A Novel Approach to Evaluate Incremental Transmission Losses

P. M. De Oliveira-De Jesus, and M. A. Alvarez
Simon Bolivar University
Conversion of Energy Department
Caracas
Venezuela
pdeoliveira@usb.ve, malvarezp@usb.ve

M. T. Ponce de Leão
Universidade do Porto
DEEC-FEUP
Oporto
Portugal
mleao@fe.up.pt

J. M. Yusta
Zaragoza University
Electrical Department
Caracas
Venezuela
jmyusta@unizar.es

Abstract: During the past thirty years, the evaluation of incremental losses in transmission systems has been widespread around the world encompassing a fundamental task in order to solve the well-known economic dispatch problem. Nowadays, incremental losses or computation of penalty factors has gained a new relevance allocating transmission losses among market participants being integrated in the spot pricing model. Nevertheless, vagueness in power injections from load demand and renewable resources has a fundamental influence on transmission incremental losses, and consequently affecting spot prices of the energy. This paper presents a new method to compute Incremental Transmission Loss (ITL) factors taking into account the effect of uncertainty on power injections due to measurement errors or future estimation. Fuzzy Set Theory is applied to deal with vagueness of information about power injections. The proposed technique provides a broad spectrum of feasibility of incremental losses in order to perform operational planning tasks by Transmission System Operators or network regulators. A 3-bus system is used to illustrate the proposed methodology.

Key–Words: Incremental transmission loss factors, loss allocation, spot pricing.

1 Nomenclature

P_k	Real power injected at bus k
P_{Gk}	Real power generated at bus k
P_{Dk}	Real power demanded at bus k
Δ	Fuzzy deviation
μ	Membership degree
$\sim$	Fuzzy operator
$\ominus$	Fuzzy difference operator
$\oplus$	Fuzzy addition operator
$\odot$	Fuzzy multiplication operator
csp	Crisp vaule
n	number of nodes or buses
P_{Gk}^{ctr}	Central value of real power generated at bus k
P_{Dk}^{ctr}	Central value of real power demanded at bus k
LMP_k	Locational Marginal Price at bus k
ITL_k	Incremental Transmission Loss factor at bus k
λ	Market System Price
B'	DC matrix
L_{DC}	Losses DC approach
L	Losses AC approach
θ	Bus angle DC approach
ψ	ITL factor DC approach
V , ϕ	Bus Voltage and angle AC approach

2 Introduction

The computation of Incremental Transmission Loss (ITL) factors has been widespread in power system analysis in the last fifty years [1], [2]. Recently, ITL factors have acquired renewed attention because they can be integrated in the spot pricing problem in order to allocate transmission losses among market participants though Locational Marginal Prices (LMPs) [3–5].

There are two ways to compute ITL factors as loss components of LMPs:

- First by means of an *ex-ante* approach where ITL factors can be directly obtained (before the course of events) from Lagrangian multipliers of the Optimal Power Flow (OPF) problem [6] [7] or aplying penalty factors as described in [4].
- Second by means of an *ex-post* approach where ITL factors can be calculated (afterwards the course of events) using a known state of the system using available data from the Supervisory Control And Data Acquisition (SCADA) system [9]. This practice is applied when the optimal dispatch of generators has been previously performed. Then, ITL factors are calculated using the SCADA's collected data related to given a operating point.

It is important to highlight that under both approaches, ITL factor calculations are strongly affected by several sources of uncertainty as measurement errors, variations on load consumption or the intermittency of natural resources [10] [8]. Some attempts have been made to achieve the effect of uncertainty upon the spot prices and ITLs under the ex-ante approach. For instance, in [11], fuzzy set theory was applied to compute fuzzy spot prices in order to dispatch power generators at minimum production cost. These analysis are based on multi-parametric Fuzzy Optimal Power Flow studies and ITL factors are obtained using a complex vertex identification and estimating branch losses from the DC model. Solving this type fuzzy optimization problems implies a great computation cost not suitable to be applied on large-scale applications.

In this paper, the effect of uncertainty of the nodal power injections upon ITL factors is assessed using an alternative approach based on an *ex-post* analysis from measured and estimated data.

Uncertainty in load and generation power injections are modeled as fuzzy numbers that can be assumed or directly obtained from the output of a *Fuzzy System State Estimator* (FSSE) [12] providing a whole spectrum of feasibility of all measured and estimated variables as shown in Figure 2. The FSSE/Fuzzy ITL calculation modules can coexist jointly with the monitoring system (SCADA) and central dispatch authority or Transmission System Operator (TSO).

This paper is organized as follows: section 2 is devoted to describe the fuzzy model applied to loads and generating buses, section 3 describes the algorithm to compute proposed fuzzy ITL factors, section 4 discusses an illustrative example and finally, section 5 draws the main conclusions and recommendations.

3 Modeling Fuzzy Power Injections in Network Analysis

Fuzzy set theory was introduced by [13] as a mathematical formulation of describing vagueness and imprecision. The basic idea of fuzzy sets is somewhat simple. Thinking in deterministic way, a specified element of the space either belongs to or doesn't belong to the specified set. After that, the membership of an element of the universe is crisp and nonfuzzy. In Fuzzy sets theory, the association it is allowed the degree of membership for each element to range over the unit interval [0,1]. Fuzzy set foundations can be reviewed in detail in [14] and a complete description of the fuzzy set concepts applied to power system applications can be found in [15]. In the following lines, a brief description of fuzzy injections is provided.

3.1 Power Injections as Fuzzy Numbers

The real power injection P_k associated to a specific bus *k*, as shown in Figure 1 depends on real power consumption P_{Dk} and real power generation P_{Gk}.

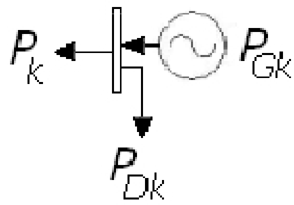

Figure 1: Real power injection at bus k

In a crisp sense, the net real power injection is defined as:

$$P_k = P_{Gk} - P_{Dk} \quad k = 1, \ldots, n \tag{1}$$

As a result, the uncertainty on a real power injection $\widetilde{P}_k$ depends strongly on uncertainties in load consumption $\widetilde{P}_{Dk}$ and the power generation $\widetilde{P}_{Gk}$.

