MONTE CARLO SIMULATION FOR VOLTAGE STABILITY INDEX EVALUATION

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Abstract: - This paper presents an alternative approach to evaluate voltage stability indices in electric power systems by using Monte Carlo simulation (MCS). Fast voltage stability index (FVSI), line stability index (L_{mn}) and line stability factor (LQP) are used as indicators to determine the capability of reactive power loading at a given bus position. Conventionally, loading the test bus with pure reactive power is not realistic because power system load changes randomly at every load bus in both real and reactive powers. In this paper, MCS was introduced to simulate load increment of all load buses with appropriate random numbers and their probability. From this approach, all the indices mentioned above can be calculated repeatedly with a large number of trials. To evaluate its use, a small five-bus, and the standard IEEE 30- and 57-bus test systems were employed for demonstration. As a result, comparing among these indices, the weakest bus of the test system can be identified with some degree of confidence.

Key-Words: - Fast voltage stability index, line stability index, line stability factor, Monte Carlo simulation

1 Introduction

In recent years, major power system failures (system blackout) have occurred more frequently around the world [1]. Some of these failures have been caused by voltage instability problems. These circumstances lead, the voltage stability issues, to an important and urgent concern for electric utilities. Solving this problem is not the right thing to be proceeded because an power system under the voltage-instability event is typically unpredictable. It is difficult or impossible to correct this problem when it occurred. Prevention of voltage instability is probably moderate one. Many researchers in power system voltage stability devote their works to identify the weakest bus of the system. The weakest bus is considered as the bus that is likely to be the first bus of the system facing voltage collapsed. The prevention of voltage instability relies on locating the system weakest bus. Some efficient actions can be employed to put an operating point of the system far from the voltage collapsing point with an acceptable margin. Static voltage stability can be determined by using continuation power flow calculations [2]. Many static voltage evaluation methods have been proposed, for example, the minimum singularity value method, mode analysis method and sensitivity method [1,3,4]. The major disadvantages of the continuation power flow-based methods are computing efforts that make difficulty of implementation in on-line applications and experiences of solution divergence due to the numerical instability in power flow calculations. Moreover, inconsistency between off-line model and real-life situation leads incapability of identifying the weakest bus that causes the system voltage collapse

In this paper, modification based on MCS for evaluation of voltage stability indices (e.g. FVSI, L_{mn} and LQP) to identify weak buses in electric power systems has been proposed. Statistical load models with appropriate random numbers and their probability are used to predict electrical demand growth of all load buses. With their positive mean and variance of the statistical load models, the power system is driven to increase its total loading with some considerable momentum. By performing the system load increment with a moderate time span and computing all the voltage stability indices, weak buses of the system can be observed. This simulation can typically involve over 10,000 trials to pretend a real-world power system operation. The results from this simulation can be used to evaluate the weakest bus with a certain degree of confidence.

In this paper, Section 2 gives a brief explanation of voltage stability indices used in this paper as mention earlier. It notes that the full Newton-Raphson power flow calculation [5] was applied to solve for system voltage solutions. It is well-known and widely-accepted as the most powerful tool for this purpose. Therefore, the method of Newton Raphson power flow calculation was not included here. Monte Carlo simulation was reviewed and summary of its use to evaluate weak buses of the system were reviewed in Section 3. For test, a small five-bus, the standard IEEE 30- and 57-bus power systems were employed. To simulate the demand growth of the system, appropriate statistical load models were assigned to all load buses. Simulation results were illustrated in Section 4. Section 5 provided adequate conclusion remark.

2 Voltage Stability Indices

Voltage stability indices can be evaluated by using a two-machine coupling model. There exist many useful indices from literature. In this paper, only FVSI, L_{mn} and LQP are reviewed as follows.

2.1 FVSI (fast voltage stability index) [6-9]

Calculation of current flowing through the line in Fig. 1 gives primary expression of this index.

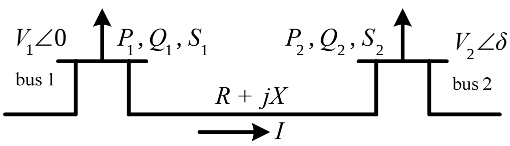

Fig. 1 Simple two-bus power system for FVSI

Given that bus 1 is the sending-end bus and bus 2 is the receiving-end bus. FVSI can be expressed in (1).

$$\mathrm{FVSI}_{12} = \frac{4Z^2Q_2}{V_1^2X} \tag{1}$$

Where Z is the line impedance
X is the line reactance
Q_2 is the reactive power at bus 2
V_1 is the voltage magnitude at bus 1

FVSI is used to indicate a stable operating region of the load. According to this index, the system becomes unstable if FVSI is equal to or greater than unity [6].

2.2 L_{mn} (line stability index) [6,9,10]

Calculation of current flowing through the line in Fig. 1 gives primary expression of this index.

