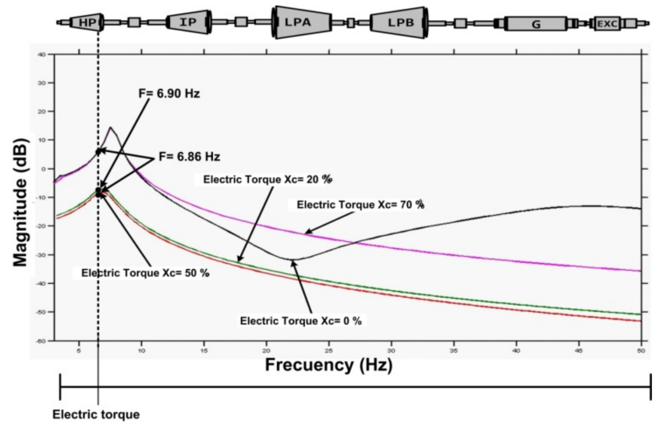


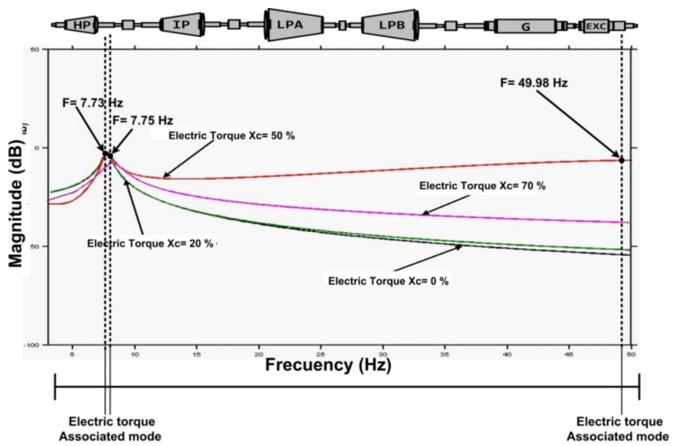
Fig. 11 Torsional Monitoring, considering Including the Static transient Loads Model Effect with Constant impedance Characteristics.

6.4 Subsynchronous content determination of the electrical torque in turbo-generator terminals considering Xc = 20, 50 and 70 % and the influence of the static transient loads model.

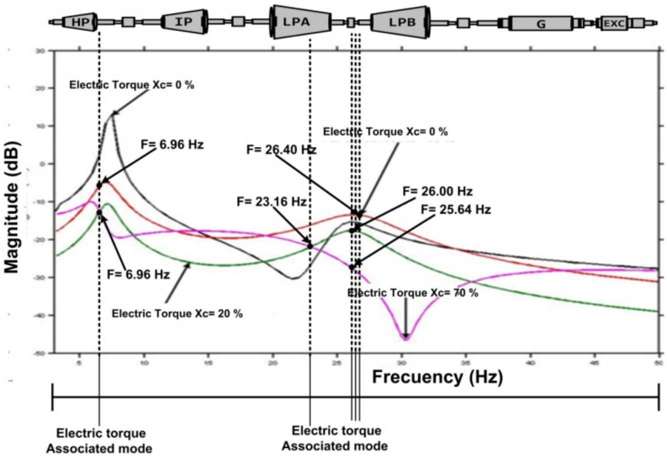
In order to determine the electrical torque subsynchronous component, the methodology described in the previous section, considering that now exists fixed series compensation in the transmission system at different compensation levels (20%, 50% and 70 %) and the static loads model with constant current characteristic. The torsional monitoring is a very fast form to evaluate the fatigue level accumulated in the turbine due to the presence of SSR and the influence of static loads in the transmission system. The level of fatigue at different compensation levels increases when reduce the natural damping of each one of the connections between the turbines, this can cause instabilities in the torsional modes, this is because an excessive fatigue has been generated in the turbo-generator mechanical system due to the torsional efforts accumulation, which can arrive producing severe damages, inclusively to the axis fracture. Figure 12 present the answer of these currents.



a) Static transient Loads Model with Constant Current Characteristic.



b) Static transient Loads Model with Constant Power Characteristic.



c) Static transient Loads Model with Constant Impedance Characteristic.

Fig. 12 Electric torque, frequencies spectral analysis.

In this point the SSR phenomenon appears with greater danger, due to the firing of the compensation in the transmission system approximately in 1.1 s. of the simulation time. The spectral representation of the electrical frequencies associated to the electrical torque, allows to determine where the coincident frequencies with the torsional oscillation frequencies appear, based on the compensation level and static transient loads model characteristic. In table 3, 4 and 5 appear coincident frequencies.

Table 3
Oscillation modes associated to electric torque with constant current characteristics.

Oscillation mode	Frecuency (Hz)	Damping (rad/s)	Phase (°)
Xc=20%			
1	0.0000	0.0000	0.0000
2	6.902621	0.140931	85.0965
Xc=50%			
1	0.0000	0.0000	0.0000
2	6.868608	0.104306	-151.4606
Xc=70%			
1	0.0000	0.0000	0.0000
2	6.860047	0.101364	-152.6992

Table 4
Oscillation modes associated to electric torque with constant power characteristics.

Oscillation mode	Frecuency (Hz)	Damping (rad/s)	Phase (°)
Xc=20%			
1	0.0000	0.0000	0.0000
2	6.976492	0.076038	-231.2818
3	**26.006041**	**0.093241**	**-106.6442**
Xc=50%			
1	0.0000	0.0000	0.0000
2	**6.967676**	**0.131994**	**-13.0971**
3	26.407171	-0.118485	-88.5360
Xc=70%			
1	0.0000	0.0000	0.0000
2	6.210644	-0.125383	77.0676
3	23.161641	0.374037	-20.9547
4	**25.644538**	**-0.747274**	**103.6168**

Table 5
Oscillation modes associated to electric torque with constant impedance characteristics.