In general, any power injection $\widetilde{P}_k$, $\widetilde{P}_{Dk}$ and $\widetilde{P}_{Gk}$ associated to a specific bus *k* can be described by a membership function relating each element $x \in \mathbb{R}$ to its compatibility or association degree according to:

$$\widetilde{P}_k = \{(x, \mu_{\widetilde{P}_k}(x)) \mid x \in \mathbb{R}\}; \mu_{\widetilde{P}_k}(x) : \mathbb{R} \rightarrow [0,1] \tag{2}$$
$$\widetilde{P}_{Dk} = \{(x, \mu_{\widetilde{P}_{Gk}}(x)) \mid x \in \mathbb{R}\}; \mu_{\widetilde{P}_{Dk}}(x) : \mathbb{R} \rightarrow [0,1]$$
$$\widetilde{P}_{Gk} = \{(x, \mu_{\widetilde{P}_{Gk}}(x)) \mid x \in \mathbb{R}\}; \mu_{\widetilde{P}_{Gk}}(x) : \mathbb{R} \rightarrow [0,1]$$

These degree ranges of membership are given by $\mu_{\widetilde{P}_k}(x)$, $\mu_{\widetilde{P}_{Dk}}(x)$ and $\mu_{\widetilde{P}_{Gk}}(x)$, respectively.

In normalized fuzzy sets, this membership value goes from 0 to 1 leading to a gradual transition between a complete belonging of *x* to $\widetilde{P}_k$ and no belonging of *x* to $\widetilde{P}_k$. Then, a normalized fuzzy number is defined on the real semi-plane such that $\mu_{\widetilde{P}_k}(x)$ is normalized between 1-cut and 0-cut and piecewise continuous. Figures 3 and 4 show the difference between a fuzzy power injection and a deterministic power injection. In the fuzzy case, power injection values *x* near the reference central point are considered compatible with the label of the set. The membership value α-cut give us a non-linear quantification of that compatibility. Then, the fuzzy number of Figure 3 means that real power injections at bus *k are close to 800kW*. On the other hand, in the deterministic case, only one value is considered compatible, the real power injections at bus *k are exactly 800kW*.

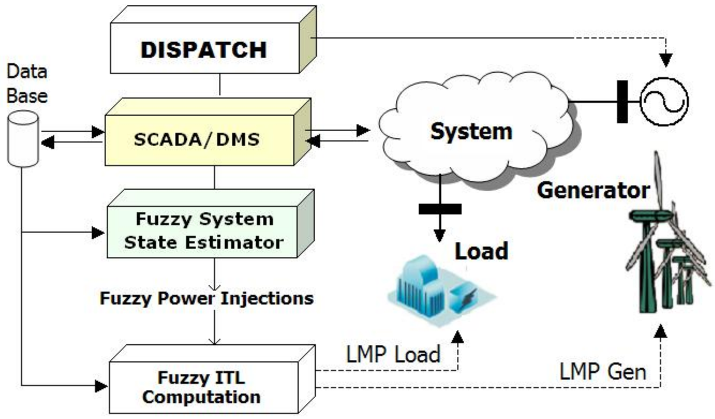

Figure 2: Fuzzy loss access-pricing scheme

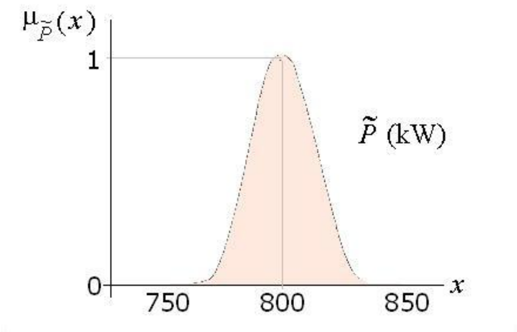

Figure 3: Fuzzy and deterministic power injections. A “more less 800kW” power injection.

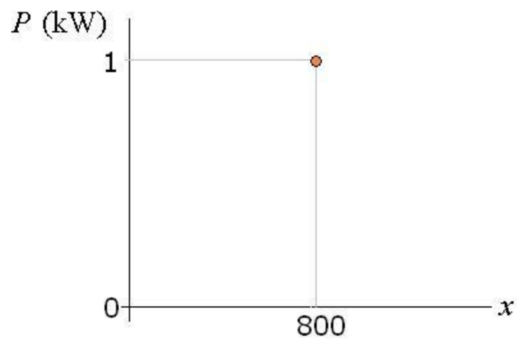

Figure 4: Fuzzy and deterministic power injections. A 800kW power injection.

This non-linear quantification can be simplified using a trapezoidal representation as shown in Figure 5. Triangular representation (see Figure 6) is a particular case of trapezoidal representation where the membership value give a piecewise linear and constant quantification of that compatibility.

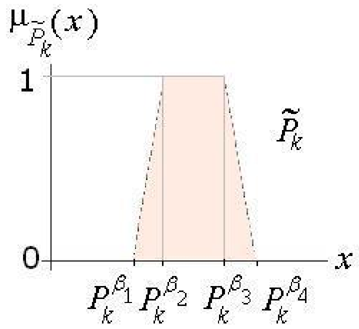

Figure 5: Fuzzy numbers. A trapezoidal representation.

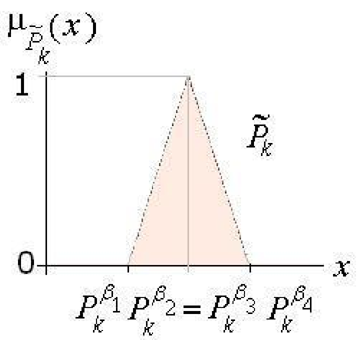

Figure 6: Fuzzy numbers. A triangular representation.