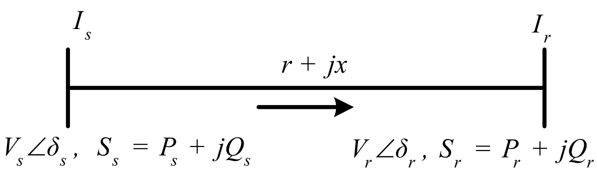

Fig. 2 Simple two-bus power system for L_{mn}

Consider Fig. 2. Using relation between voltage and current of the transmission line, line stability index (L_{mn}) can be clearly defined as follows.

$$\frac{4xQ_r}{\left[V_s\sin(\theta - \delta)\right]^2} = L_{mn} \le 1.00 \tag{2}$$

Where x is the line reactance
Q_r is reactive power at the receiving-end
V_s is voltage at the sending-end
θ is the phase angle of the line impedance
δ is the voltage phase difference of bus s and r

L_{mn} is used to indicate a stable operating region of the load. According to this index, the system becomes unstable if L_{mn} is equal to or greater than unity.

2.3 LQP (line stability factor) [6,11]

LQP uses the same concept of FVSI and L_{mn}. It is derived from power transfer equations describing the system in Fig. 3. LQP can be expressed in (3).

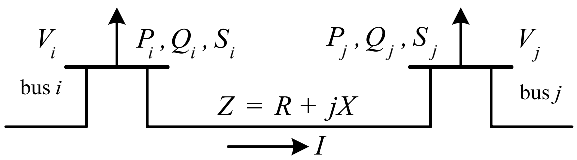

Fig. 3 Simple two-bus power system for LQP

$$LQP = 4\left(\frac{X}{V_i^2}\right)\left(\frac{X}{V_i^2}P_i^2 + Q_j\right) \tag{3}$$

Where P_i is the real power flow from bus i
X is the line reactance
Q_j is the reactive power flow to bus j
V_i is the voltage magnitude at bus i

The system is stable as long as LQP is less than 1.0 [6].

2.4 Procedure to compute voltage stability indices

All three voltage stability indices can be obtained in each power flow calculation loop driven by the incremental reactive power loading at a particular load bus. By increasing only the reactive power of this bus until the indices greater than unity, the maximum reactive power loading ability can be calculated. The following step-by-step procedure describes the computation algorithm for voltage stability indices.

START:
Step 0

Load a test power system
Initialize parameters
Reset all counters
Define reactive power increment and time step
Start the load bus counter (LBC)

Step 1
Assign the next load bus for reactive power loading

Step 2
Increase the reactive power loading
Perform the power flow calculation
Compute FVSI, L_{mn} and LQP

Step 3
Check the stopping criterion
IF (FVSI $\geq$1) & ($L_{mn}\geq$1) & (LQP $\geq$1)
Go to Step 4
OTHERWISE
Go to Step 2
END

Step 4
Evaluate the maximum reactive power loading ability
IF (LBC $\geq$Total number of load buses)
go to Step 5
OTHERWISE
LBC = LBC + 1
go to Step 1
END

Step 5
Successfully obtain voltage stability indices
Perform weak bus ranking
Identifying the weakest bus of the system

STOP

3 Applied Monte Carlo Simulation [12-15]

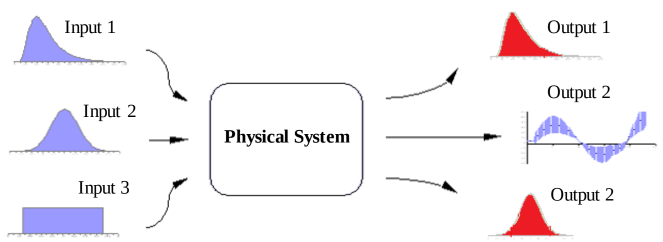

Fig. 4 Principle of Monte Carlo Simulation

A Monte Carlo simulation (MCS) is a technique that involves using random numbers and probability to solve physical problems. This technique relies on computer simulation of real-life models to predict what is likely to happen next. When the simulation going on, randomly change in some input variables can be made to drive the system to operate at a new unpredictable operating state. This method is one among several methods for analyzing uncertainty propagation, where the objective is to determine how random variation, lack of knowledge, or error affects the system performance that is being modeled. The MCS is categorized as a sampling method because the input variables are randomly generated from probability distributions to simulate the process of sampling from an actual population as shown in Fig. 4.

The procedure for Monte Carlo simulation corresponding to the uncertainty propagation shown in Fig. 4 is fairly simple, and can be easily implemented in some scientific software, such as MATLAB, MATCAD, SCILAB, etc. There are five simple steps listed below to organize the typical MCS procedure to work.

Step 1
Create a parametric model

Step 2
Generate a set of random inputs

Step 3
Evaluate the model and store the results

Step 4
Repeat Steps 2 and 3 for a large amount of trials

Step 5
Report the results, statistics, confidence intervals.

To apply the MCS for voltage stability index evaluation, classification of system variables is necessary. For this problem, electrical demand growth at each load bus can be increased randomly with a particular random distribution. Due to the random process, not only the reactive power but also the real power can be assumed as input random variables. With their positive mean and variance of the statistical load models of Gaussian distribution, the power system is driven to increase its total loading with some considerable momentum. By performing the system load increment with a moderate time span and computing all the voltage stability indices according to their associated power flow solution, weak buses of the system can be observed and ranked. This simulation can typically involve over 10,000 trials to pretend a real-world power system operation. The results from this simulation can be used to evaluate the weakest bus with a certain degree of confidence. Hence, when applying the MCS to the voltage stability index calculation, the step-by-step procedure summarizing the modified computation algorithm is described as follows.

START:

Step 0
Load a test power system
Initialize parameters
Reset all counters
Define statistical models to all loads
Start the time counter

Step 1

Randomly generate demand growth of this time step

Step 2

Perform the power flow calculation

Compute FVSI, L_{mn} and LQP

Step 3

Check the stopping criterion

IF (FVSI ≥1) & (L_{mn} ≥1) & (LQP ≥1)

Go to Step 4

OTHERWISE

Increase the time counter

Go to Step 1

END

Step 4

Evaluate the maximum power loading ability

Successfully obtain voltage stability indices

Perform weak bus ranking

Identifying the weakest bus of the system

STOP

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4 Simulation Results

To demonstrate the proposed algorithm for evaluating the voltage stability indices, three test cases: i) a simple five-bus test system [16], ii) the standard IEEE 30-bus test system [17] and iii) the standard IEEE 57-bus test system [18] were used.

4.1 5-bus test system

Its base-case loading was assumed to be 165 MW and 40 Mvar. Some information of the test system was given in Table 1 and 2 for load data and line data, respectively. Statistical models of all load buses used in this paper was Gaussian distribution in which 10% of each base-case power was set as its mean and 0.9 was a typical value of its variance.

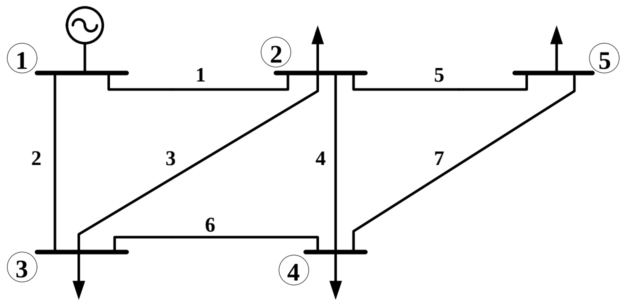

Fig. 5 Five-bus test system

The test started from the base case. Random load increasing was repeatedly performed with a uniform time step until the system voltage collapses. This operation was defined as one trial. To achieve realistic results, the test must be performed as many as possible. In this work, the test was conducted with 30,000 trials to ensure a normal distribution of the output variable outcome. Table 3 showed a summary of simulation results.

Table 1. Load at base case of the five-bus test system

bus	Load at base case (MVA)
1	-
2	20 + j10
3	45 + j15
4	40 + j5
5	60 + j10

Table 2. Line data of the five-bus test system

Line number	Form bus	To bus	Impedance (p.u.)
1	1	2	0.02 + j0.06
2	1	3	0.08 + j0.24
3	2	3	0.06 + j0.18
4	2	4	0.06 + j0.18
5	2	5	0.04 + j0.12
6	3	4	0.01 + j0.03
7	4	5	0.08 + j0.24

Table 3. Simulation results for a total of 30,000 trials

Line	FVSI	L_{mn}	LQP
1	0	1	0
2	0	9148	0
3	10	4	14
4	4	1	15
5	3	0	11
6	985	503	1074
7	22114	12003	21127

Figs 6-8 illustrated the change of each index during the random load incremental. From these figures, maximum power load ability can be evaluated. The value obtained by each index was slightly different. However, they are just 2.0% of maximum variation from the mean of 4.00. As can be seen in Tables 3 and 4, by using the FVSI, the weakest bus was bus 5 with 95.7% of confidence. The LQP also told the similar result with 95.0% of confidence. However, by using the L_{mn}, the degree of confidence was dropped to be as low as 55.4%, the weakest bus was still the bus number 5. In addition, the voltage magnitude of bus 5 was observed and depicted as shown in Fig. 9 – 10, whereas Figs 11 and 12 were the apparent power and the time counter being the *x* axis, respectively. From these figures, the voltage magnitude decreased dramatically until its collapse.

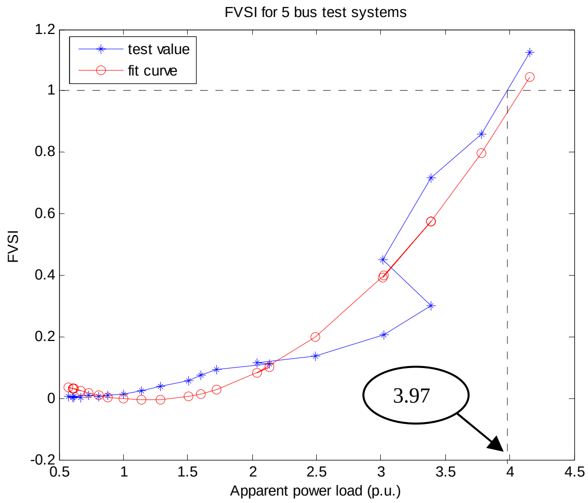

Fig. 6 FVSI results

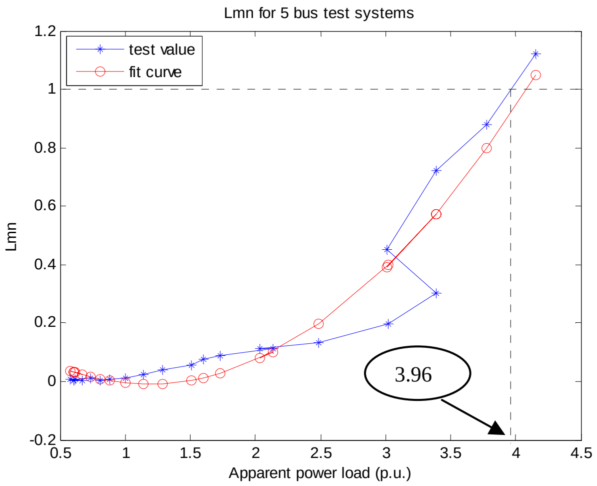

Fig. 7 L_{mn} results

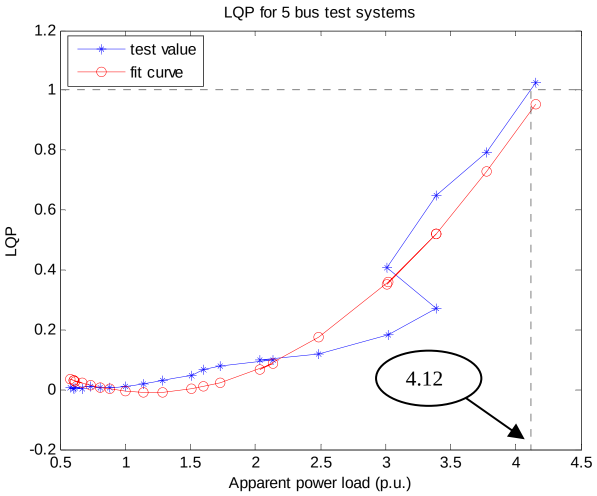

Fig. 8 LQP results

Again, Figs 11-13 illustrated the change of each index during the random load incremental. This set of figures was set the time counter as the x axis. Although the shape of curves was different, the conclusion drawn from these graph was the same as described earlier.

Table 4 Summary of simulation results

Load at bus 5	FVSI	L_{mn}	LQP
Base load	0.61	0.61	0.61
Increasing load	3.97	3.96	4.12
% increasing	550.82	549.18	575.41
% confidence	95.7	55.4	95.0

Table 5 Summary of the conventional index evaluation

Load bus	Maximum loading (p.u.)	Rank
2	9.826	4
3	4.789	2
4	3.664	1
5	8.558	3

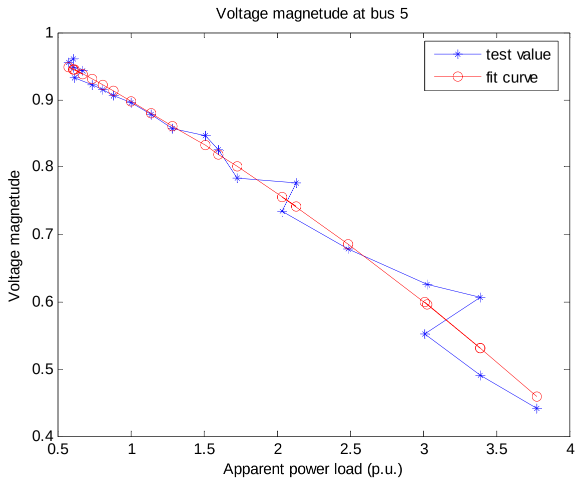

Fig. 9 Voltage magnitude of bus 5 versus load power

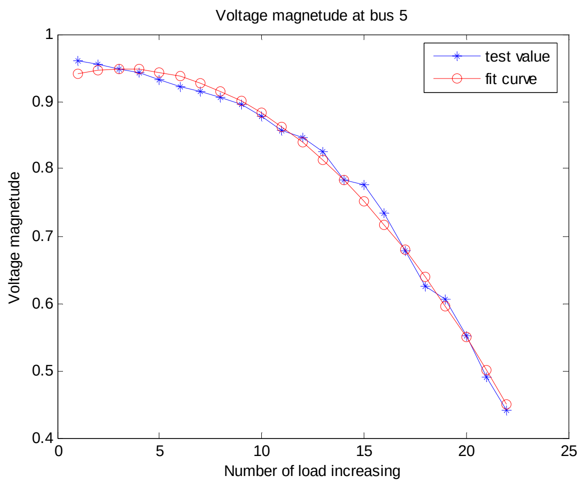

Fig. 10 Voltage magnitude of bus 5 versus time counter

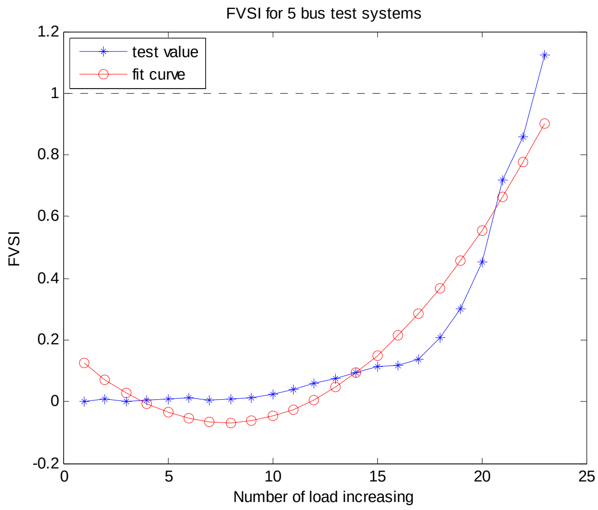

Fig. 11 FVSI results

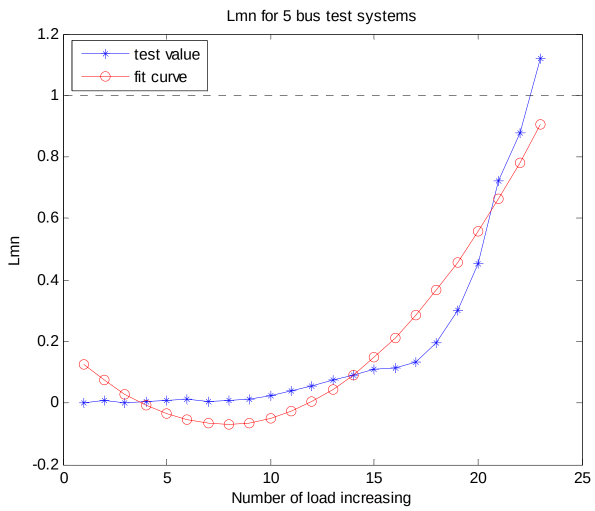

Fig. 12 L_{mn} results

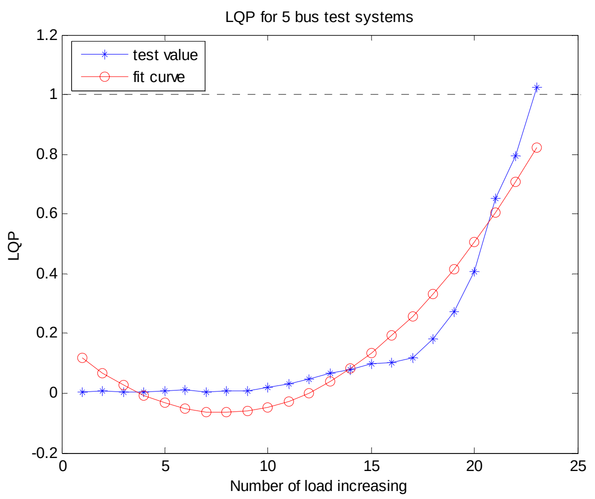

Fig. 13 LQP results

4.2 Standard IEEE 30-bus test system

In this test case, complexity of interconnection of 30 buses required spaces for explanation. Briefly, the test system was given in Fig. 14. This test case used the same procedure in order to evaluate voltage stability indices of the system. By increasing MVA loads at every single bus within a certain time step, system voltage profiles were characterized. Based on an appropriately statistical load model for each load bus, Monte Carlo simulation for voltage stability indices were carried out. Statistical models of all load buses used in this test case was Gaussian distribution in which 10% of each base-case power was set as its mean and 0.9 was a typical value of its variance. The test started from the base case. Random load increasing was repeatedly performed with a uniform time step until the system voltage collapses. This operation was defined as one trial. To achieve realistic results, the test must be performed as many as possible. In this work, the test was conducted with 30,000 trials to ensure a normal distribution of the output variable outcome. Table 6 showed a summary of simulation results for this test case. It noted that the total number of system buses was thirty, therefore it was inconvenient to present results of all buses or all lines as given in the first test case. Only two first ranks were demonstrated.

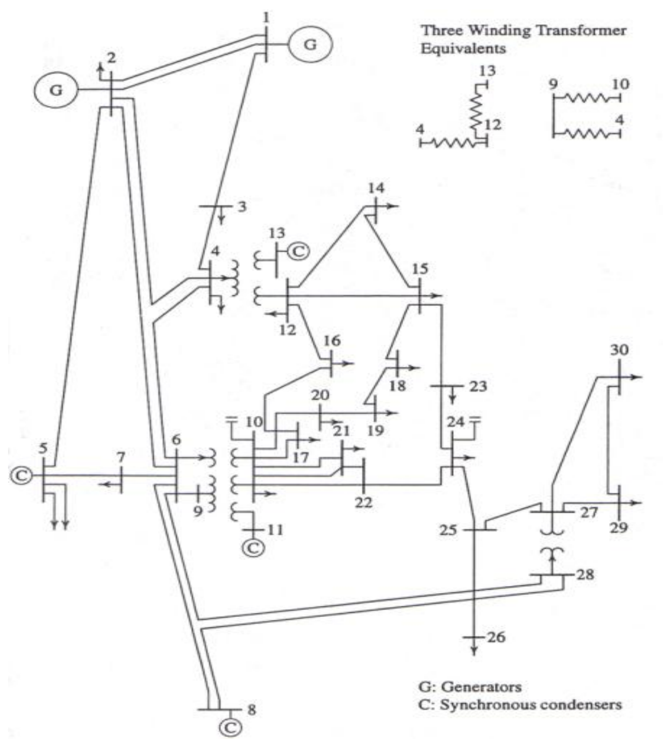

Fig. 14 Standard IEEE 30-bus test system

Table 6 Simulation results for a total of 30,000 trials

Line	FVSI	L_{mn}	LQP
2	3582	6699	10336
5	15124	13808	8180

In contrast, 30,000 trials did not give the outscore outcome. According to FVSI and L_{mn} indices line number 5 was the most critical line and resulted in bus 4 as the weakest bus. With LQP, the most critical line moved to line number 2 that pointed other bus as the weakest bus. However, FVSI and L_{mn} had higher degree of confidences (80.85% and 67.33% respectively) whereas LQP dropped to 55.82%. Figs 15-17 illustrated the change of each index during the random load incremental. From these figures, maximum power load ability can be evaluated although the weakest bus among them were not the same.

Figs 18-20 illustrated the change of each index during the random load incremental. This set of figures was set the time counter as the *x* axis. Although the shape of curves was different, the conclusion drawn from these graph was the same as described earlier.

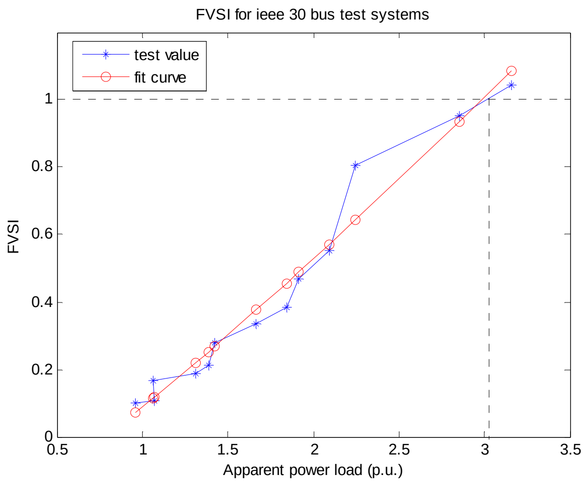

Fig. 15 FVSI results

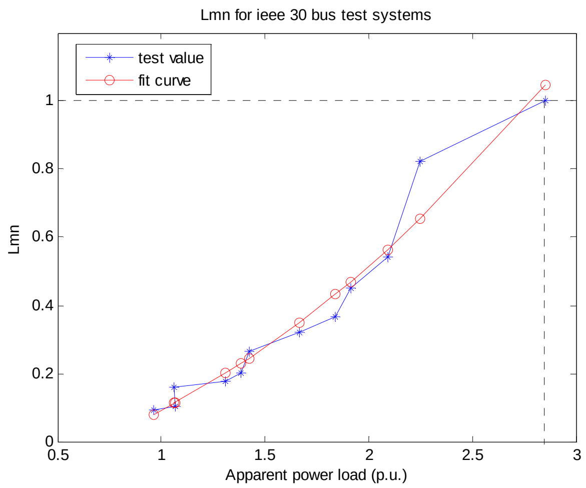

Fig. 16 L_{mn} results

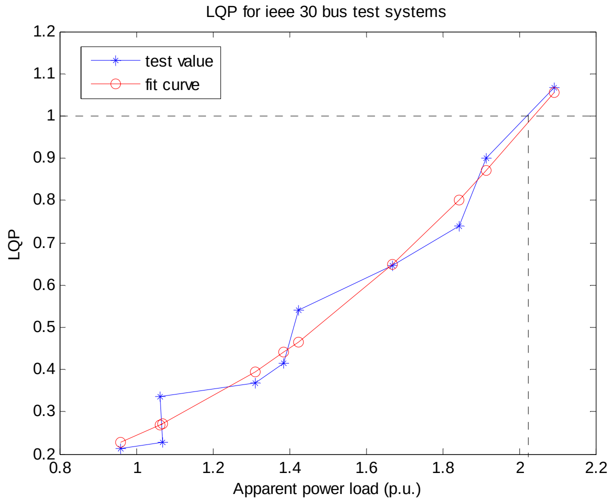

Fig. 17 LQP results

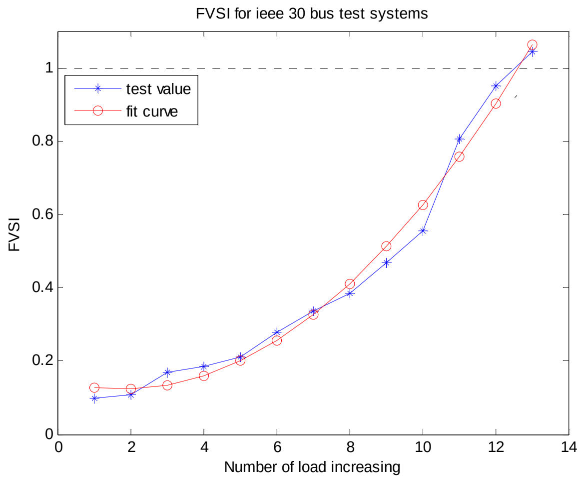

Fig. 18 FVSI results

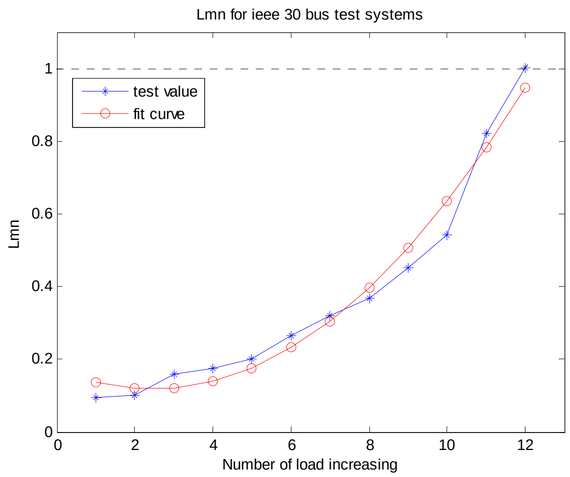

Fig. 19 L_{mn} results

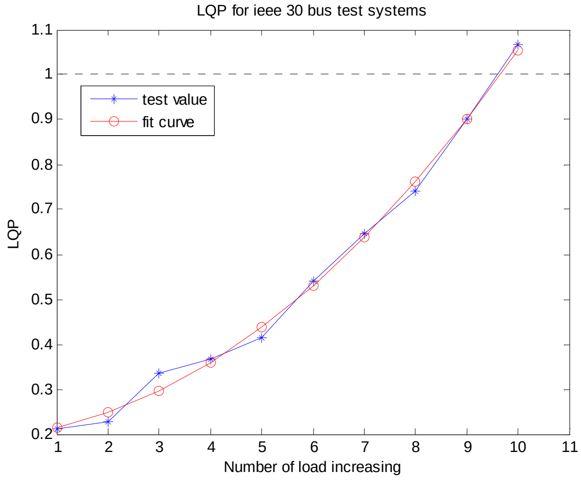

Fig. 20 LQP results

4.3 Standard IEEE 57-bus test system

The test system was given in Fig. 21. This test case used the same procedure in order to evaluate voltage stability indices of the system. By increasing MVA loads at every single bus within a certain time step, system voltage profiles were characterized. Based on an appropriately statistical load model for each load bus, Monte Carlo simulation for voltage stability indices were carried out. Statistical models of all load buses used in this test case was Gaussian distribution in which 10% of each base-case power was set as its mean and 0.9 was a typical value of its variance. The test started from the base case. Random load increasing was repeatedly performed with a uniform time step until the system voltage collapses. This operation was defined as one trial. To achieve realistic results, the test must be performed as many as possible. In this work, the test was conducted with 30,000 trials to ensure a normal distribution of the output variable outcome. Table 7 showed a summary of simulation results for this test case. It noted that the total number of system buses was thirty, therefore it was inconvenient to present results of all buses or all lines. Only two first ranks were given.

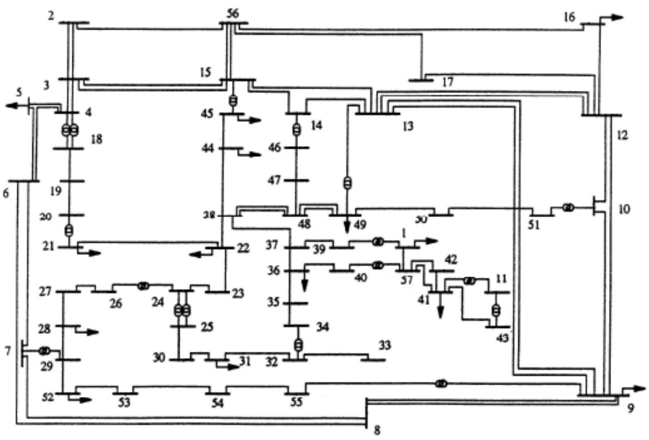

Fig. 21 Standard IEEE 57-bus test system

Table 7 Simulation results for a total of 30,000 trials

Line	FVSI	$\mathbf{L_{mn}}$	LQP
16	691	35	219
73	23083	28007	21867

Table 8 Summary of simulation results

Load at bus 5	FVSI	Lmn	LQP
Base load (p.u.)	0.8805	0.8805	0.8805
Increasing load	2.205	1.950	1.395
% increasing	150.43	121.47	58.43
% confidence	97.09%	99.88%	99.01%

As can be seen in Tables 7 and 8, by using the FVSI, the weakest bus was bus 2 with 97.09% of confidence. The LQP also told the similar result with 99.88% of confidence. By using the L_{mn}, the degree of confidence was as high as 99.01%. Figs 22-24 illustrated the change of each index during the random load incremental. From these figures, maximum power load ability can be evaluated

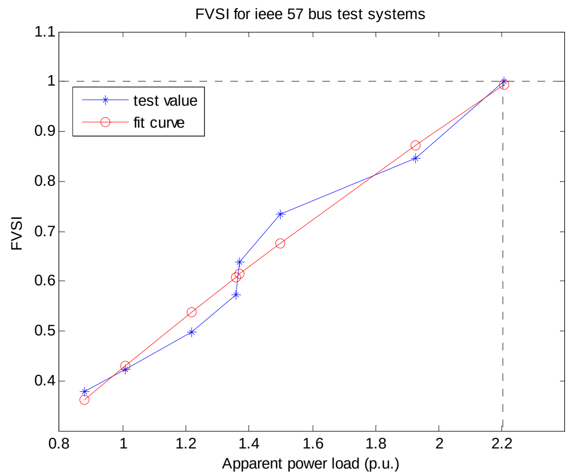

Fig. 22 FVSI results

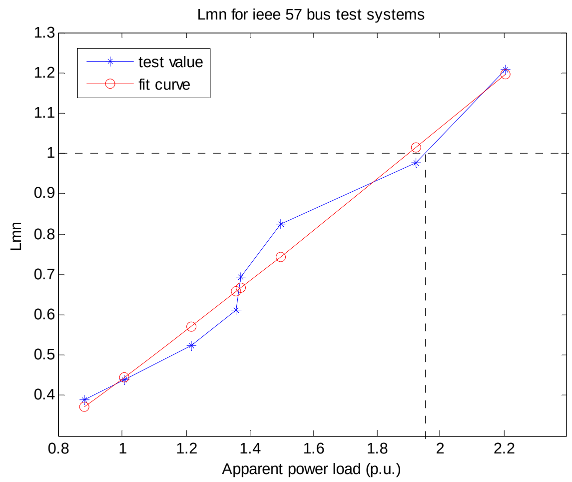

Fig. 23 L_{mn} results

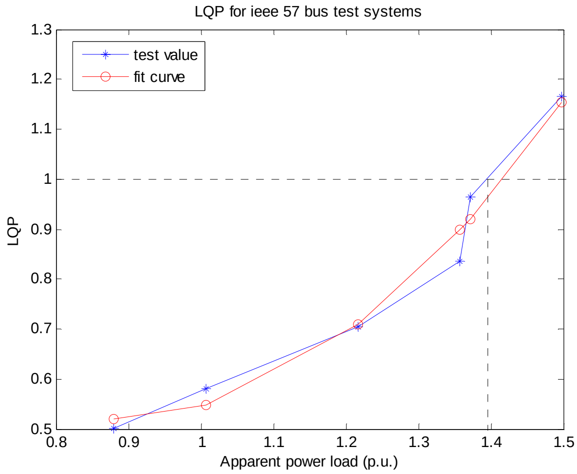

Fig. 24 LQP results

Figs 25-27 illustrated the change of each index during the random load incremental. This set of figures was set the time counter as the *x* axis. Although the shape of curves was different, the conclusion drawn from these graph was the same as described earlier.

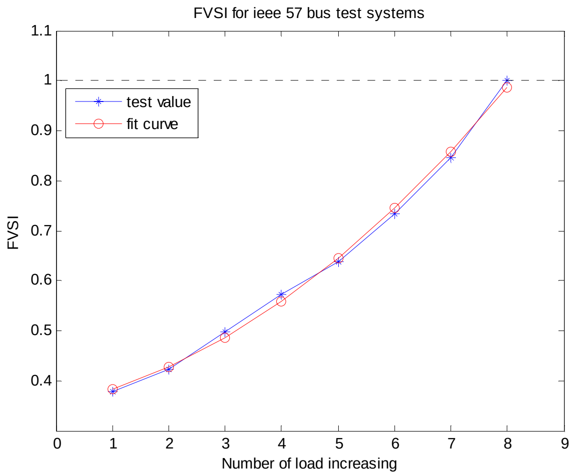

Fig. 25 FVSI results

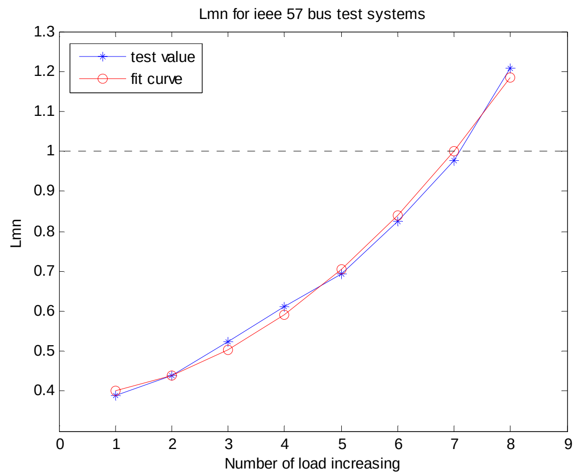

Fig. 26 L_{mn} results

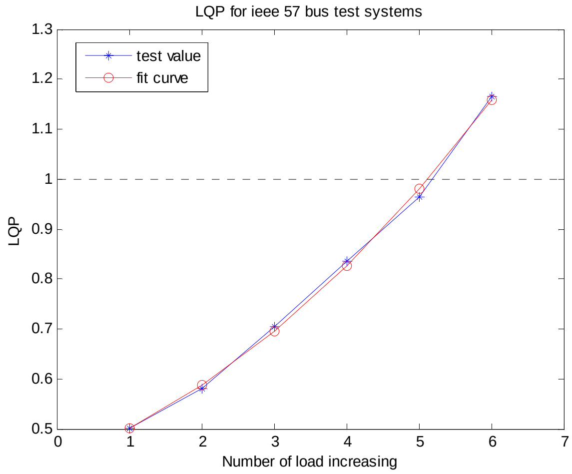

Fig. 27 LQP results

Apart from the MCS, the test power systems were also challenged with the conventional approach to determine the voltage stability indices. In this test category, only reactive power was assumed to be increased, bus-by-bus, until voltage collapsed. By measuring the maximum loading ability, all load buses can be ranked according to this value.

5 Conclusion

This paper presents a random-based approach to calculate voltage stability indices in electric power systems by using fast voltage stability index (FVSI), line stability index (L_{mn}) and line stability factor (LQP) as indicators to identify the point of voltage collapses at a given bus position. The Monte Carlo simulation was employed to situate realistic growth of electrical demand at all load buses instead of just increasing the reactive power at a single load bus. To evaluate this proposed method, a five-bus, the standard IEEE 30- and 57-bus test systems were emploed. As a result, the voltage stability margins according to each index were calculated and, therefore, the weakest bus of the system was identified. Obviously, when using the old fashion way to evaluate the weakest bus of the system by increasing only the reactive powers, the result gave the different bus number as the weakest bus. The old fashion method relied on the decouple assumption that the reactive power strongly affects the voltage change. In fact, there may be some slight interaction made by the real power that cannot be neglected.

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