MONTE CARLO SIMULATION FOR VOLTAGE STABILITY INDEX EVALUATION

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Abstract: - This paper presents an alternative approach to evaluate voltage stability indices in electric power systems by using Monte Carlo simulation (MCS). Fast voltage stability index (FVSI), line stability index (Lmn) and line stability factor (LQP) are used as indicators to determine the capability of reactive power loading at a given bus position. Conventionally, loading the test bus with pure reactive power is not realistic because power system load changes randomly at every load bus in both real and reactive powers. In this paper, MCS was introduced to simulate load increment of all load buses with appropriate random numbers and their probability. From this approach, all the indices mentioned above can be calculated repeatedly with a large number of trials. To evaluate its use, a small five-bus, and the standard IEEE 30- and 57-bus test systems were employed for demonstration. As a result, comparing among these indices, the weakest bus of the test system can be identified with some degree of confidence.

Key-Words: - Fast voltage stability index, line stability index, line stability factor, Monte Carlo simulation

1 Introduction

In recent years, major power system failures (system blackout) have occurred more frequently around the world [1]. Some of these failures have been caused by voltage instability problems. These circumstances lead, the voltage stability issues, to an important and urgent concern for electric utilities. Solving this problem is not the right thing to be proceeded because an power system under the voltage-instability event is typically unpredictable. It is difficult or impossible to correct this problem when it occurred. Prevention of voltage instability is probably moderate one. Many researchers in power system voltage stability devote their works to identify the weakest bus of the system. The weakest bus is considered as the bus that is likely to be the first bus of the system facing voltage collapsed. The prevention of voltage instability relies on locating the system weakest bus. Some efficient actions can be employed to put an operating point of the system far from the voltage collapsing point with an acceptable margin. Static voltage stability can be determined by using continuation power flow calculations [2]. Many static voltage evaluation methods have been proposed, for example, the minimum singularity value method, mode analysis method and sensitivity method [1,3,4]. The major disadvantages of the continuation power flow-based methods are computing efforts that make difficulty of implementation in on-line applications and experiences of solution divergence due to the numerical instability in power flow calculations. Moreover, inconsistency between off-line model and real-life situation leads incapability of identifying the weakest bus that causes the system voltage collapse.

In this paper, modification based on MCS for evaluation of voltage stability indices (e.g. FVSI, Lmn and LQP) to identify weak buses in electric power systems has been proposed. Statistical load models with appropriate random numbers and their probability are used to predict electrical demand growth of all load buses. With their positive mean and variance of the statistical load models, the power system is driven to increase its total loading with some considerable momentum. By performing the system load increment with a moderate time span and computing all the voltage stability indices, weak buses of the system can be observed. This simulation can typically involve over 10,000 trials to pretend a real-world power system operation. The results from this simulation can be used to evaluate the weakest bus with a certain degree of confidence.

In this paper, Section 2 gives a brief explanation of voltage stability indices used in this paper as mention earlier. It notes that the full Newton-Raphson power flow calculation [5] was applied to solve for system voltage solutions. It is well-known and widely-accepted as the most powerful tool for this purpose. Therefore, the method of Newton Raphson power flow calculation was not included here. Monte Carlo simulation was reviewed and summary of its use to evaluate weak buses of the system were reviewed in Section 3. For test, a small five-bus, the standard IEEE 30- and 57-bus power systems were employed. To simulate the demand growth
of the system, appropriate statistical load models were assigned to all load buses. Simulation results were illustrated in Section 4. Section 5 provided adequate conclusion remark.

2 Voltage Stability Indices

Voltage stability indices can be evaluated by using a two-machine coupling model. There exist many useful indices from literature. In this paper, only FVSI, Lmn and LQP are reviewed as follows.

2.1 FVSI (fast voltage stability index) [6-9]
Calculation of current flowing through the line in Fig. 1 gives primary expression of this index.

\[ FVSI = \frac{4Z^2Q_2}{V_i^2X} \]  \hspace{1cm} (1)

Where

- \( Z \) is the line impedance
- \( X \) is the line reactance
- \( Q_2 \) is the reactive power at bus 2
- \( V_i \) is the voltage magnitude at bus 1

FVSI is used to indicate a stable operating region of the load. According to this index, the system becomes unstable if FVSI is equal to or greater than unity [6].

2.2 Lmn (line stability index) [6,9,10]
Calculation of current flowing through the line in Fig. 1 gives primary expression of this index.

\[ Lmn = \frac{4xQ_r}{V_i^2 \sin(\theta - \delta)} \leq 1.00 \]  \hspace{1cm} (2)

Where

- \( x \) is the line reactance
- \( Q_r \) is reactive power at the receiving-end
- \( V_i \) is voltage at the sending-end
- \( \theta \) is the phase angle of the line impedance
- \( \delta \) is the voltage phase difference of bus \( s \) and \( r \)

\( Lmn \) is used to indicate a stable operating region of the load. According to this index, the system becomes unstable if \( Lmn \) is equal to or greater than unity.