Oscillation mode	Frecuency (Hz)	Damping (rad/s)	Phase (°)
Xc=20%			
1	0.0000	0.0000	0.0000
2	7.759052	0.403978	78.3381
Xc=50%			
1	0.0000	0.0000	0.0000
2	7.756392	0.043893	80.0744
3	49.980266	0.353146	2.7458
Xc=70%			
1	0.0000	0.0000	0.0000
2	7.739901	0.38514	92.2784

This type of monitoring should be applied for turbo-generators which are near large transient equipment to measure baseline torsional modal characteristics and after modifications to the rotor which may alter the torsional resonances

In tables 3, 4 and 5 are the associated modes of the resulting electrical torque of the Prony analysis, in this case the study incorporates the static transitory loads model, modifying the associated electrical torque modes like in the previous case coincidences with frequencies of the torsional oscillation modes were identified The proposed monitoring spectral technique indirectly evaluates the damping level of the turbo-generator mechanical system, through the torsional oscillation frequencies identification tend to increase the fatigue level (instability tendency) and the subsynchronous content analysis of the electrical torque in terminals of the generator to identify the coincident frequencies with the torsional oscillation frequencies. This methodology is it shown in figure 13.

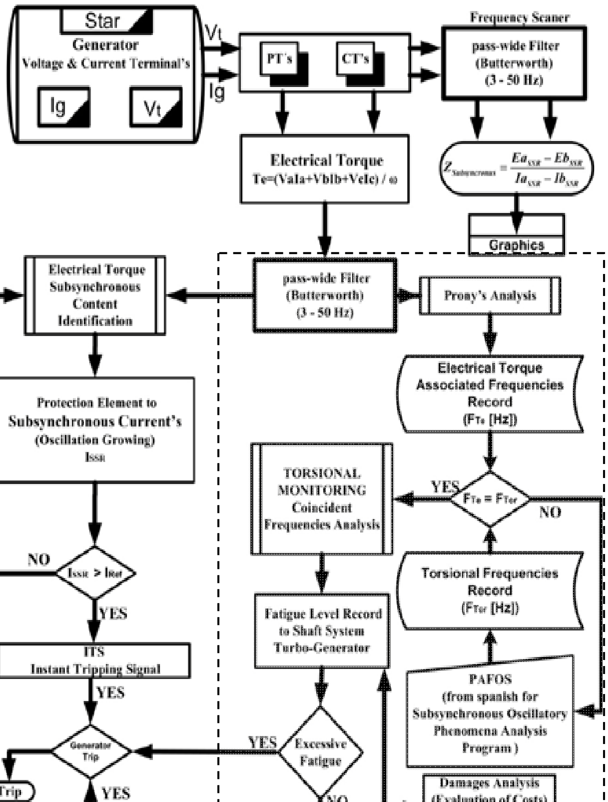


Fig. 13 Methodology proposed including electric torque monitoring.

The new modes introduced by the static transient loads model hit directly in the level of damping of the mechanical system of the generator, an image of this phenomenon can be seen reflected through the air-gap torque that can describe what is happening in the turbo-generator axis and providing information through the detected electrical torque in the terminals. Consequently, the impedance seen from generator node will be modified and also it the torsional frequency and damping characteristics, particularly, the load presents a reaction to the torsional variations of the mechanical system of the turbo-generator. An option that allows reducing the influence of the compensation and the load is the implementation of filters with dynamic characteristics. In the events happened in the plants of Mohave static filters were gotten up to the system, fit to the frequencies of torsional oscillation, filter evidently were these frequencies, reason why the system I present a reconfiguration, modifying the oscillation frequencies, and this would imply to modify the values of the filter elements. Coincidences between the modes associated to the electrical torque with the torsional modes were identified, at least exists an oscillating mode whit the same torsional frequency for a 50% and 70% of the compensation; in the second case the incorporation of the loads model, according to the tables 3, 4 and 5.

7.0 Conclusions

Turbine-generator rotor failures due to torsional induced oscillation fatigue may be catastrophic and costly. The relatively long length of time from onset of the torsional mechanism to failure allows the opportunity to detect, analyze and safely shut down the turbo-generator. This incipient failure detection is possible with adequately designed data acquisition monitoring systems.

The identification of the subsynchronous frequencies component in the electrical torque is described and analyzed with two cases a methodology to identify the coincidences between the frequencies associated to the electrical torque and the turbo-generator torsional oscillation frequencies; to evaluating the negative influence that can have the static loads transient, connected to the power system, in the presence of SSR and SSTI. If in the subsynchronous content of the electrical torque coincident frequencies with the torsional oscillation frequencies exist, these can be associated to the appearance of torques that are opposed to the natural torques, influencing the decrease of the damping levels between turbines, causing the torsional fatigue in each one of the shaft sections. The load analysis effect, show sample that the model can modify the damping characteristics; because new modes of resonance are introduced that modify the existing modes of the system.

The variation of the load model has the potential to interact with the torsional modes of the turbo-generator as it is appraised in the results of the studies for different levels from compensations and different types from load. The application of the methods of analysis to a E.P.S. shown that the load affects in important form the damping of the critical modes of the E.P.S. (torsional modes at greater compensation level), based on the characteristic of dependency of voltage of the model.

The flexible configurability of the monitoring and criteria makes it possible to adapt the measurement system to suit any generator system or measurement requirements.

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