A trapezoidal fuzzy set can be represented by: ${P_k}^{\beta_1}$, ${P_k}^{\beta_2}$, ${P_k}^{\beta_3}$, ${P_k}^{\beta_4}$ where ${P_k}^{\beta_1} \leq {P_k}^{\beta_2} \leq {P_k}^{\beta_3} \leq {P_k}^{\beta_4}$. Each one is associated to a membership degree as indicated as follows:

$$0 \leq \mu_{\widetilde{P}_k}(x) \leq \begin{cases} \frac{x - P_k{}^{\beta_1}}{P_k{}^{\beta_2} - P_k{}^{\beta_1}} & \text{if } x \in [P_k{}^{\beta_1}, P_k{}^{\beta_2}] \\ 1 & \text{if } x \in [P_k{}^{\beta_2}, P_k{}^{\beta_3}] \\ \frac{x - P_k{}^{\beta_4}}{P_k{}^{\beta_3} - P_k{}^{\beta_4}} & \text{if } x \in [P_k{}^{\beta_3}, P_k{}^{\beta_4}] \end{cases} \tag{3}$$

A more general and even useful notion is that of an α-cut or level set [14]. At given membership level $\mu_{\widetilde{P}_k}(x) = \alpha$, the hard set $\widetilde{P}_k^{\alpha}$ is defined in $\mathbb{R}$ for each cut [0,1] such that:

$$\widetilde{P}_k^{\alpha} = \{(x, \mu_{\widetilde{P}_k^{\alpha}}(x) \geq \alpha) \mid x \in \mathbb{R}\} \tag{4}$$

3.2 Fuzzy Loads and Fuzzy Power Generation Injections

In fuzzy sense, the system real power injections can be gathered in a *1xn* vector as follows:

$$\widetilde{\boldsymbol{P}} = [\widetilde{P}_1, \ldots, \widetilde{P}_k, \ldots, \widetilde{P}_n] \tag{5}$$

where *n* is the number of buses of the system and each trapezoidal fuzzy power injection is defined by four deterministic numbers as:

$$\widetilde{P}_k = (P_k^{\beta_1} : P_k^{\beta_2} : P_k^{\beta_3} : P_k^{\beta_4}) \tag{6}$$

In fuzzy sense, an equivalent net real power injection equation is given by:

$$\widetilde{\boldsymbol{P}} = \widetilde{\boldsymbol{P}}_{\boldsymbol{G}} \ominus \widetilde{\boldsymbol{P}}_{\boldsymbol{D}} \tag{7}$$

where $\ominus$ is the fuzzy difference operator for trapezoidal membership functions[1]. Fuzzy load and generation vectors are given by:

$$\widetilde{\boldsymbol{P}}_{\boldsymbol{G}} = [\widetilde{P}_{G1}, \ldots, \widetilde{P}_{Gk}, \ldots, \widetilde{P}_{Gn}] \tag{8}$$
$$\widetilde{\boldsymbol{P}}_{\boldsymbol{D}} = [\widetilde{P}_{D1}, \ldots, \widetilde{P}_{Dk}, \ldots, \widetilde{P}_{Dn}] \tag{9}$$

Each component of $\widetilde{\boldsymbol{P}}_{\boldsymbol{G}}$ and $\widetilde{\boldsymbol{P}}_{\boldsymbol{D}}$ can be represented as the quadruples:

$$\widetilde{P}_{Gk} = (P_{Gk}^{\beta_1} : P_{Gk}^{\beta_2} : P_{Gk}^{\beta_3} : P_{Gk}^{\beta_4}) \tag{10}$$
$$\widetilde{P}_{Dk} = (P_{Dk}^{\beta_1} : P_{Dk}^{\beta_2} : P_{Dk}^{\beta_3} : P_{Dk}^{\beta_4}) \tag{11}$$
$$\widetilde{P}_{Gk} = P_{Gk}^{ctr} \oplus (\Delta P_{Gk}^{\beta_1} : \Delta P_{Gk}^{\beta_2} : \Delta P_{Gk}^{\beta_3} : \Delta P_{Gk}^{\beta_4}) \tag{12}$$
$$\widetilde{P}_{Dk} = P_{Dk}^{ctr} \oplus (\Delta P_{Dk}^{\beta_1} : \Delta P_{Dk}^{\beta_2} : \Delta P_{Dk}^{\beta_3} : \Delta P_{Dk}^{\beta_4}) \tag{13}$$

[1] Because the concept of fuzzy numbers include the interval at given membership value as a particular case, fuzzy arithmetic is an extension of interval arithmetic. Hence, given two convex-normalized trapezoidal fuzzy numbers $\widetilde{a} = (a^{\beta_1} : a^{\beta_2} : a^{\beta} : a^{\beta_4})$ and $\widetilde{b} = (b^{\beta_1} : b^{\beta_2} : b^{\beta} : b^{\beta_4})$ the difference is $\widetilde{a} \ominus \widetilde{b} = (a^{\beta_1} - b^{\beta_4} : a^{\beta_3} - b^{\beta_3} : a^{\beta_3} - b^{\beta_2} : a^{\beta_4} - b^{\beta_1})$

where the symbol Δ denotes the corresponding deviation respect to the center value. The symbol $\oplus$ denotes the fuzzy addition operator for trapezoidal fuzzy sets[2]. The center values P_{Gk}^{ctr} and P_{Dk}^{ctr} are given by the mean value of the elements having 1.0 membership degree:

$$P_{Dk}^{ctr} = \left(\frac{P_{Dk}^{\beta 2} + P_{Dk}^{\beta 3}}{2}\right); P_{Gk}^{ctr} = \left(\frac{P_{Gk}^{\beta 2} + P_{Gk}^{\beta 3}}{2}\right) \tag{14}$$

4 Incremental Transmission Loss Pricing Fundamentals

Under the incremental cost allocation approach, economical signals are sent to the market agents by means of Locational Marginal Prices (LMPs). Hence, in order to allocate the cost of the losses, LMPs are applied at given bus k to remunerate or penalize the effect of each power injection (producer or load connected) upon power loss increase or decrease.