2.3 LQP (line stability factor) [6,11]
LQP uses the same concept of FVSI and Lmn. It is derived from power transfer equations describing the system in Fig. 3. LQP can be expressed in (3).

\[ LQP = 4 \left( \frac{X}{V_i^2} \right) \left( \frac{X}{V_i^2} P_i^2 + Q_j \right) \]  \hspace{1cm} (3)

Where

- \( P_i \) is the real power flow from bus \( i \)
- \( X \) is the line reactance
- \( Q_j \) is the reactive power flow to bus \( j \)
- \( V_i \) is the voltage magnitude at bus \( i \)

The system is stable as long as LQP is less than 1.0 [6].

2.4 Procedure to compute voltage stability indices
All three voltage stability indices can be obtained in each power flow calculation loop driven by the incremental reactive power loading at a particular load bus. By increasing only the reactive power of this bus until the indices greater than unity, the maximum reactive power loading ability can be calculated. The following step-by-step procedure describes the computation algorithm for voltage stability indices.

START:

Step 0
Load a test power system
Initialize parameters
Reset all counters
Define reactive power increment and time step
Start the load bus counter (LBC)

**Step 1**
Assign the next load bus for reactive power loading

**Step 2**
Increase the reactive power loading
Perform the power flow calculation
Compute FVSI, Lmm and LQP

**Step 3**
Check the stopping criterion
IF (FVSI ≥ 1) & (Lmm ≥ 1) & (LQP ≥ 1)
   Go to Step 4
OTHERWISE
   Go to Step 2
END

**Step 4**
Evaluate the maximum reactive power loading ability
IF (LBC ≥ Total number of load buses)
   go to Step 5
OTHERWISE
   LBC = LBC + 1
   go to Step 1
END

**Step 5**
Successfully obtain voltage stability indices
Perform weak bus ranking
Identifying the weakest bus of the system
STOP

3 Applied Monte Carlo Simulation [12-15]

A Monte Carlo simulation (MCS) is a technique that involves using random numbers and probability to solve physical problems. This technique relies on computer simulation of real-life models to predict what is likely to happen next. When the simulation going on, randomly change in some input variables can be made to drive the system to operate at a new unpredictable operating state. This method is one among several methods for analyzing uncertainty propagation, where the objective is to determine how random variation, lack of knowledge, or error affects the system performance that is being modeled. The MCS is categorized as a sampling method because the input variables are randomly generated from probability distributions to simulate the process of sampling from an actual population as shown in Fig. 4.

The procedure for Monte Carlo simulation corresponding to the uncertainty propagation shown in Fig. 4 is fairly simple, and can be easily implemented in some scientific software, such as MATLAB, MATCAD, SCILAB, etc. There are five simple steps listed below to organize the typical MCS procedure to work.

**Step 1**
Create a parametric model

**Step 2**
Generate a set of random inputs

**Step 3**
Evaluate the model and store the results

**Step 4**
Repeat Steps 2 and 3 for a large amount of trials

**Step 5**
Report the results, statistics, confidence intervals.

To apply the MCS for voltage stability index evaluation, classification of system variables is necessary. For this problem, electrical demand growth at each load bus can be increased randomly with a particular random distribution. Due to the random process, not only the reactive power but also the real power can be assumed as input random variables. With their positive mean and variance of the statistical load models of Gaussian distribution, the power system is driven to increase its total loading with some considerable momentum. By performing the system load increment with a moderate time span and computing all the voltage stability indices according to their associated power flow solution, weak buses of the system can be observed and ranked. This simulation can typically involve over 10,000 trials to pretend a real-world power system operation. The results from this simulation can be used to evaluate the weakest bus with a certain degree of confidence. Hence, when applying the MCS to the voltage stability index calculation, the step-by-step procedure summarizing the modified computation algorithm is described as follows.

**START:**

**Step 0**
Load a test power system
Initialize parameters
Reset all counters
Define statistical models to all loads
Start the time counter
Step 1
Randomly generate demand growth of this time step
Step 2
Perform the power flow calculation
Compute FVSI, L_{mn} and LQP
Step 3
Check the stopping criterion
IF (FVSI ≥ 1) & (L_{mn} ≥ 1) & (LQP ≥ 1)
Go to Step 4
OTHERWISE
Increase the time counter
Go to Step 1
END
Step 4
Evaluate the maximum power loading ability
Successfully obtain voltage stability indices
Perform weak bus ranking
Identifying the weakest bus of the system
STOP
If your paper deviates significantly from these specifications, our Publishing House may not be able to include your paper in the Proceedings. When citing references in the text of the abstract, type the corresponding number in square brackets as shown at the end of this sentence [1].

4 Simulation Results
To demonstrate the proposed algorithm for evaluating the voltage stability indices, three test cases: i) a simple five-bus test system [16], ii) the standard IEEE 30-bus test system [17] and iii) the standard IEEE 57-bus test system [18] were used.

4.1 5-bus test system
Its base-case loading was assumed to be 165 MW and 40 Mvar. Some information of the test system was given in Table 1 and 2 for load data and line data, respectively. Statistical models of all load buses used in this paper was Gaussian distribution in which 10% of each base-case power was set as its mean and 0.9 was a typical value of its variance.

Fig. 5 Five-bus test system

The test started from the base case. Random load increasing was repeatedly performed with a uniform time step until the system voltage collapses. This operation was defined as one trial. To achieve realistic results, the test must be performed as many as possible. In this work, the test was conducted with 30,000 trials to ensure a normal distribution of the output variable outcome. Table 3 showed a summary of simulation results.

| Table 1. Load at base case of the five-bus test system |
|---|---|---|
| bus | Load at base case (MVA) |
| 1 | - |
| 2 | 20 + j10 |
| 3 | 45 + j15 |
| 4 | 40 + j5 |
| 5 | 60 + j10 |

| Table 2. Line data of the five-bus test system |
|---|---|---|---|
| Line number | Form bus | To bus | Impedance (p.u.) |
| 1 | 1 | 2 | 0.02 + j0.06 |
| 2 | 1 | 3 | 0.08 + j0.24 |
| 3 | 2 | 3 | 0.06 + j0.18 |
| 4 | 2 | 4 | 0.06 + j0.18 |
| 5 | 2 | 5 | 0.04 + j0.12 |
| 6 | 3 | 4 | 0.01 + j0.03 |
| 7 | 4 | 5 | 0.08 + j0.24 |

| Table 3. Simulation results for a total of 30,000 trials |
|---|---|---|---|
| Line | FVSI | L_{mn} | LQP |
| 1 | 0 | 1 | 0 |
| 2 | 0 | 9148 | 0 |
| 3 | 10 | 4 | 14 |
| 4 | 4 | 1 | 15 |
| 5 | 3 | 0 | 11 |
| 6 | 985 | 503 | 1074 |
| 7 | 22114 | 12003 | 21127 |

Figs 6-8 illustrated the change of each index during the random load incremental. From these figures, maximum power load ability can be evaluated. The value obtained by each index was slightly different. However, they are just 2.0% of maximum variation from the mean of 4.00. As can be seen in Tables 3 and 4, by using the FVSI, the weakest bus was bus 5 with 95.7% of confidence. The LQP also told the similar result with 95.0% of confidence. However, by using the L_{mn} the degree of confidence was dropped to be as low as 55.4%, the weakest bus was still the bus number 5. In addition, the voltage magnitude of bus 5 was observed and depicted as shown in Fig. 9 – 10, whereas Figs 11 and 12 were the apparent power and the time counter being the x axis, respectively. From these figures, the voltage magnitude decreased dramatically until its collapse.
Again, Figs 11-13 illustrated the change of each index during the random load incremental. This set of figures was set the time counter as the x axis. Although the shape of curves was different, the conclusion drawn from these graph was the same as described earlier.

Table 4 Summary of simulation results

<table>
<thead>
<tr>
<th>Load at bus 5</th>
<th>FVSI</th>
<th>Lmn</th>
<th>LQP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base load</td>
<td>0.61</td>
<td>0.61</td>
<td>0.61</td>
</tr>
<tr>
<td>Increasing load</td>
<td>3.97</td>
<td>3.96</td>
<td>4.12</td>
</tr>
<tr>
<td>% increasing</td>
<td>550.82</td>
<td>549.18</td>
<td>575.41</td>
</tr>
<tr>
<td>% confidence</td>
<td>95.7</td>
<td>55.4</td>
<td>95.0</td>
</tr>
</tbody>
</table>

Table 5 Summary of the conventional index evaluation

<table>
<thead>
<tr>
<th>Load bus</th>
<th>Maximum loading (p.u.)</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>9.826</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>4.789</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>3.664</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>8.558</td>
<td>3</td>
</tr>
</tbody>
</table>
4.2 Standard IEEE 30-bus test system

In this test case, complexity of interconnection of 30 buses required spaces for explanation. Briefly, the test system was given in Fig. 14. This test case used the same procedure in order to evaluate voltage stability indices of the system. By increasing MVA loads at every single bus within a certain time step, system voltage profiles were characterized. Based on an appropriately statistical load model for each load bus, Monte Carlo simulation for voltage stability indices were carried out. Statistical models of all load buses used in this test case was Gaussian distribution in which 10% of each base-case power was set as its mean and 0.9 was a typical value of its variance. The test started from the base case. Random load increasing was repeatedly performed with a uniform time step until the system voltage collapses. This operation was defined as one trial. To achieve realistic results, the test must be performed as many as possible. In this work, the test was conducted with 30,000 trials to ensure a normal distribution of the output variable outcome. Table 6 showed a summary of simulation results for this test case. It noted that the total number of system buses was thirty, therefore it was inconvenient to present results of all buses or all lines as given in the first test case. Only two first ranks were demonstrated.