Then, for each bus k, LMP_k is computed as a function of the real incremental loss coefficients ITL_k and the market system price λ:

$$LMP_k = \lambda(1 - ITL_k) \tag{15}$$

The ITL factors are defined as the variation in the real power losses, due to the incremental change of the real power injections in each bus.

$$ITL_k = \frac{\partial L}{\partial P_k}(V_k, \theta_k) \tag{16}$$

$$rITL_k = \frac{\partial L}{\partial Q_k}(V_k, \theta_k) \tag{17}$$

These prices and factors are typically interpreted in crisp sense. Next section provides a procedure to interpret these economical signals under a fuzzy sense.

5 Fuzzy Incremental Transmission Loss Pricing

The idea behind this proposal is to estimate ITL factors in fuzzy sense. To do this, it is necessary to assess the state system in fuzzy sense. It is like to take a snapshot of the state of the system taking into account the intrinsic uncertainties of the power injections.

The proposed procedure has four basic steps as shown in Figure 7.

[2] Fuzzy addition of trapezoidal fuzzy sets: $\widetilde{a} \oplus \widetilde{b} = (a^{\beta_1} + b^{\beta_1} : a^{\beta_2} + b^{\beta_2} : a^{\beta_3} + b^{\beta_3} : a^{\beta_4} + b^{\beta_4})$

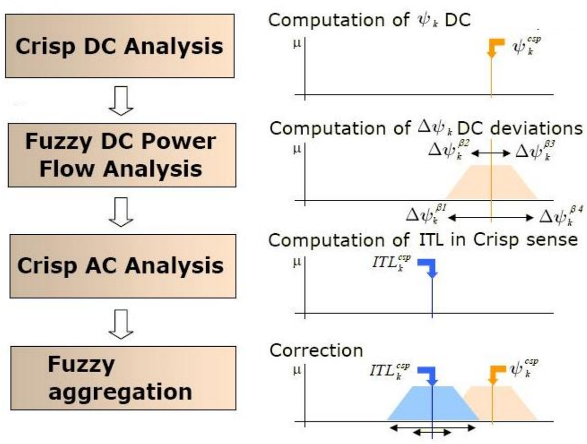

Figure 7: General Procedure to get Fuzzy ILFs

First, a crisp analysis is performed using a standard DC model in order to obtain crisp ψ factors. The state of the system is directly achieved from central values of fuzzy loads and power generation injections discussed in previous section. Second, the basic Fuzzy DC Power Flow is used to obtain angle and ψ deviations respect to the crisp values. Fuzzy trapezoidal representation requires four calculations. Third, a standard AC model is run to get ITLs in crisp sense. Fourth, the fuzzy ITL factors obtained in steps 1 and 2 are corrected using real crisp value obtained in step 3. At this point, fuzzy incremental factors computed under a DC approach can be integrated on the loss allocation process through the short-term tariffs of use of the network.

☐ STEP 1: CRISP DC ANALYSIS

The Crisp DC Incremental Transmission Loss Factors are defined as the variation in the real power losses owing to the incremental change in a nodal power injection k:

$$\psi_k^{csp} = \frac{\partial L_{DC}}{\partial (P_{Gk}^{ctr} - P_{Dk}^{ctr})} \qquad (18)$$

where losses are estimated using:

$$L_{DC}^{csp} = \sum_{i=1}^{n}\sum_{k=1}^{n} G_{ik}(1 - \cos\theta_{ik}^{csp}) \qquad (19)$$

where G_{ik} is the real part of each impedance element of matrix $\mathsf{Y_{BUS}}$.

In the DC model, power losses are calculated from the state of the system established by $\boldsymbol{\theta^{csp}} = [\theta_1^{csp}, \ldots, \theta_n^{csp}]$. A deterministic DC power flow [2] must be previously performed using the specified injected real powers associated with the central point of load a generation membership vectors $\boldsymbol{P_G^{ctr}} = [P_{G1}^{ctr}, \ldots, P_{Gk}^{ctr}, \ldots, P_{Gn}^{ctr}]$ and $\boldsymbol{P_D^{ctr}} = [P_{D1}^{ctr}, \ldots, P_{Dk}^{ctr}, \ldots, P_{Dn}^{ctr}]$ obtained from Equation 14 for each bus of the system. Thus, using classic DC formulation crisp values for the angles crisp are obtained.

$$\boldsymbol{\theta^{csp}} = \mathsf{B}'^{-1} \cdot (\boldsymbol{P_G^{ctr}} - \boldsymbol{P_D^{ctr}}) \qquad (20)$$

where B' is the DC model matrix. In this formulation column and file of slack bus must be eliminated.

ψ factors are derived from loss equation 19 applying the chain rule as follows:

$$\frac{\partial L_{DC}}{\partial \theta_i^{csp}} = \sum_{k=1}^{n} \frac{\partial L_{DC}}{\partial (P_{Gk}^{ctr} - P_{Dk}^{ctr})} \frac{\partial (P_{Gk}^{ctr} - P_{Dk}^{ctr})}{\partial \theta_i^{csp}} \qquad (21)$$

Examining this equation it is possible to rearrange it as:

$$\begin{bmatrix} \frac{\partial L_{DC}}{\partial P_1^{ctr}} \\ \vdots \\ \frac{\partial L_{DC}}{\partial P_n^{ctr}} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_1^{csp}}{\partial \theta_1^{csp}} & \cdots & \frac{\partial P_n^{csp}}{\partial \theta_1^{csp}} \\ \vdots & \ddots & \vdots \\ \frac{\partial P_1^{csp}}{\partial \theta_n^{csp}} & \cdots & \frac{\partial P_n^{csp}}{\partial \theta_n^{csp}} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial L_{DC}}{\partial \theta_1^{csp}} \\ \vdots \\ \frac{\partial L_{DC}}{\partial \theta_n^{csp}} \end{bmatrix} \qquad (22)$$

where $\frac{\partial L_{DC}}{\partial P_1^{ctr}}, \ldots, \frac{\partial L_{DC}}{\partial P_n^{ctr}}$ are the incremental transmission loss factors ψ_k^{csp} under a DC approach.

The real power injection in the DC model is given by:

$$P_i^{csp} = \sum_{k=1}^{n} (G_{ik} \cdot \cos\theta_{ik}^{csp} + B_{ik} \cdot \sin\theta_{ik}^{csp}) \qquad (23)$$

The partial derivatives of a power injection respect to the bus angles are:

$$\frac{\partial P_k^{csp}}{\partial \theta_i^{csp}} = -G_{ik} \cdot \sin\theta_{ik}^{csp} + B_{ik} \cdot \cos\theta_{ik}^{csp} \qquad (24)$$

where B_{ik} are the imaginary part of each impedance element of matrix $\mathsf{Y_{BUS}}$. As θ_{ik}^{csp} is small, then $\cos\theta_{ik}^{csp} \approx 1$ and $\sin\theta_{ik}^{csp} \approx 0$ then

$$\frac{\partial P_k^{csp}}{\partial \theta_i^{csp}} = B_{ik} \quad \text{and} \quad \frac{\partial P_i^{csp}}{\partial \theta_i^{csp}} = B_{ii} \qquad (25)$$

Assuming $r_{ik} \ll x_{ik}$, these elements become in the negative values of the DC matrix model $B'_{ik} = -x_{ik}^{-1}$ and $B'_{ii} = \sum_{k=1}^{n} x_{ik}^{-1}$. The Equation 22 can now be succinctly rewritten as:

$$\boldsymbol{\psi}^{\boldsymbol{csp}} = -\mathsf{B}'^{-1} \cdot \boldsymbol{T}^{\boldsymbol{csp}} \tag{26}$$

where $\boldsymbol{T}^{\boldsymbol{csp}}$ is a vector whose elements are derivatives of DC losses respect to bus angles. Each element T_i^{csp} is given by:

$$T_i^{csp} = \frac{\partial L_{DC}}{\partial \theta_i^{csp}} = 2\sum_{k=1}^{n} G_{ik} \sin \theta_{ik}^{csp} \tag{27}$$

The Equation 26 is important because it highlights the fact that it is possible obtain incremental factors directly from the state of the system $\boldsymbol{\theta}^{\boldsymbol{csp}}$ obtained from Equation 20.

⊡ STEP 2: FUZZY DC POWER FLOW ANALYSIS

The Fuzzy DC power flow is an extension of the classic DC power flow, introduced originally by [16] based upon the following expression

$$\widetilde{\boldsymbol{\theta}} = \mathsf{B}'^{-1} \odot \widetilde{\boldsymbol{P}} \tag{28}$$

where $\odot$ is the fuzzy multiplication operator[3], and $\boldsymbol{\theta}$ and $\boldsymbol{P}$ are bus angle and power injection vectors whose entries are fuzzy numbers. Eliminating file and column corresponding to slack bus of matrix B', the possibility distribution of bus angle deviations are written as

$$\Delta\widetilde{\boldsymbol{\theta}} = \mathsf{B}'^{-1} \odot \Delta\widetilde{\boldsymbol{P}} \tag{29}$$

In trapezoidal fuzzy set representation, it is required to compute four angle deviations vectors: $\Delta\boldsymbol{\theta}^{\boldsymbol{\beta_1}}$ and $\Delta\boldsymbol{\theta}^{\boldsymbol{\beta_4}}$ for α-cut=0 and $\Delta\boldsymbol{\theta}^{\boldsymbol{\beta_2}}$ and $\Delta\boldsymbol{\theta}^{\boldsymbol{\beta_3}}$ for α-cut=1. Using the crisp angles computed according to Equation 20, the vector of fuzzy angle membership function is aggregated as:

$$\widetilde{\boldsymbol{\theta}} = \boldsymbol{\theta}^{\boldsymbol{csp}} \oplus (\boldsymbol{\Delta\theta^{\beta 1}} : \boldsymbol{\Delta\theta^{\beta 2}} : \boldsymbol{\Delta\theta^{\beta 3}} : \boldsymbol{\Delta\theta^{\beta 4}}) \tag{30}$$

As the relationship between bus angles and power injections is linear, it is verified that maximum angle deviations take place when power injection deviations are maximum, when generation is maximum and load is minimum. This means that minimum angle deviations always occurs when power injection deviations are minimum (when generation is minimum and load is maximum). This criterion cannot be applied to find the membership functions related to power losses and line flow currents due to non-linearity. However, this problem is not relevant in the ψ membership aggregation because the relationship between the ψ factor and the bus angle for each α-cut degree is approximately linear. As a result, Equation 26 can be applied to obtain the ψ vector at each α-cut level and related points β_γ where $\gamma = 1, 2, 3, 4$ in trapezoidal fuzzy sets.

$$\boldsymbol{\psi}^{\boldsymbol{\beta_\gamma}} = \text{-}\mathsf{B}'^{-1} \cdot \boldsymbol{T}^{\boldsymbol{\beta_\gamma}} \tag{31}$$

where the element $T_i^{\beta_\gamma}$ of the vector $\boldsymbol{T}^{\boldsymbol{\beta_\gamma}}$ are:

$$T_i^{\beta_\gamma} = 2\sum_{k=1}^{n} G_{ik} \sin(\theta_{ik}^{csp} + \Delta\theta_{ik}^{\beta_\gamma}) \tag{32}$$

when $\theta_{ik}^{\beta_\gamma}$ is small.

$$T_i^{\beta_\gamma} \approx 2\sum_{k=1}^{n} G_{ik} \theta_{ik}^{\beta_\gamma} \tag{33}$$

Then, the elements $\boldsymbol{\psi}^{\boldsymbol{\beta_\gamma}}$ are written as:

$$\psi_k^{\beta_\gamma} = -2\sum_{i=1}^{n} Z_{ik} \sum_{j=1}^{n} G_{ij} \sin(\theta_{ij}^{csp} + \Delta\theta_{ij}^{\beta_\gamma}) \tag{34}$$

This equation is equivalent to:

$$\psi_k^{\beta_\gamma} = \sum_{i=1}^{n}\sum_{j=1}^{n} G_{ij} \sin(\theta_{ij}^{csp} + \Delta\theta_{ij}^{\beta_\gamma})(-Z'_{ik} + Z'_{jk}) \tag{35}$$

and approximately by

$$\psi_k^{\beta_\gamma} \approx \sum_{i=1}^{n}\sum_{j=1}^{n} G_{ij} (\theta_{ij}^{csp} + \Delta\theta_{ij}^{\beta_\gamma})(-Z'_{ik} + Z'_{jk}) \tag{36}$$

where Z'_{jk} and Z'_{ik} are elements of B'^{-1}. Note that Z'_{ij} elements of file and column related to slack bus are equal to zero.

The incremental transmission loss factors can be represented by a linear behavior for small angle differences. For example, Figure 8 shows this dependence in a simply two-bus system.

[3] The multiplication between a trapezoidal fuzzy set and a real number is shape preserving. Given a convex-normalized fuzzy number $\widetilde{a} = (a^{\beta_1} : a^{\beta_2} : a^{\beta} : a^{\beta_4})$ and a real number c the fuzzy multiplication is expressed as a trapezoidal fuzzy set $c \odot \widetilde{a} = (c \cdot a^{\beta_1} : c \cdot a^{\beta_2} : c \cdot a^{\beta_3} : c \cdot a^{\beta_4})$.

In fact, deviations $\Delta\psi$ can be expressed as the linear relationship:

$$\Delta\psi_k^{\beta_\gamma} \approx \sum_{i=1}^{n}\sum_{j=1}^{n} G_{ij}(\Delta\theta_{ij}^{\beta_\gamma})(Z'_{ik} + Z'_{jk}) \qquad (37)$$

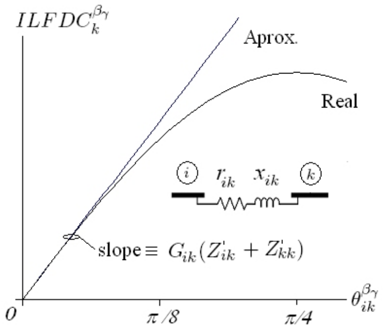

Figure 8: 2-Bus System – $\psi_k^{\beta_\gamma}$ as function of $\theta_{ik}^{\beta_\gamma}$

This approximately linear dependence implies that incremental factor deviations are reached in the same direction that bus angles and power injection deviations. This happens in the same way that angle deviations are numerically correlated with power injection deviations in the conventional DC Fuzzy Power Flow analysis. As a result, it is possible ensure the following numerical correlation among power injection deviations, angle deviations and incremental factor deviations:

$$\begin{matrix} \boldsymbol{\Delta P_G^{\beta_\gamma}} \\ \boldsymbol{\Delta P_D^{\beta_{5-\gamma}}} \end{matrix} \rightsquigarrow \boldsymbol{\Delta\theta^{\beta_\gamma}} \rightsquigarrow \boldsymbol{\Delta\psi^{\beta_\gamma}} \qquad (38)$$

The deviation value ($\boldsymbol{\Delta\psi^{\beta_4}}$) occurs at maximum-generation and minimum load scenario, $\boldsymbol{\Delta P_G^{\beta_4}}$ and $\boldsymbol{\Delta P_D^{\beta_1}}$. On the other hand, deviation ($\boldsymbol{\Delta\psi^{\beta_1}}$) occurs at minimum-generation and maximum demand scenario, $\boldsymbol{\Delta P_G^{\beta_1}}$ and $\boldsymbol{\Delta P_D^{\beta_4}}$. This result is meaningful because incremental loss factors are directly linked to bus angles and not to power losses and line currents. It allows to build the ITL membership functions from the angles membership function (see Equation 30) obtained from a conventional DC Fuzzy Power Flow analysis.

Finally, the direct computation of $\Delta\boldsymbol{\psi}^{\beta_\gamma}$ are given by :

$$\Delta\boldsymbol{\psi}^{\beta_\gamma} = \boldsymbol{\psi}^{\beta_\gamma} - \boldsymbol{\psi}^{csp} \qquad (39)$$

Fuzzy deviation $\Delta\widetilde{\boldsymbol{\psi}}$ is aggregated as

$$\Delta\widetilde{\boldsymbol{\psi}} = (\Delta\boldsymbol{\psi}^{\beta_1} : \Delta\boldsymbol{\psi}^{\beta_2} : \Delta\boldsymbol{\psi}^{\beta_3} : \Delta\boldsymbol{\psi}^{\beta_4}) \qquad (40)$$

The achievement of the fuzzy incremental transmission loss factors directly from bus angle membership functions provided by a conventional DC Fuzzy Power Flow is a fundamental contribution of this research.

⊡ STEP 3: CRISP AC ANALYSIS.

The crisp real ITL factors (AC approach) can be obtained from the current state of the system by evaluating the Jacobean of a converged Newton Raphson AC power flow. According to [2], the standard chain rule is applied to calculate the ITLs by means of intermediate variables, voltages and angles, V, ϕ:

$$\begin{bmatrix} ITL \\ rITL \end{bmatrix} = \begin{bmatrix} \frac{\partial P}{\partial\phi} & \frac{\partial Q}{\partial\phi} \\ \frac{\partial P}{\partial V} & \frac{\partial Q}{\partial V} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial L}{\partial\phi} \\ \frac{\partial L}{\partial V} \end{bmatrix} \qquad (41)$$

where system losses are given by:

$$L = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} G_{ij}[V_i^2 + V_j^2 - 2V_iV_j\cos\phi_{ij}] \qquad (42)$$

⊡ STEP 4: FUZZY ITL AGGREGATION.

The $\widetilde{\psi}$ membership function built in Step 1 and 2 is corrected with the real crisp values calculated in Step 3.

$$\widetilde{\boldsymbol{ITL}} = \boldsymbol{ITL}^{csp} \oplus \Delta\widetilde{\boldsymbol{\psi}} \qquad (43)$$

It is important to underline that, at given operating point, the state of the system and therefore the ITL factors should be considered optimal or not depending on the network optimization methodology adopted by the TSO. The adoption of an *ex-ante* methodology to optimize the power system –as an optimal reactive dispatch model for power loss minimization as proposed by [18]– can improve the incremental signals and achieve more efficient tariffs.

6 Illustrative Example

The proposed methodology is applied for illustration purposes in a simple three-bus test system as shown in Figure 9.

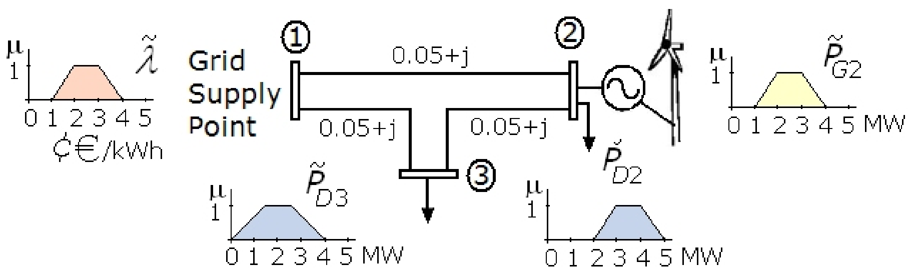

Figure 9: 3-Bus System – Test network with fuzzy injections

Bus 1 corresponds to Grid Supply Point (GSP) where the distribution system connects to the transmission system. Uncertainty at power generation and load is represented using trapezoidal fuzzy numbers. Network parameters are given in p.u. using $S_{\text{BASE}} = 10\text{MVA}$ and $V_{\text{BASE}} = 10\text{kV}$.

After some algebra, the fuzzy real power injections can be written in p.u. as:

$$\widetilde{\boldsymbol{P}} = \begin{bmatrix} 0.05 \\ -0.35 \end{bmatrix} \oplus \left(\begin{bmatrix} -0.35 \\ -0.15 \end{bmatrix} : \begin{bmatrix} -0.10 \\ -0.05 \end{bmatrix} : \begin{bmatrix} 0.10 \\ 0.05 \end{bmatrix} : \begin{bmatrix} 0.35 \\ 0.15 \end{bmatrix} \right)$$

Step 1: The elements of the DC matrix B are $B_{22} = B_{33} = 0.666$ and $B_{23} = B_{32} = 0.333$. Using Equation 20 crisp angles are $\boldsymbol{\theta^{csp}} = [-0.0833, -0.2167]$. Using Equation 26 crisp incremental transmission loss factors are given by:

$$\begin{bmatrix} \psi_2^{csp} \\ \psi_3^{csp} \end{bmatrix} = \begin{bmatrix} B_{22} & B_{23} \\ B_{32} & B_{33} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial L_{DC}}{\partial \theta_2^{csp}} \\ \frac{\partial L_{DC}}{\partial \theta_3^{csp}} \end{bmatrix} = \begin{bmatrix} -0.0083 \\ -0.0215 \end{bmatrix}$$

Note that $\frac{\partial L_{DC}}{\partial \theta_2^{csp}}$ and $\frac{\partial L_{DC}}{\partial \theta_3^{csp}}$ have been computed using the general form Equation 27. If the approximation $\sin\theta_{ik} \approx \theta_{ik}$ is applied results are basically the same $[-0.0083, -0.0216]$.

Step 2: The bus angle deviations are obtained using 28:

$$\boldsymbol{\Delta\theta^{\beta}} = \left(\begin{bmatrix} -0.2833 \\ -0.2166 \end{bmatrix} : \begin{bmatrix} -0.0833 \\ -0.0666 \end{bmatrix} : \begin{bmatrix} 0.0833 \\ 0.0666 \end{bmatrix} : \begin{bmatrix} 0.2833 \\ 0.2166 \end{bmatrix} \right)$$

Applying Equation 39

$$\boldsymbol{\Delta}\widetilde{\psi} = \left(\begin{bmatrix} -0.0273 \\ -0.0206 \end{bmatrix} : \begin{bmatrix} -0.0082 \\ -0.0065 \end{bmatrix} : \begin{bmatrix} 0.0083 \\ 0.0066 \end{bmatrix} : \begin{bmatrix} 0.0281 \\ 0.0215 \end{bmatrix} \right)$$

Step 3: The Crisp AC analysis indicate that there is no reactive injection in buses 2 and 3. Bus 2 is specified as a PV bus with $V_2^{csp} = 1.0$. The state of the system is given by and $\boldsymbol{\phi^{csp}} = [-0.0858, -0.2238]$ and $V_2^{csp} = 0.9738$. Applying Equation 41, $\boldsymbol{ITL^{csp}} = [-0.0088, -0.0239]$.

Step 4: Finally, incremental transmission loss factors are corrected using Equation 43 with the real crisp incremental values:

$$\widetilde{\boldsymbol{ITL}} = \boldsymbol{ITL^{csp}} \oplus \boldsymbol{\Delta}\psi$$

$$\widetilde{\boldsymbol{ITL}} = \left(\begin{bmatrix} -0.0362 \\ -0.0445 \end{bmatrix} : \begin{bmatrix} -0.0171 \\ -0.0304 \end{bmatrix} : \begin{bmatrix} -0.0006 \\ -0.0173 \end{bmatrix} : \begin{bmatrix} 0.0195 \\ -0.0024 \end{bmatrix} \right)$$

In Figure 10 is shown how the membership function of $\widetilde{\boldsymbol{\psi}}_{\boldsymbol{3}}$ is built through steps 1 to 4. Results show that for a confidence degree $\mu = 1$, incremental transmission loss factors goes from -0.0171 to -0.0005 in the bus 2 and for confidence degree $\mu = 0$ goes from -0.0304 to -0.0173 in the bus 3, respectively. Note that, crisp values -0.0083 and -0.0215 coincide with the center values of the membership functions displayed in Equation 44.

This means that the length of deviations are practically the same at both sides of fuzzy numbers when $\mu = 1$, $\Delta\psi^{\beta_2} = \Delta\psi^{\beta_2}$. Nevertheless, if $\mu = 0$ this behavior is not observed being $\Delta\psi^{\beta_1} \neq \Delta\psi^{\beta_4}$.

7 Limitations and Future Research

This first model acquires fuzzy ITLs from a DC Fuzzy Power Flow model [16]. This was done because, its application provides a simply and clear example of how to get fuzzy ITLs. However, more realistic membership functions can be obtained from a complete AC Fuzzy Power Flow model as reported in [17] or [18]. The application of this idea to real and large-scale transmission systems is currently matter of research.

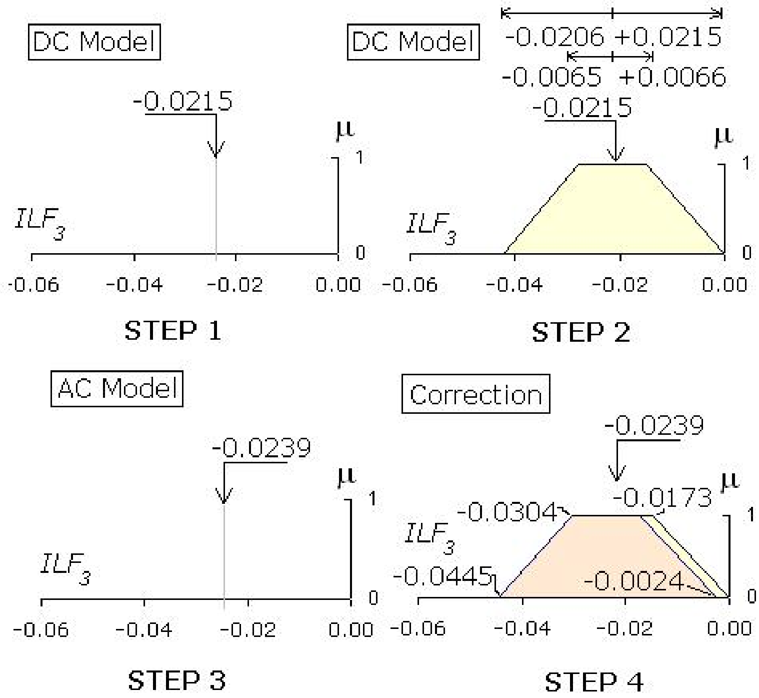

Figure 10: 3-Bus System – $\widetilde{\boldsymbol{ITL}}_{\boldsymbol{3}}$ Aggregation

8 Conclusions

This paper addresses the following conclusions:

- It is proposed a novel methodology to assess the effect of uncertainty upon Incremental Transmission Loss (ITL) factors computation.
- It was demonstrated that Incremental Transmission Loss factors can be efficiently assessed using fuzzy programming. At a given operating point, location and time specific fuzzy ITL factors are obtained from the results of running a Fuzzy Power Flow program where load and generation are represented as fuzzy sets.
- A general algorithm is developed and tested. Fuzzy set techniques provide a whole spectrum of feasibility of the spot prices applied to each market agent.
- Proposed methodology has been applied to the 3-bus test system for didactic purposes.

References:

[1] Kirchmayer, L. K. (1958). Economic operation of power systems. General Electric series. Wiley.

[2] Wood, A. J. and Wollenberg, B. F. (1996). Power generation, operation, and control. J. Wiley & Sons, New York, 2nd edition.

[3] Power System Economics: Designing Markets for Electricity. IEEE Press Wiley-Interscience, New York.

[4] Alvarez, M. and Bermudez, J. (2006). Marginal Pricing via Penalty Factors in the Venezuelan Hydrothermal System. Transmission & Distribution Conference and Exposition: Latin America 2006. IEEE/PES.

[5] Rui Li, Luonan Chen, Ryuichi Yokoyama. Valuation of the Cost of Transmission Losses Considering Wheeling Routes. WSEAS TRANSACTIONS on POWER SYSTEMS Issue 1, Volume 1, January 2006 pp. 3-11.

[6] Schweppe, Fred C. (1987). Spot pricing of electricity. Kluwer international series in engineering and computer science. Power electricity and electronics. Kluwer Academic, Boston.

[7] Takashi Mitani, Taiji Satoh, Koichi Nara. A Novel Optimal Power Flow for On-line Analysis. WSEAS TRANSACTIONS on POWER SYSTEMS Issue 8, Volume 1, August 2006 pp.1396-1403.

[8] H. Tammoja, M. Valdma, M. Keel. Optimal Load Dispatch in Power Plant under Probabilistic and Uncertain Information. WSEAS TRANSACTIONS on POWER SYSTEMS Issue 5, Volume 1, May 2006 pp.754-460.

[9] Galiana, F. D.,Conejo, A. J. and Kockar, I. (2002). Incremental transmission loss allocation under pool dispatch. IEEE Transactions on Power Systems, Vol. 17 No. 1, pp. 26-33.

[10] P. M. De Oliveira-De Jesus, E. D. Castronuovo, and M. T. Ponce de Leao. Reactive Power Response of Wind Generators under an Incremental Network-Loss Allocation Approach. IEEE Transactions on Energy Conversion, Vol. 23, pp. 612 - 621, 2008.

[11] Saraiva, J. T. (1999b). Evaluation of the Impact of Load Uncertainties in Spot Prices Using Fuzzy Set Models. In Proceedings of PSCC99-13th Power Systems Computation Conference, pp. 265-271, Trondheim, Norway.

[12] Miranda, V.,Pereira, J. and Saraiva, J. T. (2000). Load allocation in DMS with a fuzzy state estimator. IEEE Transactions on Power Systems, Vol. 15 No. 2, pp. 529-534.

[13] Zadeh, L. A. (1965). Fuzzy Sets. Information and Control, Vol. 8, No. 3 pp. 338

[14] Zimmermann, H. J. (1991). Fuzzy set theory and its applications. Kluwer Academic Publishers, Boston, 2nd rev. edition.

[15] El-Hawary, M. E. (1998). Electric power applications of fuzzy systems. Power Systems Engineering series. IEEE, New York.

[16] Miranda, V. and Saraiva, J. T. (1992). Fuzzy modelling of power system optimal load flow. IEEE Transacions on Power Systems, Vol. 7, No. 2, pp. 843-849.

[17] Saraiva, J.T.,Fonseca, N. and Matos, M.A. (2004). Fuzzy power flow - An AC model addressing correlated data. In 2004 International Conference on Probabilistic Methods Applied to Power Systems, Sep. 12-16 2004, pp. 519-524

[18] Tomsovic, K. (1992). A fuzzy linear programming approach to the reactive power/voltage control problem. IEEE Transactions on Power Systems, Vol. 7, No. 1, pp. 287-293.