<table>
<thead>
<tr>
<th>Line</th>
<th>FVSI</th>
<th>Lmn</th>
<th>LQP</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3582</td>
<td>6699</td>
<td>10336</td>
</tr>
<tr>
<td>5</td>
<td>15124</td>
<td>13808</td>
<td>8180</td>
</tr>
</tbody>
</table>
In contrast, 30,000 trials did not give the outscore outcome. According to FVSI and $L_{mn}$ indices line number 5 was the most critical line and resulted in bus 4 as the weakest bus. With LQP, the most critical line moved to line number 2 that pointed other bus as the weakest bus. However, FVSI and $L_{mn}$ had higher degree of confidences (80.85% and 67.33% respectively) whereas LQP dropped to 55.82%. Figs 15-17 illustrated the change of each index during the random load incremental. From these figures, maximum power load ability can be evaluated although the weakest bus among them were not the same.

Figs 18-20 illustrated the change of each index during the random load incremental. This set of figures was set the time counter as the x axis. Although the shape of curves was different, the conclusion drawn from these graph was the same as described earlier.
4.3 Standard IEEE 57-bus test system

The test system was given in Fig. 21. This test case used the same procedure in order to evaluate voltage stability indices of the system. By increasing MVA loads at every single bus within a certain time step, system voltage profiles were characterized. Based on an appropriately statistical load model for each load bus, Monte Carlo simulation for voltage stability indices were carried out. Statistical models of all load buses used in this test case was Gaussian distribution in which 10% of each base-case power was set as its mean and 0.9 was a typical value of its variance. The test started from the base case. Random load increasing was repeatedly performed with a uniform time step until the system voltage collapses. This operation was defined as one trial. To achieve realistic results, the test must be performed as many as possible. In this work, the test was conducted with 30,000 trials to ensure a normal distribution of the output variable outcome. Table 7 showed a summary of simulation results for this test case. It noted that the total number of system buses was thirty, therefore it was inconvenient to present results of all buses or all lines. Only two first ranks were given.

Table 7 Simulation results for a total of 30,000 trials

<table>
<thead>
<tr>
<th>Line</th>
<th>FVSI</th>
<th>Lmn</th>
<th>LQP</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>691</td>
<td>35</td>
<td>219</td>
</tr>
<tr>
<td>73</td>
<td>23083</td>
<td>28007</td>
<td>21867</td>
</tr>
</tbody>
</table>

As can be seen in Tables 7 and 8, by using the FVSI, the weakest bus was bus 2 with 97.09% of confidence. The LQP also told the similar result with 99.88% of confidence. By using the Lmn, the degree of confidence was as high as 99.01%. Figs 22-24 illustrated the change of each index during the random load incremental. From these figures, maximum power load ability can be evaluated.
Figs 25-27 illustrated the change of each index during the random load incremental. This set of figures was set the time counter as the $x$ axis. Although the shape of curves was different, the conclusion drawn from these graphs was the same as described earlier.

Apart from the MCS, the test power systems were also challenged with the conventional approach to determine the voltage stability indices. In this test category, only reactive power was assumed to be increased, bus-by-bus, until voltage collapsed. By measuring the maximum loading ability, all load buses can be ranked according to this value.

5 Conclusion

This paper presents a random-based approach to calculate voltage stability indices in electric power systems by using fast voltage stability index (FVSI), line stability index ($L_{mn}$) and line stability factor (LQP) as indicators to identify the point of voltage collapse at a given bus position. The Monte Carlo simulation was employed to situate realistic growth of electrical demand at all load buses instead of just increasing the reactive power at a single load bus. To evaluate this proposed method, a five-bus, the standard IEEE 30- and 57-bus test systems were employed. As a result, the voltage stability margins according to each index were calculated and, therefore, the weakest bus of the system was identified. Obviously, when using the old fashion way to evaluate the weakest bus of the system by increasing only the reactive powers, the result gave the different bus number as the weakest bus. The old fashion method relied on the decouple assumption that the reactive power strongly affects the voltage change. In fact, there may be some slight interaction made by the real power that cannot be neglected.

References:


